1)
$$f_D(x) = \begin{cases} +1 & \text{if } \mathbb{P}[y=+1 \mid x] > 1/2 \\ -1 & \text{if } \mathbb{P}[y=-1 \mid x] > 1/2 \end{cases}$$

goal: show $L_D(f_D) \leq L_D(h)$ → The bayes classifier is the best

True error of a classifier:

$$L_D(h) = \mathbb{E}\ \mathbb{P}[h(x) \neq y]$$

$$= \mathbb{E} \begin{cases} \mathbb{P}[y \neq -1 \mid x] & \text{if } h(x) = -1 \\ \mathbb{P}[y \neq 1 \mid x] & \text{if } h(x) = 1 \end{cases}$$

we can rewrite as

$$\mathbb{E} \begin{cases} \mathbb{P}[y=1 \mid x] & \text{if } h(x) = -1 \\ 1 - \mathbb{P}[y=1 \mid x] & \text{if } h(x) = 1 \end{cases}$$

If $\mathbb{P}[y=1 \mid x] < 1 - \mathbb{P}[y=1 \mid x]$ → choose $h(x) = -1$

$$\mathbb{P}[y=1 \mid x] + \mathbb{P}[y=1 \mid x] < 1$$

$$2\,\mathbb{P}[y=1 \mid x] < 1$$

$$\mathbb{P}[y=1 \mid x] < 1/2$$

→ Choose $h(x) = -1$
This is the same as for the bayes predictor as listed above