

Question

Consider the ODE IVP

$$\frac{dy}{dt} = y + t \log(y), \quad y(0) = 1.$$

With step size 0.1, the forward Euler approximation of $y(0.1)$ equals

Correct Answer

- ☐ 1.1
- ☐ 1
- ☐ 0.9
- ☐ 0.2

Question

To numerically solve the second order ODE

$$5 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \sin(x)y = 0$$

with appropriate initial conditions, we need to re-write it as a first order vector ODE

Correct Answer

- ☐ $\frac{d}{dx} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ -\frac{(y_2 x + y_1 \sin(x))}{5} \end{bmatrix}.$
- ☐ $\frac{d}{dx} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ \frac{y_2 x + y_1 \sin(x)}{5} \end{bmatrix}.$
- ☐ $\frac{d}{dx} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} y_2 \\ -\frac{(y_1 x + y_2 \sin(x))}{5} \end{bmatrix}.$

Question

The $n \times n$ matrix

$$\begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n} & \frac{1}{n} & \dots & \frac{1}{n} \end{bmatrix}$$
 has spectral radius

Correct Answer

- ☐ 1
- ☐ $\frac{1}{n}$
- ☐ n

Question

The power iteration applied to the 2×2 diagonalizable matrix $\begin{bmatrix} -b & a \\ -c & b \end{bmatrix}$ is guaranteed to converge

Correct Answer

- ☐ for any values of a, b, c
- ☐ when $b^2 > ac$
- ☐ when $b^2 < ac$
- ☐ when $b^2 = ac$

Question

When applying the inverse power iteration for computing the smallest magnitude eigenvalue, we can

Correct Answer

- ☐ use the LU decomposition to avoid explicit computation of matrix inverse.
- ☐ never apply LU decomposition.
- ☐ never guarantee convergence.

Question

Given a diagonalizable $n \times n$ matrix, the power iteration can be used to compute the dominant eigenvalue λ_1 where $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \dots \geq |\lambda_n|$.

The above inequality implies that λ_1

Correct Answer

- ☐ must be a real eigenvalue.
- ☐ must be a complex eigenvalue.
- ☐ maybe real or complex depending on the matrix.

Question

The backward Euler method for solving ODE IVP is

Correct Answer

- ☐ an implicit method.
- ☐ an explicit method.

Question

Suppose we compute the dominant eigenvector $\mathbf{v}^{(1)}$ for a diagonalizable matrix \mathbf{A} using the power iteration.

What is the most efficient way to compute the dominant eigenvector of the transposed matrix \mathbf{A}^\top ?

Correct Answer

- ☐ It has to be computed by applying the power iteration all over again to the transposed matrix.
- ☐ It is the same as $\mathbf{v}^{(1)}$ and therefore, we don't need to compute anything else.
- ☐ It is a known nonlinear function of $\mathbf{v}^{(1)}$ and therefore, can be computed by writing some additional code.

Question

Consider an adjacency matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$

The corresponding column-stochastic matrix \mathbf{S} is given by

Correct Answer

- ☐ $\mathbf{S} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$
- ☐ $\mathbf{S} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$
- ☐ $\mathbf{S} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} \end{bmatrix}$
- ☐ $\mathbf{S} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \end{bmatrix}$

Question

MATLAB ode45 is

Correct Answer

- ☐ is a variable step-size solver.
- ☐ is a fixed step-size solver.