Gabriel Maayan - FOCS Assignment 2

1 DMC 3.3

Liamsi has more RAM than Kilam = p : Pigs can fly = q

p = T, q = F

(a) $\neg p \rightarrow q = T$

(b) $p \rightarrow q = F$

(c) $\neg p \wedge q = F$

(d) $\neg p \vee q = F$

(e) $p \wedge q = F$

(f) $p \vee q = T$

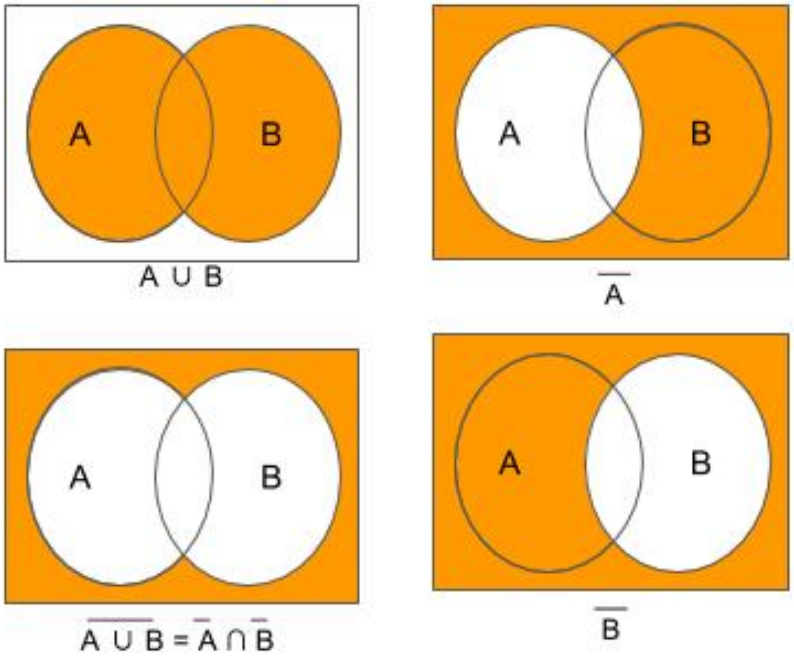
2 DMC 3.21

(c) $\neg p \wedge q \wedge \neg r$

3 DMC 4.7

(a) Prove "x **is irrational** $\rightarrow \sqrt{x}$ **is irrational."**
Let $\sqrt{x}$ **be rational.**
$\sqrt{x} = a/b, a \in \mathbb{Z}, b \in \mathbb{N}$
$x = a^2/b^2$
a^2/b^2 **is rational. So x is rational.**
$\therefore x$ **is irrational** $\rightarrow \sqrt{x}$ **is irrational, is True.**

4 DMC 4.25 and 4.26



(b)

$(A \cup B)'$
by DeMorgan's Thm
$A' \cap B'$
$\therefore (A \cup B)' = A' \cap B'$

5 DMC 4.13

(l) Prove: If x, y are irrational, then y^x is irrational.

Let $x = \log_2(9)$ and $y = \sqrt{2}$
$y^x = \sqrt{2}^{\log_2(9)}$
$2^{\log_2(3)}$
$= 3$

$\therefore$ **There can be an irrational** x, y **s.t.** y^x **is rational.**

(o) Prove $\exists x, y \in \mathbb{Z} : 2x^2 + 5y^2 = 14$

Let $y = 0$**, for** $x = 0, 1, 2, 3, \cdots$, $2x^2 = \{0, 2, 8, 18, \cdots\}$
Let $y = 1$**, for** $x = 0, 1, 2, 3, \cdots$, $2x^2{+}5 = \{5, 7, 13, 23, \cdots\}$
Let $y = 2$**, for** $x = 0, 1, 2, 3, \cdots$, $2x^2{+}20 = \{20, 22, 28, 38, \cdots\}$
$\therefore \exists x, y \in \mathbb{Z} : 2x^2 + 5y^2 = 14$ **is False.**