Gabriel Maayan - FOCS Assignment 1

1 DMC 1.3

This comparison is wrong because net worth is a measurement of the cost of something, and GDP is a measurement of profit.

2 DMC 2.5

(b) $(A \cap \overline{(B \cup C)}) \cup (A \cap B \cap C)$

3 DMC 2.14

(a) $A \times B = \{(1, a), (1, b), (1, c), (1, d), (2, a), (2, b), (2, c), (2, d),$
$(3, a), (3, b), (3, c), (3, d)\}$
$|A \times B| = 12$

(b) $B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3),$
$(c, 1), (c, 2), (c, 3), (c, 1), (c, 2), (c, 3), \}$
$|B \times A| = 12$

(c) $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3),$
$(3, 1), (3, 2), (3, 3)\}$
$|A \times A| = 9$

(d) $B \times B = \{(a, a), (a, b), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d),$
$(c, a), (c, b), (c, c), (c, d), (d, a), (d, b), (d, c), (d, d)\}$
$|B \times B| = 16$

$.A \times B \times C = \{(a, b, c) | a \in A, b \in B, c \in C\}$

4 DMC 3.33

$6 : 1, 2, 3, 6$
$8 : 1, 2, 4, 8$

$12 : 1, 2, 3, 4, 6, 12$
$15 : 1, 3, 5, 15$
$18 : 1, 2, 3, 6, 9, 18$
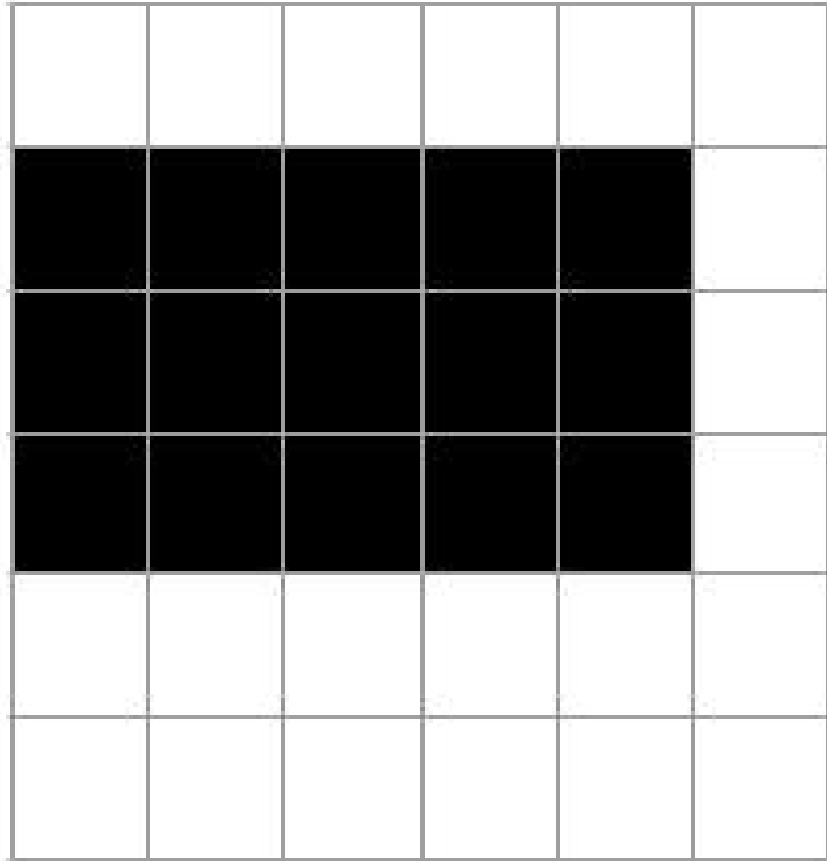$30 : 1, 2, 3, 5, 6, 10, 15, 30$
$4 : 1, 2, 4$
$9 : 1, 3, 9$
$16 : 1, 2, 4, 8, 16$
$25 : 1, 5, 25$
$36 : 1, 2, 3, 4, 6, 9, 12, 18, 36$

Let d(n) be the number of positive divisors of n.
d(n) is odd if $n = x^2$**, where x** $\in \mathbb{Z}$**.**
Otherwise d(n) is even.

5 DMC 3.34

(a)

(b) It is not possible to fill the entire 6×6 grid from **5** original infections. **6** original infections are able to fill the grid.
For a 4×4 and 5×5 grid, you need at least **4** and **5** infections respectively.

(c) For an $n \times n$ grid, $n \in \mathbb{N}$, at least n original infections are needed to infect the entire grid.

6 DMC 1.2

(f) An 8×8 grid gives **32** black squares and **32** white squares. If you remove one of each color, you have **31** of each. However, as long as there is an equal number of white and black squares, you will be able to cover the entire board. Each square removed shifts a domino over a square but since the dominoes are shifted twice, you can just remove one domino and cover the board.