Gabriel Maayan - FOCS Assignment 3

1 DMC 13.3

There are two cases:

(1) Start with a hot meal. Then there are two options for breakfast, one for lunch, and two for dinner. This gives a total of four options.

(2) Start with a cold meal. There are two options for breakfast, two for lunch, and two for dinner, giving eight options.

In total, this gives **12** options for daily menus.

2 DMC 13.4

Let each day be a digit in a binary sequence. A day where you walk a mile is a 1 and a day where you sleep in is a 0. This gives a binary sequence of length **20**, with **12** one's. The number of combinations is $\binom{20}{12} = 125970$ possible ways to walk **12** miles in **20** days.

3 DMC 13.20

(c) The total number of ways to make the digits add up to **27** is $\binom{32}{5}$, but that includes digits that are larger than 9, so subtracting those cases and adding back in the ones that were correct, you get $\binom{32}{5} - \binom{6}{1}\binom{22}{5} + \binom{12}{5}\binom{6}{2} = 55252$ possible ways.

4 DMC 13.40

$x_1 = z_1, x_2 = z_2 - z_1, \cdots, x_k = z_k - z_{k-1}$

$x_1 + x_2 + \cdots + x_k = \underline{z_1} + \underline{z_2} - \underline{z_1} + \cdots + \underline{z_k - z_{k-1}} = z_k \leq n$

Gives a sequence of length $n + k$ with k ones, so the number of sequences is $\binom{n+k}{k}$.

5 DMC 14.32

n

	0	1	2	3	4
0	1	1	1	1	1
1	1	2	3	4	5
2	1	3	6	10	15
3	1	4	10	20	35
4	1	5	15	35	70

m

(a)

As shown by the table, any number is the sum of the number above it and the number to the right of it. Thus, $P(n,m) = P(n, m-1) + P(n-1, m)$

(b) Also shown by the table above, $P(0, m) = P(n, 0) = 1$.

(c) As seen in the table, $P(4, 4) = 70$.

(d) A diagonal on the table cooresponds to a row in Pascal's triangle. For instance, the second diagonal is the same as the second row in Pas-

cal's triangle. The constant on a diagonal is $n + m$, so $n_p = n + m$, and $k_p = n$. Therefore, $P(n, m) = \binom{n+m}{n}$.

<div align="center">n</div>

	0	1	2	3	4
0	1	1	1	1	1
1	1	2	x	1	2
2	1	3	3	4	6
3	1	4	7	x	6
4	1	5	12	12	18

(m on the left)

(e)

The new table would look like the above table, so the new $P(4, 4) = 18$.

(f) The number of combinations is equal to the ways to get to $(4, 4)$, minus the ways to get to the crossed off spaces, plus the ways to get to the spaces in the middle. This is $\binom{8}{4} - \binom{3}{1}\binom{3}{2} - \binom{6}{3}\binom{2}{1} + \binom{3}{1}\binom{3}{1}\binom{2}{1}$.

6 DMC 12.19

By Theorem 12.4, there is a stable matching with a group of n men and n women. Suppose there is a group of n men and n women, that is in a stable matching. If any two men or any two women are switched, there is an unstable matching, because those matchings would prefer to switch back. Therefore, there does not exist an $n \geq 3$ for which there

are preferences of n men and n women such that
every matching is stable.