Singularities, Schwarzschild Radii, and Spaghettification:

The Extreme Physics of Black Holes

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June 16, 2024
Vanderbilt University

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- Responsible for many extreme astrophysical phenomena.
- Fully understanding them requires new theories of physics (quantum gravity).

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This calculation was done by English natural philosopher John Michell in 1784. He theorized that a sufficiently massive object would gravitationally pull the light back towards it. He called such objects “dark stars.”

General Relativity

- Special Relativity (1905)
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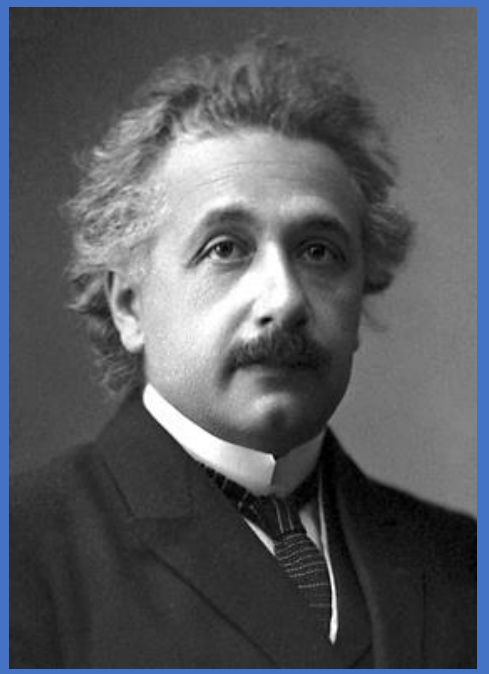
  - Space and time are interwoven into a single continuum known as "spacetime".
  - Energy and mass are equivalent ($E = mc^2$).



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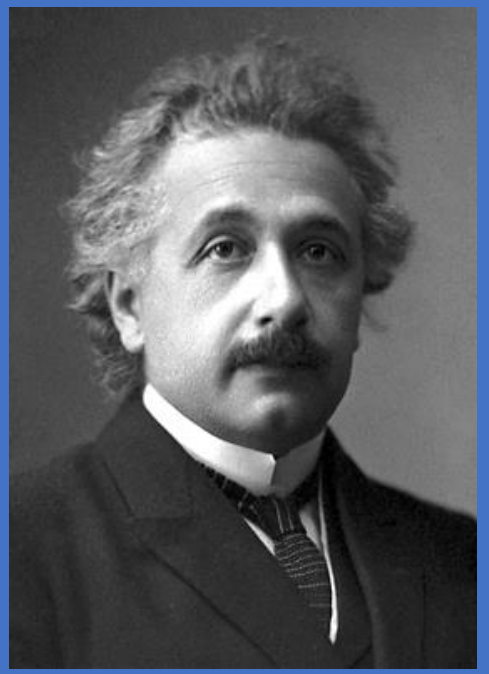

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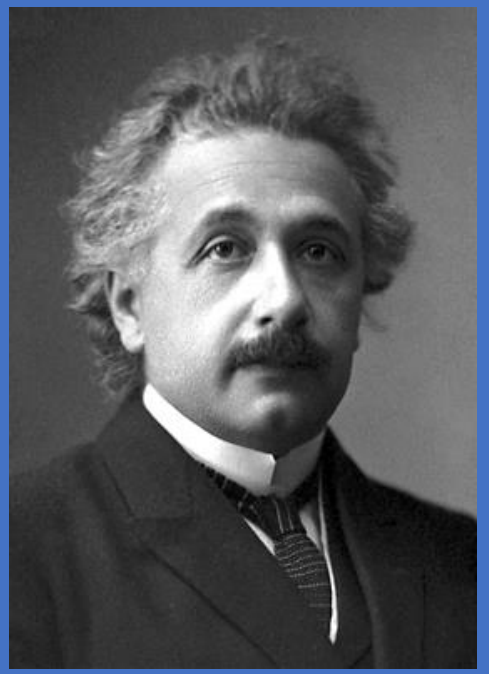
- This led him to formulate gravity in terms of curved spacetime in 1915.



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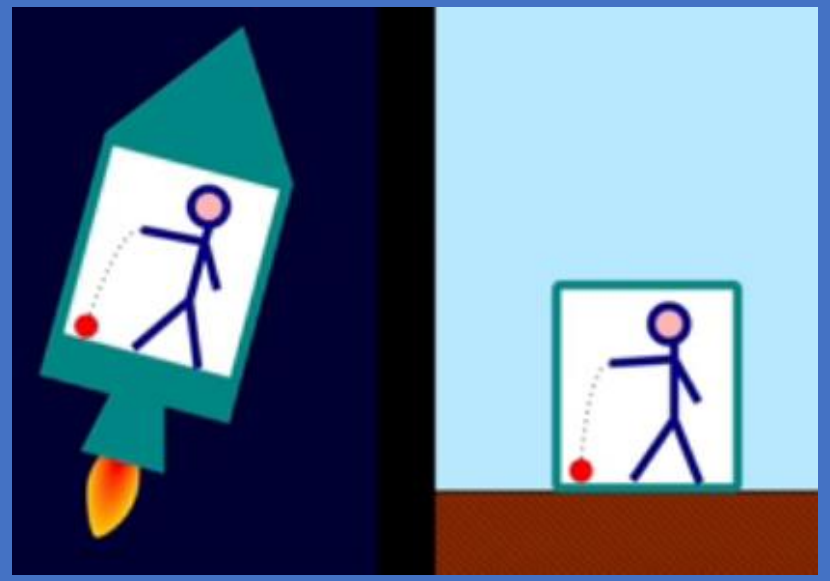

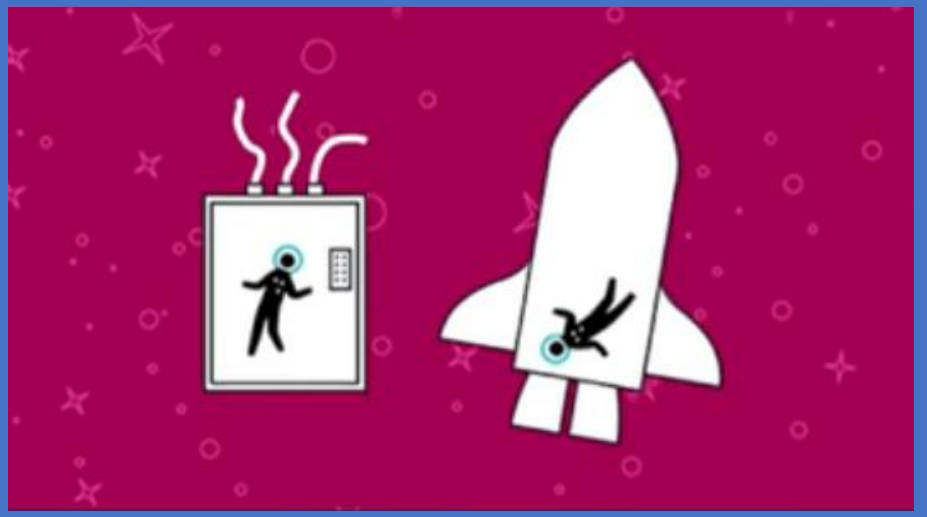

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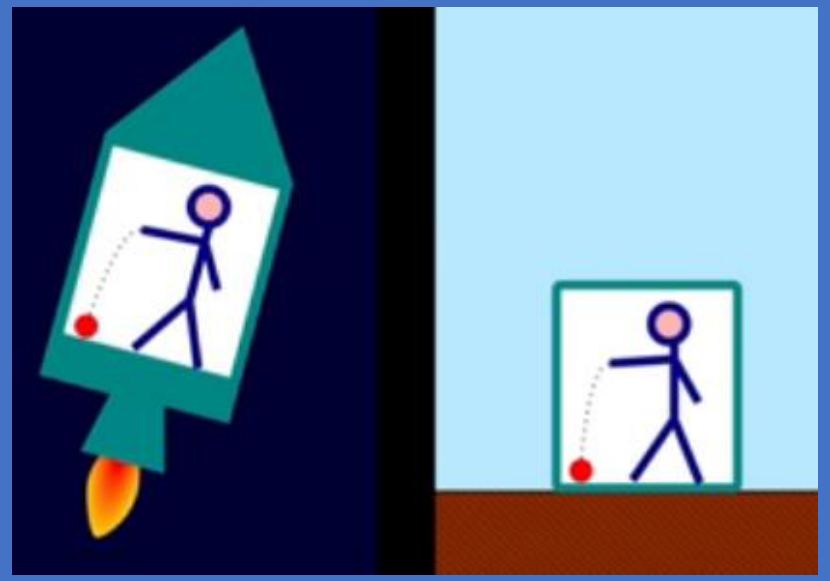

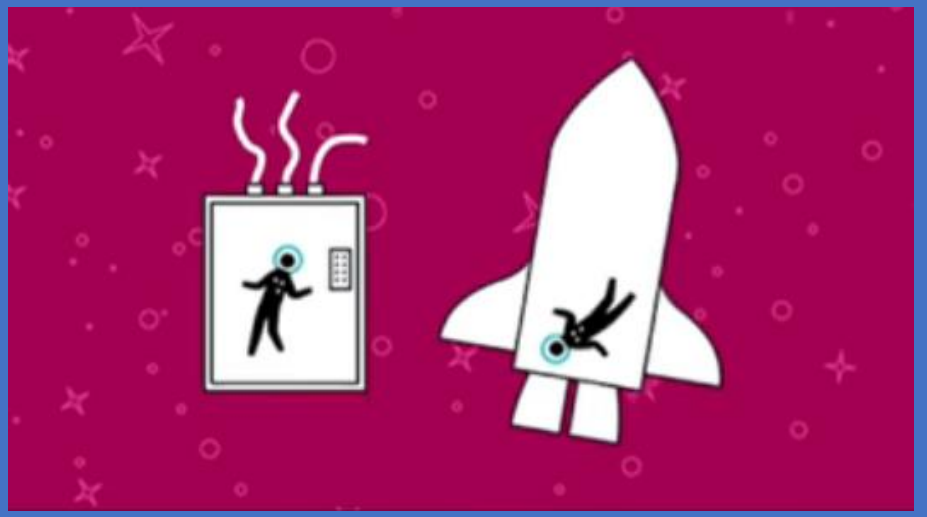

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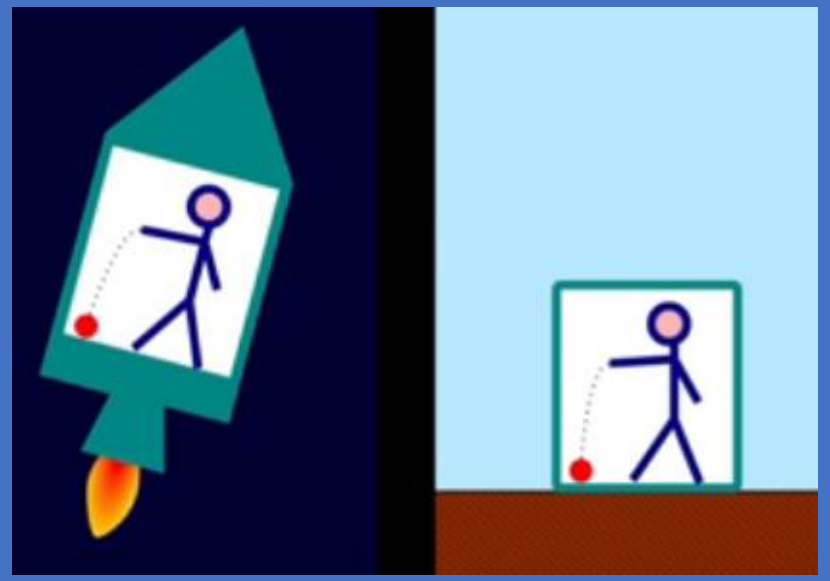
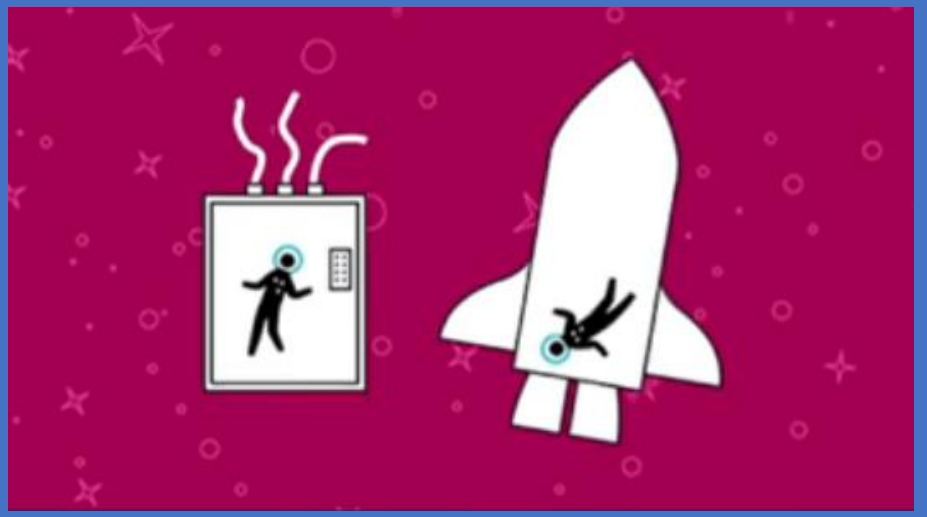
- Einstein called this insight "the happiest thought of his life."





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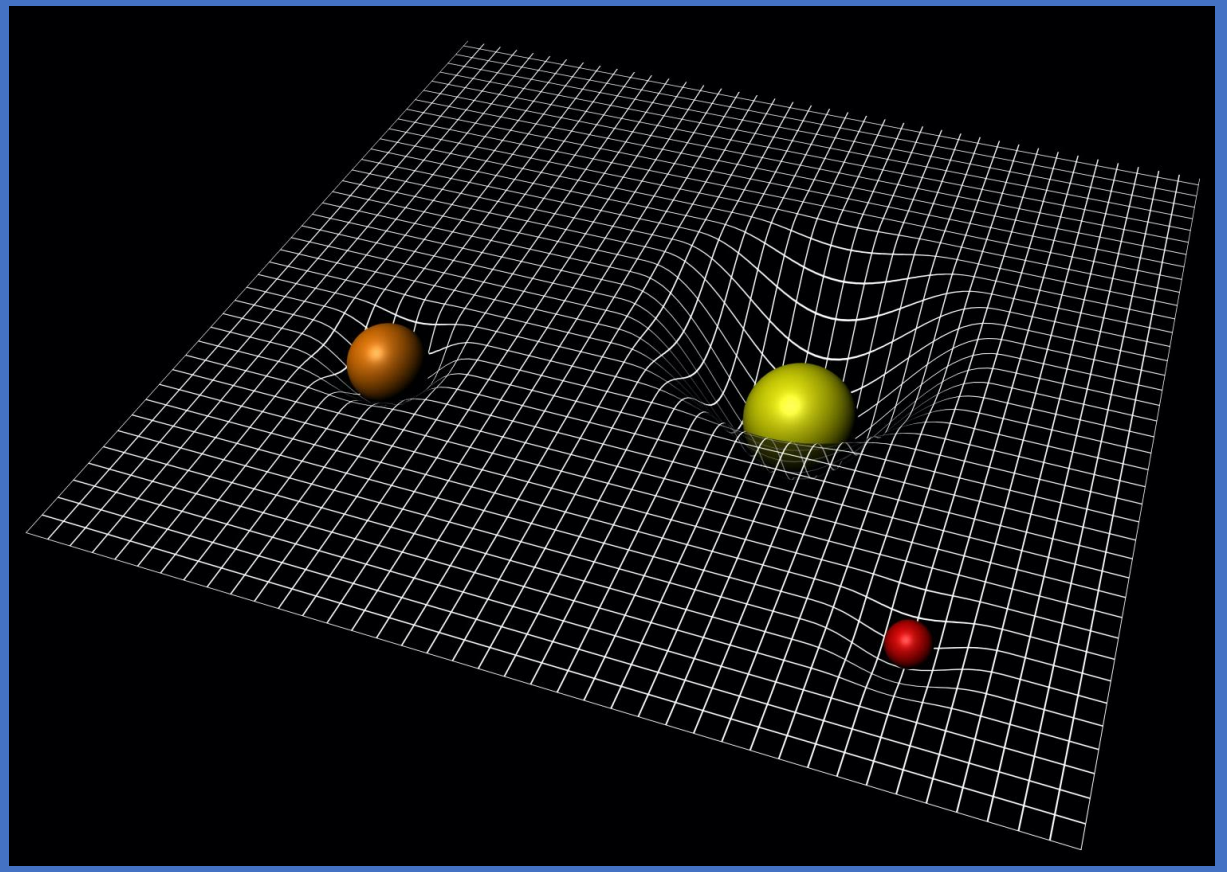

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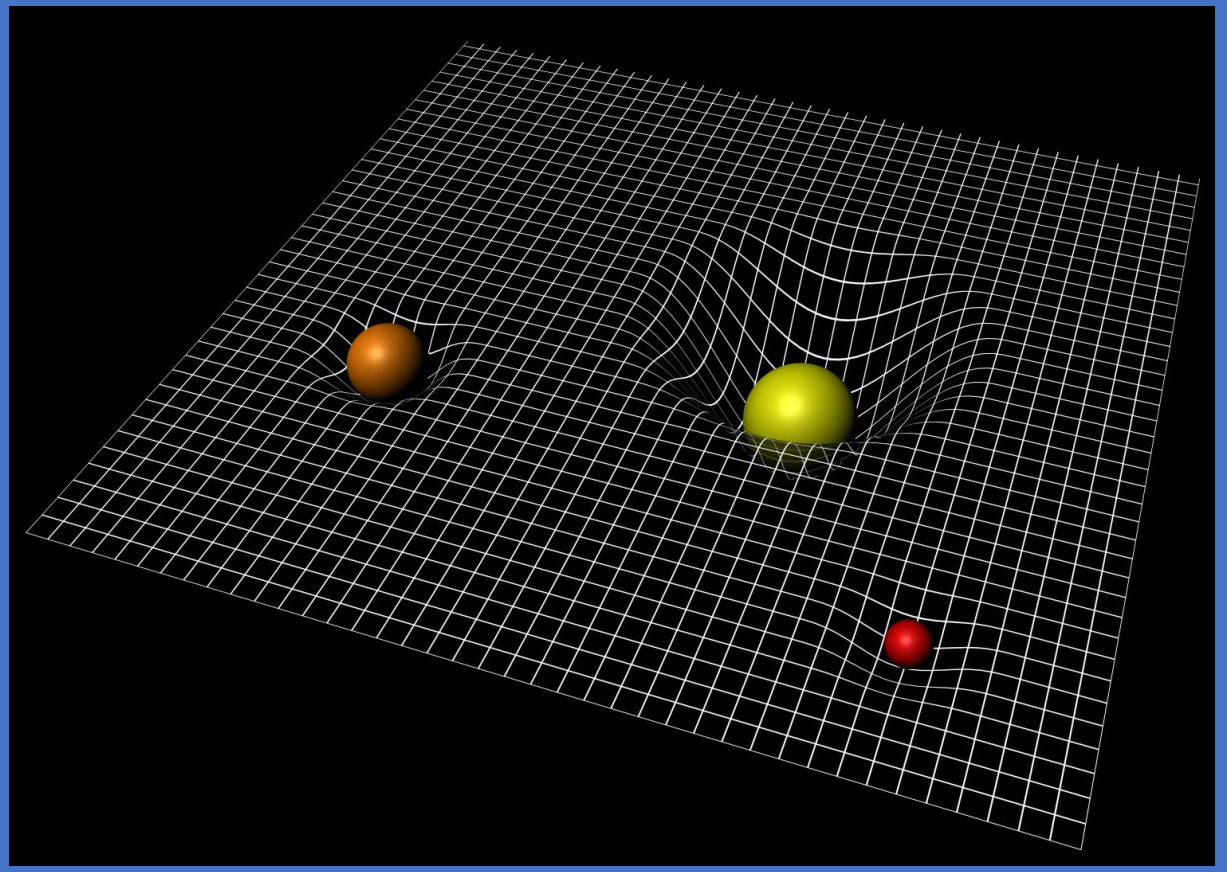

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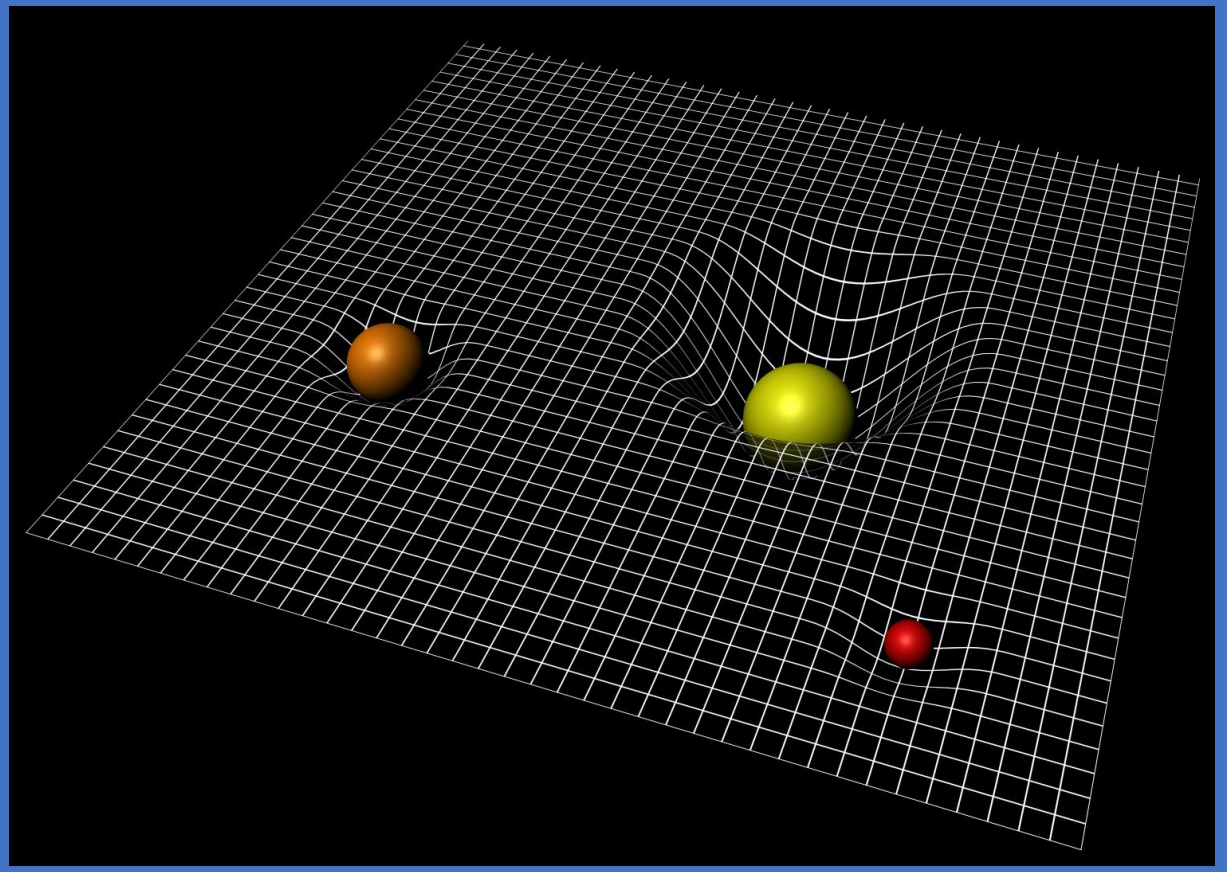

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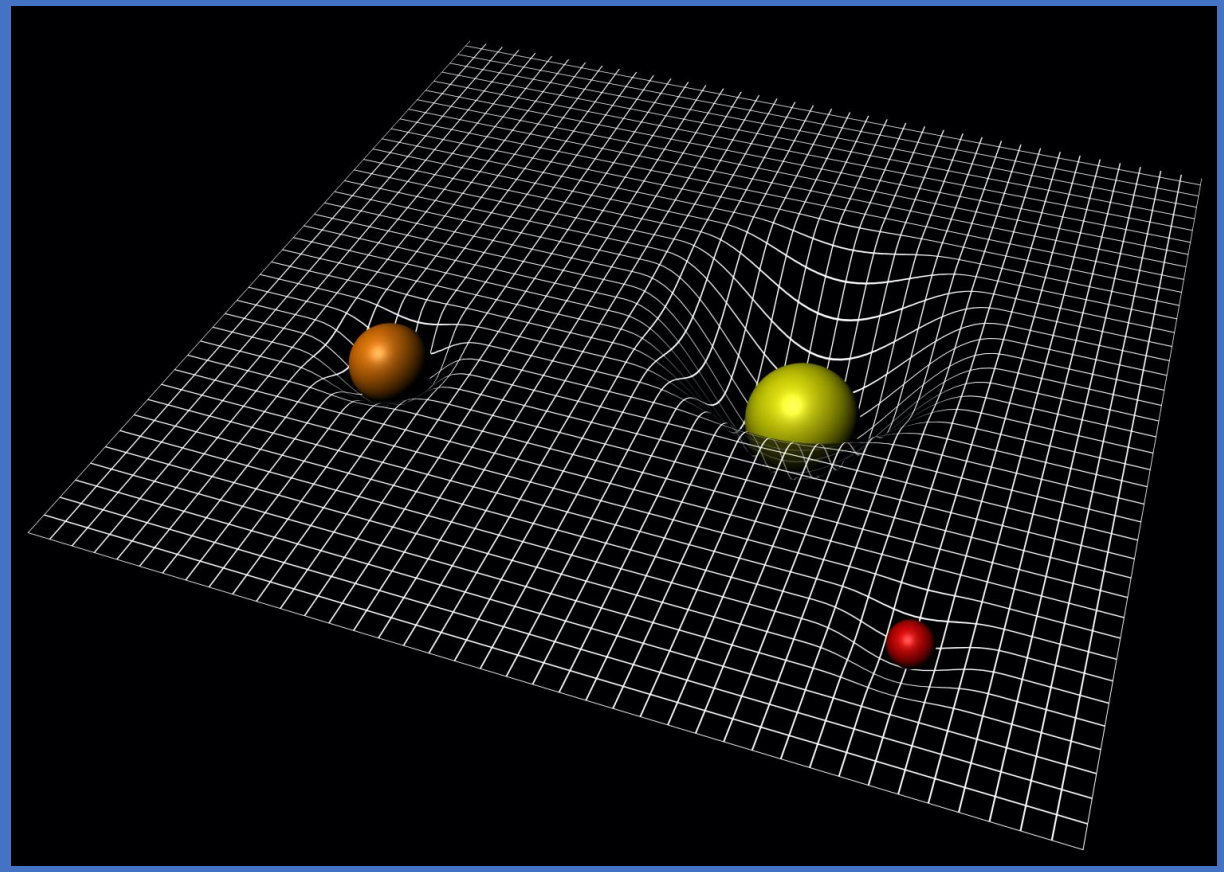
- This new theory has a variety of predictions, including gravitational time dilation and the bending of light in a gravitational field.



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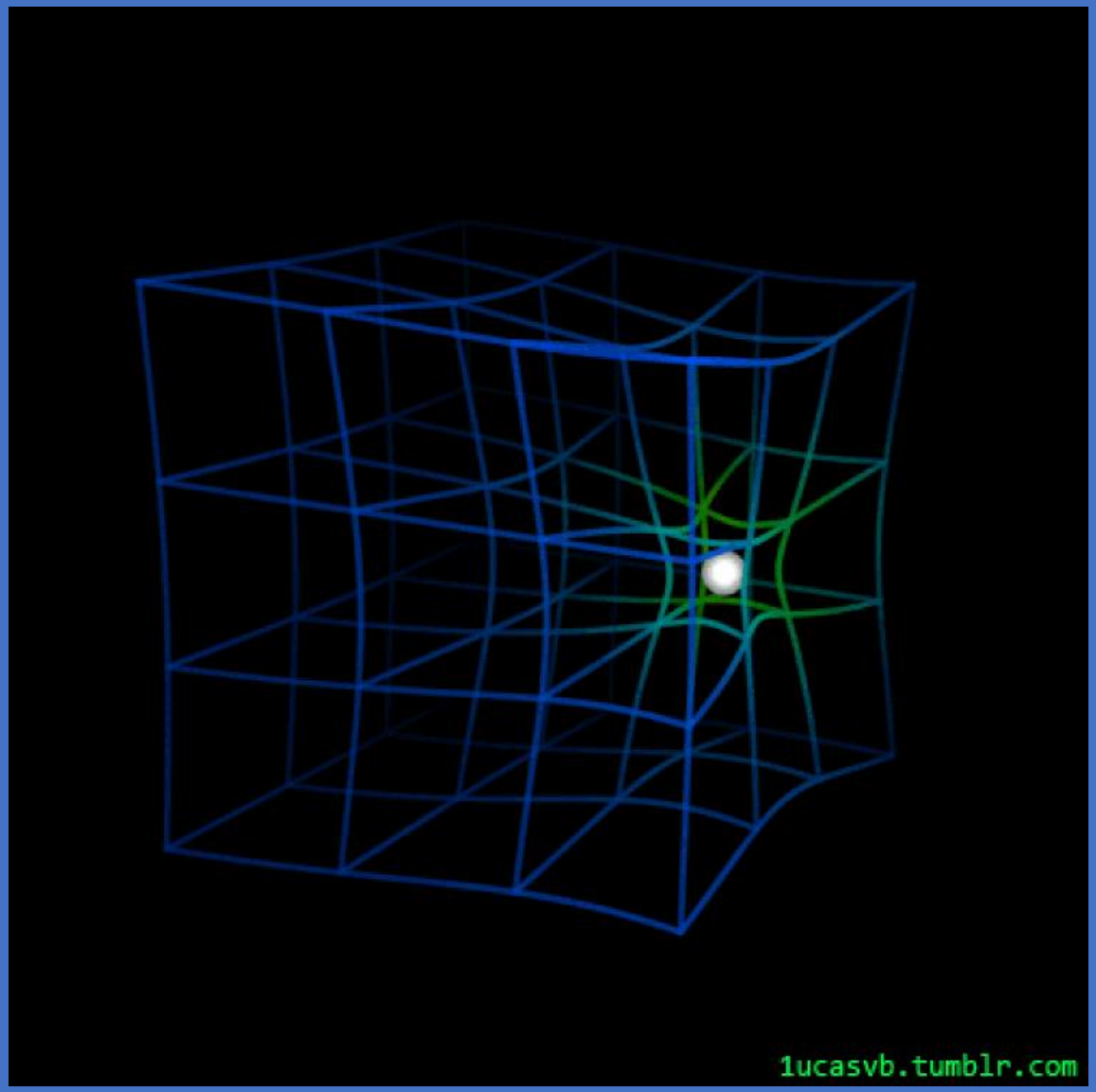

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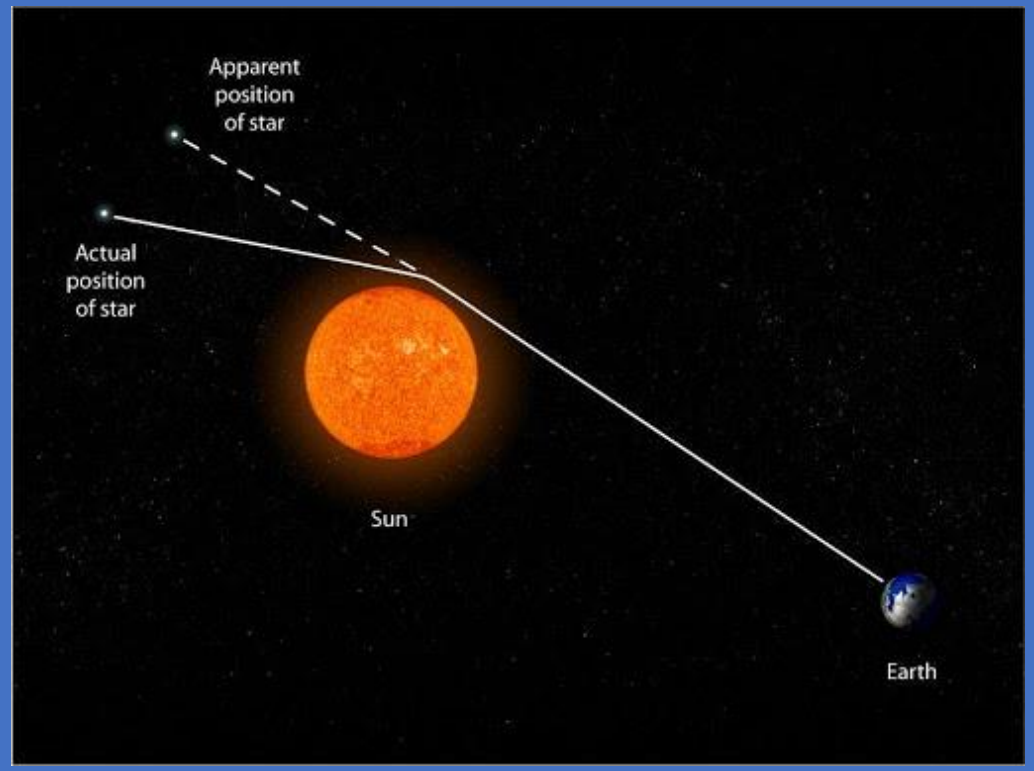

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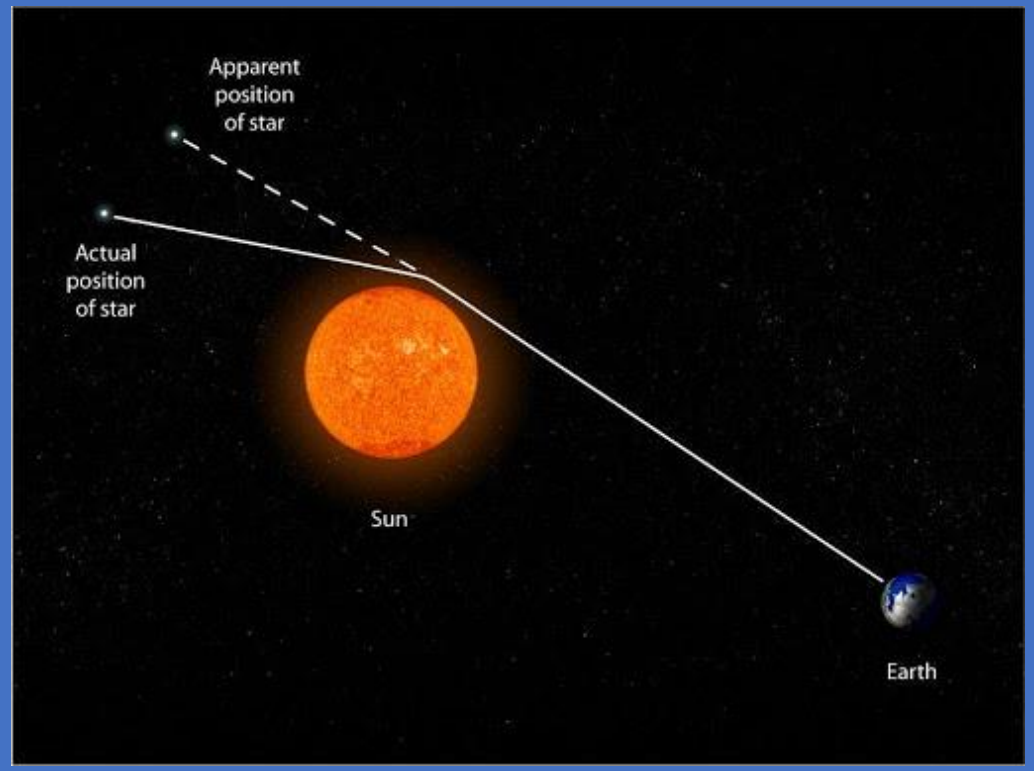

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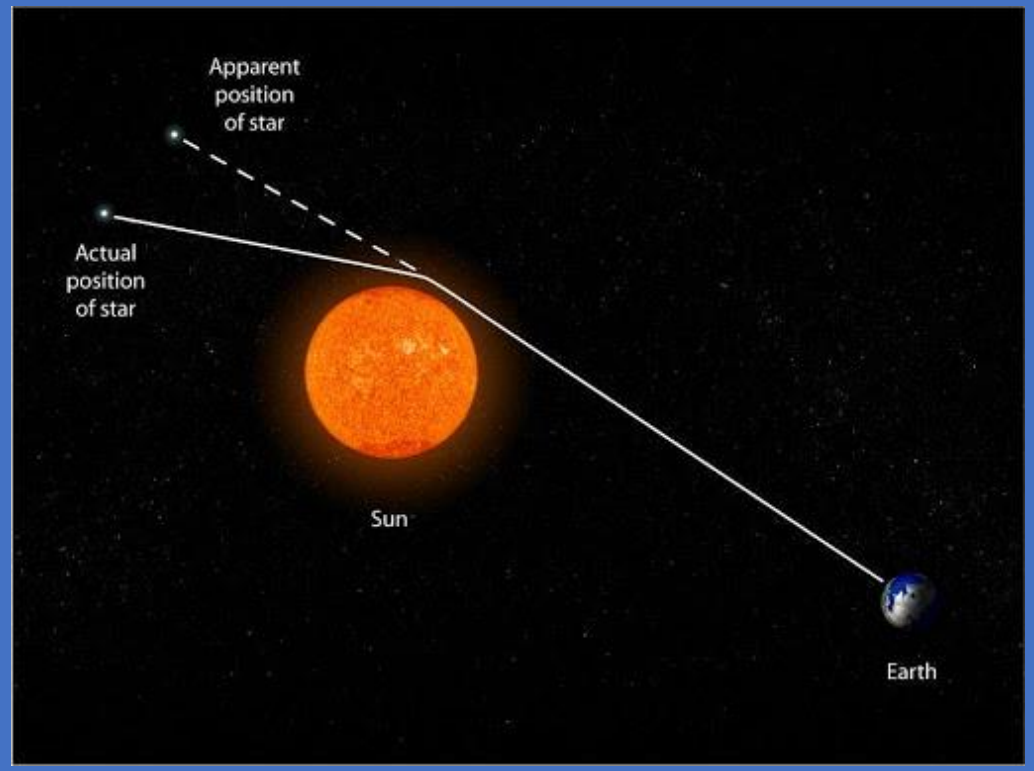
- This effect is the basis of gravitational lensing, a useful observational tool (more later).



The Einstein Field Equations

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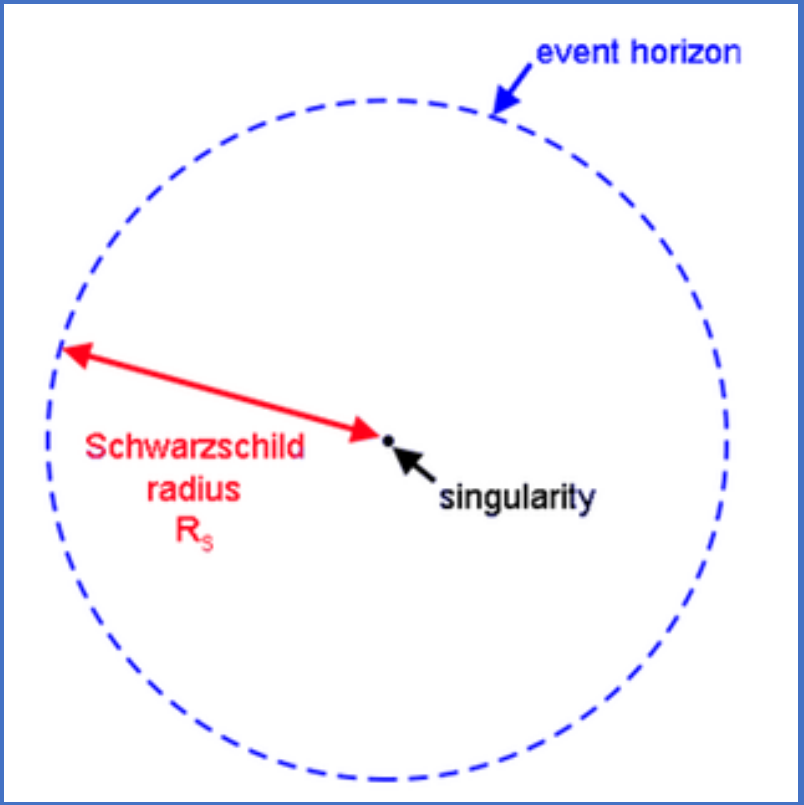
“Spacetime tells matter how to move, and matter tells spacetime how to curve.”

Black Hole Anatomy

- Despite the complexity of the Einstein equations, within a month of their publication a solution was found by Karl Schwarzschild.



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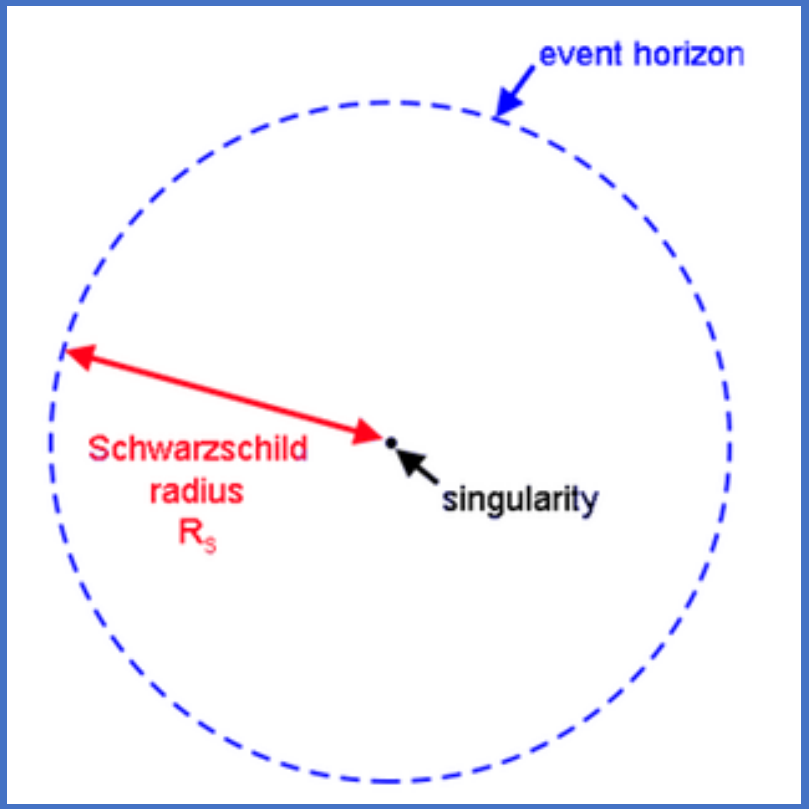

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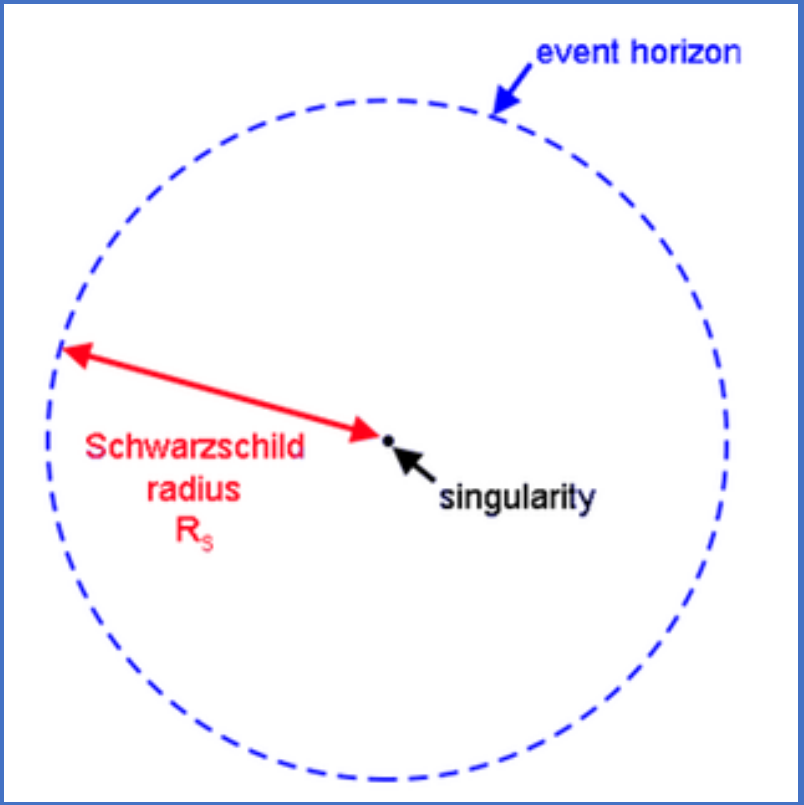

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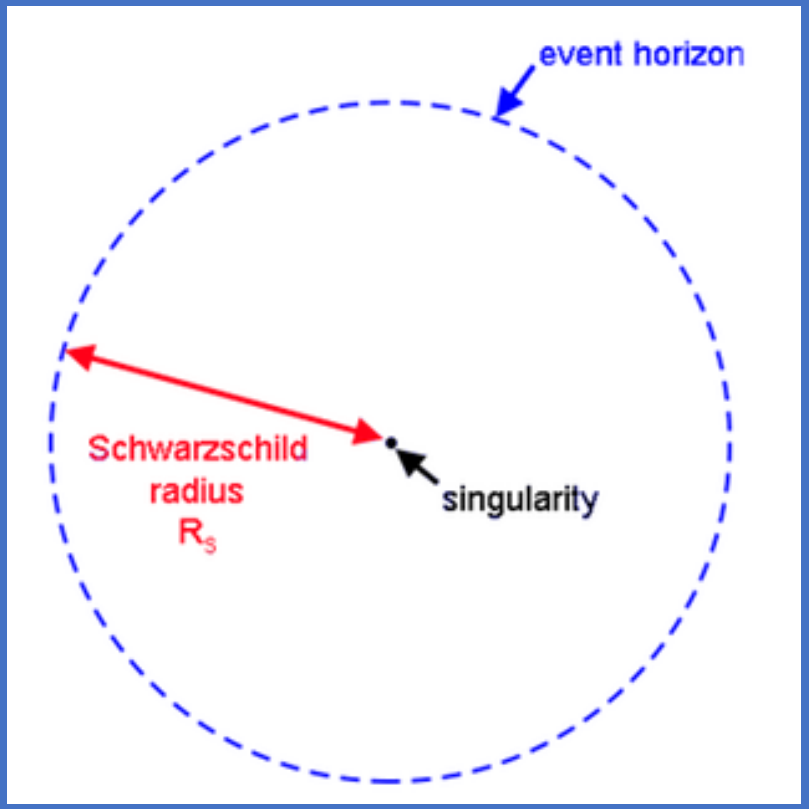
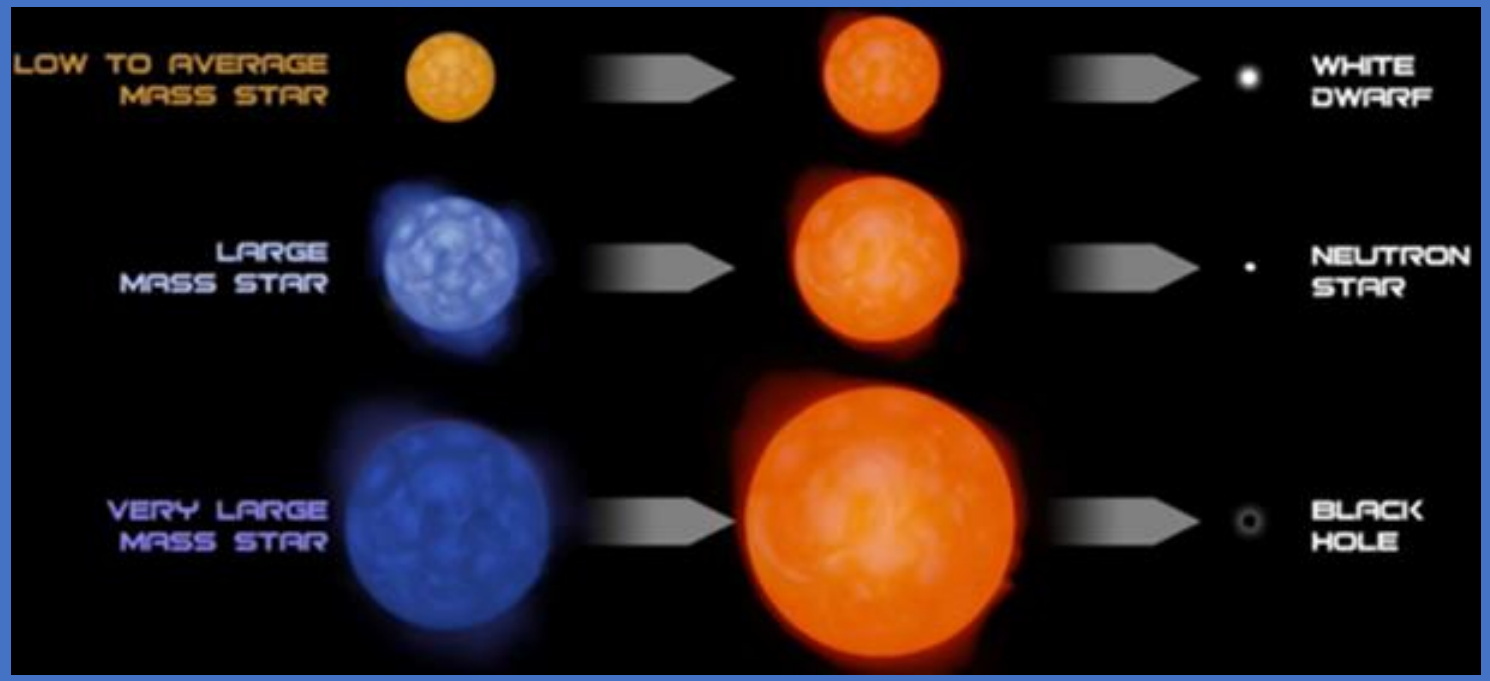
- Black holes can also have charge and angular momentum.



Black Hole Formation

- Anything can become a black hole if you could force it inside its Schwarzschild radius ($R = \frac{2GM}{c^2}$).



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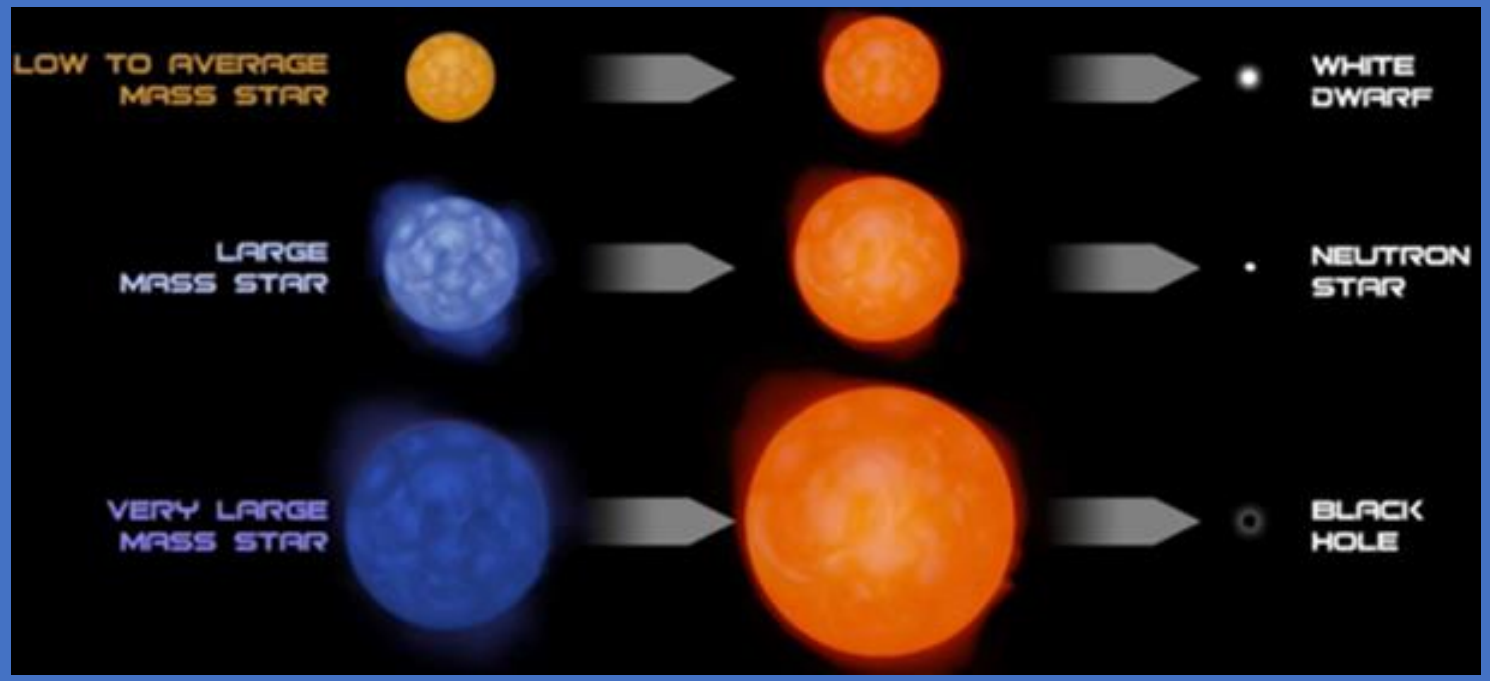
- When a sufficiently large star exhausts its fuel, the gravitational attraction will collapse it into a black hole.



Sizes of Black Hole

Class	Mass	Schwarzschild Radius
Stellar Mass	$5 – 10\ M_{Sun}$	15 – 30 km
Intermediate Mass	$10^2 – 10^5\ M_{Sun}$	$10^2 – 10^5$ km
Supermassive	$10^5 – 10^{10}\ M_{Sun}$	$10^5 – 10^{10}$ km

- $R_{Earth} \approx 6400$ km
- $R_{Sun} \approx 7 \cdot 10^5$ km
- 1 AU $\approx 1.5 \cdot 10^8$ km

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- For reasons not fully understood, this process produces jets of particles and radiation that blast out from the poles.

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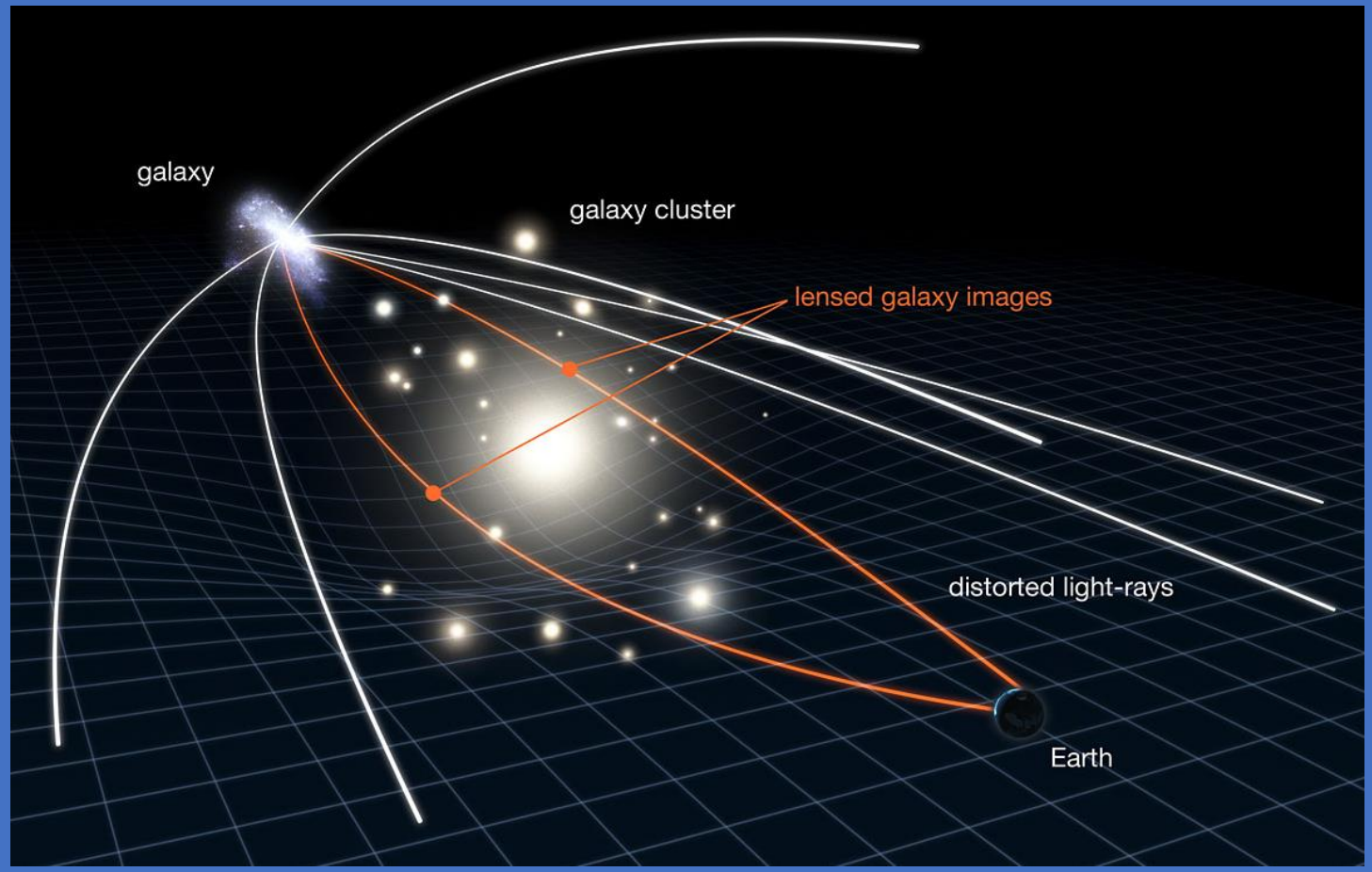
- Once you pass the event horizon, there is no escape, and no way to send a signal to the outside world.

Gravitational Lensing

- Because light is affected by curved spacetime, light coming from distant galaxies can be bent by large objects, which affects the image like a lens.
- This gives one way to study black holes.



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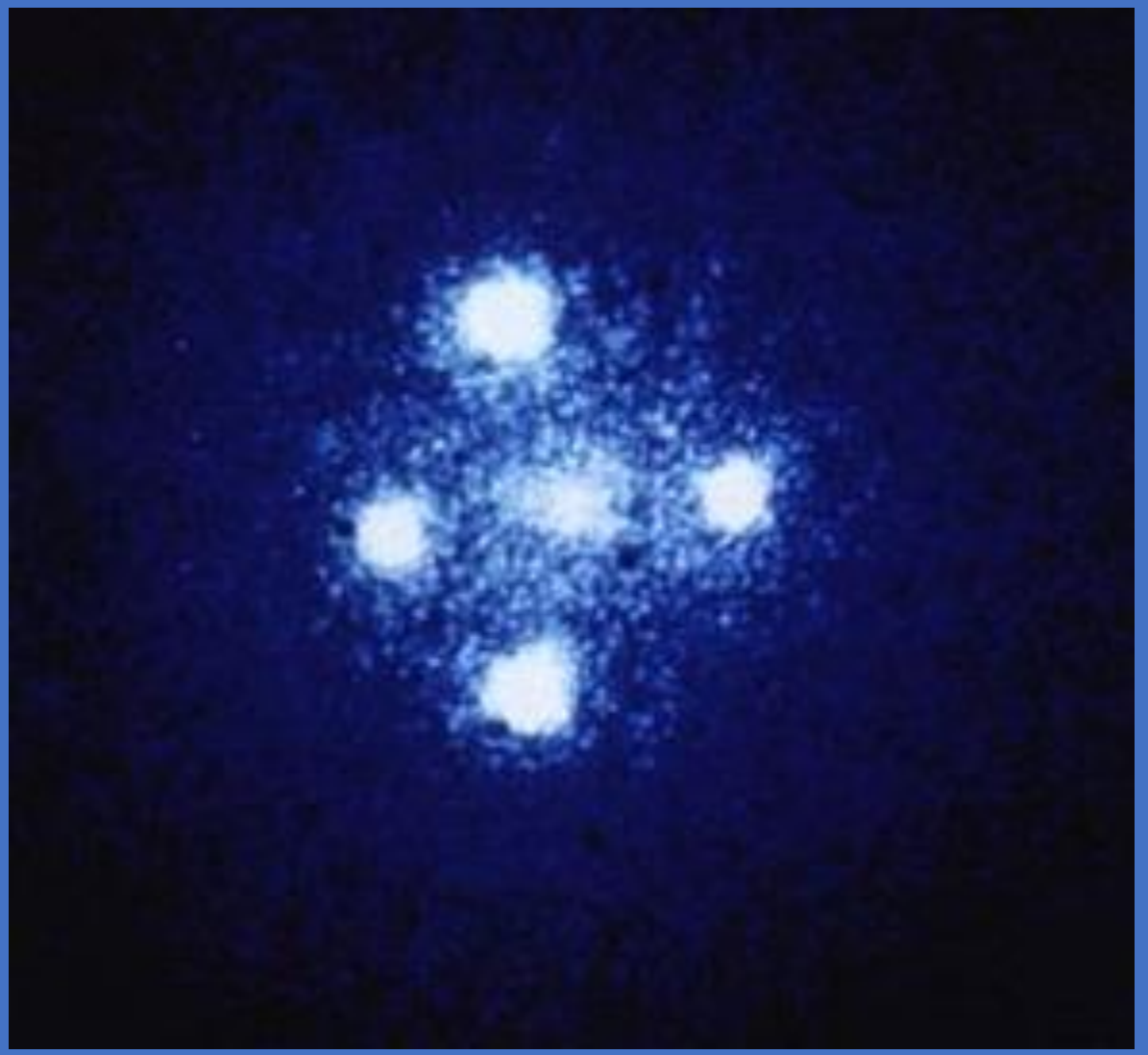

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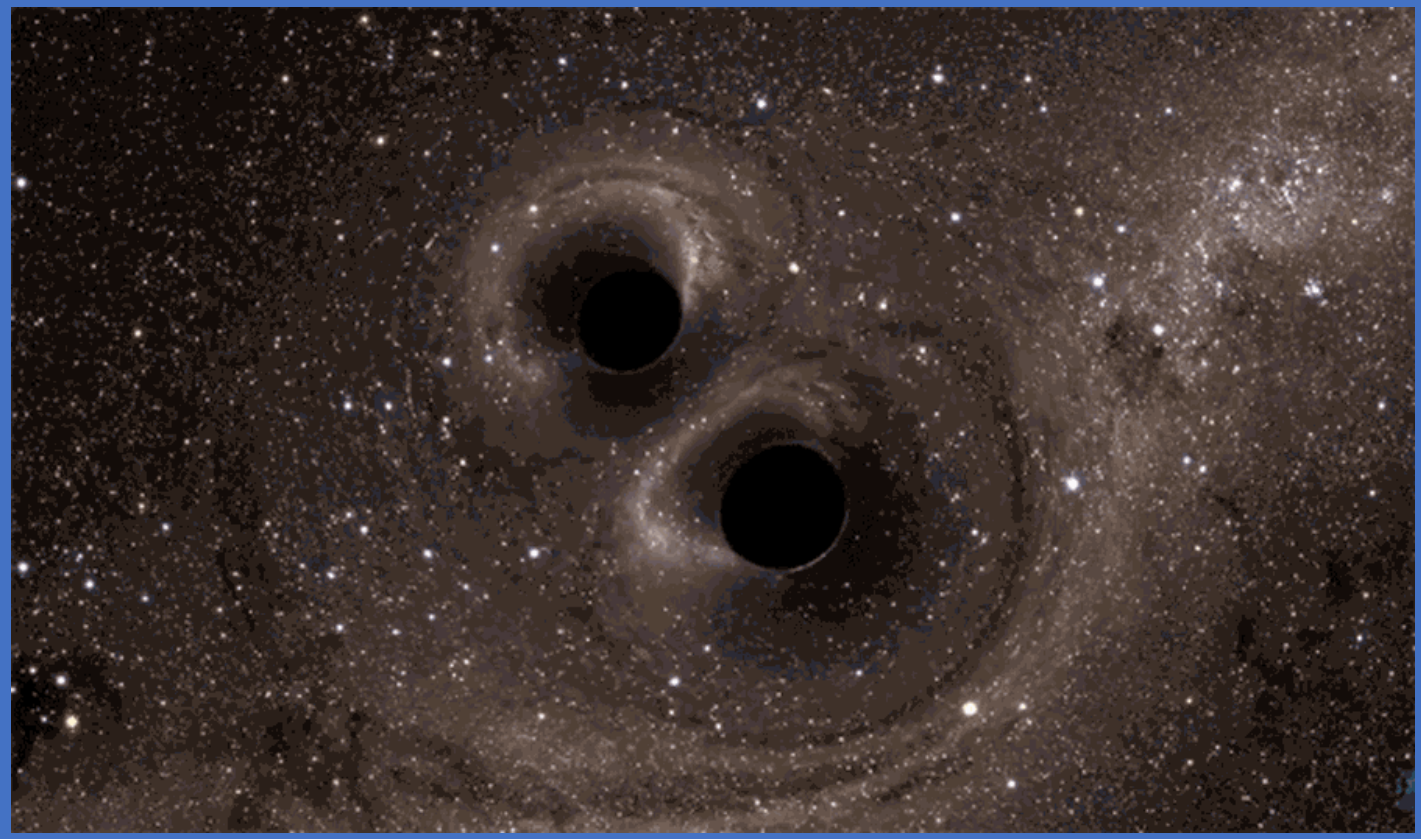
- When black holes collide, they create ripples in spacetime that propagate outward at the speed of light.



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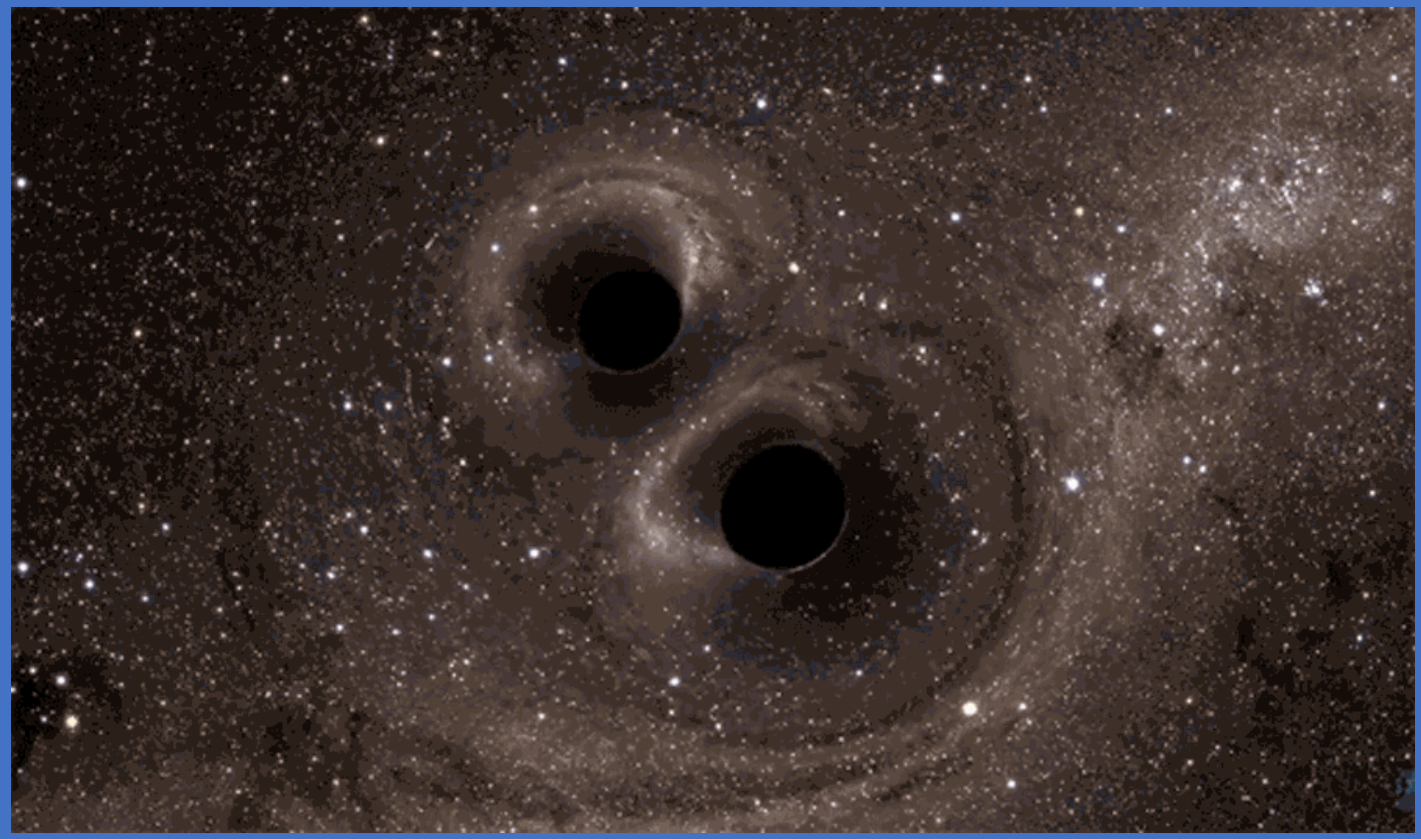
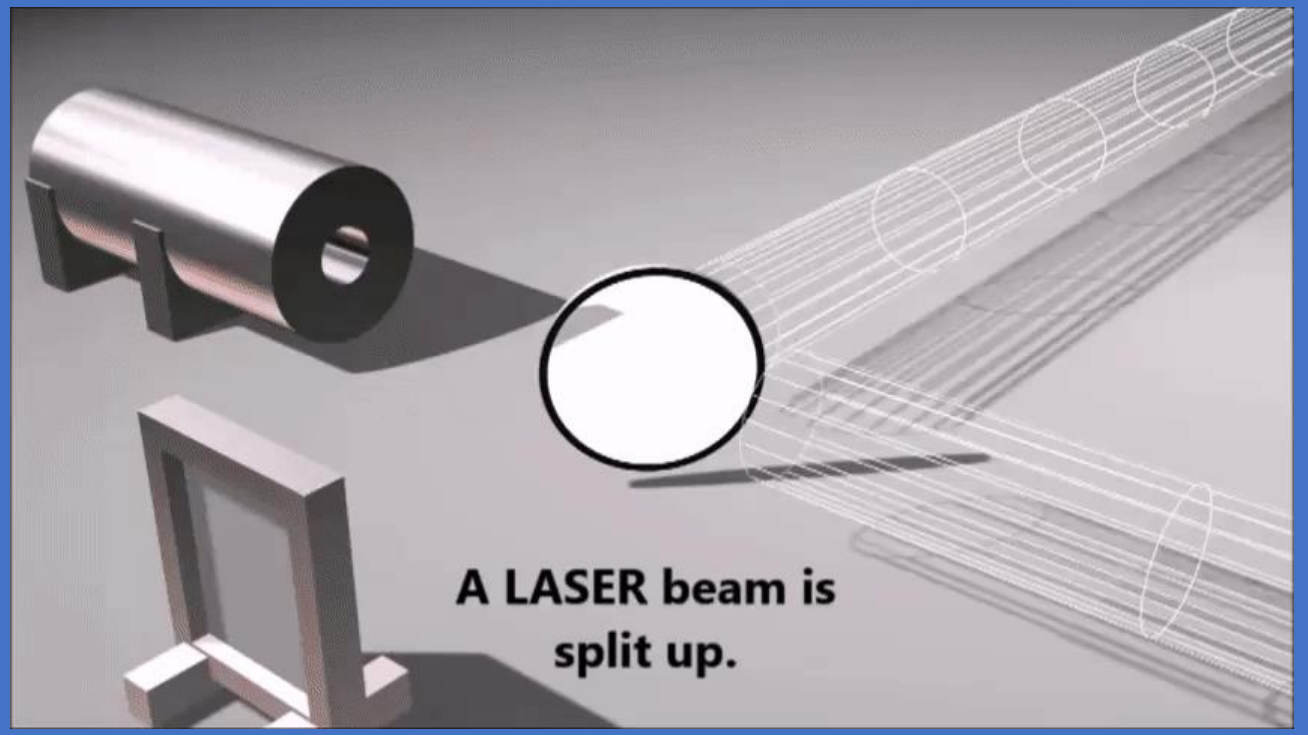
- They were predicted by Einstein in 1916, but he was skeptical they could ever be measured.



Detecting Gravitational Waves

- The Laser Interferometer Gravitational-Wave Observatory (LIGO) first detected gravitational waves in Sept. 2015.



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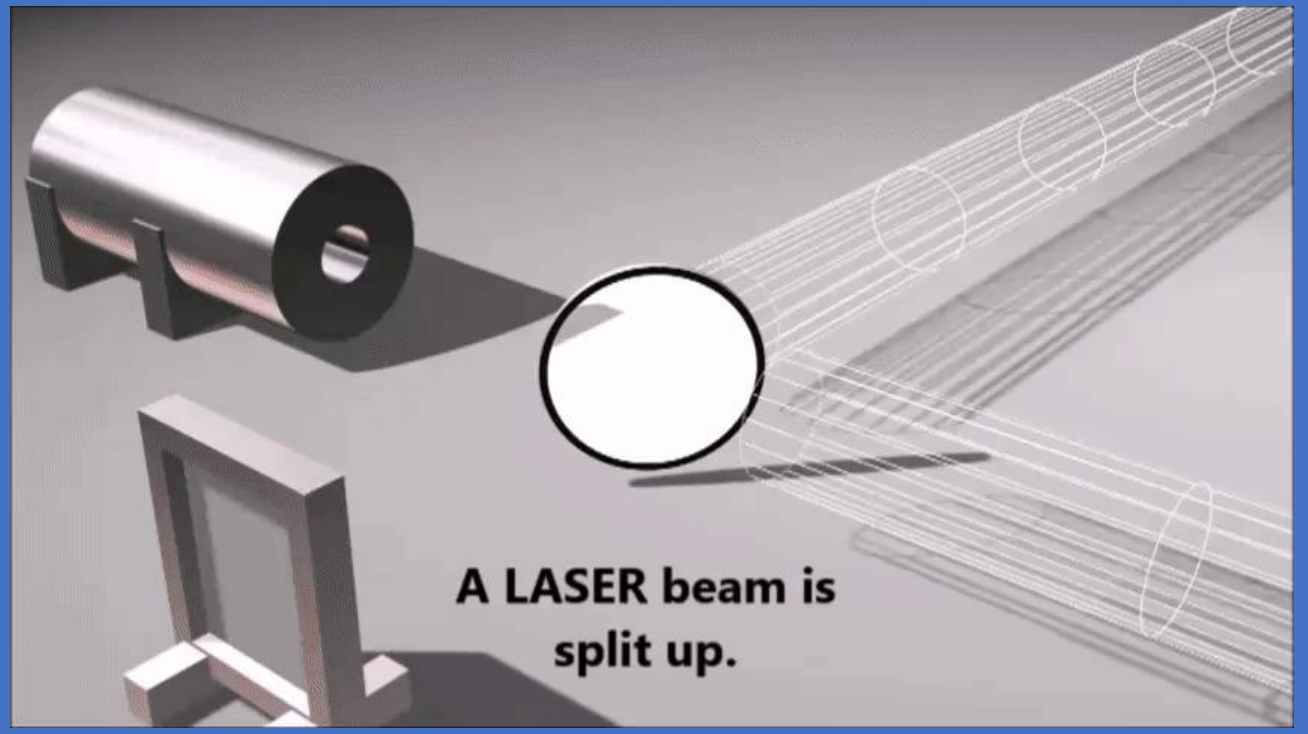

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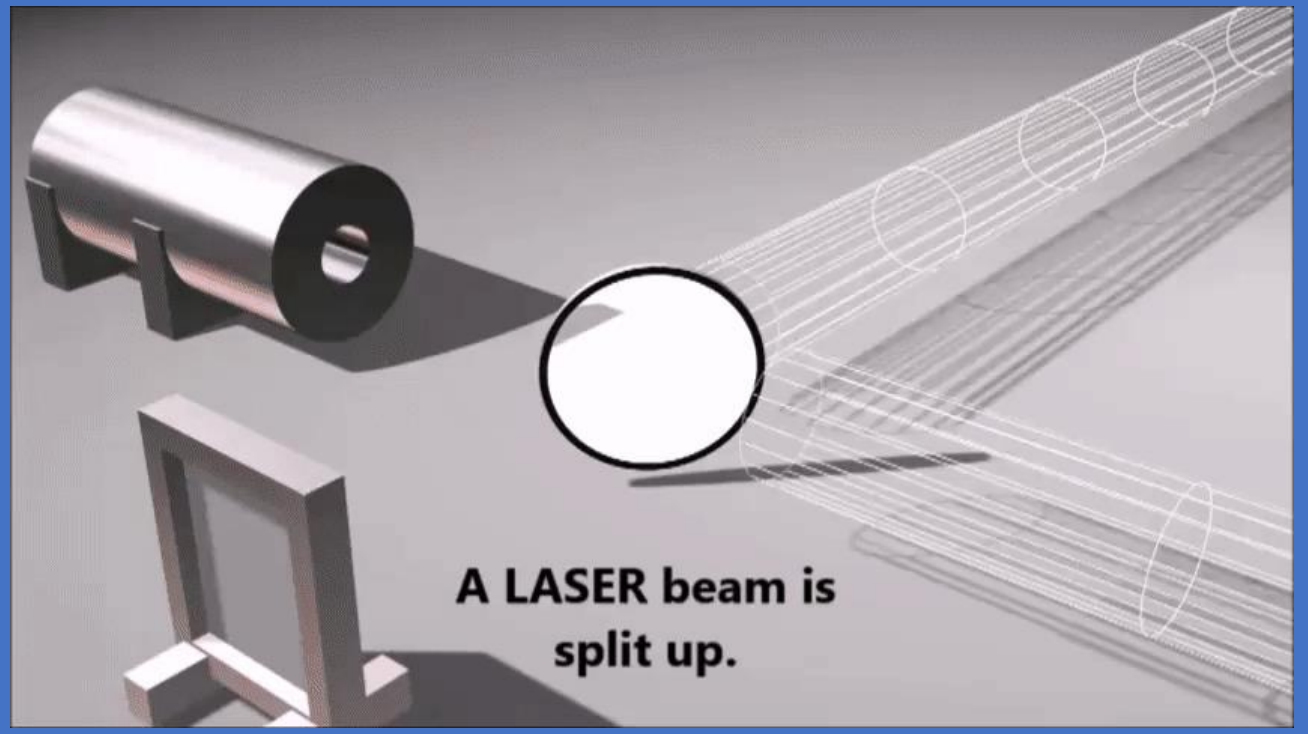
- This won the 2017 Nobel Prize.



The Event Horizon Telescope

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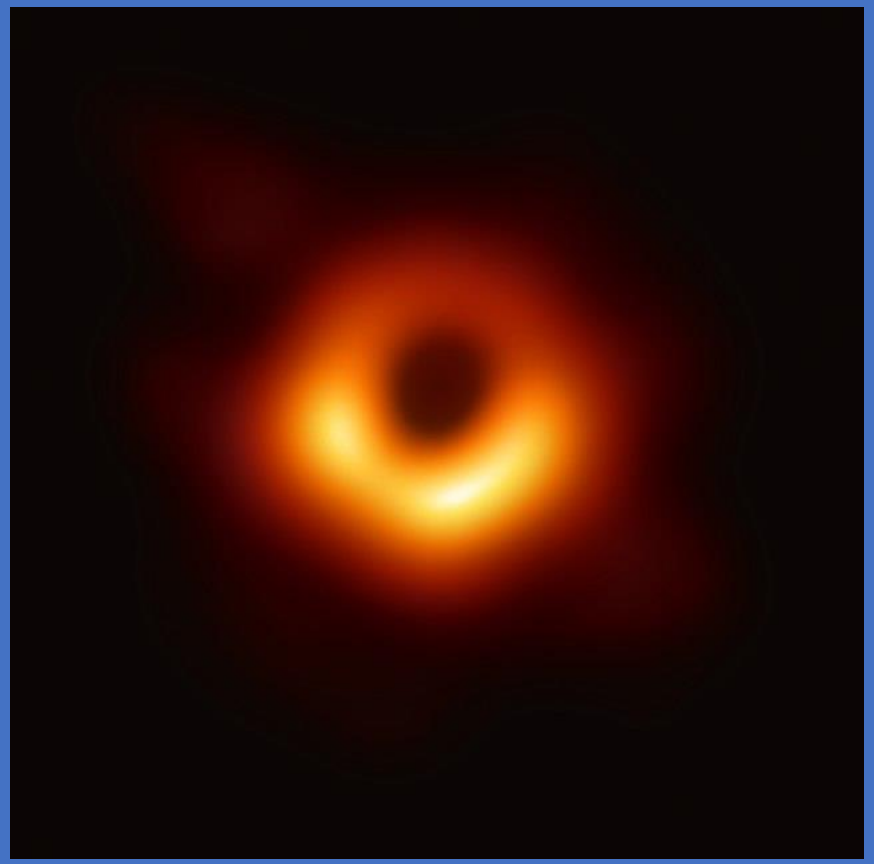

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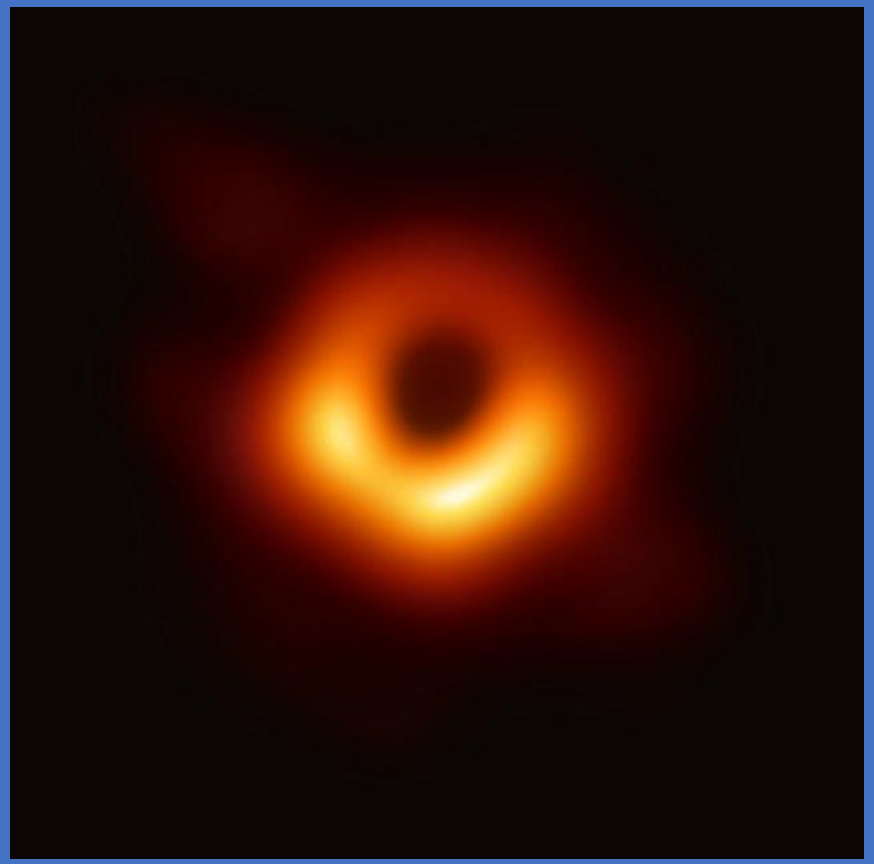

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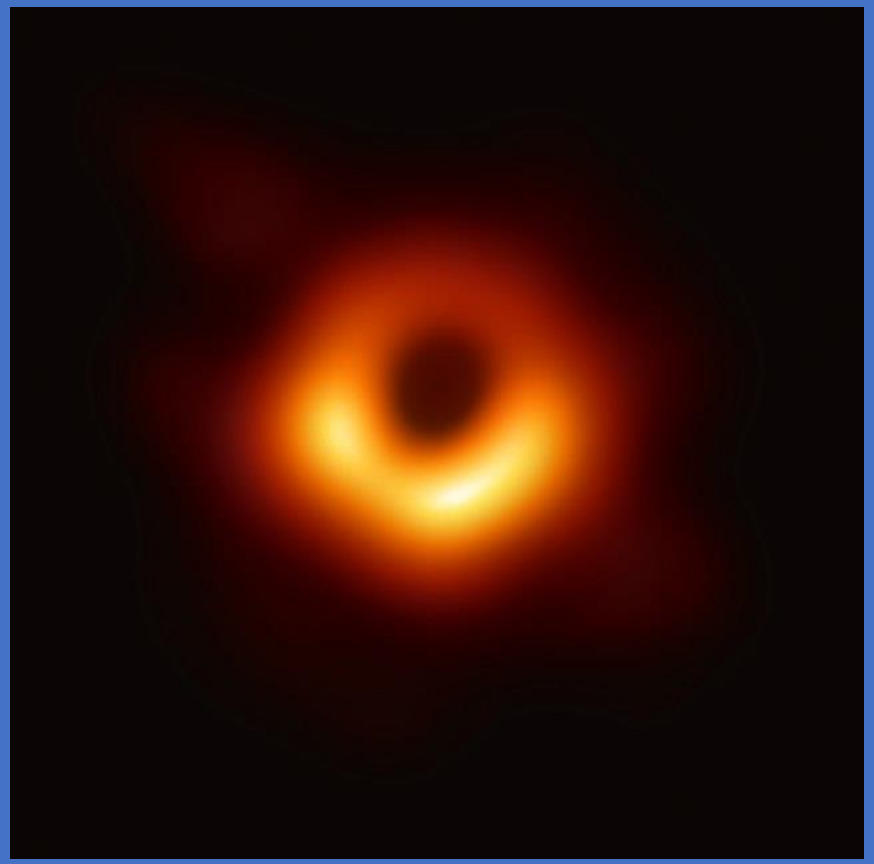
- This black hole is at the center of a galaxy called Messier 87, over 50 million light-years away.



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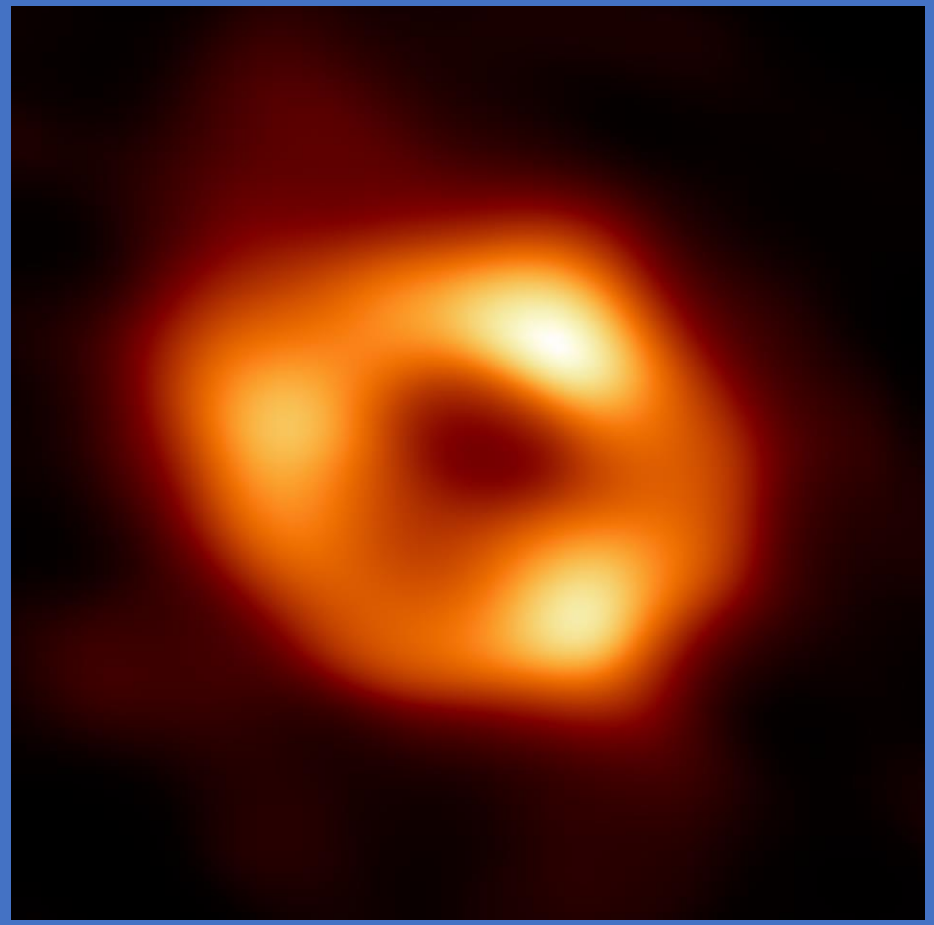
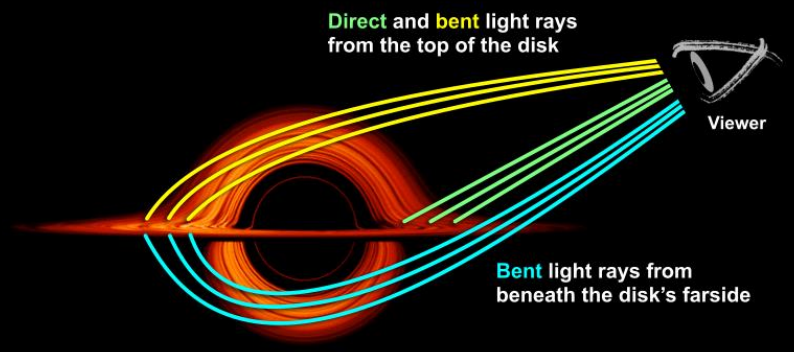
- In 2022, they did it again, but this time for Sagittarius A*, the black hole at the center of our galaxy, 27,000 light-years away.

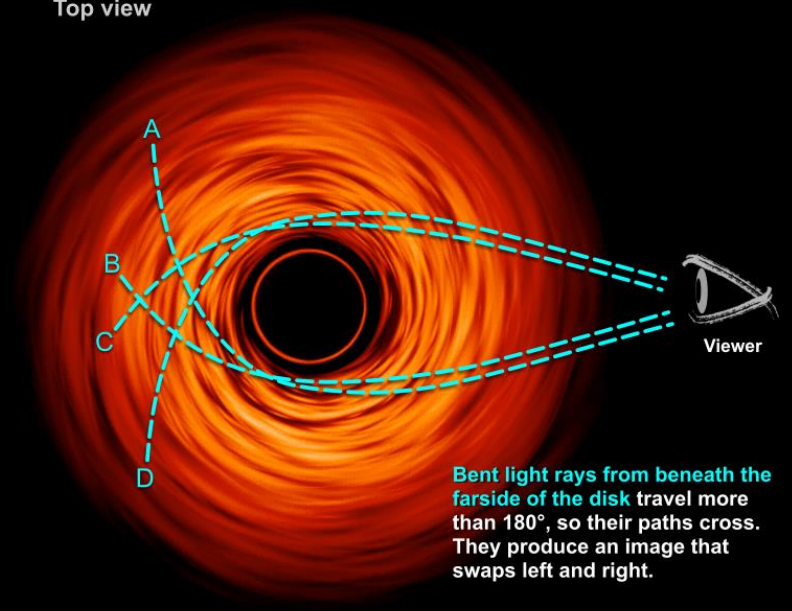


A Warped Look at Black Hole Optics

Side view



Top view



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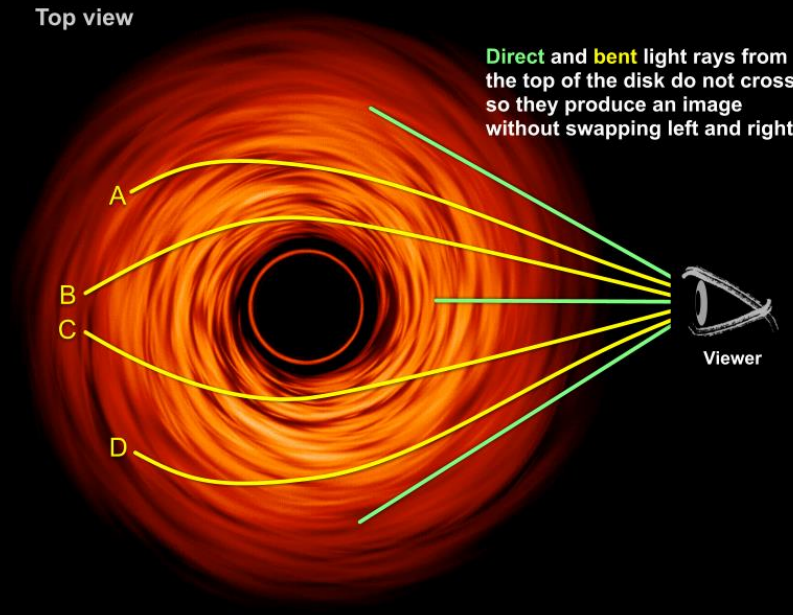
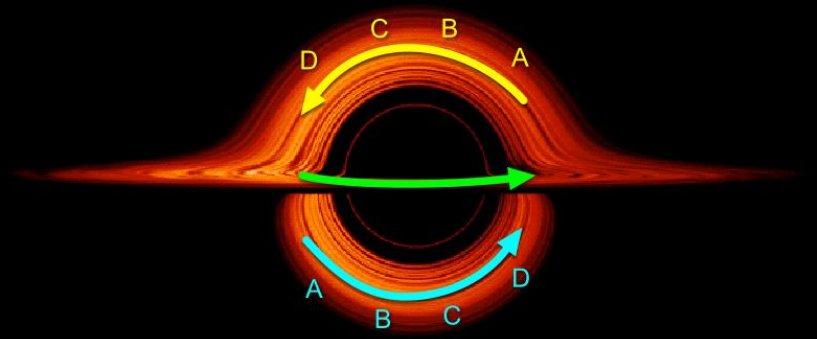
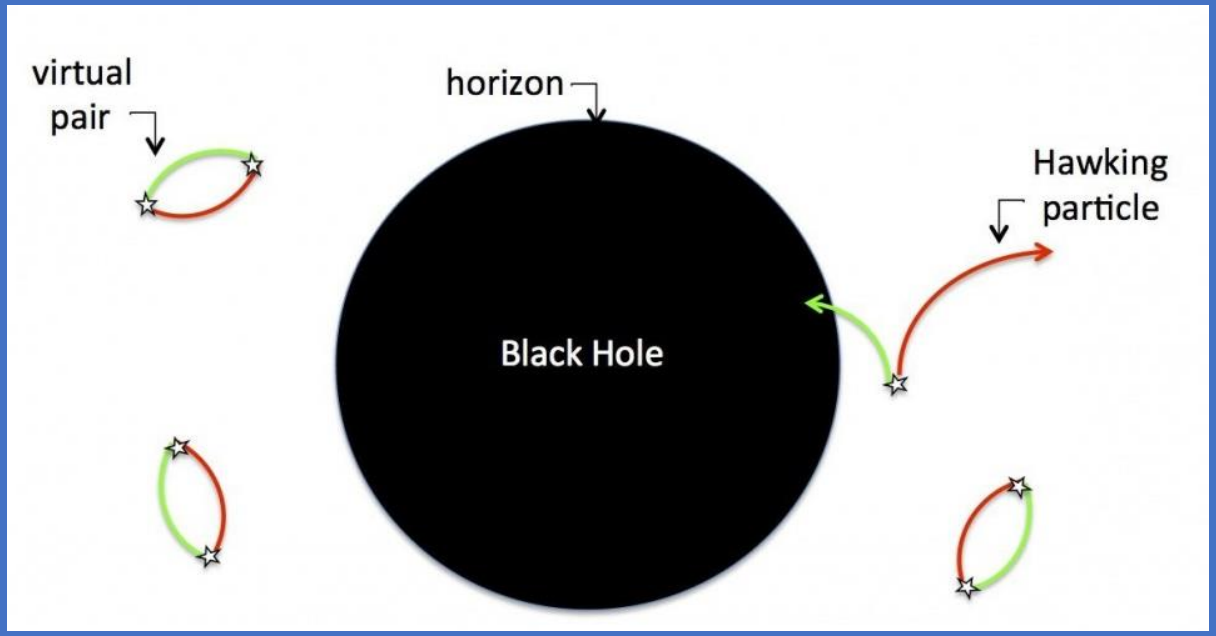


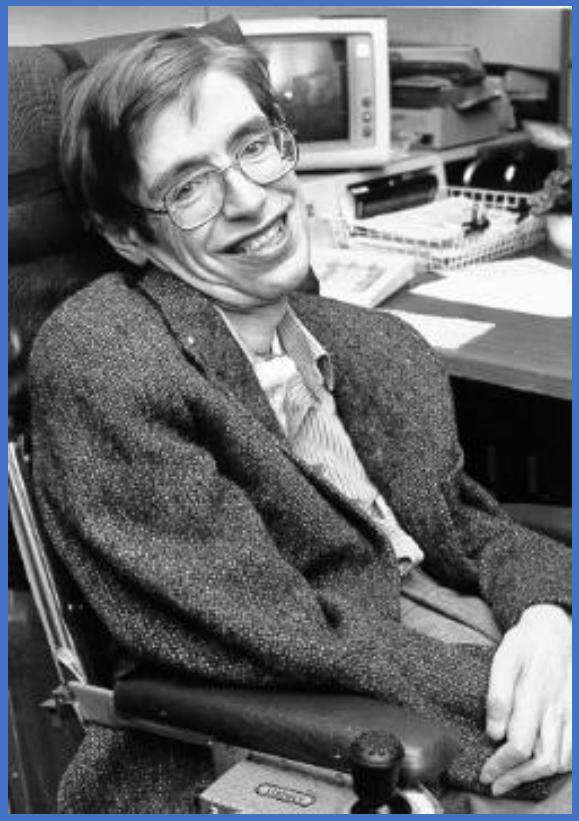


Apparent image and disk motion

Hawking Radiation

- Virtual particle-antiparticle pairs are constantly being created and destroyed.

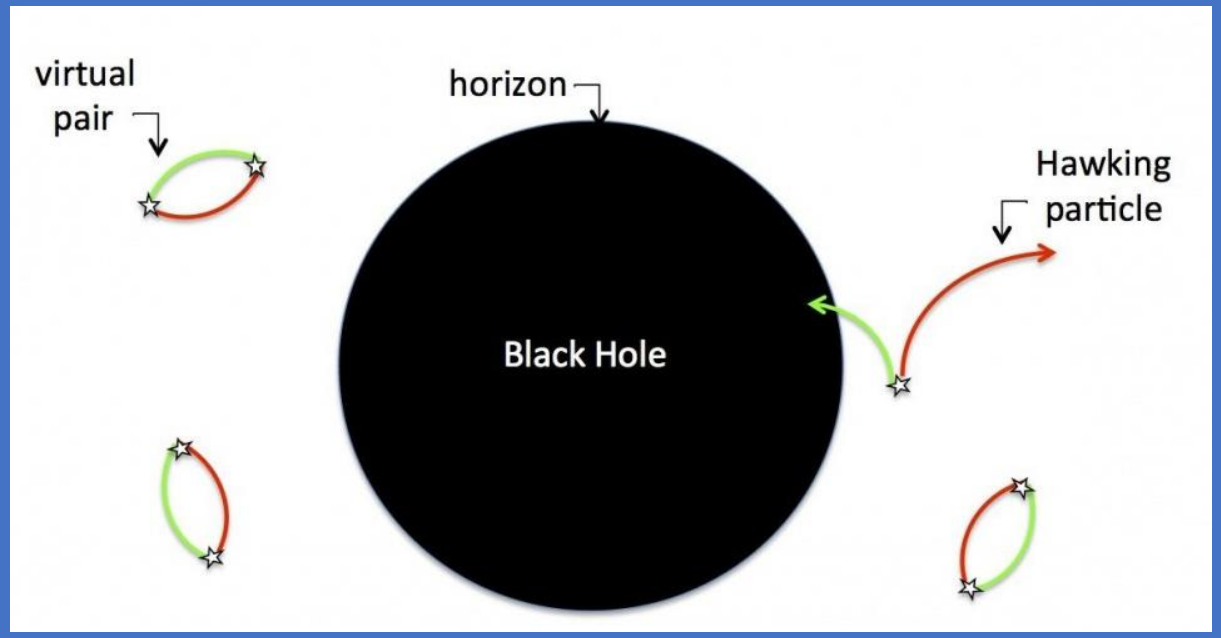


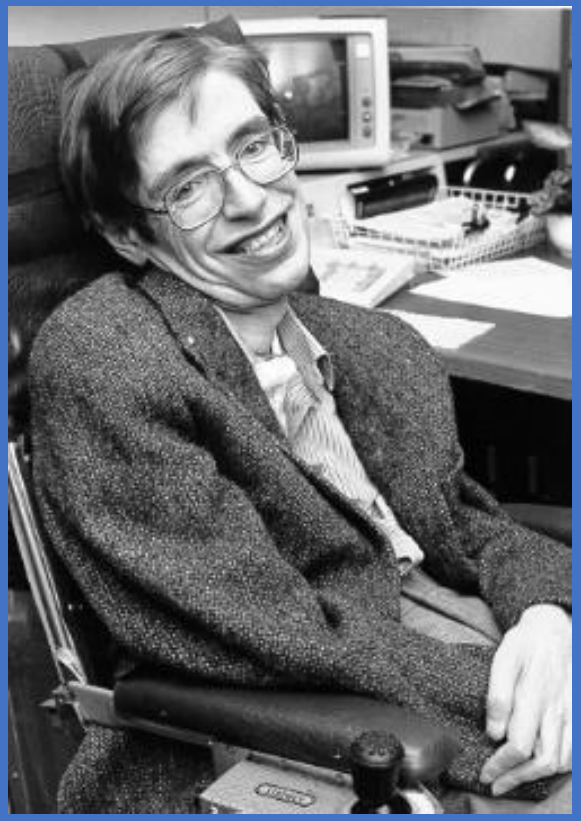


Stephen Hawking

Hawking Radiation

- Virtual particle-antiparticle pairs are constantly being created and destroyed.
- Near the horizon, one member can fall in while the other escapes. This particle is observed as radiation.

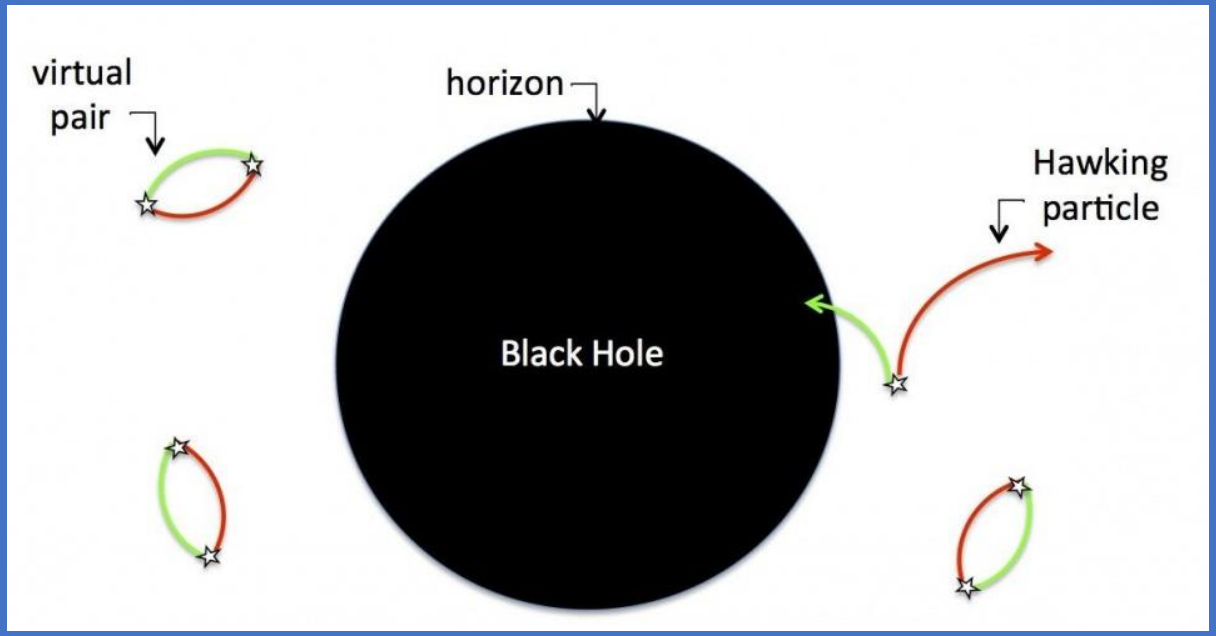


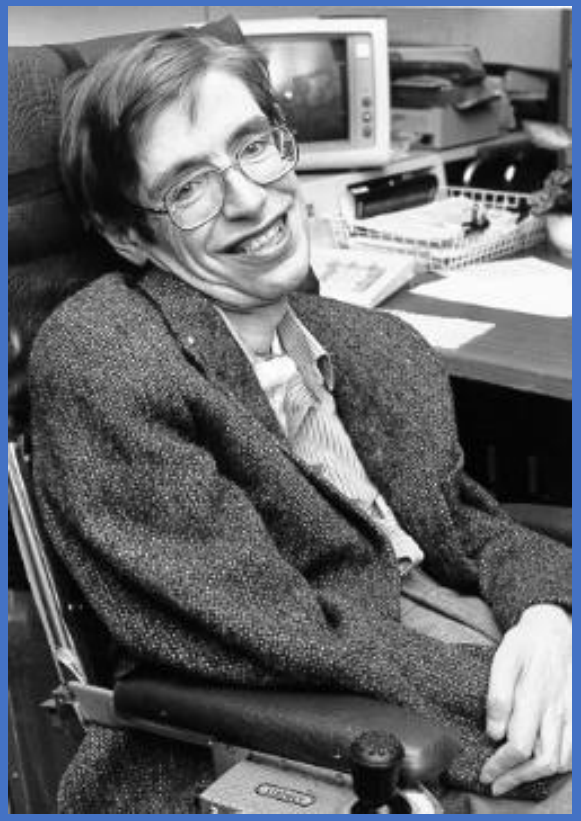


Stephen Hawking

Hawking Radiation

- Virtual particle-antiparticle pairs are constantly being created and destroyed.
- Near the horizon, one member can fall in while the other escapes. This particle is observed as radiation.
- Thus black holes have a very small temperature.





Stephen Hawking

The Information Paradox

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The Information Paradox

- Black holes all look the same, regardless of what went into them. They are simply determined by their mass (and charge and angular momentum).
- The Hawking radiation thus doesn't contain any information about what went into the black hole.
- However, a fundamental principle of quantum mechanics is that information cannot be destroyed.
- Either our understanding of black holes is wrong, or quantum mechanics itself is wrong (or both!).
- The solution may be related to the AdS/CFT correspondence.

Thank You!

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