If degree d spline: $S_i(x)$ is a degree d polynomial
each such polynomial has
(different) $(d+1)$ coefficients:
$(a_0^i, a_1^i, \dots, a_d^i)$
$a_0^i + a_1^i x + a_2^i x^2 + \dots + a_d^i x^d$

$x_0 < x_1 < x_2 < \dots < x_n \leftarrow (n+1)$ datapoints
$\Rightarrow n$ intervals
$\Rightarrow (n-1)$ breakpoints

$\underbrace{\text{every interval fits } (a_0, a_1, \dots, a_d)}_{\text{(a different degree } d \text{ polynomial)}}$: $(d+1)$ coefficients

$\Rightarrow \boxed{(d+1)n \text{ unknowns}}$

How many equations:

data: 2 endpoints $\Rightarrow$ $(n+1) - 2 = n-1$ interior/break points

Each interior point generates how many eq$\underline{\underline{n}}$s:

$\underbrace{\text{match } 0^{th}, 1^{st}, 2^{nd} \ldots, (d-1)^{th} \text{ order derivatives}}_{(n-1)d \text{ equations from matching requirements}}$:

@ each interior pt. also satisfy given data: $(n-1)$ equations

$\therefore$ All the interior points contribute to: $\boxed{(n-1)(d+1) \text{ equations}}$

2 Boundary points generate: 2 equations (for continuity)

$\therefore$ Total # of equations: $\boxed{(d+1)(n-1) + 2 \text{ equations.}}$

$\therefore$ we have a deficit of: $(d+1)n - \{(d+1)(n-1) + 2\}$

$= (d+1) - 2 = (d-1)$ equations.

$d=1$ (linear spline)	$(1+1)(n-1)+2$ $= 2n - \cancel{2} + \cancel{2}$ $= 2n$ equations	$2n$ unknowns
$d=2$ (quadratic spline)	$3(n-1)+2$ $= (3n-1)$ equations	$3n$ unknowns. (1 equation shortage)
$d=3$ (cubic spline)	$4n-2$ equations	$4n$ unknowns, (2 equation shortage)

Natural Boundary conditions

For $d=2$, enforce @ any one boundary

e.g., $S_1'(x_0) = 0$.

For $d=3$, enforce $S_1''(x_0)=0$, $S_n''(x_{n+1}) = 0$.