Gabriel Maayan - FOCS Assignment 3

1 DMC 5.4

Only (ii) is a valid way of proving $F(n) = P(n) \to P(n+1)$ **because when proving** $P \to Q$**, proving** $Q = f \to P = f$ **is the only way to prove** $F(n) = t$

2 DMC 5.3

(c) $P(n) \to P(n^2) \wedge P(n-2)$
$P(2) \to P(4) \wedge P(0)$
$P(4) \to P(16) \wedge P(2)$
$P(16) \to P(256) \wedge P(14)$
$\vdots$
P **is true for** $n = 2k, k \in \mathbb{Z}$

3 5.11

(b) For $0 \leq x \leq \frac{1}{2}$**,** $-2x \leq ln(1-x) \leq -x$
$x = \frac{1}{n+1}$
$\frac{-2}{n+1} \leq ln(\frac{n}{n+1}) \leq \frac{-1}{n+1}$
$\frac{-2}{n+1} \leq ln(n) - ln(n+1) \leq \frac{-1}{n+1}$
$\therefore$ **(a)** $ln(n+1) - \frac{2}{n+1} \leq ln(n)$ **and (b)** $ln(n) \leq ln(n+1) - \frac{1}{n+1}$

$P(n) : 1 + \frac{1}{2}ln(n) \leq H_n \leq 1 + ln(n)$
$P(n+1) : 1 + \frac{1}{2}ln(n+1) \leq H_{n+1} \leq 1 + ln(n+1)$

By (a) : $1 + \frac{1}{2}(ln(n+1) - \frac{2}{n+1}) \leq H_n$

$1 + \frac{1}{2}ln(n+1) - \frac{1}{n+1} \leq H_n$
$1 + \frac{1}{2}ln(n+1) \leq H_{n+1}$

By (b) : $ln(n) \leq ln(n+1) - \frac{1}{1+n}$
$H_n \leq 1 + ln(n+1) - \frac{1}{n+1}$
$H_{n+1} \leq 1 + ln(n+1)$
$\therefore 1 + \frac{1}{2}ln(n) \leq H_n \leq 1 + ln(n)$

4 DMC 5.43

(a) **The robot is at position (3, 3), it wants to get to position (3, 4)**
Let $n = x + y$**, position** $= (x, y)$
So the robot starts at a position of $n = 6$ **and wants to get to a position of** $n = 7$**. However, since it can only move diagonal, the only possible** Δn **is** ± 0 **or** ± 2
Therefore, it is only ever possible for the robot to move onto even n-numbered squares and thus can never reach (3, 4).

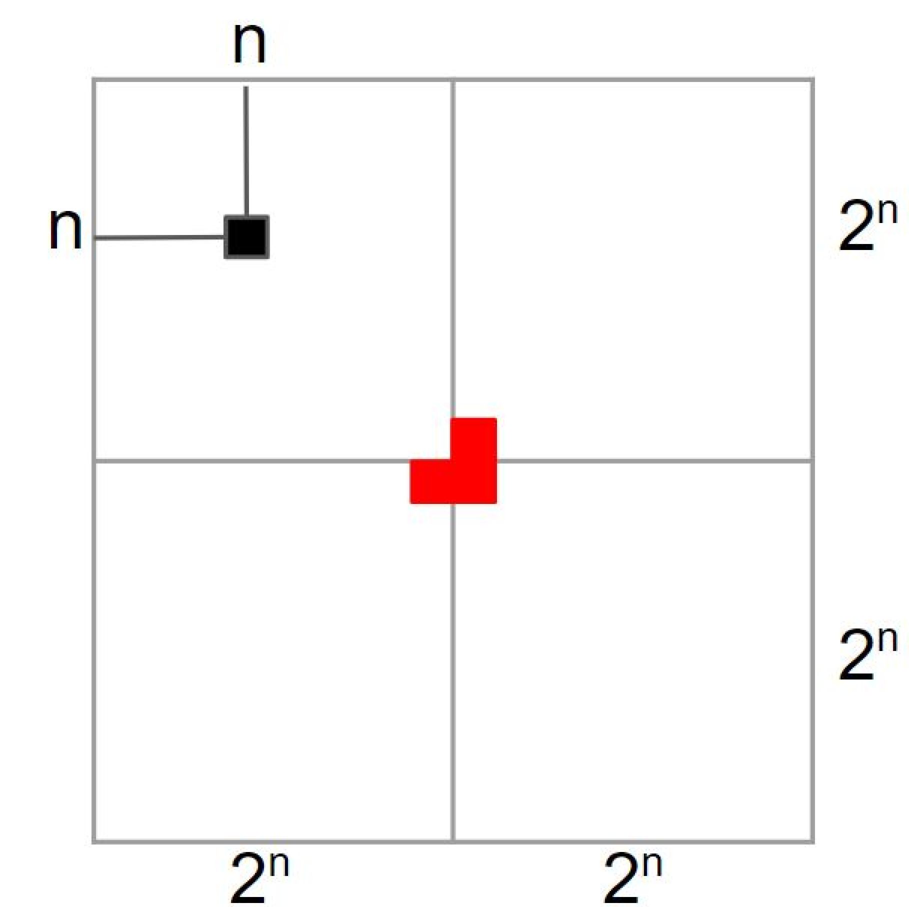
(b) **Following the same logic as above, the new possible** Δn**'s are** $\pm 0, +3, -2$**, so now the robot can reach odd n-numbered squares, as well as even n-numbered squares, and thus can reach all squares in a finite number of moves.**

5 DMC Exercise 6.2

Assume $P(n) : n^3 < 2^n, n \geq 10$
Prove $n^3 + 3n^2 + 3n + 1 < 2^{n+1}$

$n^3 < 2^n$
$3n^2 + 3n + 1 < n^3 < 2^n$ **for** $n \geq 10$
$\therefore n^3 + 3n^2 + 3n + 1 < 2^n + 3n^2 + 3n + 1 < 2^n + 2^n$
$\therefore (n+1)^3 < 2^{n+1}$

6 DMC Exercise 6.4



Assume that if a missing square is at position (n, n) on a 2^nx2^n patio, you can still L-tile the patio.
A 2^{n+1}x2^{n+1} patio with a missing square at (n, n),

with a tile placed in the center, as shown in the Figure above, can be broken up into 4 2^nx2^n patios, 1 with a center tile missing and 3 with a corner tile missing. By the proof given in Chapter 6.1 : L-Tile Land of DMC, both of these types of 2^nx2^n patios can be tiled individually, therefore the 2^{n+1}x2^{n+1} patio can be tiled.