AM 147: Computational Methods and Applications: Winter 2023

Homework #8

Instructor: Abhishek Halder

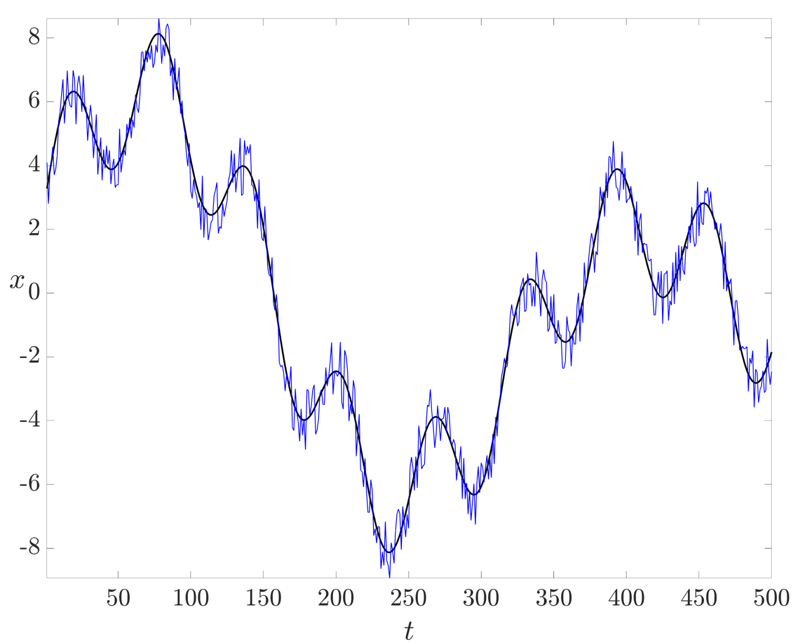
Due: March 08, 2023

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW8.zip` via CANVAS. For example, `HalderAbhishekHW8.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m (MATLAB script) and .pdf files for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Estimate true signal from measured noisy signal (50 points)



Download the starter code `W23HW8.m` from CANVAS Files section folder "HW Problems and Solutions" to your computer, and rename it as `YourlastnameYourfirstnameHW8.m`. The starter code plots a true signal $x(t)$ (in solid black) that is NOT known. Only the noise corrupted signal $x_{\text{noisy}}(t)$ (in solid blue), as shown in the plot above, is available as a sensor measurement. As data scientist, your job is to recover an estimate $\widehat{x}(t)$ of the true signal from the noisy measured signal $x_{\text{noisy}}(t)$. We think of t as time; the noisy signal was measured at 500 different time instances.

This problem can be formulated as

$$\widehat{\boldsymbol{x}} = \underset{\boldsymbol{x}\in\mathbb{R}^{500}}{\arg\min} \left\{ \|\boldsymbol{x} - \boldsymbol{x}_{\text{noisy}}\|_2^2 + \beta \sum_{k=1}^{499} (x_{k+1} - x_k)^2 \right\}, \quad \beta > 0, \tag{1}$$

where x_k denotes the k^{th} entry of the vector $\boldsymbol{x}$.

We can interpret the above optimization problem as follows. Since we want the recovered signal to be close to the measured signal $\boldsymbol{x}_{\text{noisy}}$, it makes sense to minimize $\|\boldsymbol{x} - \boldsymbol{x}_{\text{noisy}}\|_2^2$. This justifies the first summand in the objective. We also want the estimated signal to be "smooth" in the sense it should not change too rapidly between consecutive times. To mathematically capture this, the second summand penalizes rapid changes in the signal between consecutive times. The known parameter $\beta > 0$ is fixed. Therefore, the first summand in the objective represents data fidelity; the second summand in the objective promotes regularization/smoothness. Large β implies smoother but larger mismatch with the measured signal. Similarly, small β implies better match with the measured but more "spiky" signal. For numerical solution, you need to rewrite problem (1) in the standard least squares form

$$\widehat{\boldsymbol{x}} = \underset{\boldsymbol{x}\in\mathbb{R}^{500}}{\arg\min} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|_2^2$$

for some appropriately defined tall matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$.

Your job is to complete the starter code by typing in some lines between lines 25 and 30. Then uncomment lines 32–43 and complete line 37 within the for loop. The completed code should make a plot of the estimated signals for $\beta = 1, 10, 100$ together with the blue and black curves shown above, all in the same figure.

In the zip file containing your MATLAB code, please also include a file named `YourlastnameYourfirstnameHW8.pdf` that clearly shows your hand calculations in deriving the tall matrix $\boldsymbol{A}$ and vector $\boldsymbol{b}$ appearing in the standard least squares form.

Hint: You may find the MATLAB commands `eye` and `zeros` useful. Please look them up in MATLAB documentation in case you are not familiar with them.