Fatigue Homework 5

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[1]:
```python
# Notebook Preamble
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1 Preliminary Questions

What are the expressions of the plastic zone size for plane stress and plane strain?

For plane stress:

$$2r_y = \frac{1}{\pi}\left(\frac{K}{S_y}\right)^2$$

For plane strain:

$$2r_y = \frac{1}{3\pi}\left(\frac{K}{S_y}\right)^2$$

where K is the stress intensity factor, r_y is the plastic zone radius, and S_y is the yield strength.

What are the restrictions on the use of LEFM?

The following restrictions on the use of the LEFM are:

- The plastic zone size at the crack tip must be small relative to the crack length.
- The net nominal stresses in the crack plane must be less than $0.8S_y$
- Under monotonic loading, $r_y \leq (1/8)a$
- $r_y \leq (1/8)t$ and $r_y \leq (1/8)(w-a)$
- For cyclic loading, $r_y \leq a/4$

What are the restrictions for the plane strain fracture toughness K_{IC} value to be considered valid?

In order for a plane strain fracture toughness value to be considered valid, it is required that:

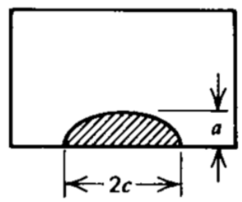
$$a \geq 2.5\left(\frac{K_{IC}}{S_y}\right)^2$$

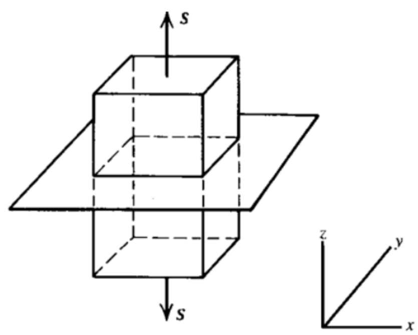
$$t \geq 2.5\left(\frac{K_{IC}}{S_y}\right)^2$$

2

2.1 Given

A gas turbine component is made of recrystallized, annealed Ti-6A1-4V with $K_{IC} = 85\ MPa\sqrt{m}$ and $S_y = 815\ MPa$. A surface semi-circular crack ($a/c = 1$) (shown in the figure below) is found during a routine maintenance inspection. If the component thickness is 25 mm, comment on the stress state (plane stress or plane strain).





a/c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Φ	1.0	1.016	1.051	1.097	1.151	1.211	1.277	1.345	1.418	1.493	1.571

2.2 Find

a. If a stress is applied normal to the crack plane like that in the second figure, what maximum stress is required to cause fracture if $a = 8\ mm$ and $K_c = 105\ MPa\sqrt{m}$?
b. If the thickness were doubled, what maximum stress would cause fracture?
c. Comment on the conditions required for fracture at each thickness and whether LEFM is valid for each case.

2.3 Solution

The plane stress or plane stress condition can be deduced from the following relationships:

$$a \geq 2.5 \left(\frac{K_{IC}}{S_y}\right)^2$$

$$t \geq 2.5 \left(\frac{K_{IC}}{S_y}\right)^2$$

where if true, the condition is plane strain. If not true, then the condition is either a mixed mode or a plane stress condition.

[2]:
```
K_IC = 85  # MPa sqrt(m)
S_y = 815  # MPa
a = 8  # mm
t = 25  # mm

# Apply conditions
expr = 2.5*(K_IC/S_y)**2
a >= expr*1000, t >= expr*1000
```

[2]:
```
(False, False)
```

Therefore, the condition can be considered to be **plane stress**.

2.3.1 Part A

The relationship for a semi-elliptical surface crack is

$$K = \frac{1.12S\sqrt{\pi a}}{\Phi}\sqrt{\sec(\pi a/2t)}$$

[3]:
```
K_c = 105
S_t = lambda t_: K_c*1.571/(1.12*np.sqrt(np.pi*a/1000))*np.sqrt(np.cos(np.pi*a/
↪(2*t_)))
S_t(25)  # MPa
```

[3]:
```
869.6722102981946
```

2.3.2 Part B

[4]:
```
S_t(50)  # MPa
```

[4]:
```
914.3153034270322
```

2.3.3 Part C

The LEFM method can be tested by finding the plastic zone size.

[5]:
```
# For plane stress
r_y = 1/(2*np.pi)*(K_c/S_y)**2*1000
r_y  # mm
```

[5]:
```
2.6417000979911123
```

[6]:
```
# Monotonic test
r_y <= (1/8)*a
```

[6]: False

[7]:
```
r_y <= (1/8)*t
```

[7]: True

[8]:
```
# For cyclic loading
r_y <= a/4
```

[8]: False

The LEFM method is not a good approximation. This is because the plastic zone size (r_y) is not significantly less than the crack length (a), and the calculated stress values exceed the yield strength of 815 MPa.