Question

What is the solution of the following least norm problem

$$\min_{\boldsymbol{x}\in\mathbb{R}^3} \|\boldsymbol{x}\|_2$$

subject to $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$

where $\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$?

Correct Answer

- $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$
- $\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Question

Let $f_f'(x_0)$ denote the **two point forward difference** approximation of $f'(x_0)$ with step size h . Let $f_b'(x_0)$ denote the the **two point backward difference** approximation of $f'(x_0)$ with the same step size.

Then the **three point central difference** approximation of $f''(x_0)$ with the same step size h can be written as

- $\frac{f_f'(x_0) - f_b'(x_0)}{h^2}$
- $\frac{f_b'(x_0) - f_f'(x_0)}{h}$

Correct Answer

- $\frac{f_f'(x_0) - f_b'(x_0)}{h}$
- $\frac{f_b'(x_0) - f_f'(x_0)}{h^2}$

Question

For the function $f(x) = x \ln x$, the **two point central difference approximation** for $f'(1)$ with step size h is

Correct Answer

- $\frac{1}{2h} \ln \frac{(1+h)^{1+h}}{(1-h)^{1-h}}$
- $\frac{1}{h} \ln \frac{(1+h)^{1+h}}{(1-h)^{1-h}}$
- $\frac{1}{2} \ln \frac{(1+h)^{1+h}}{(1-h)^{1-h}}$
- $\ln \frac{(1+h)^{1+h}}{(1-h)^{1-h}}$

Question

The **central difference approximation** for $f''(x_0)$ with step size $0 < h << 1$, is

Correct Answer

- $\frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$.
- $\frac{f(x_0+h) - f(x_0-h)}{2h}$.
- $\frac{f(x_0+h) - f(x_0-h)}{h^2}$.

Question

Consider the function $f(x) = \cos(x)$.

The **two point central difference approximation** of $f'\left(\frac{\pi}{2}\right)$ with step size h satisfying $0 < h << 1,$ is

Correct Answer

- $-\frac{\sin(h)}{h}$
- zero
- $\frac{\cos(h)}{h}$
- $-\frac{\cos(h)}{2h}$

Question

What is the solution of the following least norm problem

$$\min_{\boldsymbol{x}\in\mathbb{R}^3} \|\boldsymbol{x}\|_2$$

subject to $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$

where $\boldsymbol{A} = \begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$?

- $\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$
- $\begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$

Correct Answer

- The solution does not exist since the rows of the coefficient matrix are linearly dependent.

Question

Consider the definite integral

$$\int_0^1 x^x \,\mathrm{d}x = 0.7834305107 \qquad \text{(up to 10 significant digits).}$$

The midpoint method approximation for this integral with x axis partition $[0, 1/2], [1/2, 1]$, equals

Correct Answer

- $\frac{1}{2}\left(\left(\frac{1}{4}\right)^{1/4} + \left(\frac{3}{4}\right)^{3/4}\right)$.
- $\frac{1}{2}\left(\left(\frac{1}{4}\right)^{3/4} + \left(\frac{3}{4}\right)^{1/4}\right)$.
- $\left(\frac{1}{4}\right)^{1/4} + \left(\frac{3}{4}\right)^{3/4}$.

Question

Suppose we want to compute the integral

$$I = \int_{-1}^{1} \frac{1}{1+x^2} \,\mathrm{d}x$$

using the **trapezoid method** and the **midpoint method**, both by partitioning $[-1, 1]$ into two subintervals $[-1, 0]$ and $[0, 1]$. Then

Correct Answer

- $I_{\text{trapezoid}} = \frac{3}{2}, \quad I_{\text{midpoint}} = \frac{8}{5}$.
- $I_{\text{trapezoid}} = \frac{1}{2}, \quad I_{\text{midpoint}} = \frac{8}{5}$.
- $I_{\text{trapezoid}} = \frac{5}{2}, \quad I_{\text{midpoint}} = \frac{9}{5}$.

Question

Suppose we want to approximate $\int_a^b f(x)\,\mathrm{d}x$ using **trapezoid method** by partitioning $[a, b]$ into two sub-intervals $[a, (a+b)/2], [(a+b)/2, b]$.

If $f(a) = f(b) = 0$, then the trapezoidal approximation of the integral reduces to computing the area of

Correct Answer

- an isosceles triangle.
- a circle.
- a rectangle.

Question

The **Simpson's three point method** to approximate a one dimensional definite integral with uniform discretization Δx, has error

- $O\left((\Delta x)^3\right)$

Correct Answer

- $O\left((\Delta x)^4\right)$
- $O\left((\Delta x)^2\right)$