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HW 6

6.2 $Z_i = \frac{1}{j\omega C} + \frac{j\omega L R}{R + j\omega L} = \frac{R - \omega^2 RLC + j\omega L}{-\omega^2 LC + j\omega RC}$

$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC)^2 + (\omega RC)^2}$

$= \frac{(R - \omega^2 RLC)(-\omega^2 LC) + \omega L(\omega RC) - j((\omega L)(\omega^2 LC) + \omega RC(R - \omega^2 RLC))}{(-\omega^2 LC)^2 + (\omega RC)^2}$

take imaginary part & equate to 0

$(\omega L)(\omega^2 LC) + \omega RC(R - \omega^2 RLC) = 0$

$\cancel{\omega} RC(R - \omega^2 RLC) = -(\cancel{\omega} L)(\omega^2 LC)$

$R^2 C - \omega^2 R^2 LC^2 = -\omega^2 L^2 C$

$R^2 C = \omega^2 R^2 LC^2 - \omega^2 L^2 C$

$\sqrt{\frac{R^2 C}{R^2 LC^2 - L^2 C}} = \omega$ input given values

$\boxed{\omega = 20\,\text{krad/s}}$ $|a+bi| = \sqrt{a^2+b^2}$

6.8 b) $H(\omega) = \frac{0.4(50 + j\omega)^2}{(j\omega)^2} = \frac{1000(1 + \frac{j\omega}{50})^2}{(j\omega)^2} = \frac{-1000(1 + \frac{j\omega}{50})^2}{\omega^2}$

$M(\omega) = \frac{1000\left(\sqrt{1 + (\omega/50)^2}\right)^2}{(\sqrt{\omega^2})^2}$ $1 - (\frac{\omega}{50})^2 + \frac{2j\omega}{25}$

$M(\omega) = \frac{1000(1 + (\omega/50)^2)}{\omega^2}$

$M[dB] = 20\log 1000 + 20\log(1 + (\frac{\omega}{50})^2) - 40\log\omega$

$\phi(\omega) = \tan^{-1}(0/(1000)) + 2\tan^{-1}(\frac{\omega}{50}) - 2\tan^{-1}(\frac{\omega}{0})$

$= 2\tan^{-1}(\frac{\omega}{50}) - 180°$

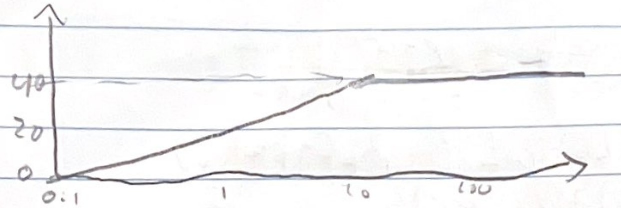
$\frac{0.4(50 + j\omega)^2}{(j\omega)^2} = \frac{-0.4(2500 + 100j\omega + j^2\omega^2)}{\omega^2}$

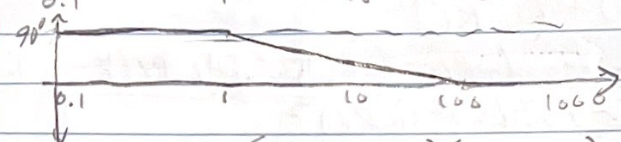
$= \frac{-1000 - 40j\omega + \omega^2}{\omega^2}$

$\lim_{\omega \to \infty} M[dB] = 20\log(1000) + 40\log(\frac{\omega}{50}) - 40\log\omega$

$= 20\log(1000) + \cancel{40\log\omega} - 40\log 50 - \cancel{40\log\omega}$

$= -7.96$





d.) $H(\omega) = \dfrac{(20+j5\omega)(20+j\omega)}{j\omega} = \dfrac{400\left(1+\frac{j\omega}{4}\right)\left(1+\frac{j\omega}{20}\right)}{j\omega}$

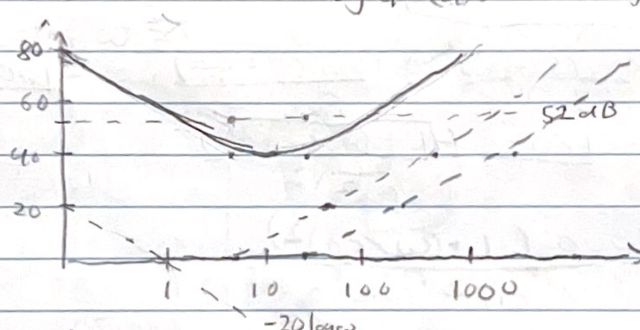
$M(\omega) = \dfrac{400\left(\sqrt{1+\frac{\omega^2}{16}}\right)\left(\sqrt{1+\frac{\omega^2}{400}}\right)}{\sqrt{\omega^2}}$

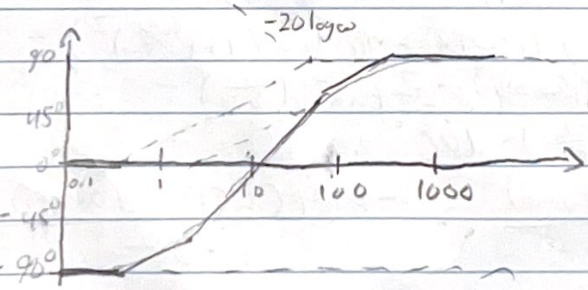
$M[dB] = 20\log 400 + 10\log\left(1+\left(\frac{\omega}{4}\right)^2\right) + 10\log\left(1+\left(\frac{\omega}{20}\right)^2\right) - 20\log\omega$

$\Phi(\omega) = \tan^{-1}\left(\frac{\omega}{4}\right) + \tan^{-1}\left(\frac{\omega}{20}\right) - 90°$

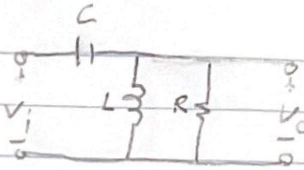
As $\omega \Rightarrow 0$ $\quad 10\log\left(1+\left(\frac{\omega}{4}\right)^2\right) \Rightarrow 0 \quad 10\log\left(1+\left(\frac{\omega}{20}\right)^2\right) \Rightarrow 0$

As $\omega \Rightarrow \infty$ $\quad 10\log\left(1+\left(\frac{\omega}{4}\right)^2\right) \Rightarrow 20\log\left(\frac{\omega}{4}\right) \quad 10\log\left(1+\left(\frac{\omega}{20}\right)\right)^2 \Rightarrow 20\log\left(\frac{\omega}{20}\right)$





6.17a)



$$H(\omega) = \frac{V_o}{V_i} = \frac{(R \| j\omega L)}{\frac{1}{j\omega C} + (R \| j\omega L)}$$

$$= \left(\frac{jR\omega L}{R+j\omega L}\right) \Big/ \left(\frac{1}{j\omega C} + \frac{jR\omega L}{R+j\omega L}\right)$$

$$= \left(\frac{jR\omega L}{\cancel{R+j\omega L}}\right) \Big/ \left(\frac{R+j\omega L + j\omega C(jR\omega L)}{(j\omega C)\cancel{(R+j\omega L)}}\right)$$

$$= jR\omega L \cdot \frac{j\omega C}{R+j\omega L+j\omega C(jR\omega L)}$$

$$= \frac{-R\omega^2 LC}{R + j\omega L - R\omega^2 CL}$$

$$H(\omega) = \boxed{\frac{-\omega^2 LC}{1 + \frac{j\omega L}{R} - \omega^2 CL}}$$

b) $$M(\omega) = \frac{\omega^2 LC}{\sqrt{(1-\omega^2 CL)^2 + (\frac{\omega L}{R})^2}}$$

$$M[dB] = 20\log(\omega^2 LC) - 10\log\left((1-\omega^2 CL)^2 + \left(\frac{\omega L}{R}\right)^2\right)$$

$$= 20\log(LC) + 40\log(\omega) - 10\log\left((1-\omega^2 CL)^2 + \left(\frac{\omega L}{R}\right)^2\right)$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{\omega L}{R} \Big/ (1-\omega^2 CL)\right)$$

as $\omega \Rightarrow \infty$ $M[dB] \Rightarrow 20\log(LC) + 40\log\omega - 40\log(\omega\sqrt{CL})$

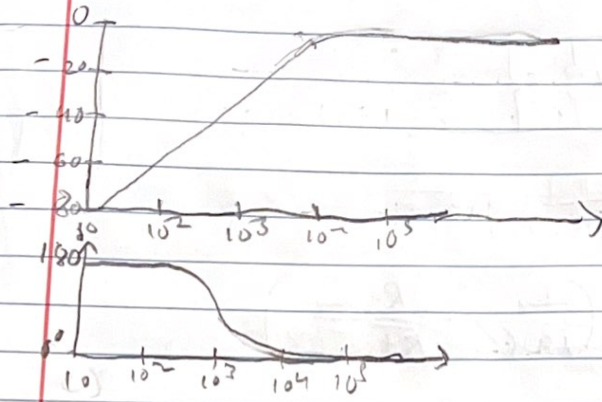
$\Rightarrow 20\log(LC) - 20\log(LC)$

$= 0$

$\phi(\omega) \Rightarrow 0$

as $\omega \Rightarrow 0$ $M[dB] \Rightarrow 20\log(LC) + 40\log\omega - \overset{0}{\cancel{10\log(1)}}$

$\phi(\omega) \Rightarrow 180°$



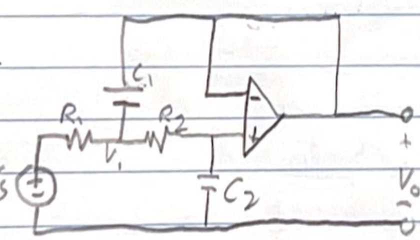
c) $H(\omega) = \dfrac{-(\omega/\omega_c)^2}{1 + j2\xi\omega/\omega_c + (j\omega/\omega_c)^2}$

$\omega_c = 10^4$ rad/s

$= 40$

slope when $\omega/\omega_c << 1$ = 40 dB/decade

6.22



a) $\dfrac{V_s - V_1}{R_1} = \dfrac{V_1 - V_o}{R_2} + \dfrac{V_1 - V_o}{1/j\omega C_1}$

$\dfrac{R_2}{j\omega C_1}(V_s - V_1) = \dfrac{R_1}{j\omega C_1}(V_1 - V_1) + R_1 R_2 (V_1 - V_o)$

$\dfrac{R_2}{j\omega C_1} V_s + \left[\dfrac{R_1}{j\omega C_1} + R_1 R_2\right] V_o = \left[\dfrac{R_1}{j\omega C_1} + \dfrac{R_2}{j\omega C_1} + R_1 R_2\right] V_1$

$V_1 = j\omega V_o R_2 C_2 + V_o$

$V_1 = V_o (j\omega R_2 C_2 + 1)$

$\dfrac{V_o}{V_s} = \boxed{\dfrac{1}{-\omega^2 R_1 R_2 C_1 C_2 + j\omega C_2 (R_1 + R_2) + 1} = H(\omega)}$

b)

b) $H(s) = \dfrac{1}{j^2\omega^2 R_1R_2C_1C_2 + j\omega C_2(R_1+R_2)+1}$

$= \dfrac{1}{s^2R_1R_2C_1C_2 + sC_2(R_1+R_2)+1}$

$= \dfrac{1/R_1R_2C_1C_2}{s^2 + \dfrac{s(R_1+R_2)}{R_1R_2C_1} + \dfrac{1}{R_1R_2C_1C_2}}$

$= \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$\zeta = \frac{1}{2}(R_1+R_2)\sqrt{\frac{C_2}{R_1R_2C_1}}$

$Q = 1/2\zeta = 2.5$

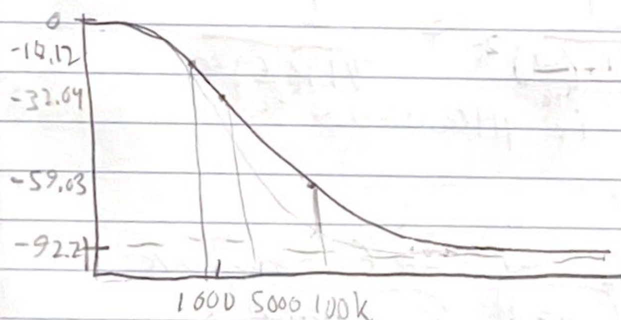
$M[dB] = -10\log[(1-\omega^2(4\cdot10^{-8}))^2+\omega^2(64\cdot10^{-6})]$

$\omega \Rightarrow 0 \quad M[dB] \Rightarrow 0$

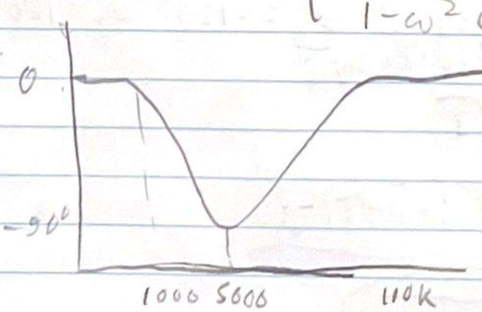
$\omega \Rightarrow 10^6 \quad M[dB] \Rightarrow -92.21\,dB$

at $\omega = \omega_n = 5\cdot10^3$

$M[dB] = -32.04\,dB$



$\phi(\omega) = -\tan^{-1}\left(\dfrac{\omega(80\cdot10^{-6})}{1-\omega^2(4\cdot10^{-8})}\right)$



c) low-pass filter circuit with cutoff frequency $250\cdot 10^5$ r/s
max gain $G=1$ R_1 and R_2 are equal

6.28 $|H(f)| = |H_1(f)| + |H_2(f)|$

$h_{LP_1}(t) = 4 sinc(4\pi t)$
$h_{LP_2}(t) = 2 sinc(2\pi t)$
$\boxed{h(t) = 4 sinc(4\pi t) + 2 sinc(2\pi t)}$

6.35 $\omega_0 = 2\pi f_0 = 200\pi$ rad
$h(t) = \delta(t) - Ae^{-at}\cos(\omega_0 t + \theta)u(t)$
$\theta = \tan^{-1}\left(\frac{a}{2\omega_0}\right)$
$H(s) = 1 - \frac{2as + a^2}{(s+a)^2 + \omega_0^2}$
$= \boxed{1 - \frac{200(s+50)}{s^2 + 200s + 404784.176}}$

6.44 a) $|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^{2n}}}$
at $f = 10$ Hz is $|H(\omega)| = 0.9$
$\left(\frac{10}{f_c}\right)^{2n} = \frac{19}{81}$
$f_c = 12.733$ Hz $\omega_c = 2\pi f_c = 80$ rad/s
$\boxed{n = 3}$ $\boxed{\omega_c = 80 \text{ rad/s}}$

b) $\theta_i = \pm 120°, 180°$ $\theta_1 = 120°$ $\theta_2 = -120°$ $\theta_3 = 180°$
$p_1 = \omega_c e^{j\theta_1} = -40 + j40\sqrt{3}$
$p_2 = -40 - j40\sqrt{3}$
$p_3 = -80$
$\boxed{-40 + j40\sqrt{3}, -40 - j40\sqrt{3}, -80}$

c) $= \boxed{\frac{1}{s^3 + 160s^2 + 12800s + 512000}}$

d) $H(s) = \dfrac{1}{s^3 + 160s^2 + 12800s + 512000}$

$s^3 Y(s) + 160s^2 Y(s) + 12800s Y(s) + 512000\, Y(s) = X(s)$

$\dfrac{d^3 y(t)}{dt^3} + 160\dfrac{d^2 y(t)}{dt^2} + 12800\dfrac{dy(t)}{dt} + 512000\, y(t) = x(t)$

6.54. $x(t) = A\cos(2\pi f_T t)$ $\quad x_c(t) = \cos(2\pi f_c t)$

a) $s_{DSB}(t) = x(t)x_c(t) = A\cos(2\pi f_T t)\cos(2\pi f_c t)$
$= \frac{A}{2}\cos(2\pi(f_T + f_c)t) + \frac{A}{2}\cos(2\pi(f_c - f_T)t)$

$\boxed{\text{power} = \frac{A^2}{4}}$

b) $s_{Am}(t) = 1(1 + k_a A\cos 2\pi f_T t)\cos 2\pi f_c t$
$= \cos 2\pi f_c t + \frac{1}{2}\cos 2\pi(f_T + f_c)t + \frac{\cos 2\pi(f_c - f_T)t}{2}$

$P = \frac{1^2}{2} + \frac{(\frac{A}{2})^2}{2} + \frac{(\frac{A}{2})^2}{2}$
$= \boxed{\frac{3A^2}{4} \text{ watts}}$

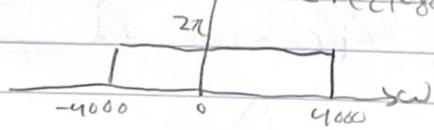
6.57

a) $h(t) = \dfrac{\sin(2\pi 2000 t)}{\pi t} 2\cos[2\pi(f_c + 2000)t]$

$= m(t)n(t)$

$m(t) = 8000\left[\dfrac{\sin(2\pi 2000t)}{2\pi 2000t}\right] = 8000\,\text{sinc}(4000t)$

$M(\omega) = 2\pi\left[\text{rect}\left(\frac{\omega}{8000}\right)\right]$

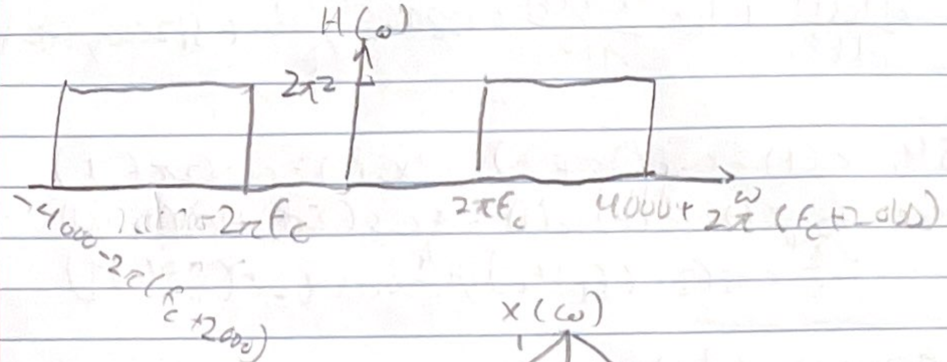


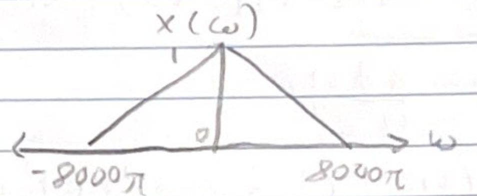
$n(t) = \cos(2\pi(f_c + 2000)t]$

$N(\omega) = \pi[\delta(\omega - 2\pi(f_c + 2000)] + \delta\{\omega + 2\pi(f_c + 2000)\}]$

$H(\omega) = M(\omega) \cdot N(\omega)$

$= 2\pi^2 \text{rect}\left(\frac{\omega - 2\pi(f_c + 2000)}{8000}\right) + 2\pi^2 \text{rect}\left(\frac{\omega + 2\pi(f_c + 2000)}{8000}\right)$

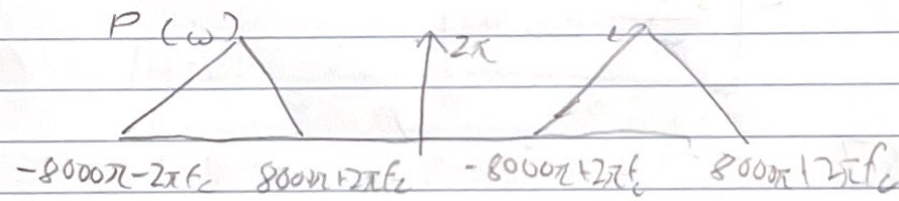




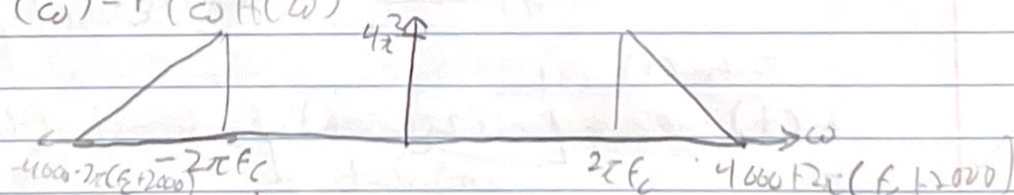
$p(t) = x(t) 2\cos(2\pi f_c t)$

$P(\omega) = 2X(\omega) * \pi(\delta(\omega - 2\pi f_c) + \delta(\omega + 2\pi f_c))$

$= 2\pi[X(\omega - 2\pi f_c) + X(\omega + 2\pi f_c)]$

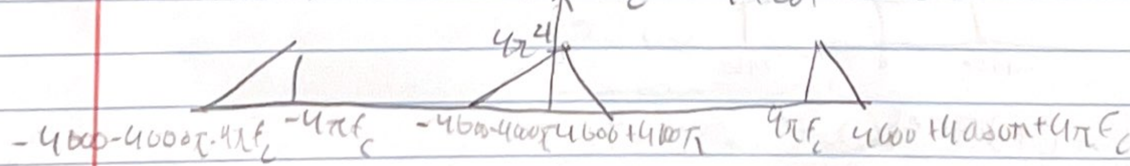


$Y(\omega) = P(\omega) H(\omega)$



b) $r(t) = y(t)\cos(2\pi f_c t)$

$R(\omega) = \pi[Y(\omega - 2\pi f_c) + Y(\omega + 2\pi f_c)]$



pass r(t) throug low pass w/ cutoff 4kHz

c) $B_{SSB} = 4000 + 2\pi(f_c + 2000) - 2\pi f_c$

$= 4000 \text{ Hz}$

$= B$

d) $B_{SSB} = 4000 + 2\pi(f_c + 2000) - [-4000 + 2\pi(f_c + 2000)]$

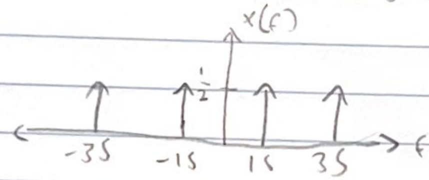
$= 8000 \text{ Hz}$

$= B$

$P_{SSB} = \frac{P_c m_a^2}{4}$ SSB requires half the power and bandwidth of DSB-SC

6.64 $x(t) = \sin(30\pi t) + \sin(70\pi t)$

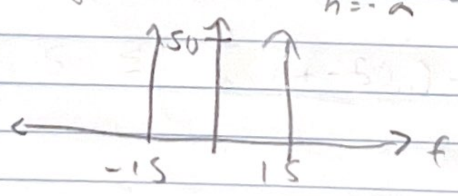
$X(f) = \frac{1}{2}(\delta(f-15) + \delta(f+15)) + \frac{1}{2}(\delta(f-35) + \delta(f+35))$



$x(t)\,\delta(t) = x_\delta(t)$

$x_\delta(t) \longleftrightarrow f_s \sum_{n=-\infty}^{\infty} X(f - nf_s)$

$x_\delta(t) \longleftrightarrow 50 \sum_{n=-\infty}^{\infty} X(f - 50n)$



6.67 $x(n) = x(nT_s) = x\left(\frac{k}{f_s}\right)$

If $x(t) \longleftrightarrow \hat{x}(f)$

$x(n) \longleftrightarrow \frac{1}{T_s} \sum_{k=\infty} X(f - kf_s)$