Question

If the LU-decomposition of a square real matrix $\boldsymbol{A}$ exists, and the matrix $\boldsymbol{L}$ happens to be a diagonal matrix, then

Correct Answer

- $\boldsymbol{A}$ is always an upper triangular matrix.
- $\boldsymbol{A}$ is always a lower triangular matrix.
- The information is not sufficient to conclude whether $\boldsymbol{A}$ is upper or lower triangular.

Question

Which of the following is a valid LU decomposition of the matrix $\begin{bmatrix} 8 & 6 & 12 \\ 4 & 5 & 8 \\ 6 & 3 & 10 \end{bmatrix}$?

- $\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/4 & -3/4 & 1 \end{bmatrix} \begin{bmatrix} 8 & 6 & 12 \\ 0 & 2 & 2 \\ 0 & 1 & 5/2 \end{bmatrix}$
- $\begin{bmatrix} 8 & 6 & 12 \\ 0 & 2 & 2 \\ 0 & 0 & 5/2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/4 & -3/4 & 1 \end{bmatrix}$

Correct Answer

- $\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/4 & -3/4 & 1 \end{bmatrix} \begin{bmatrix} 8 & 6 & 12 \\ 0 & 2 & 2 \\ 0 & 0 & 5/2 \end{bmatrix}$

Question

Consider 2-by-2 matrix $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 \\ 3/5 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 5 & 1 \\ 0 & 17/5 \end{bmatrix}$$

The determinant of matrix A is

- 1

Correct Answer

- 17
- Not enough information available to compute the determinant.

Question

A nonzero matrix that is BOTH upper triangular and lower triangular, must be a diagonal matrix.

Correct Answer

- TRUE
- FALSE

Question

The system of linear equations

$$\begin{aligned} x_1 + 4x_2 + \alpha x_3 &= 6, \\ 2x_1 - x_2 + 2\alpha x_3 &= 3, \\ \alpha x_1 + 3x_2 + x_3 &= 5, \end{aligned}$$

has unique solution (x_1, x_2, x_3) for

Correct Answer

- $\alpha = 0.$
- $\alpha = 1.$
- $\alpha = -1.$

Question

The square linear system $\boldsymbol{Ax} = \boldsymbol{b}$, where vector $\boldsymbol{b}$ is nonzero, and matrix $\boldsymbol{A} = \begin{bmatrix} 2 & 6 \\ 3 & 9 \end{bmatrix}$

Correct Answer

- will either have no solution or multiple solutions depending on the choice of $\boldsymbol{b}$.
- will always have a unique solution independent of the choice of $\boldsymbol{b}$.
- will have either no solution or a single solution depending on the choice of $\boldsymbol{b}$.

Question

For applying the contraction mapping theorem to the fixed point equation $x = g(x)$, the mapping g

Correct Answer

- need not be differentiable.
- must be differentiable.

Question

Which of the following is a correct way to rewrite the nonlinear equation $f(x) := x^5 + x - 7 = 0$, as a fixed point equation $x = g(x)$?

Correct Answer

- $g(x) = \dfrac{7}{x^4 + 1}$
- $g(x) = \dfrac{7}{x^4 - 1}$
- $g(x) = 7 + x^5$

Question

The fixed point recursion $x_{k+1} = ax_k$, where $k = 0, 1, 2, \ldots$, will converge to $x = 0$ for any initial guess $x_0 \in (-\infty, +\infty)$ provided

- $-1 \leq a \leq 1.$

Correct Answer

- $-1 < a < 1.$
- a is any real number.

Question

Which of the following is TRUE for the fixed point recursion algorithm?

- If it converges then it must converge for ANY initial guess.
- It is guaranteed to converge but may converge to the wrong solution.

Correct Answer

- If it converges then the convergence may only be local.