ME 8253 Spring 2023
Homework #6
Due Date: Monday May 1st, 2023

Please submit your homework through CANVAS as a PDF file. For all problems with calculation, present all calculation details.

Problem 1 (*30 points*)

A stepped circular rod of 4340 steel (with $S_u = 1468$ MPa) with diameters of 60 and 45 mm has a root radius of 3 mm at the stepped section. The rod is to be subjected to axial cyclic loading.

The cyclic yield strength S_y' is estimated from:

$$S_y' = K'(0.002)^{n'}$$

where K' and n' are given in Table A.2.

For the purpose of constructing Haith diagram, exact value of σ_f is not needed, as the diagram is not very sensitive to its value. Since σ_f is not listed in Table A.2 for this material, we use Eq. 5.20 in the textbook to approximate it as

$$\sigma_f \approx S_u + 345 \text{ (in MPa)}.$$

Using a Haigh diagram, determine the following for an approximate median fatigue life of 10^6 cycles:

(a) What fully reversed alternating force, P_a, can be applied?
(b) What is the maximum value of P_a, if proper compressive residual stresses are present at the notch root? What is the magnitude of the compressive residual stress needed to obtain this maximum alternating stress? (Use **Figure 3** next page to plot the Haigh diagram)
(c) What value of P_a, can be applied if the residual stress calculated in (b) is tensile? What fully reversed alternating fore, P_a, can be applied?

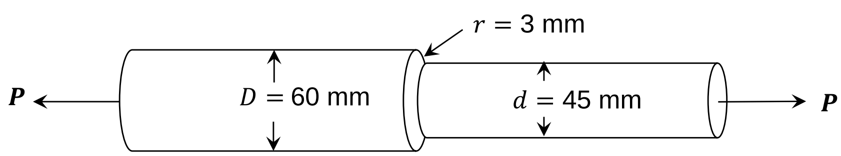

Figure 1 – A stepped circular rod.

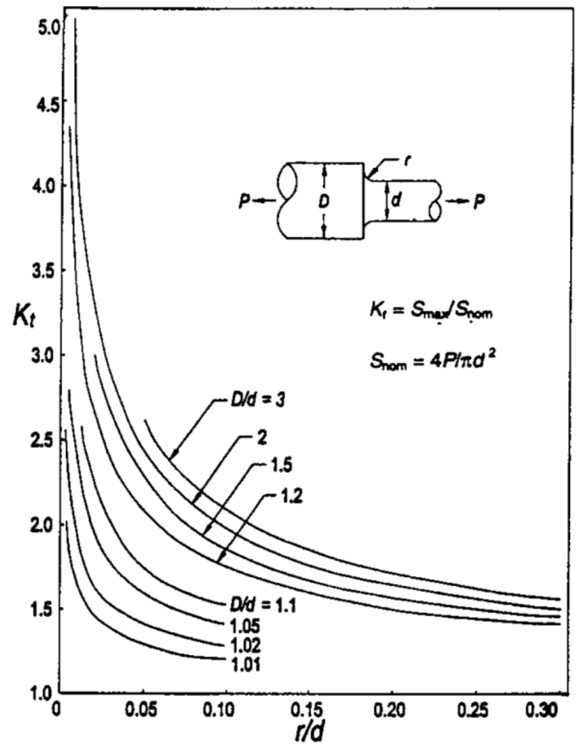

Figure 2 – Stress concentration factors for a stepped shaft in tension.

Answer:

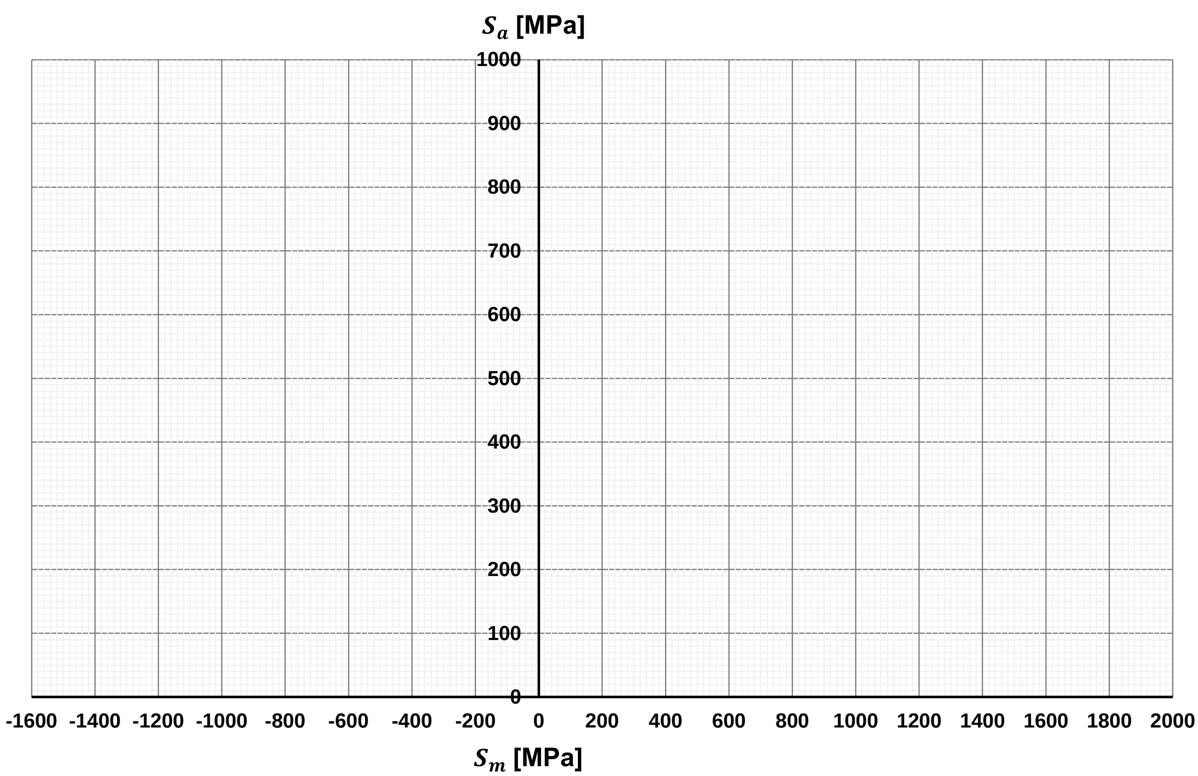

Figure 3 – Haigh diagram.

Problem 2 (*30 points*)

An axially loaded member made of 2024-T3 aluminum is repeatedly subjected to the block of stress history shown (see Figure 4).

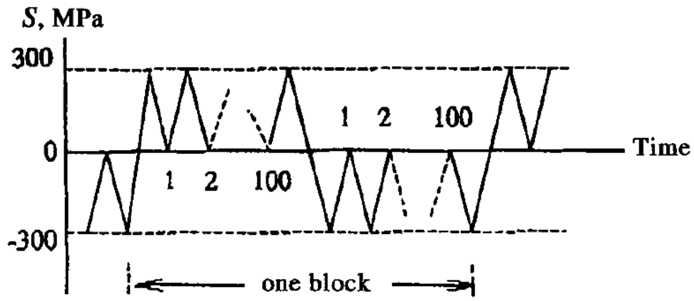

Figure 4 – Block of stress history.

Using the **S-N approach**:

(a) Complete the summary of the loading block in Table 1 below.

(b) Using the Basquin equation

$$S_{Nf} = \sigma_f'(2N_f)^b$$

and fatigue parameters from Table A.2, determine the fatigue strength S_{Nf}, the fatigue life N_f and the damage ratio n/N_f for each load segment and estimate the expected life if the **member is smooth**. Report your results in **Table 2**.

(c) Estimate the expected life if the member has a notch with $K_t = 2$ and the notch root radius is 1 mm. For **the notched member**, assume that the given nominal stress block and K_t are based on net stress. Report your results in **Table 3**.

Answer:

Table 1 – Loading summary

Load Segment	$S_{\min}$ **(MPa)**	$S_{\max}$ **(MPa)**	S_a **(MPa)**	S_m **(MPa)**	n
1	0	300			
2	−300	300			
3	−300	0			

Table 2 – Fatigue strengths and damage ratios for the smooth member.

Load Segment	S_{NF} **(MPa)**	N_f	n	n/N_f
1				
2				
3				
			Total	

Table 3 – Fatigue strengths and damage ratios for the notched member

Load Segment	S_{NF} **(MPa)**	N_f	n	n/N_f
1				
2				
3				
			Total	

Problem 3 (***40 points***)

Repeat Problem 2 using the **strain-life approach (Use Excel file to solve nonlinear equations).**

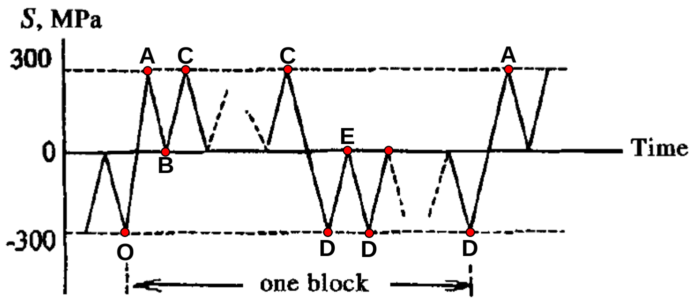

Figure 5 – Block of stress history

For the **smooth member:**

(a) Explain why the behavior is linear elastic and the strain amplitude can be calculated using

$$\varepsilon_a = S_a/E.$$

(b) Use the Smith-Watson-Topper (SWT) equation

$$\sigma_{\max}\varepsilon_a E = \left(\sigma_f'\right)^2\left(2N_f\right)^{2b} + \sigma_f'\varepsilon_f' E\left(2N_f\right)^{b+c}$$

to account for the mean stress effect, and the fatigue properties from Table A.2 to determine the fatigue life and calculate the damage ratio for each load segment. Report your results in **Table 4**. **Tip:** Load segments are same as in **Table 1** and for the smooth member $\sigma_{\max} = S_{\max}$.

(c) Estimate the **total fatigue life in terms of blocks**.

Table 4 – Stress amplitude, strain amplitude, maximum stress, etc., and damage ratios (smooth member).

Load Segment	S_a **(MPa)**	ε_a	$\sigma_{\max}$ **(MPa)**	$\sigma_{\max}\varepsilon_a$	n	N_f	n/N_f
1							
2							
3							
						Total	

For the **notched member** with $K_t = 2$ and a root radius of 1mm:

(d) Explain why the behavior is inelastic.

(e) Determine the fatigue notch factor K_f.

(f) Determine the stress range S from the beginning of the block (**point O**) to **point A**. Using the Neuber's rule

$$\frac{\sigma_A^2}{E} + \sigma_A\left(\frac{\sigma_A}{K}\right)^{1/n} = \varepsilon_A\sigma_A = \frac{\left(K_f S_A\right)^2}{E}$$

with the cyclic stress-strain equation

$$\varepsilon_A = \frac{\sigma_A}{E} + \left(\frac{\sigma_A}{K'}\right)^{1/n'}$$

find the notch stress σ_A and strain ε_A at point A (see Figure 5).

(g) For **load segment 1 (point A to point B)**:

1. Determine the stress range ΔS from point A to point B. Using the Neuber's rule
$$\frac{(\Delta\sigma)^2}{E} + 2\Delta\sigma\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} = \frac{\left(K_f \Delta S\right)^2}{E}$$
with the cyclic stress-strain equation
$$\Delta\varepsilon = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}$$
find the notch stress $\sigma_B = \sigma_A - \Delta\sigma$ and strain $\varepsilon_B = \varepsilon_A - \Delta\varepsilon$ at point B (see Figure 5).
2. Determine the strain amplitude ε_a and the maximum stress $\sigma_{\max}$ and calculate the fatigue life N_f using the Smith-Watson-Topper (SWT) equation. Report your results in **Table 5**.

(h) For **load segment 2 (point C to point D)**:

1. Determine the stress range ΔS from point C to point D. Using the Neuber's rule
$$\frac{(\Delta\sigma)^2}{E} + 2\Delta\sigma\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} = \frac{\left(K_f \Delta S\right)^2}{E}$$
with the cyclic stress-strain equation
$$\Delta\varepsilon = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}}$$
find the notch stress $\sigma_D = \sigma_C - \Delta\sigma$ and strain $\varepsilon_D = \varepsilon_C - \Delta\varepsilon$ at point D (see Figure 5).
Note: The notch stress and notch strain at C are the same as at A ($\sigma_C = \sigma_A$ and $\varepsilon_C = \varepsilon_A$).
2. Determine the strain amplitude ε_a and the maximum stress $\sigma_{\max}$ and calculate the fatigue life N_f using the Smith-Watson-Topper (SWT) equation. Report your results in **Table 5**.

(i) For **load segment 3 (point D to point E)**:

1. Explain why the notch stress range $\Delta\sigma$ and the notch strain range $\Delta\varepsilon$ in load segment 3 are the same as in load segment 1 (see Figure 5).
2. Find the notch stress $\sigma_E = \sigma_D + \Delta\sigma$ and strain $\varepsilon_E = \varepsilon_D + \Delta\varepsilon$ at point E.
3. Determine the strain amplitude ε_a and the maximum stress $\sigma_{\max}$ and calculate the fatigue life N_f using the Smith-Watson-Topper (SWT) equation. Report your results in **Table 5**.

(j) Estimate the **total fatigue life in terms of blocks**.

Table 5 – Stress amplitude, strain amplitude, maximum stress, etc., and damage ratios (notched member).

Load Segment	**ΔS (MPa)**	**ε_a**	**$\sigma_{\max}$ (MPa)**	**$\sigma_{\max}\varepsilon_a$**	**n**	**N_f**	**n/N_f**
1							
2							
3							
						Total	