AM 147: Computational Methods and Applications: Winter 2023

Homework #7

Instructor: Abhishek Halder

Due: March 01, 2023

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW7.zip` via CANVAS. For example, `HalderAbhishekHW7.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) and .pdf files for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Least squares solution and round-off errors **(25 points)**

Let k be any positive integer, and $\boldsymbol{A} = \begin{pmatrix} 1 & 1 \\ 10^{-k} & 0 \\ 0 & 10^{-k} \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} -10^{-k} \\ 1 + 10^{-k} \\ 1 - 10^{-k} \end{pmatrix}$.

Write a MATLAB code `YourlastnameYourfirstnameHW7p1.m` that for each $k = 5, 6, 7, 8$, computes the least squares solution for the above $\boldsymbol{A}, \boldsymbol{b}$, in three different ways: (i) analytically solve the normal equation by hand and hard code that analytical solution `xhat_analytical`, (ii) `xhat_QR` that numerically solves the normal equation using QR decomposition via MATLAB `A\b`, (iii) `xhat_normal` that numerically solves the normal equation as a square linear system. In other words, your code should output 3 different least squares solutions for each k, where the integer k increments in a for loop from 5 to 8.

The goal of this exercise is to compare the effectiveness of different numerical solutions for the least squares problem. For instance, you will notice that `xhat_normal` may return strange

outputs for some k even though the math and code are correct.

For computing the analytical solution in part (i), you may use the formula for the inverse of a 2×2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Please also submit a file `YourlastnameYourfirstnameHW7p1.pdf` showing all the steps in your hand calculations for part (i).

Problem 2

MathWorks tutorial for ordinary differential equations **(25 points)**

We will soon cover numerical algorithms to solve the ordinary differential equation (ODE) initial value problems. Since some of you may not have seen ODEs before, this exercise will prepare you to understand what they are and what does it mean to solve ODEs.

In your browser, go to

`https://matlabacademy.mathworks.com/details/solving-ordinary-differential-equations-with-matlab/odes`

On the top right corner of that page, login to your UCSC MathWorks account that you used to install MATLAB, and finish the ODE tutorial. This is similar to the OnRamp tutorial you did earlier in HW1. When finished, generate the certificate pdf file using "View/Print certificate". Submit this certificate as `YourlastnameYourfirstnameHW7p2.pdf`