Fatigue Homework 6

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Gabe Morris

[1]:
```python
# Notebook Preamble
import matplotlib.pyplot as plt
import numpy as np
import sympy as sp
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

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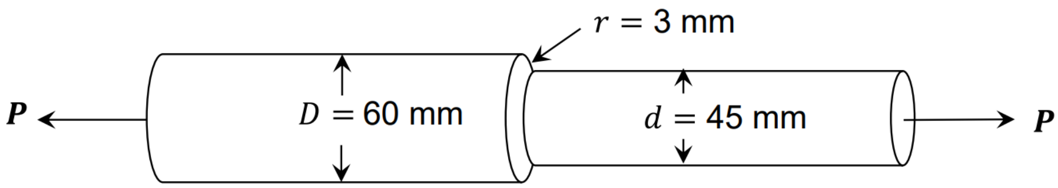
1.1 Given

A stepped circular rod of 4340 steel (with $S_u = 1468$ MPa) with diameters of 60 and 45 mm has a root radius of 3 mm at the stepped section. The rod is to be subjected to axial cyclic loading.



The cyclic yield strength (S'_y) is estimated from:

$$S'_y = K'(0.002)^{n'}$$

where K' and n' are given in Table A.2

For the purpose of constructing Haigh diagram, exact value of σ_f is not needed, as the diagram is not very sensitive to its value. Since σ_f is not listed in Table A.2 for this material, we use Eq. 5.20 in the textbook to approximate it as

$$\sigma_f \approx S_u + 345\ (MPa)$$

1.2 Find

Using a Haigh diagram, determine the following for an approximate median fatigue life of 10^6 cycles:

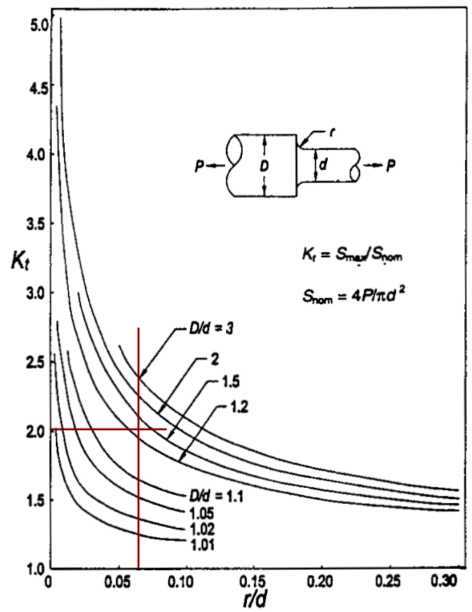
a. What fully reversed alternating force, P_a, can be applied?
b. What is the maximum value of P_a, if proper compressive residual stresses are present at the notch root? What is the magnitude of the compressive residual stress needed to obtain this maximum alternating stress?
c. What value of P_a can be applied if the residual stress calculated in (b) is tensile? What fully reversed alternating force, P_a, can be applied?

1.3 Solution

According to equation 4.3b, the endurance limit is 700 MPa for materials with an ultimate strength greater than 1400 MPa. With the size effect, the endurance limit becomes

$$S_f = 0.85(700) = 595\ MPa$$

1.3.1 Part A



[2]:
```
D, d, r = 60, 45, 3  # mm
Su = 1468   # MPa
size_effect = 0.85

if Su <= 1400:
    Sf = 0.5*Su*size_effect
else:
    Sf = 700*size_effect

D/d
```

[2]: 1.3333333333333333

[3]:
```
r/d
```

[3]: 0.06666666666666667

From above the stress concentration factor is $K_t = 2$. The fatigue notch factor for the fully reversed condition is

$$K_f = 1 + \frac{K_t - 1}{1 + a/r}$$

where $a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8}$ with a in mm and S_u in MPa.

[4]:
```
Kt = 2
a = 0.0254*(2070/Su)**1.8
```

```
a  # mm
```

[4]: 0.047149103389883054

[5]:
```
Kf = 1 + (Kt - 1)/(1 + a/r)
Kf
```

[5]: 1.9845268144780213

Since the stress is maximized at the smaller diameter,

$$P_a = \frac{S_f}{K_f}\left(\frac{\pi}{4}\right)(d)^2$$

[6]:
```
Pa = Sf/Kf*np.pi/4*d**2
Pa  # N
```

[6]: 476842.4418454644

1.3.2 Part B

From Table A.2,

Property	Value
S_y	1371 MPa
S'_y	863 MPa
S_f	595 MPa
σ_f	1813 MPa
K_f	1.98
S_{cat}	70 MPa

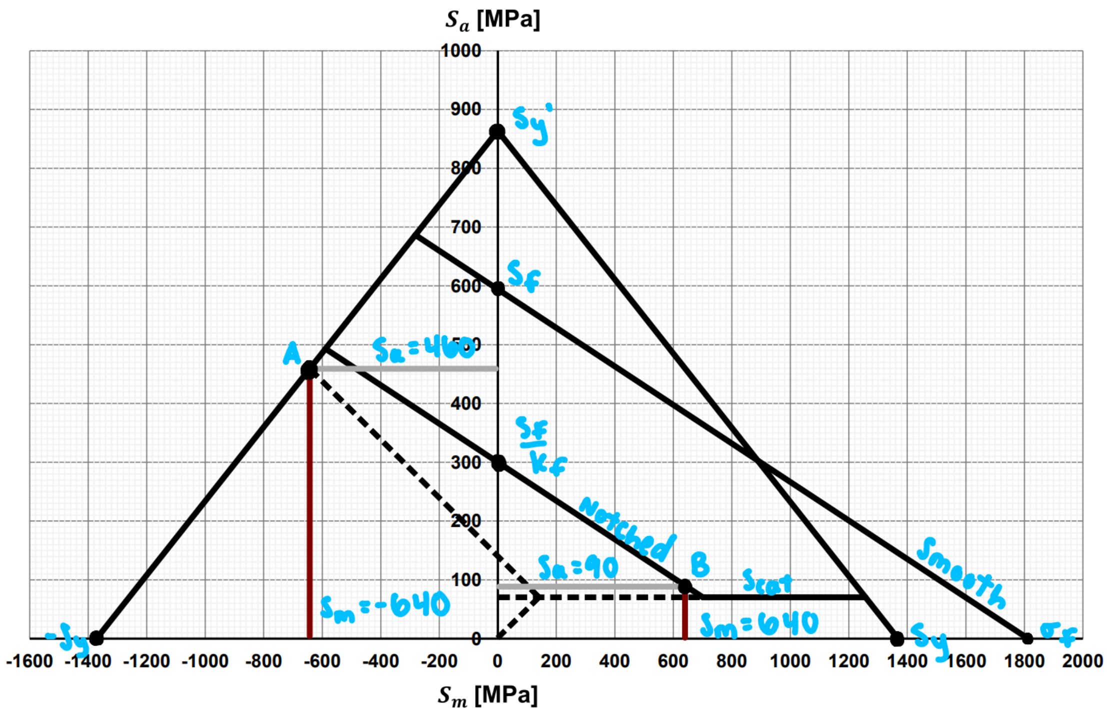
S_{cat} comes from the fact that this is a hard steel. The other calculations are shown below.

[7]:
```
sig_f = Su + 345
sig_f  # MPa
```

[7]: 1813

[8]:
```
K_prime, n_prime = 1996, 0.135
Sy_prime = K_prime*0.002**n_prime
Sy_prime  # MPa
```

[8]: 862.5804014077875




```
[9]: Sa = 460   # MPa
     Pa = Sa*np.pi/4*d**2
     Pa  # N
```

[9]: 731598.3892047232

1.3.3 Part C

From above, point B is when the mean stress is tensile.

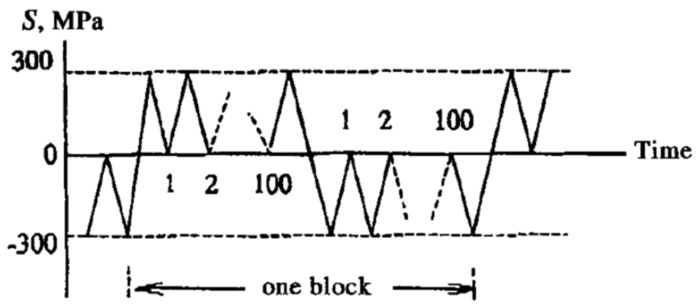
```
[10]: Sa = 90
      Pa = Sa*np.pi/4*d**2
      Pa  # N
```

[10]: 143138.81527918496

2

2.1 Given

An axially loaded member made of 2024-T3 aluminum is repeatedly subjected to the block of stress history shown below.



2.2 Find

a. Complete a summary of the loading block.
b. Using the Basquin equation, $S_{Nf} = \sigma'_f(2N_f)^b$, determine the fatigue strength S_{Nf}, the fatigue life N_f and the damage ratio n/N_f for each load segment and estimate the expected life if the member is smooth.
c. Estimate the expected life if the member has a notch with $K_t = 2$ and the notch root radius is 1 mm. For the notched member, assume that the given nominal stress block and K_t are based on net stress.

2.3 Solution

2.3.1 Part A

Load Segment	S_{min} (MPa)	S_{max} (MPa)	S_a (MPa)	S_m (MPa)	n
1	0	300	150	150	100
2	-300	300	300	0	1
3	-300	0	150	-150	100

2.3.2 Part B

Use the modified Goodman to find S_{Nf} with $S_u = 469$ MPa.

$$\frac{S_a}{S_{Nf}} + \frac{S_m}{S_u} = 1 \rightarrow S_{Nf} = \frac{S_a S_u}{S_u - S_m}$$

$$S_{Nf} = \sigma'_f(2N_f)^b \rightarrow N_f = \frac{1}{2}\left(\frac{S_{Nf}}{\sigma'_f}\right)^{1/b}$$

[11]:
```
Su, sig_prime, b = 469, 1100, -0.124

SNf_lamb = lambda Sa_, Su_, Sm_: Sa_*Su_/(Su_ - Sm_)
Nf_lamb = lambda SNf_, sig_prime__, b__: 0.5*(SNf_/sig_prime__)**(1/b__)

# Load 1
SNf1 = SNf_lamb(150, Su, 150)
SNf1  # MPa
```

[11]: 220.53291536050156

[12]:
```
Nf1 = Nf_lamb(SNf1, sig_prime, b)
Nf1
```

[12]: 212496.20843121517

[13]:
```
d1 = 100/Nf1
d1
```

[13]: 0.0004705966319976477

[14]:
```
# Load 2
SNf2 = SNf_lamb(300, Su, 0)
SNf2  # MPa
```

[14]: 300.0

[15]:
```
Nf2 = Nf_lamb(SNf2, sig_prime, b)
Nf2
```

[15]: 17764.216450750755

[16]:
```
d2 = 1/Nf2
d2
```

[16]: 5.629294164324021e-05

[17]:
```
# Load 3
SNf3 = SNf_lamb(150, Su, -150)
SNf3  # MPa
```

[17]: 113.65105008077545

[18]:
```
Nf3 = Nf_lamb(SNf3, sig_prime, b)
Nf3
```

[18]: 44578464.41972726

[19]:
```
d3 = 100/Nf3
d3
```

[19]: 2.24323563634792e-06

Now the expected life is calculated as the reciprocal of the summation of the damage ratios.

[20]:
```
1/sum([d1, d2, d3])  # Blocks
```

[20]: 1889.8846990152454

Load Segment	S_{Nf} (MPa)	N_f	n	n/N_f
1	221	212496	100	$4.706 \cdot 10^{-4}$
2	300	17764	1	$5.630 \cdot 10^{-5}$
3	114	44,578,464	100	$2.243 \cdot 10^{-6}$
Total	-	-	-	1890

2.3.3 Part C

Everything remains the same, but now the endurance limit is changed to S_f/K_f, and a new b is found by assuming that the endurance limit is reached at 10^6 cycles.

[21]:
```
a = 0.0254*(2070/Su)**1.8
a  # mm
```

[21]: 0.3676793350247542

[22]:
```
Kt, r = 2, 1
Kf = 1 + (Kt - 1)/(1 + a/r)
Kf
```

[22]: 1.7311655403361788

[23]:
```
Sf = sig_prime*2e6**b
Sf  # MPa
```

[23]: 181.9973086280446

[24]:
```
Sf_new = Sf/Kf
B = np.log(Sf_new/sig_prime)/np.log(2e6)
B
```

[24]: -0.16182533948270703

The S_{Nf} values stay the same, but the N_f values change.

[25]:
```
Nf1 = Nf_lamb(SNf1, sig_prime, B)
Nf1
```

[25]: 10274.56557852558

[26]:
```
d1 = 100/Nf1
d1
```

[26]: 0.009732771593672595

[27]:
```
Nf2 = Nf_lamb(SNf2, sig_prime, B)
Nf2
```

[27]: 1534.2156409133563

[28]:
```
d2 = 1/Nf2
d2
```

[28]: 0.0006517988562577001

[29]:
```
Nf3 = Nf_lamb(SNf3, sig_prime, B)
Nf3
```

[29]: 617791.4135432595

[30]:
```
d3 = 100/Nf3
d3
```

[30]: 0.00016186693082453747

[31]:
```
1/sum([d1, d2, d3])  # Blocks
```

[31]: 94.81874910904062

Load Segment	S_{Nf} (MPa)	N_f	n	n/N_f
1	221	10275	100	$9.732 \cdot 10^{-3}$
2	300	1534	1	$6.518 \cdot 10^{-4}$
3	114	617791	100	$1.619 \cdot 10^{-4}$
Total	-	-	-	95

3

3.1 Given

Repeat Problem 2 using the strain-life approach.

3.2 Find

a. Explain why the behavior is linear elastic and the strain amplitude can be calculated using $\epsilon_a = S_a/E$.
b. Use the Smith-Watson-Topper (SWT) equation, $\sigma_{\max}\varepsilon_a E = \left(\sigma_f'\right)^2 \left(2N_f\right)^{2b} + \sigma_f'\varepsilon_f' E \left(2N_f\right)^{b+c}$, to account for the mean stress effect, and the fatigue properties from Table A.2 to determine the fatigue life and calculate the damage ratio for each load segment.
c. Estimate the total fatigue life in terms of blocks.

For a notched member with $K_t = 2$ and a root radius of 1 mm:

d. Explain why the behavior is inelastic.
e. Determine the fatigue notch factor K_f.
f. Determine the stress range S from the beginning of the block (Point O) to point A. Using Neuber's rule, $\frac{\sigma_A^2}{E} + \sigma_A \left(\frac{\sigma_A}{K}\right)^{1/n} = \varepsilon_A \sigma_A = \frac{\left(K_f S_A\right)^2}{E}$, with the cyclic stress-strain equation, $\varepsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{K'}\right)^{1/n'}$, find the notch stress σ_A and the strain ϵ_A at point A.
g. For load segment 1 (point A to point B), determine the stress range ΔS from point A to point B. Using the Neuber's rule, $\frac{(\Delta\sigma)^2}{E} + 2\Delta\sigma \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} = \frac{\left(K_f \Delta S\right)^2}{E}$, with the cyclic stress equation, $\Delta\varepsilon = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}$, find the notch stress $\sigma_B = \sigma_A - \Delta\sigma$ and strain $\epsilon_B = \epsilon_A - \Delta\epsilon$ at point B. Determine the strain amplitude ϵ_a and the maximum stress σ_{max} and calculate the fatigue life N_f using the SWT equation.
h. Repeat (g) for load segment 2 (point C to point D).
i. Repeat (g) for load segment 3 (point D to point E).
j. Estimate the fatigue life in terms of blocks.

3.3 Solution

Recall the following,

Load Segment	S_{min} (MPa)	S_{max} (MPa)	S_a (MPa)	S_m (MPa)	n
1	0	300	150	150	100
2	-300	300	300	0	1
3	-300	0	150	-150	100

3.3.1 Part A

The behavior is mostly elastic because the stress amplitude is 150 MPa for most of the loading duration. Since the stress amplitude is less than half of the material's yield strength ($S_y = 370$

MPa), the duration is of the load is mostly elastic. Also, even the peak stress values (300 MPa) is less than the yield strength.

3.3.2 Part B

The below code cell is a contains a function that will numerically solve for N_f using the Smith-Watson-Topper equation.

[32]:
```
# Define properties
E, sig_prime = 70_000, 1100  # MPa
eps_prime, b, c = sp.S('0.22'), sp.S('-0.124'), sp.S('-0.59')

# Define symbols
sig_max_, eps_a_, E_, sig_prime_, Nf_, eps_prime_, b_, c_= sp.
↪symbols(r"\sigma_{max} \epsilon_a E \sigma^'_f N_f \epsilon^'_f b c")

def SWT_elastic(S_max, eps_a, Nf_guess=10_000):
    sub_list = [(sig_max_, S_max), (eps_a_, eps_a), (E_, E), (sig_prime_,␣
↪sig_prime), (eps_prime_, eps_prime), (b_, b), (c_, c)]
    eq = sp.Eq(sig_max_*eps_a_*E_, sig_prime_**2*(2*Nf_)**(2*b_) +␣
↪sig_prime_*eps_prime_*E_*(2*Nf_)**(b_ + c_))
    display(eq)

    with sp.evaluate(False):
        eq_sub = sp.Eq(eq.lhs.subs(sub_list), eq.rhs.subs(sub_list))
        display(eq_sub)

    try:
        sol = sp.nsolve(eq_sub, Nf_guess)
    except ValueError:
        sol = sp.oo
    display(sp.Eq(Nf_, sol))
    return sol

# Load Segment 1
S_max1, eps_a1 = 300, sp.S('0.0021429')
Nf1 = SWT_elastic(S_max1, eps_a1)
d1 = 100/Nf1
```

$$E\epsilon_a\sigma_{max} = E\epsilon'_f\sigma'_f\left(2N_f\right)^{b+c} + \left(\sigma'_f\right)^2\left(2N_f\right)^{2b}$$

$$70000\cdot 0.0021429\cdot 300 = \frac{1100^2}{\left(2N_f\right)^{(-0.124)(-2)}} + 70000\cdot 0.22\cdot 1100\left(2N_f\right)^{-0.59-0.124}$$

$$N_f = 324108.948236852$$

[33]:
```
# Load Segment 2
S_max2, eps_a2 = 300, 2*eps_a1
```

```python
Nf2 = SWT_elastic(S_max2, eps_a2)
d2 = 1/Nf2
```

$$E\epsilon_a\sigma_{max} = E\epsilon_f^{\prime}\sigma_f^{\prime}\left(2N_f\right)^{b+c} + \left(\sigma_f^{\prime}\right)^2\left(2N_f\right)^{2b}$$

$$70000 \cdot 0.0042858 \cdot 300 = \frac{1100^2}{\left(2N_f\right)^{(-0.124)(-2)}} + 70000 \cdot 0.22 \cdot 1100\left(2N_f\right)^{-0.59-0.124}$$

$$N_f = 25160.2548136955$$

[34]:
```python
S_max3, eps_a3 = 0, eps_a1
Nf3 = SWT_elastic(S_max3, eps_a3)
d3 = 100/Nf3
```

$$E\epsilon_a\sigma_{max} = E\epsilon_f^{\prime}\sigma_f^{\prime}\left(2N_f\right)^{b+c} + \left(\sigma_f^{\prime}\right)^2\left(2N_f\right)^{2b}$$

$$70000 \cdot 0.0021429 \cdot 0 = \frac{1100^2}{\left(2N_f\right)^{(-0.124)(-2)}} + 70000 \cdot 0.22 \cdot 1100\left(2N_f\right)^{-0.59-0.124}$$

$$N_f = \infty$$

Here is a table of results.

Load Segment	S_a (MPa)	ϵ_a	σ_{max}	$\sigma_{max}\epsilon_a$	n	N_f	n/N_f
1	150	0.0021429	300	0.64287	100	324109	$3.09 \cdot 10^{-4}$
2	300	0.0042858	300	1.28571	1	25160	$3.97 \cdot 10^{-5}$
3	150	0.0021429	0	0	100	∞	0
Total	-	-	-	-	-	-	2871

3.3.3 Part C

[35]:
```python
print([d1, d2, d3])
1/sum([d1, d2, d3])  # Blocks
```

```
[0.000308538226247682, 3.97452254519963e-5, 0]
```

[35]: 2871.22455896151