CSCI-1200 Data Structures
Final Exam — Practice Problem Solutions

1 Short Answer [/17]

1.1 Comparing Vectors & Arrays [/5]

The statements below can be used to compare and contrast arrays and vectors. For each statement, specify **"ARRAY"** if it is only true for arrays, **"VECTOR"** if it is only true for vectors, **"BOTH"** if it is true for both types, and **"NEITHER"** if it is true for neither type.

Solution: VECTOR Knows how many elements it contains.

Solution: BOTH Can be used to store elements of any type.

Solution: NEITHER Prevents access of memory beyond its bounds.

Solution: VECTOR Is dynamically re-sizable.

Solution: BOTH Can be passed by reference.

1.2 Limited Looping [/3]

True or False There are some algorithms that must be written using a `for` loop and *cannot* be written using a `while` or `do – while` loop.

Solution: False. All looping constructs are equivalent and any algorithm written using one looping construct can be re-written with the others.

2 Superhero Division [/14]

In this problem you will add a new operator to the `Superhero` class from lab. Remember that a superhero has a name, a true identity, and a power, but we *cannot* access the true identity of a `Superhero` object from the public interface. Here is the basic `Superhero` class declaration:

```
class Superhero {
public:
  // ACCESSORS
  const string& getName() const { return name; }
  const string& getPower() const { return power; }
  // INPUT STREAM OPERATOR
  friend istream& operator>>(istream &istr, Superhero &hero);
private:
  // REPRESENTATION
  string name;
  string true_identity;
  string power;
};
// OUTPUT STREAM OPERATOR
ostream& operator<<(ostream &ostr, const Superhero &hero);
```

And here is part of the `Superhero` class implementation:

```
ostream& operator<<(ostream &ostr, const Superhero &hero) {
  if (hero.getPower() == "")
    ostr << hero.getName() << " has no power" << endl;
  else
    ostr << "Superhero " << hero.getName() << " has power " << hero.getPower() << endl;
  return ostr;
}
```

Now let's define the `/=` operator on `Superhero`. This operator can be used to defeat a hero by dividing them from their true identity. If an attacker learns a hero's true identity and uses it against them, the superhero loses his power. A superhero must carefully guard his true identity to prevent this attack. If the attacker does not know and just incorrectly guesses the superhero's true identity, this `/=` operation does nothing. For example, suppose `elastigirl` is a `Superhero` object with name equal to "Elastigirl", true identity equal to "Zoe", and power equal to "Flexible". Then the statement:

```
cout << elastigirl;
```

would print this on the screen:

```
Superhero Elastigirl has power Flexible
```

But after executing the statement:

```
elastigirl /= ("Zoe");
```

the output of the variable `elastigirl` would print on the screen as:

```
Elastigirl has no power
```

2.1 Implementation Choices [/5]

Name the three different ways we can implement operator overloading. Which of these three is the most appropriate choice for the `/=` operator described above? Why?

Solution: The three methods are non-member function, member function, and friend function. It is usually preferable to consider the methods in that order. In this case we cannot implement the `/=` operator as a non-member because it requires read access to `true_identity` and write access to `power`. We can implement it as a member function of the `Superhero` class because the first argument is of type `Superhero`.

2.2 /= operator implementation [/9]

Now implement the `/=` operator. Part of your job is to carefully define the prototype for this function. What should be added or changed in the `superhero.h` class declaration file? And what should be added or changed in the `superhero.cpp` class implementation file? Be specific.

Solution: The following prototype for the member function should be added to the *public* portion of the `Superhero` class declaration:

```
Superhero& operator/=(const string &id);
```

And the function definition is added to the class implementation file:

```
Superhero& Superhero::operator/=(const string &id) {
  if (id == true_identity) {
    power = "";
  }
  return *this;
}
```

3 Valet Parking Maps [/38]

You have been asked to help with a valet parking system for a big city hotel. The hotel must keep track of all of the cars currently stored in their parking garage and the names of the owners of each car. *Please read through the entire question before working on any of the subproblems.* Here is the simple `Car` class they have created to store the basic information about a car:

```
class Car {
public:
  // CONSTRUCTOR
  Car(const string &m, const string &c) : maker(m), color(c) {}
  // ACCESSORS
  const string& getMaker() const { return maker; }
  const string& getColor() const { return color; }
private:
  // REPRESENTATION
  string maker;
  string color;
};
```

The hotel staff have decided to build their parking valet system using a map between the cars and the owners. This map data structure will allow quick lookup of the owners for all the cars of a particular color and maker (e.g., the owners of all of the silver Hondas in the garage). For example, here is their data structure and how it is initialized to store data about the six cars currently in the garage.

```
map<Car,vector<string> > cars;
cars[Car("Honda","blue")].push_back("Cathy");
cars[Car("Honda","silver")].push_back("Fred");
cars[Car("Audi","silver")].push_back("Dan");
cars[Car("Toyota","green")].push_back("Alice");
cars[Car("Audi","silver")].push_back("Erin");
cars[Car("Honda","silver")].push_back("Bob");
```

The managers also need a function to create a report listing all of the cars in the garage. The statement:

```
print_cars(cars);
```

will result in this report being printed to the screen (`std::cout`):

```
People who drive a silver Audi:
  Dan
  Erin
People who drive a blue Honda:
  Cathy
People who drive a silver Honda:
  Fred
  Bob
People who drive a green Toyota:
  Alice
```

Note how the report is sorted alphabetically by maker, then by car color, and that the owners with similar cars are listed chronologically (the order in which they parked in the garage).

3.1 The `Car` class [/6]

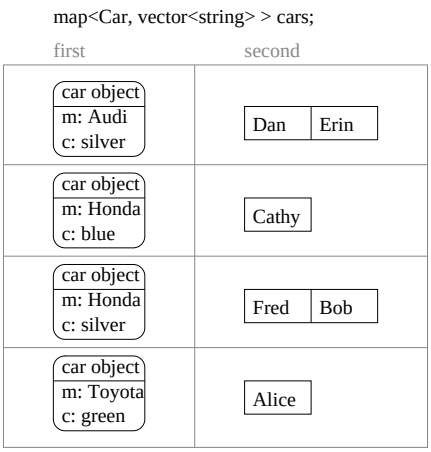
In order for the `Car` class to be used as the first part of a map data structure, what additional non-member function is necessary? Write that function. Carefully specify the function prototype (using const & reference as appropriate). Use the example above as a guide.

Solution: We must define `operator<` for `Car` objects so that we can sort the keys of the map.

```
bool operator<(const Car &a, const Car &b) {
  return (a.getMaker() < b.getMaker() ||
          (a.getMaker() == b.getMaker() && a.getColor() < b.getColor()));
}
```

3.2 Data structure diagram [/10]

Draw a picture of the map data structure stored by the `cars` variable in the example. As much as possible use the conventions from lecture for drawing these pictures. Please be neat when drawing the picture. *Optional: You may also write a few concise sentences to explain your picture.*



Solution:

3.3 print_cars [/9]

Write the `print_cars` function. Part of your job is to correctly specify the prototype for this function. Be sure to use const and pass by reference as appropriate.

Solution:

```
void print_cars(const map<Car,vector<string> > &cars) {
  map<Car,vector<string> >::const_iterator itr = cars.begin();
  while (itr != cars.end()) {
    Car c = itr->first;
    cout << "People who drive a " << c.getColor() << " " << c.getMaker() << ":" << endl;
    vector<string>::const_iterator itr2 = itr->second.begin();
    while (itr2 != itr->second.end()) {
      cout << "  " << *itr2 << endl;
      itr2++;
    }
    itr++;
  }
}
```

3.4 remove_cars [/13]

When guests pick up their cars from the garage, the data structure must be correctly updated to reflect this change. The `remove_car` function returns true if the specified car is present in the garage and false otherwise.

```
bool success;
success = remove_car(cars, "Erin", "silver", "Audi");
assert (success == true);
success = remove_car(cars, "Cathy", "blue", "Honda");
assert (success == true);
success = remove_car(cars, "Sally", "green", "Toyota");
assert (success == false);
```

After executing the above statements the `cars` data structure will print out like this:

```
People who drive a silver Audi:
  Dan
People who drive a silver Honda:
  Fred
  Bob
People who drive a green Toyota:
  Alice
```

Note that once the only blue Honda stored in the garage has been removed, this color/maker combination is completely removed from the data structure.

Specify the prototype and implement the `remove_car` function.

Solution:

```
bool remove_car(map<Car,vector<string> > &cars,
                const string &name, const string &color, const string &maker) {
  map<Car,vector<string> >::iterator itr = cars.find(Car(maker,color));
  if (itr == cars.end()) return false;
  if (itr->second.size() == 1 && itr->second[0] == name) {
    cars.erase(Car(maker,color));
    return true;
  }
  for (int i = 0; i < itr->second.size(); i++) {
    if (itr->second[i] == name) {
      itr->second.erase(itr->second.begin() + i);
      return true;
    }
  }
  return false;
}
```

4 Computational Desert Island [/]

Suppose that a monster is holding you captive on a computational desert island, and has a large file containing double precision numbers that he needs to have sorted. If you write correct code to sort his numbers he will release you and when you return home will be allowed to move on to DSA. If you don't write correct code, he will eventually release you, but only under the condition that you retake CS 1. The stakes indeed are high, but you are quietly confident — you know about the standard library sort function. (Remember, you are supposed to have forgotten all about bubble sort.) The monster startles you by reminding you that this is a computational desert island and because of this the only data structure you have to work with is a queue.

After panicking a bit (or a lot), you calm down and think about the problem. You realize that if you maintain the values in the queue in increasing order, and insert each value into the queue one at a time, then you can solve the rest of the problem easily. Therefore, you must write a function that takes a new double, stored in `x`, and stores it in the queue. Before the function is called, the values in the queue are in increasing order. After the function ends, the values in the queue must also be in increasing order, but the new value must also be among them.

Here is the function prototype:

```
void insert_in_order(double x, queue<double>& q)
```

You may only use the public queue interface (member functions) as specified in lab. You may use a second queue as local variable scratch space or you may try to do it in a single queue (which is a bit harder). Give an "O" estimate of the number of operations required by this function.

Solution: Here is a version with a scratch queue:

```
void insert_in_order(double x, queue<double>& q) {
  if (q.empty())
    q.push(x);
  else {
    queue<double> temp(q);           // copy q;
    while (!q.empty()) q.pop();      // empty q out
    while (!temp.empty() && x > temp.front()) {
      double item = temp.front();
      temp.pop();
      q.push(item);
    }
    q.push(x);    // insert x in its proper position
    while (!temp.empty()) {
      double item = temp.front()
      temp.pop();
      q.push(item);
    }
  }
}
```

This function requires $O(n)$ operations. Copying the queue initially and emptying the queue requires $O(n)$ time each. The second and third while loops, combined, touch each entry in the queue and therefore require $O(n)$ operations. Since none of these loops are nested we add the results and get $O(n)$ time overall.

Here is a version without a scratch queue:

```
void insert_in_order(int x, queue<double>& q) {
  int n = q.size();
  int position = 0;
  // Find the position for x in the queue, copying all values
  // less than x to the back of the queue
  while (position < n && x < q.front()) {
    q.push(q.front());      // copy the front to the back
    q.pop();                // remove the front
    ++position;
  }
  q.push(x);                // put x in position

  // The first n-position entries on the queue haven't been
  // touched and are greater than or equal to the value
  // stored in x.  They need to move to the back of the queue
  int i = position;
  while (i < n) {
    q.push(q.front());      // copy the front to the back
    q.pop();                // remove the front
    ++i;
  }
}
```

The two while loops, combined touch each entry in the queue once and therefore require $O(n)$ operations. Since none of these loops are nested we add the results and get $O(n)$ time overall.

5 Operations on Lists [/]

5.1 Reversing a dslist [/]

Write a `dslist<T>` member function called `reverse` that reverses the order of the nodes in the list. The head pointer should point to what was the tail node and the tail pointer should point to what was the head node. All directions of pointers should be reversed. The function prototype is:

```
template <class T> void dslist<T>::reverse();
```

The function must NOT create ANY new nodes.

Solution:

```
template <class T>
void cs2list<T>::reverse() {
  // Handle empty or single node list
  if (head_ == tail_) return;
  // Swap pointers at each node of the list, using a temporary
  // pointer q to remember where to go next.
  Node<T>* p = head_;
  while (p) {
    Node<T>*q = p->next_;
    p->next_ = p->prev_;
    p->prev_ = q;
    p = q;
  }
  // Swap head and tail pointers
```

```
  p = head_;
  head_ = tail_;
  tail_ = p;
}
```

5.2 Sublists [/]

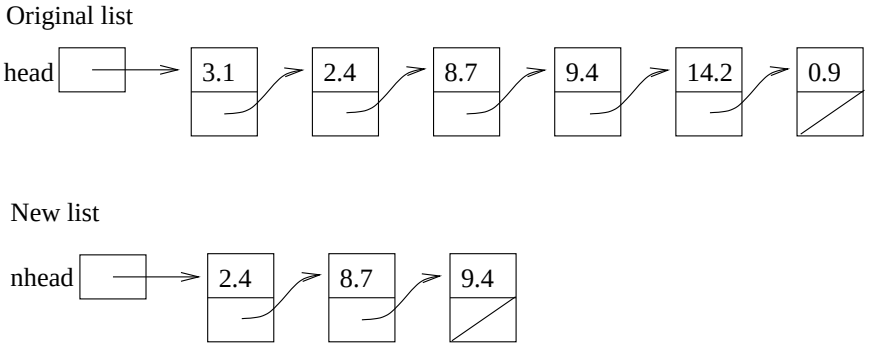
Write a function to create a new singly-linked list that is a *copy* of a sublist of an existing list. The prototype is:

```
Node<T>* Sublist(Node<T>* head, int low, int high)
```

The `Node` class is:

```
template <class T>
class Node {
public:
   T value;
   Node* next;
};
```

The new list will contain `high-low`+1 nodes, which are copies of the values in the nodes occupying positions `low` up through and including `high` of the list pointed to by `head`. The function should return the pointer to the first node in the new list. For example, in the following drawing the original list is shown on top and the new list created by the function when `low==2` and `high==4` is shown below.



A pointer to the first node of this new list should be returned. (In the drawing this would be the value of `nhead`.) You may assume the original list contains at least `low` nodes. If it contains fewer than `high` nodes, then stop copying at the end of the original list.

Solution:

```
Node<T>* Sublist(Node<T>* head, int low, int high) {
  // Skip over the first low-1 nodes in the existing list
  Node<T>*p = head;
  int i;
  for (i=1; i<low; ++i) p = p-> next;
  // Make the new head node and make a pointer to the last node in list
  Node<T>* new_head = new Node<T>;
  new_head->value = p->value;
  Node<T> * last = new_head;
  // Copy the remaining nodes, one at a time
  for (++i, p = p->next; i<=high && p; ++i, p = p->next) {
    last->next = new Node<T>;
    last->next->value = p->value;
    last = last->next;
  }
  last->next = 0;
}
```

5.3 Splicing into a cs2list[/]

Write a `cs2list<T>` member function called `splice` that takes an iterator and a second `cs2list<T>` object and splices the entire contents of the second list between the node pointed to by the iterator and its successor node. The second list must be completely empty afterwards. The function prototype is:

```
template <class T>
void cs2list<T>::splice(iterator itr, cs2list<T>& second);
```

No new nodes should be created by this function AND it should work in $O(1)$ time (i.e. it should be independent of the size of either list).

Solution:

```
template <class T>
void cs2list<T>::splice(iterator itr, cs2list<T>& second) {
  if (second.empty()) return;
  second.head_->prev_ = itr.ptr_;
  second.tail_->next_ = itr.ptr_->next_;
  if (itr.ptr_->next_) {
    itr.ptr_->next_->prev_ = second.tail_;
  } else { // itr.ptr_ is the tail, so it must be reset
    this->tail_ = second.tail_;
  }
  itr.ptr_->next_ = second.head_;
  this->size_ += second.size_;
  second .size_ = 0;
  second.head_ = second.tail_ = 0;
}
```

6 Concurrency and Asynchronous Computing [/3]

Why might a group of dining philosophers starve?

A) Because it's impossible to eat spaghetti with chopsticks.

B) Because they are all left-handed.

C) Because due to a bank error they didn't have enough money in their joint account.

D) Because they didn't all want to eat at the same time.

Solution: B

7 Garbage Collection [/12]

For each of the real world systems described below, choose the *most appropriate* memory management technique. Each technique should be used exactly once.

A)	Explicit Memory Management (C++)	B)	Reference Counting
C)	Stop & Copy	D)	Mark-Sweep

7.1 Student Registration System [/3]

Must handle the allocation and shuffling of pointers as students register and transfer in and out of classes. Memory usage will not be a deciding factor. Fragmentation of data should be minimized.

Solution: C

7.2 Playing Chess [/3]

Implementation of a tree-based algorithm for searching the game space. Remember that a *tree* is a *graph* with no cycles.

Solution: B

7.3 Webserver [/3]

A collection of infrequently changing interconnected webpages. Any memory usage overhead should be low. Pauses in service are tolerable.

Solution: D

7.4 Hand Held Game (e.g., GameBoy or PSP, etc.) [/3]

Performance critical application with extremely limited memory resources.

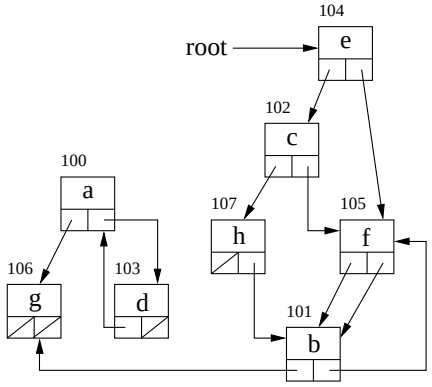
Solution: A

8 Short Answer [/22]

8.1 Garbage Identification [/7]

To which address in the memory below should the root variable point so that exactly 2 cells are garbage? Draw a *box and pointer diagram* to justify your answer and state which 2 cells are garbage.

address	100	101	102	103	104	105	106	107
value	a	b	c	d	e	f	g	h
left	106	106	107	100	102	101	0	0
right	103	105	105	0	105	101	0	101



Solution: The root should point to cell 104.
Cells 100 and 103 are garbage.

8.2 Stop and Copy Garbage Collection [/4]

What is the purpose of the *forwarding address* in Stop and Copy garbage collection? What will go wrong if you neglect to record this value? Write 2 or 3 concise and well-written sentences.

Solution: When a non-garbage cell is copied from the old memory partition to the new memory partition, a forwarding address is left to indicate that the cell has been copied and to provide the address of the new location. All references to the old location will see the forwarding address and can then be updated as appropriate. If we don't record this forwarding address cyclical data structures will not be copied correctly: the garbage collector will repeatedly copy the same cell resulting in an infinite loop!

8.3 Concurrency and Asynchronous Computing [/4]

When programming with multiple threads or processes, the correct use of mutexes (locks) and condition variables will ensure that:

A) The program always returns the exact same answer.

B) The program returns an answer that was not possible if the program ran sequentially.

C) The entire program is atomic.

D) Each student in a large class will be able to successfully copy a complete set of lecture notes (with no repetitions), even if there are multiple professors.

E) Deadlock will be avoided if there are multiple mutexes, but may still happen in systems with a single lock.

Solution: D

8.4 Perfect Hashing for Image Compression [/7]

For the last homework, you implemented a compression scheme for 2D images. What are the drawbacks of using this format as the underlying representation for an image editing program? What types of edits to the image are simple? What types of edits will be comparatively inefficient to process? Write 3-4 *concise and well-written* sentences.

Solution: The underlying representation is geared towards a static image because the computation of the perfect hash depends on the occupancy data. Simply changing the color of some or all of the non-white pixels is cheap. Removing non-white pixels (painting them white) or adding a non-white pixel that happens to hash to an empty spot in the hash data table is also cheap. Adding non-white pixels that collide with other pixels requires recomputation of the offset table, and possibly resizing of the hash data and/or offset tables.

9 Data Structures [/18]

Indicate by letter the data structure(s) that have each characteristic listed below.

A) vector	B) list	C) map	D) set
E) priority queue	F) hash table	G) leftist heap	

Solution:

B C D	allows efficient (sublinear) removal of the first and last elements (or the minimum and maximum elements)
A E F	uses an array or vector as the underlying representation
B C D (F) G	uses a network of nodes connected by pointers as the underlying representation
C D (E) F	the underlying data structure must be "balanced" or well-distributed to achieve the targeted performance
C D E G	requires definition of `operator<` or `operator>`
C D E F G	entries cannot be modified after they are inserted (requires re-insertion or re-processing of position)
C D (F)	duplicates are not allowed
B G	allows sublinear merging of two of instances of this data structure

10 Order Notation [/16]

Match the order notation with each fragment of code. Two of the letters will not be used.

A) $O(n)$	B) $O(1)$	C) $O(n^n)$	D) $O(n^2)$
E) $O(2^n)$	F) $O(\log n)$	G) $O(n \log n)$	H) $O(\sqrt{n})$

Solution: D

```
vector<int> my_vector;
// my_vector is initialized with n entries
// do not include initialization in performance analysis
for (int i = 0; i < n; i++) {
  my_vector.erase(my_vector.begin());
}
```

Solution: F

```
map<string,int> my_map;
// my_map is initialized with n entries
// do not include initialization in performance analysis
my_map.find("hello");
```

Solution: E

```
int foo(int n) {
  if (n == 1 || n == 0) return 1;
  return foo(n-1) + foo(n-2);
}
```

Solution: A

```
int k = 0;
for (int i = 0; i < sqrt(n); i++) {
  for (int j = 0; j < sqrt(n); j++) {
    k += i*j;
  }
}
```

Solution: G

```
set<string> my_set;
for (int i = 0; i < n; i++) {
  string s;
  cin >> s;
  my_set.insert(s);
}
```

Solution: B

```
float* my_array = new float[n];
// do not include memory allocation in performance analysis
my_array[n/2] = sqrt(n);
```

11 Office Demolition [/31]

In this problem we will explore a simple class to manage the assignment of people to offices and desks. Each `Office` object stores its name, the number of desks it can hold, and the names of the people assigned to those desks. An office also stores a *reference* to a master queue of all the people who still need to be assigned to desks. When an office is constructed, people are assigned to the office from the front of this master queue. When an office is demolished, the people who were assigned to that office should be added to the end of the queue while they wait for a new office assignment. Here is the *partial* declaration of the `Office` class:

```
class Office {
public:
  Office(const string& name, int num_desks, queue<string> &unassigned);
  friend ostream& operator<<(ostream &ostr, const Office &office);
private:
  // representation
  string _name;
  int _num_desks;
  string* _desks;
  queue<string>& _unassigned;  // a reference to the master queue
};
```

In the example below we create the master `queue` of people who need to be assigned to desks in offices, and create and delete several `Office` objects:

```
queue<string> unassigned;
unassigned.push("Alice");
unassigned.push("Bob");
unassigned.push("Cathy");
unassigned.push("Dan");
unassigned.push("Erin");
unassigned.push("Fred");
unassigned.push("Ginny");
```

```
Office *red = new Office("red", 4, unassigned);
Office *green = new Office("green", 2, unassigned);
cout << *red << *green;

delete red;
cout << "After deleting the red office, "
     << unassigned.size() << " people are waiting for desks." << endl;

Office *blue = new Office("blue", 3, unassigned);
cout << *blue;

cout << "Before deleting the blue & green offices, "
     << unassigned.size() << " people are waiting for desks." << endl;
delete green;
delete blue;
cout << "After deleting all of the offices, "
     << unassigned.size() << " people are waiting for desks." << endl;
```

Here is the desired output from this example:

```
The red office has 4 desks:
  desk[0] = Alice
  desk[1] = Bob
  desk[2] = Cathy
  desk[3] = Dan
The green office has 2 desks:
  desk[0] = Erin
  desk[1] = Fred
After deleting the red office, 5 people are waiting for desks.
The blue office has 3 desks:
  desk[0] = Ginny
  desk[1] = Alice
  desk[2] = Bob
Before deleting the blue & green offices, 2 people are waiting for desks.
After deleting all of the offices, 7 people are waiting for desks.
```

Here is the implementation of the constructor, as it appears in the `office.cpp` file:

```
Office::Office(const string& name, int num_desks, queue<string> &unassigned)
  : _name(name), _num_desks(num_desks), _unassigned(unassigned) {
  _desks = new string[_num_desks];  // allocate the desk space
  for (int i = 0; i < _num_desks; i++) {
    if (_unassigned.size() > 0) {   // assign from the master queue
      _desks[i] = _unassigned.front();
      _unassigned.pop();
    } else {  // if there are no unassigned people, leave the desk empty
      _desks[i] = "";
    }
  }
}
```

11.1 Classes and Memory Allocation [/10]

Anytime you write a new class, especially those with *dynamically allocated memory*, it is very important to consider the member functions that the compiler will automatically generate and determine if this default behavior is appropriate. List these 4 important functions by their generic names, *AND* write their prototypes as they would appear within the `Office` class declaration.

Solution:

default constructor	`Office();`
destructor	`~Office();`
copy constructor	`Office(const Office &office);`
assignment operator	`Office& operator=(const Office &office);`

11.2 Declaring a Destructor [/3]

The `Office` class is incomplete and requires implementation of a custom destructor so that people assigned to demolished offices are returned to the master queue and memory is deallocated as appropriate to avoid memory leaks. What line needs to be added to the header file to declare the destructor? Be precise with syntax. Where should this line be added: within the `public`, `protected`, or `private` interface?

Solution: Anywhere in the `public:` interface,

```
~Office();
```

11.3 Implementing a Destructor [/12]

Implement the destructor, as it would appear in the `office.cpp` file.

Solution:

```
Office::~Office() {
  for (int i = 0; i < _num_desks; i++) {
    if (_desks[i] != "") {
      _unassigned.push(_desks[i]);
    }
  }
  delete [] _desks;
}
```

11.4 Operator Overloading [/6]

Here is the implementation of the `<<` stream operator as it appears within the `office.cpp` file:

```
ostream& operator<<(ostream &ostr, const Office &o) {
  ostr << "The " << o._name << " office has "
       << o._num_desks << " desks:" << endl;
  for (int i = 0; i < o._num_desks; i++) {
    ostr << "  desk[" << i << "] = " << o._desks[i] << endl;
  }
  return ostr;
}
```

There are three different ways to overload an operator: as a non-member function, as a member function, and as a friend function. Which method was selected for the `Office` object `<<` stream operator? What are the reasons for this choice? Discuss why the other two methods are inappropriate or undesirable. Write 3 or 4 concise and thoughtful sentences.

Solution: This operator has been implemented as a friend function, which is necessary to gain access to the private member variables of the `Office` object. If we had implemented it as a non-member function the function wouldn't have access to these variables and accessor functions would need to be added to the public interface, which is undesirable since that would expose the private representation unnecessarily. We cannot implement the stream operator as a member function of the `Office` class because the `<<` syntax requires the `ostream` object to be the first argument of the operator. In order to be written as a member operator of a particular class, the first argument must be of that class type.

12 Dynamically-Allocated Arrays [/17]

Write a function that takes an STL list of integers, finds the even numbers, and places them in a dynamically-allocated array. Only the space needed for the even numbers should be allocated, and no containers other than the given list and the newly-created array may be used. As an example, given a list containing the values:

```
3 10 -1 5 6 9 13 14
```

the function should allocate an array of size 3 and store the values 10, 6 and 14 in it. It should return, via arguments, both the pointer to the start of the array and the number of values stored. No subscripting may be used — not even `*(a+i)` in place of `a[i]`. Here is the function prototype:

```
void even_array(const list<int>& b, int* & a, int& n);
```

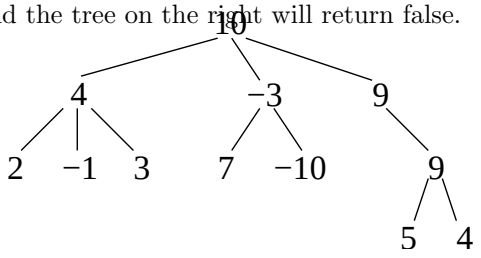
Solution:

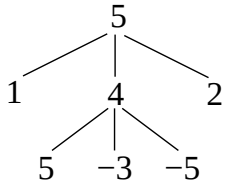
```
void even_array(const list<int>& b, int* & a, int& n) {
  n = 0;    // count
  for (list<int>::const_iterator p = b.begin(); p!=b.end(); ++p)
    if (*p % 2 == 0) ++n;
  a = new int[n];   // allocate
  int* q = a;     // store in array
  for (list<int>::const_iterator p = b.begin(); p!=b.end(); ++p)
    if (*p % 2 == 0) {
      *q = *p;
      ++q;
    }
}
```

13 Ternary Tree Recursion [/17]

A *ternary tree* is similar to a binary tree except that each node has at most 3 children. Write a *recursive* function named `EqualsChildrenSum` that takes one argument, a pointer to the root of a ternary tree, and returns true if the value at each non-leaf node is the sum of the values of all of its children and false otherwise. In the examples below, the tree on the left will return true and the tree on the right will return false.

```
class Node {
public:
  int value;
  Node* left;
  Node* middle;
  Node* right;
};
```





Solution:

```
bool EqualsChildrenSum(Node *node) {
  if (node == NULL) return true;
  if (node->left == NULL && node->middle == NULL && node->right == NULL)
    return true;
  int sum = 0;
  if (node->left != NULL) sum += node->left->value;
  if (node->middle != NULL) sum += node->middle->value;
  if (node->right != NULL) sum += node->right->value;
  if (sum != node->value) return false;
  return
    EqualsChildrenSum(node->left) &&
    EqualsChildrenSum(node->middle) &&
    EqualsChildrenSum(node->right);
}
```

14 Priority Queues [/16]

```
template <class T> class priority_queue {
public:
  // CONSTRUCTOR
  priority_queue() {}
  // ACCESSORS
  int size() { return m_heap.size(); }
  bool empty() { return m_heap.empty(); }
  const T& top() const { assert(!m_heap.empty()); return m_heap[0]; }
  // MODIFIERS
  void push(const T& entry) {
    m_heap.push_back(entry);
    this->percolate_up(int(m_heap.size()-1));
  }
  void pop() {  // find and remove the element with the smallest value
    assert(!m_heap.empty());
    m_heap[0] = m_heap.back();
    m_heap.pop_back();
    this->percolate_down(0);
  }
  void pop_max() { /* YOU WILL IMPLEMENT THIS FUNCTION */ }

private:
  // HELPER FUNCTIONS
  void percolate_up(int i) {
    T value = m_heap[i];
    while (i > 0) {
      int parent = (i-1)/2;
      if (value >= m_heap[parent]) break;  // done
      m_heap[i] = m_heap[parent];
      i = parent;
    }
    m_heap[i] = value;
  }
  void percolate_down(int i) {
    T value = m_heap[i];
    int last_non_leaf = int(m_heap.size()-1)/2;
    while (i <= last_non_leaf) {
      int child = 2*i+1, rchild = 2*i+2;
      if (rchild < m_heap.size() && m_heap[child] > m_heap[rchild])
        child = rchild;
      if (m_heap[child] >= value) break;  // found right location
      m_heap[i] = m_heap[child];
      i = child;
    }
    m_heap[i] = value;
  }

  // REPRESENTATION
  vector<T> m_heap;
};
```

14.1 Implementing pop_max [/12]

Write the new priority queue member function named pop_max that finds and removes from the queue the element with the *largest* value. Carefully think about the efficiency of your implementation. Remember that a standard priority queue stores the smallest value element at the root.

Solution:

```
  void pop_max() {
    assert(!m_heap.empty());
```

```
    int tmp = (size()+1)/2;
    for (int i = tmp+1; i < size(); i++) {
      if (m_heap[tmp] < m_heap[i])
        tmp = i;
    }
    m_heap[tmp] = m_heap.back();
    m_heap.pop_back();
    this->percolate_up(tmp);
  }
```

14.2 Analysis [/4]

If there are n elements in the priority queue, how many elements are visited by the `pop_max` function in the worst case? What is the order notation for the running time of this function?

Solution: $n + \log n$ elements are visited. Running time is $\mathbf{O}(n)$.

15 Inheritance & Polymorphism [/10]

What is the output of the following program?

```
class A {
public:
  virtual void f() { cout << "A::f\n"; }
  void g() { cout << "A::g\n"; }
};

class B : public A {
public:
  void g() { cout << "B::g\n"; }
};

class C : public B {
public:
  void f() { cout << "C::f\n"; }
  void g() { cout << "C::g\n"; }
};

int main() {
  A* a[3];
  a[0] = new A();
  a[1] = new B();
  a[2] = new C();

  for (int i = 0; i < 3; i++) {
    cout << i << endl;
    a[i]->f();
    B* b = dynamic_cast<B*>(a[i]);
    if (b) b->g();
  }
}
```

Solution:

```
0
A::f
1
A::f
B::g
2
C::f
B::g
```

16 Types & Values [/15]

For the *last expression* in each fragment of code below, give the *type* (`int`, `vector<double>`, `Foo*`, etc.) and the *value*. If the value is a legal address in memory, write "`memory address`". If the value hasn't been properly initialized, write "`uninitialized`". If there is an error in the code, write "`error`". You may want to draw a picture to help you answer each question, but credit will only be given for what you've written in the boxes.

```
double a = 5.2;
double b = 7.5;
a+b
```

Solution: **Type:** `double` **Value:** `12.7`

```
int *d;
int e[7] = { 15, 6, -7, 19, -1, 3, 22 };
d = e + e[5];
*d
```

Solution: **Type:** `int` **Value:** `19`

```
bool *f = new bool;
*f = false;
f
```

Solution: **Type:** `bool*` **Value:** ***memory address***

```
int g = 10;
int *h = new int[g];
h[0]
```

Solution: **Type:** `int` **Value:** ***uninitialized***

```
map<string, int> m;
m.insert(make_pair(string("bob"),5551111));
m.insert(make_pair(string("dave"),5552222));
m.insert(make_pair(string("alice"),5553333));
m.insert(make_pair(string("chris"),5554444));
(++m.find("bob"))->second
```

Solution: **Type:** `int` **Value:** `5554444`