AM 147: Computational Methods and Applications: Winter 2023

Homework #2

Instructor: Abhishek Halder

Due: January 25, 2023

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW2.zip` via CANVAS. For example, `HalderAbhishekHW2.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

In this course, all codes and computation are numerical, as opposed to symbolic computation. Please do not use symbolic or any other toolbox.

Problem 1

Sum of some entries **(20 points)**

Consider the outer product $\boldsymbol{X} := \boldsymbol{x}\boldsymbol{x}^\top$, $\boldsymbol{x} \in \mathbb{R}^n$, where $\mathbb{R}^n$ denotes the set of all $n \times 1$ vectors with real entries. Without using any loops (such as for, while, if else), we want to write a MATLAB function named `SumOuterProduct.m` that takes an arbitrary vector $\boldsymbol{x} \in \mathbb{R}^n$ as input and outputs the scalar s, equal to the sum of all entries on or above the main diagonal of the matrix $\boldsymbol{X}$.

Write a MATLAB executable file `YourlastnameYourfirstnameHW2p1.m` that calls your function `SumOuterProduct.m` for a random vector $\boldsymbol{x} \in \mathbb{R}^{100}$ to compute s as follows.

```
close all; clear; clc;
x = rand(100,1);
s = SumOuterProduct(x)
```

The file `YourlastnameYourfirstnameHW2p1.m` should have exactly the above three lines and nothing else. Please make sure to keep both the `.m` files in the same folder/directory so that the executable file can "see" the function it calls. Submit both `.m` files.

Hint: For your MATLAB function file, use the command `sum` (look it up in MATLAB documentation) and basic arithmetic operations such as addition (`+`), multiplication ($*$), power (`.^`). You will not need any built-in commands other than these.

Problem 2

A function and its approximations **(30 points)**

Consider $\sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}$, that is, the power series obtained by keeping every third term in the Taylor expansion of $\exp(x)$. This power series corresponds to the true function

$$f_{\text{true}}(x) = \frac{1}{3}\left(\exp(x) + 2\exp(-x/2)\cos\left(\frac{\sqrt{3}x}{2}\right)\right).$$

For any positive integer k, define the k term power series approximation $f_{\text{approx}}(x,k) := \sum_{n=0}^{k-1} \frac{x^{3n}}{(3n)!}$.

Submit a MATLAB code (.m file) named `YourlastnameYourfirstnameHW2p2.m` that plots 2D line plots for the functions $f_{\text{true}}(x)$ (in black solid line) versus $x \in [-5,5]$ (in the horizontal axis). In the same figure window, plot $f_{\text{approx}}(x,k)$ for $k = 2$ (in red dashed line), $k = 3$ (in green dashed line), $k = 4$ (in blue dashed line).

We shared a starter code `YourlastnameYourfirstnameHW2p2.m` inside the CANVAS File section folder: HW Problems and Solutions. You only need to complete lines 11 and 20 in that starter code, then rename the file appropriately with your first and last names.

Hint: Look up `sqrt`, `exp`, `cos`, `sum`, `factorial`, and power (`.^`) in MATLAB documentation. Also, intuition suggests that as k increases, f_{approx} should get close to f_{true}.