Want to understand the AHL proof of Theorem 6.4

Define

$$a_i = \mu^{i-1}\lambda_2$$

$$b_i = a_i \lambda_1$$

$$|a_i| = 2p^2 - 1 + 2(i-1)p^2$$

$$|b_i| = |a_i| + 2p-1$$

$$= 2p^2 - 1 + 2p - 1 + 2(i-1)p^2.$$

$$|a_{p^n}| = 2(p^n - 1)p^2 + 2p^2 - 1 = 2p^{n+2} - 1.$$

<u>Theorem</u>: In the spectral sequence

$$\overline{THH}_*(\ell; H\mathbb{Z}_{(p)})[v_1] \Rightarrow \overline{THH}_*(\ell)$$

the differentials are determined by

$$d_{p^n + \cdots + p}(p^{n-1} a_{kp^{n-1}}) \doteq (k-1)\, v_1^{p^n + \cdots + p}\, b_{(k-1)p^{n-1}}.$$

$n \geq 1$, $k \geq 1$

<u>Pf</u>: <u>Step 1</u>: Prove this for $k = p$.

We need to establish that $\quad (m = p^n + \cdots + p)$

$$d_m(p^{n-1} a_{p \cdot p^{n-1}}) \doteq (p-1)\, v_1^m\, b_{(p-1)p^{n-1}}.$$

$$\parallel$$

$$p^{n-1} a_{p^n}$$

proceed by induction. For $n = 1$, we want to show

$$d_p(a_p) \doteq (p-1)\, v_1^p\, b_{p-1}$$

lives in degree $2p(p-1) + 2p^2 - 1 + 2p - 1 + 2(p-2)p^2$

In our calculation of $THH(\ell; k(1))$, we found

that $THH_{2p^3-2}(\ell; k(1)) = 0.$

$$2p(p-1) + 2p^2 - 1 + 2p - 1 + 2(p-2)p^2$$

$$= \cancel{2p^2} - \cancel{2p} + \cancel{2p^2} - 1 + \cancel{2p} - 1 + 2p^3 - \cancel{4p^2}.$$

$$= 2p^3 - 2.$$

In $THH(\ell; \mathbb{Z}_p)[v_1]$ this is non zero in stem $2p^3 - 2$ & has a single generator $v_1^p b_{p-1}$. There is only one diff that can have $v_1^p b_{p-1}$ as its target.

$$\Rightarrow \quad d_p(a_p) \doteq v_1^p b_{p-1}.$$

assume for n, i

$$d_{p^n + \cdots + p}\left(p^{n-1} a_{p \cdot p^{n-1}}\right) \doteq (p-1) v_1^{p^n + p^{n-1} + \cdots + p} \cdot b_{(p-1)p^{n-1}}$$

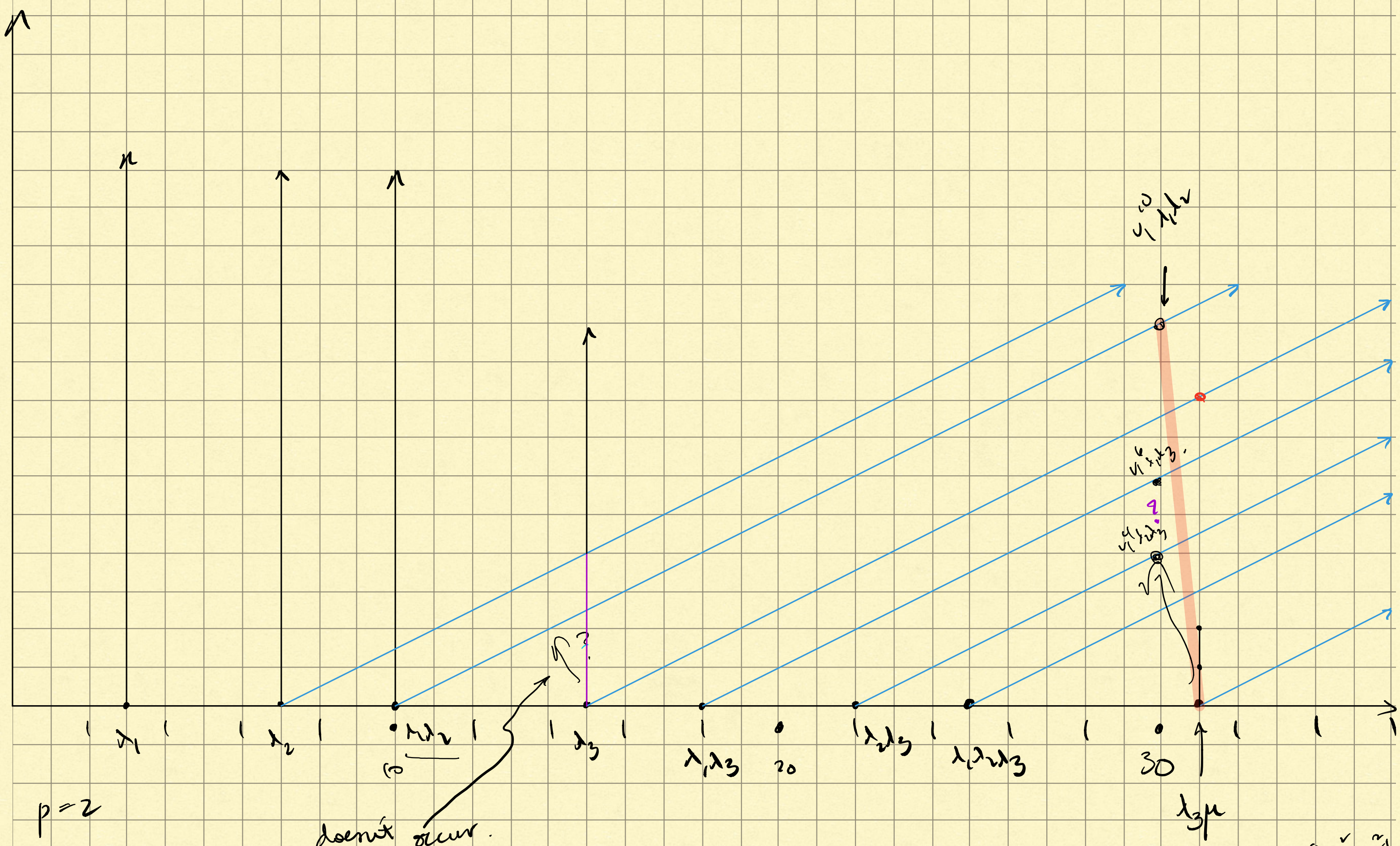
what are the non zero classes in

$$|p^{n-1} a_{p^n}| - 1 = 2p^{n+2} - 2$$

only classes in even degree

$$a_1^{(5)} = \lambda_1 \mu_3^{i-1}.$$

$$d(\lambda_1 \lambda_3) \doteq$$



$p = 2$

$THH(BP\langle 2\rangle; \mathbb{Z}_{(p)})[v_1] \Rightarrow THH(BP\langle 2\rangle; \mathbb{Q})$

$|\lambda_1| = 2p - 1 = 3$

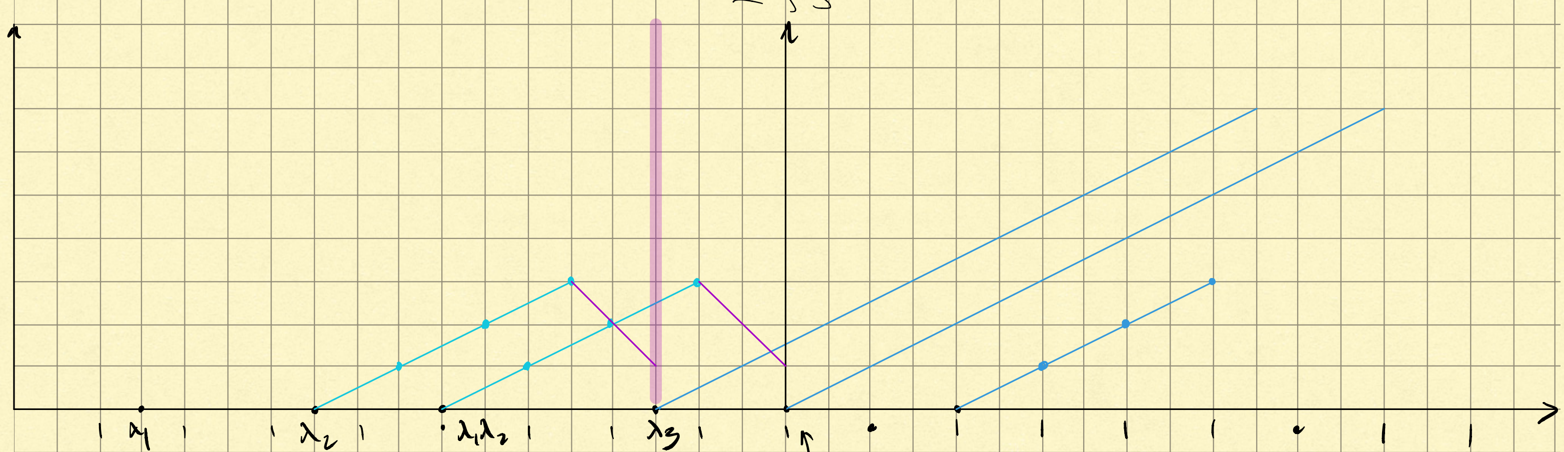
$|\lambda_2| = 2p^2 - 1 = 7$

$|\lambda_3| = 2p^3 - 1 = 15$

$d_{p^2}(\mu) = v_1^{p^2} \lambda_2$

$|\lambda_2 \mu| = 2p^2 + 2p^2 - 1 = 4p^2 - 1 = 15$

$d_{p^2}(\lambda_3 \mu) = v_1^{p^2} \lambda_3 \lambda_2$



$\lambda_2 \tau_{3,0}$. $\quad$ $\lambda_2\lambda_3$

$\parallel$ $\quad$ $\parallel$

$\lambda_2\lambda_3$. $\quad$ $\tau_{2,1,0}$

nothing here to receive hidden ext'n

$$\Rightarrow \quad d_{p^2}(\mu\lambda_3) = v_1^{p^2}\lambda_2\lambda_3$$

At Brewlab now:

$$d_r(x) \overset{?}{=} \overset{?}{\lambda_1} y \qquad x_1 \nmid x \quad ?$$

<u>WTS</u>: to effect that if x s.t. $\lambda_1 \nmid x$ & x supports a differential, then

1). in $|x|-1$ all but one possible target of x is λ_1-divisible. ☹ false. (Salvation?: incorporate diffs from earlier pages.

2) the unique y in stem $|x|-1$ w/ $\lambda_1 \nmid y$ is the target.

We have shown $(p=2)$

$$d_p(a_p) \doteq v_1^{p^2} b_{p-1}$$

$$a_i := \lambda_3 \mu^{i-1}$$

$$b_i := \lambda_2\lambda_3\mu^{i-1}.$$

<u>To do</u>: 1) Show b_i are permanent cycles in last v_0-BSS.

2)

- $v_1^{2^2} b_k$
- $v_1^{n} b_n$

$$|a_i| = 2p^3 - 1 + (i-1)2p^3$$
$$= \cancel{2p^3} - 1 + 2p^3 i - \cancel{2p^3} = 2p^3 i - 1.$$

$$|b_i| = 2p^2 - 1 + 2p^3 - 1 + (i-1)2p^3$$
$$= 2p^2 - 1 + \cancel{2p^3} - 1 + 2p^3 i - \cancel{2p^3}$$
$$= 2p^3 i + 2p^2 - 2.$$

$\exists\ k, m, n, l$ s.t.

$$\underbrace{|v_1^k b_m|}_{=\,2k(p-1) + 2p^3 m + 2p^2 - 2} = \underbrace{|v_1^l b_n|}_{=\,2(p-1)l + 2p^3 n + 2p^2 - 2}$$

$$k(p-1) + p^3 m + p^2 - 1 \overset{?}{=} l(p-1) + p^3 n + p^2 - 1$$

$$\Longleftrightarrow \quad k(p-1) + p^3 m = l(p-1) + p^3 n.$$

Suppose

$p^3 \mid k \qquad k = p^3 \cdot k'$

& $\quad p-1 \mid m \qquad m = p-1 \cdot m'$

then

$$k(p-1) + p^3 m = (k' + m')p^3(p-1)$$

then let $l = p^3 \cdot m'$

and

$n = (p-1)(k')$

then

$$l(p-1) + p^3 n$$

$$= m'\left(p^3(p-1)\right) + (p-1)\, p^3 k'$$

$$= (m' + k')\, p^3 (p-1)$$

$k = 2p^3$ $\rightsquigarrow$ $l = p^3$

$m = (p-1)$ $\qquad n = 2(p-1)$

$$2p^3 \cdot (p-1) + (p-1)p^3 = p^3 \cdot (p-1) + 2(p-1)p^3.$$

$$|v_1^k b_m| = |v_1^l b_n|$$

$$|v_1^{2p^3} b_{(p-1)}| = |v_1^{p^3} b_{2(p-1)}|.$$

gets killed by $d_p(v_1^{2p} a_p)$

$$d_{p^3+p^2}(a_{p^2}) = \underbrace{v_1^{p^3+p^2} b_{(p-1)p}}$$ does this happen? (

$b^3 = 2p^3 \cdot p^2 - 1$

$$2p^3(p-1)p + 2p^2 - 2 + 2(p-1)k \overset{?}{=} \cancel{2p^5} - \cancel{2}$$

$$\|$$

$$\cancel{2p^5} - 2p^4 + 2p^2 - \cancel{2} + 2(p-1)k$$

$d_{12}(a_4) = v_1^{12} b_2$?.

$$\cancel{2(p-1)}k = 2p^4 - 2p^2 = 2p^2(p^2-1)$$

$$= \cancel{2}p^2(p+1)\cancel{(p-1)}$$

$$k = p^2(p+1) = p^3 + p^2.$$

what else is in stem $2p^5-2$.

$$|v_1^l b_m| = 2(p-1)l + 2p^3 m + 2p^2 - \cancel{2} = 2p^5 - \cancel{2}$$

$$(p-1)l = p^5 - p^3 m - p^2 = p^2(p^3 - pm - 1)$$

$p-1 \mid p^3 - pm - 1 \Rightarrow p-1 \mid m.$

$p^2 \mid l$

$m = p-1$, $p^3 - p(p-1) - 1$

$= p^3 - p^2 + p - 1$

$$\begin{aligned}(p-1)l &= p^5 - p^3(p-1) - p^2 \\ &= p^5 - p^4 + p^3 - p^2 \\ &= p^4(p-1) + p^2(p-1) \\ &= (p^4 + p^2)(p-1)\end{aligned}$$

Todo (Dom) 1) Make charts to play with.

(Gabe) 2) Write up ~~the~~ argument for $d_{p^2}(a_p) = v_1^{p^2}$, identify?

3) # of generators in $THH_?(BP\langle 2\rangle; l)$? Lemma in AHL.

(Dom) 4) Double check first v_0, v_1 - BSS.

(Gabe) 5) $THH(BP\langle 2\rangle; ?) \longrightarrow THH(l; ?)$

6) Hidden extensions?

7) Journals for submissions.