

Given  $S = \{(\vec{x}_1, y_1), \dots, (\vec{x}_m, y_m)\}$

The loss function of logistic regression is

$$L_{\ell_2}(h_w, S) = \sum_{i=1}^m (h_w(\vec{x}_i) - y_i)^2 + \lambda \|w\|_2^2$$

Show

Solution is

$$w = (A + \lambda I)^{-1} b$$

when  $A + \lambda I$  is invertible

$$\text{where } A = \sum_{i=1}^m \vec{x}_i \vec{x}_i^\top \quad \text{and } b = \sum_{i=1}^m y_i \vec{x}_i$$

$$\begin{aligned} L_{\ell_2}(h_w, S) &= \sum_{i=1}^m h_w(\vec{x}_i)^2 - 2h_w(\vec{x}_i)y_i + y_i^2 + \lambda \|w\|^2 \\ &= \sum_{i=1}^m h_w(\vec{x}_i)^2 - 2 \sum_{i=1}^m h_w(\vec{x}_i) y_i + \sum_{i=1}^m y_i^2 + \lambda \|w\|^2 \end{aligned}$$