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$$L(h_S, S) = \frac{1}{m}\sum_{i=1}^{m} \log(1 + \exp(-y_i\langle w, x_i\rangle))$$

$$\frac{dL(h_S, S)}{dw} = \frac{1}{m}\frac{d}{dw}\sum_{i=1}^{m} \log(1 + \exp(-y_i\langle w, \vec{x}_i\rangle))$$

$$= \frac{1}{m}\sum_{i=1}^{m} \frac{1}{(1 + \exp(-y_i\langle w, \vec{x}_i\rangle))} \cdot \frac{d}{dw}(1 + \exp(-y_i\langle w, \vec{x}_i\rangle))$$

$$= \frac{1}{m}\sum_{i=1}^{m} \frac{\exp(-y_i\langle w, \vec{x}_i\rangle) \cdot -y_i\vec{x}_i}{1 + \exp(-y_i\langle w, \vec{x}_i\rangle)}$$

double checking

$$a = (-y_i\langle w, \vec{x}_i\rangle)$$

$$\frac{da}{dw} = -y_i\vec{x}_i \rightarrow \frac{d}{da}\left[\frac{1}{m}\sum_{i=1}^{m} \log(1 + \exp(a))\right]$$

$$da = -y_i\vec{x}_i\, dw$$

$$= \frac{1}{m}\sum_{i=1}^{m} \frac{1}{1 + \exp(a)} \cdot \exp(a) \cdot \frac{da}{dw}$$

$$= \frac{1}{m}\sum_{i=1}^{m} \frac{\exp(a) \cdot \frac{da}{dw}}{1 + \exp(a)}$$

$$\boxed{= \frac{1}{m}\sum_{i=1}^{m} \frac{\exp(-y_i\langle w, \vec{x}_i\rangle)}{1 + \exp(-y_i\langle w, \vec{x}_i\rangle)}}$$