AM 147: Computational Methods and Applications: Winter 2023

Homework #9

Instructor: Abhishek Halder

Due: March 15, 2023

NOTE: Please submit your Homework as a single zip file named `YourlastnameYourfirstnameHW9.zip` via CANVAS. For example, `HalderAbhishekHW9.zip`. Please strictly follow the capital and small letters in the filename of the zip file you submit. You may not receive full credit if you do not follow the file-naming conventions. Your zip file should contain all .m files (MATLAB scripts) for the questions below.

Your zip file must be uploaded to CANVAS by 11:59 PM Pacific Time on the due date. The uploads in CANVAS are time-stamped, so please don't wait till last moment. Late homework will not be accepted.

Problem 1

Model of Virus Spread **($25 + 25 = 50$ points)**

A well-known way to model the spread of virus over time t is the so-called SIRD model. Suppose the total residential population of a county is N that remains constant over $t \in [0, 100]$. At time t, the number of *susceptible* individuals is $S(t)$, the number of *infected* individuals is $I(t)$, the number of *recovered* individuals (after being infected) is $R(t)$, the number of *deceased* individuals is $D(t)$. The SIRD model is given by the nonlinear ODE

$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta}{N} IS,$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta}{N} IS - \gamma I - \mu I,$$
$$\frac{\mathrm{d}R}{\mathrm{d}t} = \gamma I,$$
$$\frac{\mathrm{d}D}{\mathrm{d}t} = \mu I,$$

where the parameters $\beta, \gamma, \mu > 0$ denote rates of infection, recovery, and mortality, respectively.

Notice that the above ODE satisfies $\frac{\mathrm{d}}{\mathrm{d}t}(S(t) + I(t) + R(t) + D(t)) = 0$ as it should, since $S(t) + I(t) + R(t) + D(t) = N$ constant.

(a) [10 + 10 + 5 = 25 points]

Write a MATLAB function `SIRD_dynamics.m` that specifies the above dynamics of virus spread for infection rate $\beta = 0.45$, recovery rate $\gamma = 0.04$, mortality rate $\mu = 0.01$, and total population $N = 1000$.

Then write another file `YourlastnameYourfirstnameHW9p1.m` that calls your MATLAB function `SIRD_dynamics.m` to solve the above ODE using `ode45` over time $t \in [0, 100]$ with initial conditions $S(0) = 995$, $I(0) = 5$, $R(0) = 0$, $D(0) = 0$.

Your file `YourlastnameYourfirstnameHW9p1.m` should then plot the time series for the variables $S(t), I(t), R(t), D(t)$ as solid lineplots all in the same figure window. Use different linecolors for different variables' solid lineplots: blue for $S(t)$, red for $I(t)$, green for $R(t)$, black for $D(t)$.

(b) [10 + 10 + 5 = 25 points]

Write a MATLAB function `FixedStepRK4.m` that implements the fixed step-size fourth order Runge-Kutta method (RK4) for any vector ODE

$$\frac{\mathrm{d}\boldsymbol{y}}{\mathrm{d}t} = \boldsymbol{f}(t, \boldsymbol{y}).$$

Call this MATLAB function in the file `YourlastnameYourfirstnameHW9p1.m` that you wrote in part (a) to solve for the variables $S(t), I(t), R(t), D(t)$ with the same parameters and initial conditions as in part (a) for the same time horizon $[0, 100]$. Use step-size $\Delta t = 0.5$. Inside `YourlastnameYourfirstnameHW9p1.m`, you need to pass the function `SIRD_dynamics` as argument to the function `FixedStepRK4`.

Plot these time-series computed using RK4 on the same figure window generated in part (a). To avoid visual clutter, plot the RK4 results using circular markers but no lines, with the same color scheme as in part (a). For example, to plot $S(t)$ from RK4, use `plot(...,...,'bo')`. To plot $I(t)$ from RK4, use `plot(...,...,'ro')` etc.

Submit all your codes/scripts from parts (a)-(b) within `YourlastnameYourfirstnameHW9.zip`.