Comparing Three Riemann Solvers for Relativistic Hydrodynamics

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Background

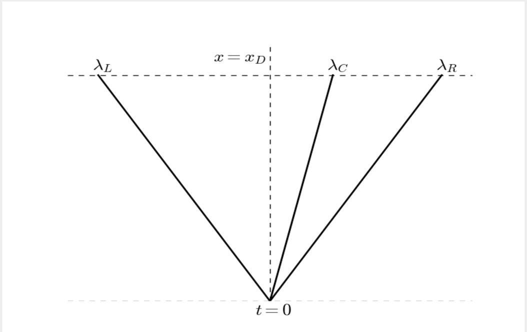
Understanding the mechanism behind core-collapse supernovae is a fascinating question in and of itself, but its solution also has implications for topics such as metal distribution in the universe. This complex problem involves several moving parts, one of which is the transport, mixing, and redistribution of **relativistic** fluids, hydrodynamics. The governing equations are complex and non-linear, and therefore require numerical solutions. Finding stable and robust methods is very important because in order to draw a meaningful conclusion we need to know that the results we obtain are accurate and reliable.

Discontinuous Galerkin Approach

- Discretize the spatial domain
- Assume that the solution can be represented by a polynomial
- Assume a number of points, or nodes, in each spatial element, say N_P
- These polynomials are local, i.e. they are discontinuous at the cell interfaces with the polynomials in the adjacent cells
- This sets up a Riemann problem at each interface

Riemann Problem

- Fluid-filled tube separated into two states by a divider
- Different values of density, velocity, and pressure on either side of the divider – a discontinuity
- At t = 0 the divider is removed and the fluid evolves according to the conservation of mass, energy, and momentum
- How do the variables evolve—Riemann Fan (see figure)



The Three Solvers

Local Lax-Friedrichs (LLF)

- Consider left and right moving waves to move with same speed
  - Requires an estimate for the wave speed
- Ignores contact discontinuity
- Works for all hyperbolic conservative systems

Harten-Lax-van-Leer (HLL)

- Consider left and right moving waves to move with different speeds
  - Requires two estimates, one for each wave
- Ignores contact discontinuity
- Works for all hyperbolic conservative systems

Harten-Lax-van-Leer Contact (HLLC)

- Consider left and right moving waves to move with different speeds
  - Requires two estimates, one for each wave
- Takes contact discontinuity into account
  - Requires estimate for the contact wave speed
  - This adds computational expense
- Uses the fact that we are solving hydrodynamics equations

Test Cases: Riemann Problems

Sod's Shock Tube

Density: ρ
Velocity: v
Pressure: p
Γ: 4/3
Initial Conditions:

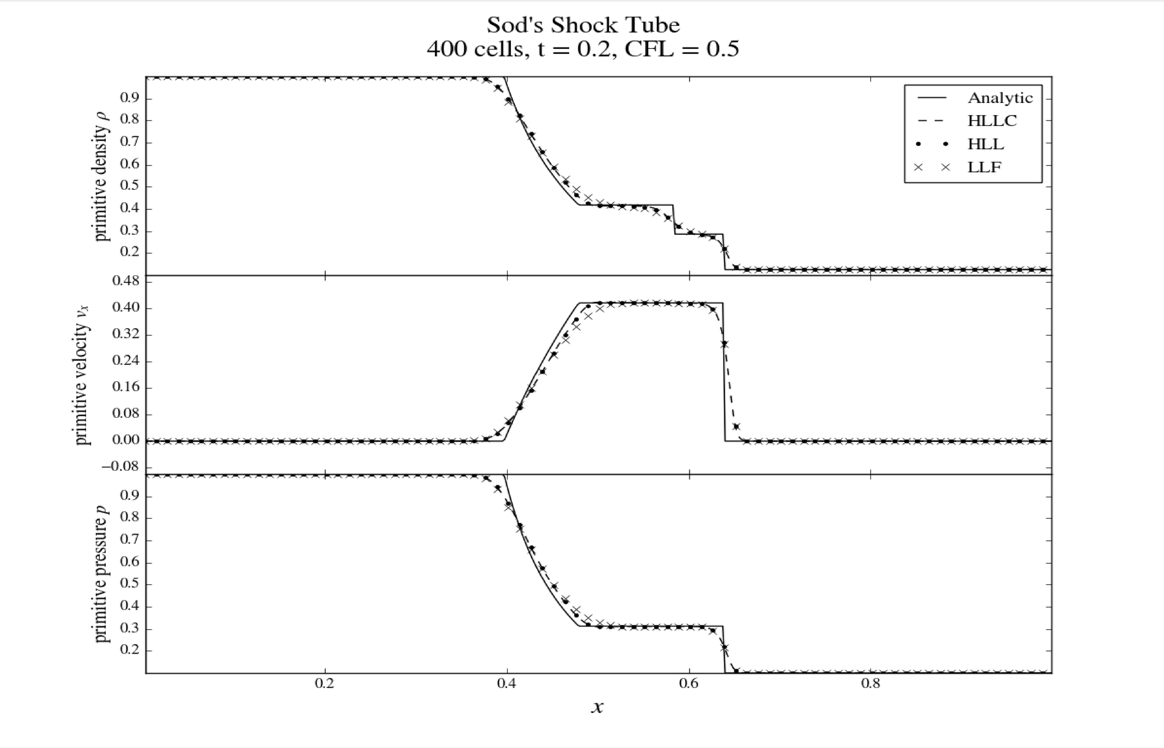
$\rho_L = 1$, $v_L = 0$, $p_L = 1$
$P_R = 0.125$, $v_R = 0$, $p_R = 0.01$

Shocks around x = 0.4 and x = 0.65

The shock at x = 0.65 is a shock front, a point where the primitive variables change abruptly over a small spatial region.

The shock at x = 0.4 is the head of a rarefaction wave

Looking at the density around x = 0.57 we see another discontinuity. This is the contact discontinuity. Its defining feature is that it is discontinuous in the density but continuous in the velocity and pressure



Contact Discontinuity

Density: ρ
Velocity: v
Pressure: p
Γ: 4/3
Initial Conditions:

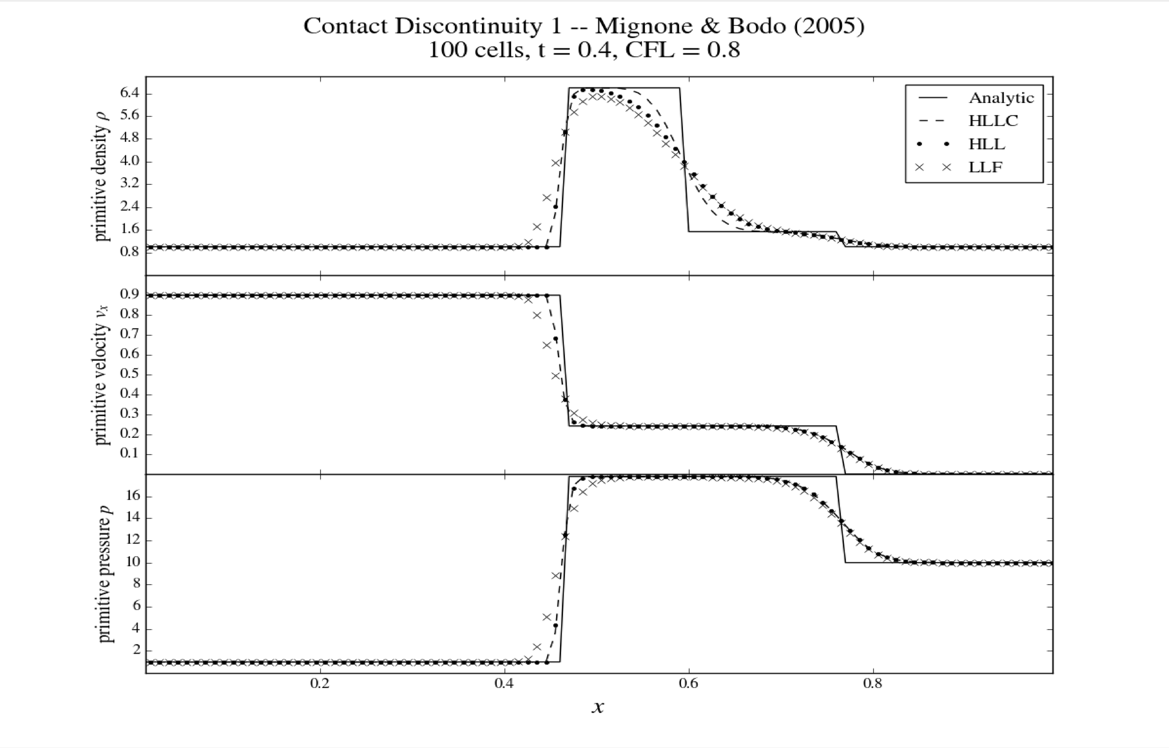
$\rho_L = 1$, $v_L = 0.9$, $p_L = 1$
$\rho_R = 1$, $v_R = 0$, $p_R = 10$

Very relativistic, v = 0.9c

Shock front around x = 0.8

The HLLC solver clearly captures the contact discontinuity better than both the LLF and HLL solvers

Approximate solvers do not capture the shock very well. This diffusiveness improves with a finer grid spacing



Conclusions and Future Work

Since the HLLC solver clearly does a better job of capturing the contact discontinuity, and because the contact discontinuity plays an important role in the physics of supernovae, we conclude that despite its added computational expense it is certainly the best choice for a Riemann solver.

With a Riemann solver chosen the next step will be to implement it in the code, which will involve deriving the appropriate equations from general relativistic hydrodynamics.

References

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Background image from: https://taliasworld.files.wordpress.com/2013/07/p0320ay.jpg

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