AM 147: Computational Methods and Applications: Winter 2023

Homework #3

Instructor: Abhishek Halder

Due: February 01, 2023

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW3.zip` via CANVAS. For example, `HalderAbhishekHW3.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Bisection method

($15 + 35 = 50$ points)

(a) Write a MATLAB .m function named `bisection.m` that computes a real root for any nonlinear equation of the form $f(x) = 0$ within numerical tolerance ε, where f is continuous in the interval $[a, b]$. Your MATLAB function `bisection.m` should take the inputs: a, b, f, ε and return an approximation for the root x_{approx}. It is a good practice (but not mandatory) to pass an additional input for maximum number of iterations. The continuous function f should be defined in another MATLAB function `func.m`.

(b) Consider a circular field with center P and radius 1. As shown in the Figure, a goat is tethered at point Q located on the circumference. We want to compute the length of the rope r that allows the goat to graze on exactly one half of the circular field's area (i.e., the goat should be able to graze in the white area shown in the Figure).

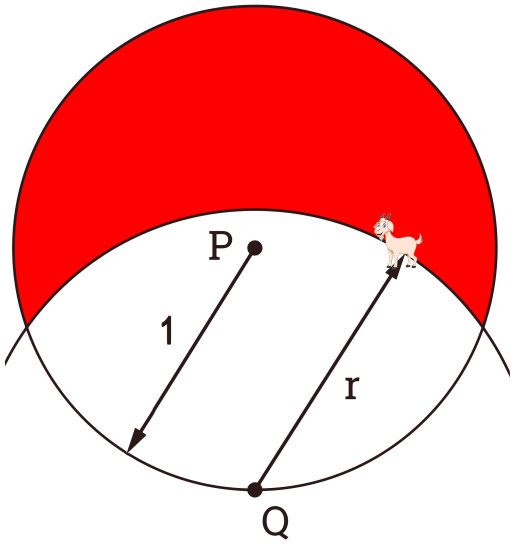

Figure 1: Goat tethered at point Q

It can be shown that r solves a nonlinear equation:

$$r^2 \arccos\left(\frac{r}{2}\right) + \arccos\left(1 - \frac{r^2}{2}\right) - \frac{r\sqrt{4 - r^2}}{2} = \frac{\pi}{2}.$$

Write a MATLAB code `YourlastnameYourfirstnameHW3.m` to numerically solve the above equation for r using your `bisection.m` from part (a) within numerical tolerance $\varepsilon = 10^{-4}$. Recall that `bisection.m` in turn calls appropriately defined `func.m`.

Executing your code `YourlastnameYourfirstnameHW3.m` should print the approximate root r found via bisection method in MATLAB command window.

Hint for part (b): For the inverse cosine $\arccos(\cdot)$, look up the built in command `acos` in MATLAB documentation. To provide appropriate interval for the bisection method, you may find it useful to think about the geometry of the problem as shown in Figure 1. Also relevant:
`https://www.mathworks.com/help/matlab/matlab_prog/pass-a-function-to-another-function.html`