

$$\hat{x} = \underset{x \in \mathbb{R}^{500}}{\operatorname{argmin}} \left\{ \|x - x_{\text{noisy}}\|_2^2 + \beta \sum_{k=1}^{499} (x_{k+1} - x_k)^2 \right\}$$

$$= \underset{x \in \mathbb{R}^{500}}{\operatorname{argmin}} \left\{ \|I x - x_{\text{noisy}}\|_2^2 + \left\| \beta \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & & \\ \vdots & & \ddots & \ddots & \\ 0 & & & 1 & -1 \end{pmatrix} x \right\|_2^2 \right\}$$

$$= \underset{x \in \mathbb{R}^{500}}{\operatorname{argmin}} \left\{ \left\| \begin{pmatrix} I \\ \beta \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & & \\ \vdots & & \ddots & \ddots & \\ 0 & & & 1 & -1 \end{pmatrix} \end{pmatrix} x - x_{\text{noisy}} \right\|_2^2 \right\}$$

$\downarrow$   $Ax = b$