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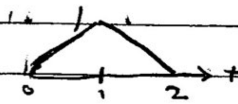
SID: 1696580

HW2

2.2

a) linear, not time-invariant
b) linear, not time-invariant
c) linear, time-invariant
d) linear, time-invariant
e) linear, time-invariant
f) linear, not time-invariant
g) linear, time-invariant

2.5 

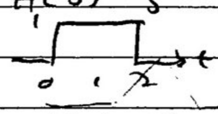
$s(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$

$h(t) = \frac{ds(t)}{dt} = \begin{cases} 1 & 0 \le t \le 1 \\ -1 & 1 \le t \le 2 \end{cases}$

$\Rightarrow u(t) = [u(t) - u(t-1)] - [u(t-1) - u(t-2)]$

$= u(t) - 2u(t-1) + u(t-2)$

$H(s) = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}$

a) 

$x(t) = u(t) - u(t-2) \quad X(s) = \frac{1}{s} - \frac{2e^{-2s}}{s}$

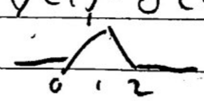
$Y(s) = H(s) \cdot X(s)$

$= \left(\frac{1}{s} - \frac{e^{-2s}}{s}\right)\left(\frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}\right)$

$= \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2} - \frac{e^{-2s}}{s^2} - \frac{2e^{-3s}}{s^2} - \frac{e^{-4s}}{s}$

$= \frac{1}{s^2} - \frac{2e^{-s}}{s^2} - 2\frac{e^{-3s}}{s^2} - \frac{e^{-4s}}{s^2}$

$y(t) = \delta(t) - 2\delta(t-1) + 2\delta(t-3) - \delta(t-4)$

b) 

$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 \le t \le 2 \end{cases}$

$x(t) = t(u(t) - u(t-1)) + (2-t)[u(t-1) - u(t-2)]$

$= tu(t) - tu(t-1) + (2-t)u(t-1) - (2-t)u(t-2)$

$= tu(t) - (2-2t)u(t-1) - (2-t)u(t-2)$

$= \delta(t) - 2\delta(t-1) + \delta(t-2)$

$X(s) = \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}$

$$y(s) = x(s) \cdot H(s)$$

$$= \left[\frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2}\right]\left[\frac{1}{s} - \frac{e^{-2s}}{s} - \frac{2e^{-s}}{s}\right]$$

$$= \frac{1}{s^3} - \frac{e^{-2s}}{s^3} - \frac{2e^{-s}}{s^3} - \frac{2e^{-s}}{s^3} + \frac{2e^{-3s}}{s^3} + \frac{4e^{-2s}}{s^3} + \frac{e^{-2s}}{s^3}$$

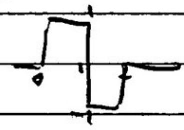
$$- \frac{e^{-4s}}{s^3} - \frac{2e^{-3s}}{s^3}$$

$$= \frac{e^{-4s}}{s^3} - \frac{4e^{-3s}}{s^3} + \frac{6e^{-2s}}{s^3} - \frac{4e^{-s}}{s^3} + \frac{1}{s^3}$$

$$y(t) = \frac{t^2}{2}u(t) - \frac{4}{2}(t-1)^2 u(t-1) + \frac{6}{2}(t-2)^2 u(t-2)$$

$$- \frac{4}{2}(t-3)^2 u(t-3) + \frac{(t-4)^2}{2} u(t-4)$$

$$\boxed{y(t) = \frac{t^2}{2} u(t) - 2u(t-1)(t-1)^2 + 3(t-2)^2 u(t-2) - 2(t-3)^2 u(t-3) + \frac{(t-4)^2}{2} u(t-4)}$$

c) 

$$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ -1 & 1 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = [u(t) - u(t-1)] - [u(t-1) - u(t-2)]$$

$$= u(t) - 2u(t-1) + u(t-2)$$

$$x(s) = \frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}$$

$$Y(s) = x(s) \cdot H(s)$$

$$= \left[\frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}\right]\left[\frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}\right]$$

$$= \frac{1}{s^2} - \frac{2e^{-s}}{s^2} + \frac{e^{-2s}}{s^2} - \frac{2e^{-s}}{s^2} + \frac{2e^{-2s}}{s^2} - \frac{2e^{-3s}}{s^2} + \frac{e^{-2s}}{s^2} - \frac{2e^{-3s}}{s^2}$$

$$+ e^{-4s}/s^2$$

$$y(t) = tu(t) - 2(t-1)u(t-1) + (t-2)u(t-2) - 2(t-1)u(t-1) + u(t-2)(t-2)$$

$$- 2(t-3)u(t-3) + (t-2)u(t-2) - 2(t-3)u(t-3) + (t-4)u(t-4)$$

$= t\,u(t) - (t-1)u(t-1) + 6u(t-2)(t-2) - u(t-3)\,u(t-3) + (t-4)u(t-4)$

$= r(t) - 4r(t-1) + 6r(t-2) - 4r(t-3) + r(t-4)$

d) $x(t) = \begin{cases} t & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$

$= t[u(t) - u(t-1)]$

$= t\,u(t) - t\,u(t-1) = t\,u(t) - (t-1)u(t-1) - u(t-1)$

$= r(t) - r(t-1) - u(t-1)$

$x(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} + 1\frac{e^{-s}}{s^2}$

$y(s) = \left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s^2}\right]\left[\frac{1}{s} - \frac{2e^{-s}}{s} + \frac{e^{-2s}}{s}\right]$

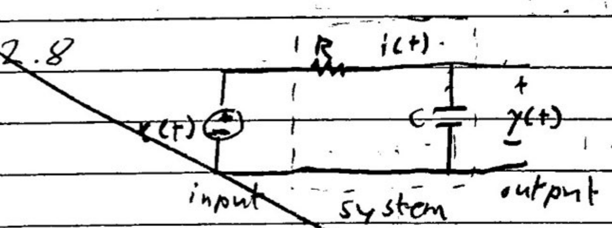
$= \frac{1}{s^3} - \frac{e^{-s}}{s^3} + \frac{e^{-s}}{s^2} - \frac{2e^{-s}}{s^3} + \frac{2e^{-2s}}{s^3} - \frac{2e^{-2s}}{s^2} + \frac{e^{-2s}}{s^3} + \frac{e^{-3s}}{s^2} - \frac{e^{-3s}}{s^3}$

$= \frac{1}{s^3} - \frac{3e^{-s}}{s^3} + \frac{e^{-s}}{s^2} + \frac{3e^{-2s}}{s^3} - \frac{2e^{-2s}}{s^2} - \frac{e^{-3s}}{s^3} + \frac{e^{-3s}}{s^2}$

$y(t) = \frac{t^2}{2}u(t) - \frac{3}{2}(t-1)^2u(t-1) + (t-1)u(t-1) + \frac{3}{2}(t-2)^2u(t-2)$
$- 2(t-2)u(t-2) - \frac{(t-3)^2}{2}u(t-3) + (t-3)u(t-3)$

$\boxed{y(t) = \frac{t^2}{2}u(t) - \frac{3}{2}(t-1)^2u(t-1) + \frac{3}{2}(t-2)^2u(t-2) - \frac{(t-3)^2}{2}u(t-3) + r(t-1) - 2r(t-2) + r(t-3)}$

2.8



KVL

$x(t) = R\,i(t) + \frac{1}{C}\int i(t)\,dt$ $\qquad y(s) = \frac{1}{C}\int \frac{s\,x(s)}{R(s+\frac{1}{RC})}\,d(s)$

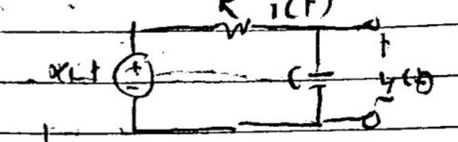
$y(t) = \frac{1}{C}\int i(t)\,dt$ $\qquad = \frac{1}{RC}\int \frac{s\,x(s)}{s+\frac{1}{RC}}\,d(s)$

$\frac{d}{dt}x(t) = R\frac{d}{dt}i(t) + \frac{1}{C}(i(t))$

$s\,x(s) = Rs\,i(s) + \frac{i(s)}{C}$

$i(s) = \frac{s\,x(s)}{Rs + \frac{1}{C}} = \frac{s\,x(s)}{R(s+\frac{1}{RC})}$

2.9



$$Y(s) = \frac{\frac{1}{RC}}{s+\frac{1}{RC}} X_1(s)$$

a) $x_1(t) = -2t\left[u(t)-u(t-2)\right] + 2(t-4)\left[u(t-2)-u(t-4)\right]$

$= -2tu(t) + (4t-8)u(t-2) - 2(t-4)u(t-4)$

$= -2tu(t) + 4(t-2)u(t-2) - 2(t-4)u(t-4)$

$$X_1(s) = -\frac{2}{s^2} + \frac{4e^{-2s}}{s^2} - \frac{2e^{-4s}}{s^2}$$

$$Y_1(s) = \frac{1}{RC}\left(\frac{1}{s+\frac{1}{RC}}\right)\left(-\frac{2}{s^2} + \frac{4e^{-2s}}{s^2} - \frac{2e^{-4s}}{s^2}\right)$$

$$= \frac{1}{RC}\left(-4+4e^{-2s}-2e^{-4s}\right)\left(\frac{1}{s^2(s+\frac{1}{RC})}\right)$$

$$= \frac{1}{RC}\left(-2+4e^{-2s}-2e^{-4s}\right)\left[\frac{RC}{s^2} + R^2C^2\left(\frac{1}{s+\frac{1}{RC}}\right) - \frac{R^2C^2}{s}\right]$$

$y_1(t) = -2t - 2RCe^{-\frac{1}{RC}(t-2)} + 2RC + 4(t-2)$
$+ 4RCe^{-\frac{1}{RC}(t-2)} - 4RC - 2(t-4) - 2RCe^{-\frac{1}{RC}(t-2)}$
$+ 2RC$

$$\boxed{y_1(t) = -2tu(t) + 4(t-2)u(t-2) - 2(t-4)u(t-4)}$$

b) $x_2(t) = 2t\left[u(t)-u(t-2)\right] + 4\left[u(t-2)-u(t-4)\right]$
$- 2(t-6)\left[u(t-4)-u(t-6)\right]$

$= 2tu(t) + (-2t+4)u(t-2) - 2(t-6+2)u(t-4) +$
$2(t-6)u(t-6)$

$= 2tu(t) - 2(t-2)u(t-2) - 2(t-4)u(t-4) + 2(t-6)u(t-6)$

$$X_2(s) = \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{2e^{-4s}}{s^2} + \frac{2e^{-6s}}{s^2}$$

$$= \frac{2}{s^2}\left(1 - e^{-2s} - e^{-4s} + e^{-6s}\right)$$

$$Y_2(s) = \frac{\frac{1}{RC}}{s+\frac{1}{RC}} X_2(s) = \frac{1}{RC}\left(\frac{1}{s+\frac{1}{RC}}\right)\frac{2}{s^2}\left(1-e^{-2s}-e^{-4s}+e^{-6s}\right)$$

$$= \frac{2}{RC}\left(1-e^{-2s}-e^{-4s}+e^{-6s}\right)\left(\frac{1}{s^2(s+\frac{1}{RC})}\right)$$

$$= \frac{2}{RC}\left(1-e^{-2s}-e^{-4s}+e^{-6s}\right)\left[\frac{RC}{s^2}+R^2C^2\left(\frac{1}{s+\frac{1}{RC}}\right)-R^2C^2\right]$$

$$\boxed{y_2(t) = 2tu(t) - 2(t-2)u(t-2) - 2(t-4)u(t-4) + 2(t-6)u(t-6)}$$

c) $x_3(t) = 2t[u(t)-(t-2)] + 2(t-4)[u(t-2)-u(t-4)]$

$$= 2tu(t) - 8u(t-2) + 2(t-4)u(t-4)$$

$$X_3(s) = \frac{2}{s^2} - \frac{8e^{-2s}}{s^2} + \frac{2e^{-4s}}{s^2}$$

$$= \frac{2}{s^2}\left(1-4e^{-2s}-e^{-4s}\right)$$

$$Y_3(s) = \frac{1}{RC}\left(\frac{1}{s+\frac{1}{RC}}\right)\frac{2}{s^2}\left(1-4e^{-2s}-e^{-4s}\right)$$

$$= \frac{2}{RC}\left(1-4e^{-2s}-e^{-4s}\right)\left(\frac{1}{s^2(s+\frac{1}{RC})}\right)$$

$$= \frac{2}{RC}\left(1-4e^{-2s}-e^{-4s}\right)\left[\frac{RC}{s^2}+R^2C^2\left(\frac{1}{s+\frac{1}{RC}}\right)-R^2C^2\right]$$

$$y_3(t) = 2tu(t) - 4(t-2)u(t-2) - 2(t-4)u(t-4)$$

2.12 $x(t) = \begin{cases} 2 \; ; & 0<t<1 \\ 0 & \text{otherwise} \end{cases}$

$h(t) = \begin{cases} 2t \; ; & 0<t<1 \\ -2t+4 \; ; & 1<t<2 \end{cases}$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)\,d\tau$$

a) $y(t) = \int_0^t 2(2\tau)\,d\tau = 4\left[\frac{\tau^2}{2}\right]_0^t = 2t^2$ for $0<t<1$

$$y(t) = \int_{t-1}^{1} 2(2\tau)\,d\tau + \int_{1}^{2} 2(-2\tau+4)\,d\tau = 4\left[\frac{\tau^2}{2}\right]_{t}^{1} - 4\left[\frac{\tau^2}{2}\right]_{t}^{2} + 8[\tau]_t^2$$

$$= 2(t^2-1) - 2(4-t^2) + 8(2-t)$$

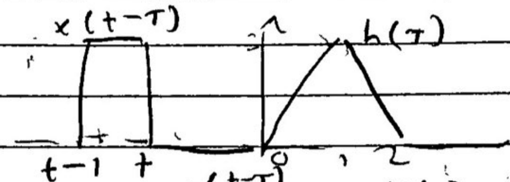
$$= 4t^2 - 8t + 6 \qquad \text{for } 1<t<2$$

$$y(t) = \int_2^t 2(-2\tau+4)\,d\tau = -4\left[\frac{\tau^2}{2}\right]_2^t + 8[\tau]_2^t$$

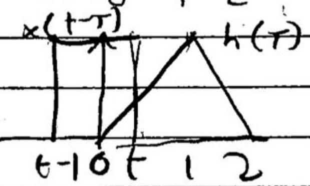
$$= -2(t^2-4) + 8(t-2) = -2t^2+8t-8 \quad \text{for } 2 \le t \le 3$$

$$y(t) = \begin{cases} 2t^2 & ; \quad 0<t<1 \\ 4t^2-8t+6 & ; \quad 1<t<2 \\ -2t^2+8t-8 & ; \quad 2<t<3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

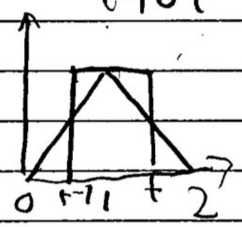
b)



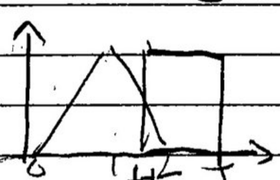
$t<0 : y(t) = 0$



$0<t<1 : y(t) = 2t^2$



$1<t<2 : y(t) = 4t^2-8t+6$



$2<t<3 : y(t) = -2t^2+8t-8$

2.16 a) $\delta(t-2)[u(t)-3u(t-1)+2u(t-2)]$

$\delta(t) * u(t) = u(t) \quad \delta(t-a) * u(t-b) = u(t-a-b)$

$(\delta(t-2) * u(t)) - (\delta(t-2) * 3u(t-1)) + (\delta(t-2) * 2u(t-2))$

$= \boxed{u(t-2) - 3u(t-3) + 2u(t-4)}$

b) $[\delta(t) + 2\delta(t-1) + 3\delta(t-2)] * [4\delta(t) + 5\delta(t-1)]$

$= (\delta(t) * 4\delta(t)) + (\delta(t) * 5\delta(t-1))$

$+ 8(\delta(t-1) * \delta(t)) + 10(\delta(t-1) * \delta(t-1))$

$+ 12(\delta(t-2) * \delta(t)) + 15(\delta(t-2) * \delta(t-1))$

$= \boxed{4\delta(t) + 5\delta(t-1) + 8\delta(t-1) + 10\delta(t-2) + 12\delta(t-2) + 15\delta(t-3)}$

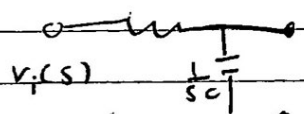
c) $u(t) * [u(t) - u(t-2) - 2\delta(t-2)]$

$= (u(t) * u(t)) - (u(t) * u(t-2)) - (u(t) * 2\delta(t-2))$

$= t\,u(t) - (t-2)\,u(t-2) - 2u(t-2)$

$= \boxed{r(t) - r(t-2) - 2u(t-2)}$

2.21 a)

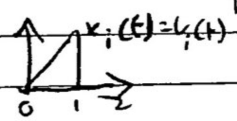


$h(t) = e^{-t}u(t)$

$V_o(s) = V_i(s)\left(\frac{1/sC}{R + \frac{1}{sC}}\right) \quad \frac{1}{RsC+1} \quad RC = 1$

$= \frac{V_i(s)}{s+1}$

$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{s+1}$



$v_i(t) = r(t) - r(t-1) - u(t-1)$

$V_i(s) = \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s^2}$

$v_o(t) = v_i(t) * h(t)$

$V_o(s) = \frac{1}{s+1}\left[\frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}\right]$

$V_o(s) = \frac{1}{s+1}\left[\frac{1 - e^{-s} - se^{-s}}{s^2}\right] = \frac{1-s+s}{(s+1)(s^2)}\left[\frac{1 - e^{-s} - se^{-s}}{1}\right]$

$= \frac{1}{s^2} - \frac{1}{s(s+1)}\left[1 - e^{-s} - se^{-s}\right]$

$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} + \left[\frac{1}{s+1} - \frac{1}{s}\right]\left[\frac{1 - e^{-s} - se^{-s}}{1}\right]$

$= \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} + \frac{1}{s+1} - \frac{1}{s} - \frac{e^{-s}}{s+1} + \frac{e^{-s}}{s} + \frac{e^{-s}}{s+1}$

$v_o(t) = r(t) - r(t-1) - u(t-1) + e^{-t}u(t) - u(t) - e^{-t}[u(t-1)]$

$+ u(t-1) + e\,u(t-1)$

$\boxed{v_o(t) = r(t) - r(t-1) - u(t) + e^{-t}u(t)}$

b) $\boxed{v_i(t) = r(t) - r(t-1) - u(t-1)}$

2.22 a) $h(t)=e^{-|t|}$

$h(t)\neq 0$ for $t<0$

$\boxed{\text{non-causal}}$

$\int_{-\infty}^{\infty} h(t)dt<\infty$ $\boxed{\text{BIBO stable}}$

b) $h(t)=(1-|t|)\left[u(t+1)-u(t-1)\right]$

$h(t)=0,\ t<0$ $\boxed{\text{causal}}$

$\int_0^1 h(t)dt<\infty$ $\boxed{\text{BIBO stable}}$

c) $e^{2t}u(-t)$

$h(t)\neq 0$ for $t<0$ $\boxed{\text{non causal}}$

$\int_{-\infty}^{0} h(t)dt<\infty$ $\boxed{\text{BIBO stable}}$

d) $e^{2t}u(t)$

$h(t)=0,\ t<0$ $\boxed{\text{causal}}$

$\int_0^{\infty} h(t)dt$ $\boxed{\text{BIBO unstable}}$

e) $\cos^2(2t)u(t)$ $h(t)=0,\ t<0$ $\boxed{\text{causal}}$

$\boxed{\text{BIBO stable}}$

f) $\frac{1}{t+1}u(t)$ $h(t)=0$ $t<0$ $\boxed{\text{Causal}}$

$\int_0^{\infty} h(t)dt$ $\boxed{\text{BIBO stable}}$

~~10000~~

~~526.3 gallons = 23685 19~~

~~285.7 gallons 4.50 1285.7~~

~~32 40~~

~~35~~