ORF 409
Two simulations.

1. For independent and identical random variables $U_1, U_2, \ldots$ with a uniform distribution on the until interval $(0,1)$, define

$$N := \min \left\{ n \ : \ \sum_{i=1}^{n} U_i > 1 \right\}.$$

That is, N is the least number of random numbers that must be summed to exceed 1.

a. Estimate $\mathbb{E}[N]$ by generating 100 values of N;

b. Estimate $\mathbb{E}[N]$ by generating $1,000$ values of N;

c. Estimate $\mathbb{E}[N]$ by generating $10,000$ values of N;

d. What do you think is the value of $\mathbb{E}[N]$ based on these simulations.

2. In a weekly lottery five distinct numbers between 1 and 59 are chosen randomly. In one year (52 weeks) number 13 was chosen in 17 weeks. We want to investigate whether this is a normal random fluctuation or it is a very unusual event that needs to be further investigated.

We define the following random variables:

$$\begin{aligned} A(n) &= \text{the number of times number } n \text{ is chosen in one year}, \ n = 1, \ldots, 59 \\ M &= \text{the number of times number } n \text{ is chosen in one year} \\ &= \max_{n=1,\ldots,59} A(n). \end{aligned}$$

Note that $A(n) \leq M$ can be any integer between 0 and 52.

a. Write a python code to simulate the lottery described above;

b. Simulate a one year long lottery and calculate the number of appearances of number 13, i.e., $A(13)$;

c. Generate N many one year simulations as in part b (with a large N) and approximate the probability of number 13 appearing at least 20 times, i.e., $\mathbb{P}(A(13) \geq 17)$.

d. Generate N many one year simulations as in part b (with a large N) and approximate $\mathbb{P}(M \geq 17)$.