

1. A time domain real-signal $x(t)$ has a Fourier Transform property of $X(\omega) = X^*(-\omega)$. Consider the following frequency domain description of a signal $G(\omega)$:

$$G(\omega) = \begin{cases} 2, & 5 \leq |\omega| \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Evaluate $g(t)$ using the definition of Inverse Fourier Transformation

$$\left(g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega \right)$$

Plot $G(\omega)$, $\text{Re}(g(t))$, and $\text{Im}(g(t))$ in a 3x1 subplot for the interval $\omega = -31.4:0.01:31.4$ and $t = -100:0.1:100$.

- (b) Now consider $Y(\omega) = G(\omega - 5)$. Plot $Y(\omega)$, $\text{Re}(y(t))$, and $\text{Im}(y(t))$ in a 3x1 subplot with the same intervals.

- (c) Are $g(t)$ and $y(t)$ real-signal or complex signal?

2. When the signal $g(t)$ goes through a filter $h(t)$ where the frequency domain definition of the filter is:

$$H(\omega) = \begin{cases} 5|\omega|, & |\omega| \leq 20 \\ 0, & \text{elsewhere} \end{cases}$$

the results in a time domain output signal: $m(t)$.

- (a) Using convolution theorem, calculate the frequency domain output signal $M(\omega)$. Plot the magnitude and phase of $M(\omega)$ in a 2x1 subplot for the interval $\omega = -31.4:0.01:31.4$.

- (b) Evaluate $m(t)$ using the definition of Inverse Fourier Transformation. Plot $\text{Re}(m(t))$ and $\text{Im}(m(t))$ in a 2x1 subplot for the interval $t = -100:0.1:100$.

3. Calculate the energy of the output signal $m(t)$ for the time range $t = -100:0.1:100$. Also evaluate the energy of the output signal in frequency domain using Parseval's theorem (use the frequency range $\omega = 31.4:0.01:31.4$).