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Question

Consider a tall linear system in QR factorization form  $QRx = b$ , given by

$$\begin{bmatrix} 2/3 & 2/3 \\ -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix} \begin{bmatrix} 3 & 3 \\ 0 & 6 \end{bmatrix} x = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}.$$

The least squares solution vector  $\hat{x}$  equals

Correct Answer

⋮

Question

Following are three mathematically equivalent ways of solving the standard least squares problem

$$\arg \min_{\theta} \|A\theta - y\|_2^2$$

in MATLAB. Which of these is least prone to round-off errors, that is, numerically most accurate?

Correct Answer

⋮

Question

Determine the matrix  $A$  and the vector  $b$  such that the problem

$$\arg \min_{x_1, x_2} \left\{ (-6x_2 + 4)^2 + (-4x_1 + 3x_2 - 1)^2 + (x_1 + 8x_2 - 3)^2 \right\}$$

can be expressed in standard least squares form

$$\arg \min_{x \in \mathbb{R}^2} \|Ax - b\|_2^2$$

where  $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ .

Correct Answer

⋮

Question

Consider  $500 \times 3$  matrix  $A$  with linearly independent columns, and its QR decomposition  $A = QR$ , where

$$R = \begin{pmatrix} 1 & -3 & -2 \\ 0 & 4 & -5 \\ 0 & 0 & 7 \end{pmatrix}.$$

The vector 2 norm of the first column of  $A$  equals

Correct Answer

⋮

Least squares function approximation

Pick 2 questions, 1 pts per question

⋮

Question

For a given set of data points, the least squares function approximation

Correct Answer

⋮

Question

Given datapoints  $(x_i, y_i)$  for  $i = 1, 2, 3$ , suppose we want to determine the least square approximation

$$\hat{f}(x) = \theta_1 \cos x + \theta_2 \sin x.$$

Let  $\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix}$ ,  $y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$ . The matrix  $A$  in the standard form  $\arg \min_{\theta} \|A\theta - y\|_2^2$  must be

Correct Answer

⋮

Question

Consider the following data:

$x_i$	$y_i$
0	0
1	6
2	6

What is the best linear function approximation of this data in the least squares sense?

Correct Answer

⋮

Question

A parabola of the form  $y = cx^2$  is used to approximate a set of  $n + 1$  data points  $\{(x_i, y_i)\}_{i=0}^n$ .

Find the value of  $c$  for least squares approximation, i.e., minimizes the sum  $\sum_{i=0}^n (y_i - cx_i^2)^2$ .

Correct Answer

⋮

Question

Solving the linear system

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 1 & 0 & 0.5 & 0.5 \end{bmatrix} x = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

for vector  $x$  requires us to solve for

Correct Answer

⋮

Question

Consider  $5 \times 1$  vector  $x$  whose entries  $x_1, x_2, \dots, x_5$  denote the amount in grams of five different types of food (for example: apples, beans, rice, bread, milk) to be consumed by a human per day.

It is recommended to intake 150 grams of carbohydrate, 112 grams of protein, and 50 grams of fat per day.

Consider a known matrix  $A$  whose (i,j)-th entry denotes how many grams of i-th nutrient (carbohydrate, protein or fat) is present in the j-th type of food (apples, beans, rice, bread or milk).

Designing daily diet then amounts to solving for  $x$  satisfying a linear system

$$Ax = \begin{pmatrix} 150 \\ 112 \\ 50 \end{pmatrix}$$

which requires us to solve for

Correct Answer