

Lecture # 3

Date: 01/13/2023

Course Roadmap

Topic # 0 (Not really algorithms)

- How to represent real numbers in a computer?

- Errors in numerical calculations

How to quantify errors?

→ Absolute errors

→ Relative errors

what types of errors?

→ Truncation error

→ Round-off error

Topic # 1 (Solving single nonlinear equation for real roots)

Solve $f(x) = 0$ for real x

- If $f(x)$ was linear, then no need for computational algorithms:

example: $2x + 3 = 0 \Rightarrow x = -3/2$

But need computational algorithm for:

- $2x^5 - 5x^4 + 20x^3 - 10x^2 + 10x - 1 = 0$

4 complex conjugate roots, 1 real root
⚡ How to compute it?

- $\underbrace{\cos(x) - x}_{f(x)} = 0 \Rightarrow \underbrace{x \approx 0.7391 \dots}_{\text{How to compute it?}}$

Topic #2 (Solving a system of linear equations)

Example:

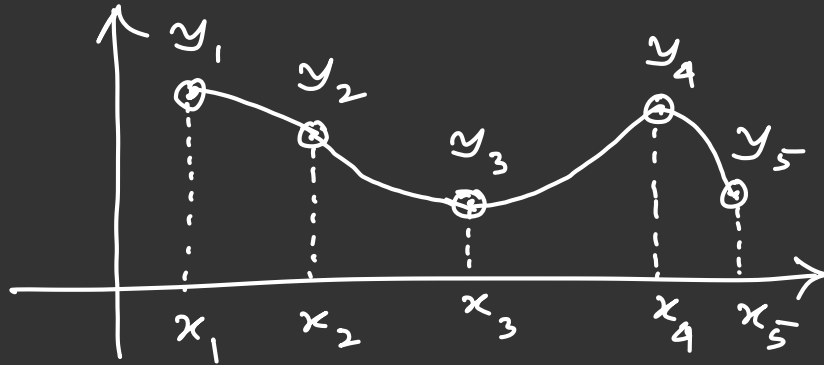
$$\left. \begin{array}{rcl} 2x + 3y - 5z & = & 0 \\ 11x - 18y + 12z & = & 0 \\ x - y - z & = & 0 \end{array} \right\} \begin{array}{l} \text{want to} \\ \text{solve for } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \end{array}$$

Equivalent to solving single
matrix-vector equation:

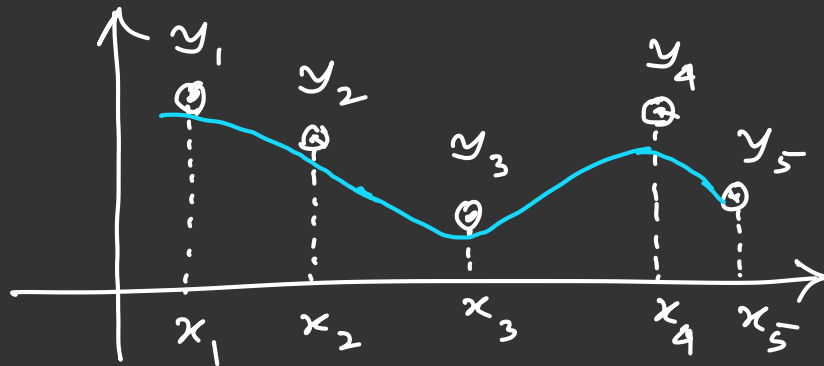
$$\underbrace{A}_{3 \times 3} \underbrace{x}_{3 \times 1} = \underbrace{b}_{3 \times 1}$$

Solve for the unknown vector $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
with given matrix A and vector b

Topic #3 (Function interpolation)



Topic #4 (Function approximation/Regression)



Topic #5 (Calculus on computer: Numerical differentiation and integration)

Topic #6 (Solving ordinary differential equation initial value problems)

Topic #7 (Matrix eigenvalue problems)

- Compute largest/smallest magnitude eigenvalue of a large square matrix
- Compute all eigenvalues of a square matrix