

Machine Design Homework 1

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```
[1]: # Notebook Preamble
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display, Markdown

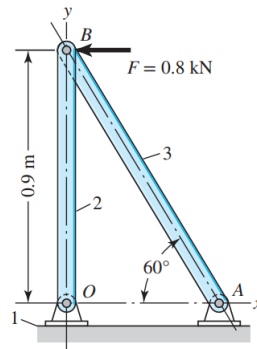
plt.style.use('maroon_ipynb.mplstyle')
```

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1 Problem 3

1.1 Given



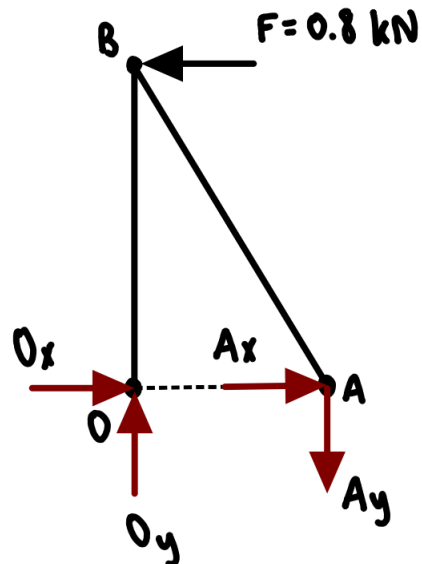
Problem 3-3

1.2 Find

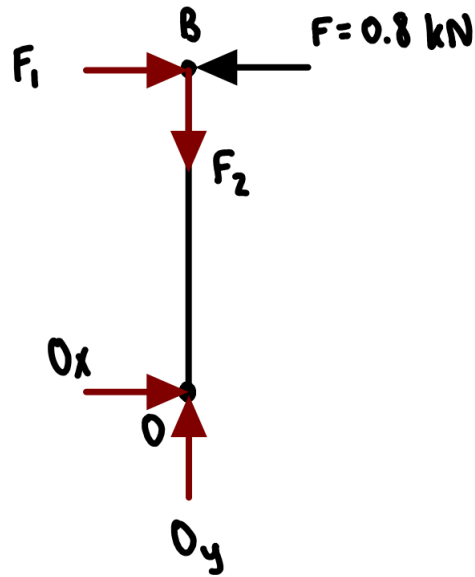
Sketch a free-body diagram of each element in the figure above. Compute the magnitude and direction of each force using an algebraic or vector method.

1.3 Solution

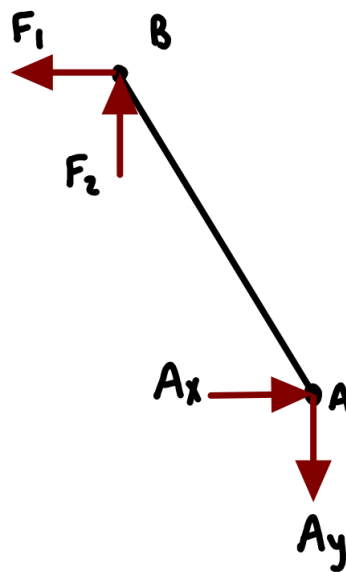
The overall structure has a free body diagram like so,



Separating the structure into individual elements would yield more equations to solve the system,



Link 2



Link 3

The 800 N force is a point load, which means it can only be applied to one link. This is why the load is shown in one FBD and not the other. Changing the link the point load is applied to will change the values for F_1 and F_2 , but it will not change the values of the reactions at O and A.

```
[2]: v = Ox, Oy, Ax, Ay, F1, F2 = sp.symbols('O_x O_y A_x A_y F_1 F_2')
y, x = sp.S(0.9), sp.S(0.9)/sp.tan(60*sp.pi/180) # Vertical and horizontal
           ↪ distance of the structure

# Overall structure equations
eq1 = sp.Eq(Ox + Ax, 800) # Sum in x
eq2 = sp.Eq(Oy - Ay, 0)  # Sum in y
```

```

eq3 = sp.Eq(-Ay*x + 800*y, 0) # Sum the moments about O

# Link 2 equations
eq4 = sp.Eq(Ox + F1, 800) # Sum in x
eq5 = sp.Eq(Oy, F2) # Sum in y
eq6 = sp.Eq(F1*y, 800*y) # Sum the moments about O

# Link 3 equations (don't need to use and still get the same answer)
# eq4 = sp.Eq(Ax, F1)
# eq5 = sp.Eq(Ay, F2)
# eq6 = sp.Eq(F2*x, F1*y)

eqs = [eval(f'eq{i}') for i in range(1, 7)]

display(Markdown('**Equations:**'))
for eq in eqs:
    display(eq)
display(Markdown('---'))

A, b = sp.linear_eq_to_matrix(eqs, v)
matrix_eqn = sp.Eq(sp.MatMul(A, sp.Matrix(v)), b)
display(matrix_eqn)
display(Markdown('---'))

sol = sp.solve(matrix_eqn)
display(Markdown('**Solution:**'))
for val in v:
    display(sp.Eq(val, sol[val]))

```

Equations:

$$A_x + O_x = 800$$

$$-A_y + O_y = 0$$

$$-0.3\sqrt{3}A_y + 720.0 = 0$$

$$F_1 + O_x = 800$$

$$O_y = F_2$$

$$0.9F_1 = 720.0$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -0.3\sqrt{3} & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0.9 & 0 \end{bmatrix} \begin{bmatrix} O_x \\ O_y \\ A_x \\ A_y \\ F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} 800 \\ 0 \\ -720.0 \\ 800 \\ 0 \\ 720.0 \end{bmatrix}$$

Solution:

$$O_x = 0.0$$

$$O_y = 1385.6406460551$$

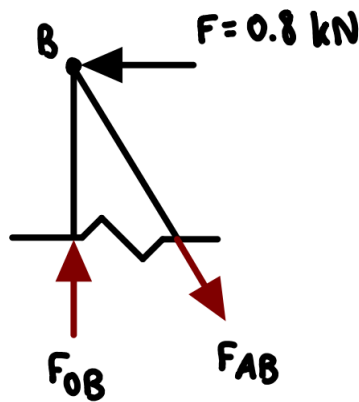
$$A_x = 800.0$$

$$A_y = 1385.6406460551$$

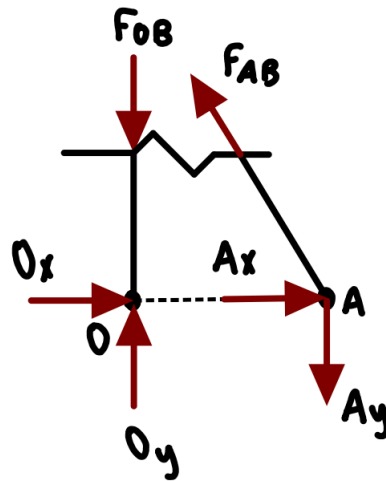
$$F_1 = 800.0$$

$$F_2 = 1385.6406460551$$

The internal stresses of each link may be found by using the method of sections.



Upper Section Cut



Lower Section Cut

```
[3]: F_OB, F_AB = sp.symbols(r'F_{OB} F_{AB}')
```

```
# Upper section equations
```

```
eq7 = sp.Eq(F_AB*sp.cos(sp.pi/3), 800) # Sum in x
```

```

eq8 = sp.Eq(F_OB, F_AB*sp.sin(sp.pi/3))  # Sum in y

display(eq7)
display(eq8)
display(Markdown('---'))

sol = sp.solve([eq7, eq8])

for val in [F_OB, F_AB]:
    display(sp.Eq(val, sol[val].n()))

```

$$\frac{F_{AB}}{2} = 800$$

$$F_{OB} = \frac{\sqrt{3}F_{AB}}{2}$$

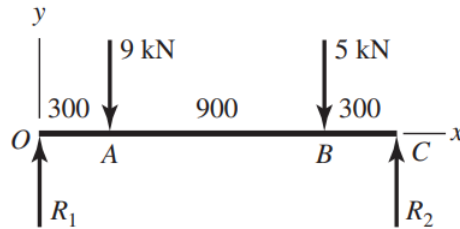
$$F_{OB} = 1385.6406460551$$

$$F_{AB} = 1600.0$$

Link 2 is in compression and link 3 is in tension.

2 Problem 5

2.1 Given



Problem 3–5

Dimensions in millimeters

2.2 Find

Find the reactions at the supports and plot the shear and bending moment diagrams.

2.3 Solution

Due to the large amount of practice, this problem is quite simple and can be plotted instantly without showing calculation.

```
[4]: O, A, B, C = 0, 300, 300 + 900, 300 + 900 + 300
V_0, V_A, V_B, V_C = 8.2, 8.2 - 9, 8.2 - 9 - 5, 0 # Values shortly after cross_
↳ the point
M_0, M_A, M_B, M_C = 0, V_0*300, V_0*300 + V_A*900, V_0*300 + V_A*900 + V_B*300

x = [O, A, A, B, B, C, C]
V = [V_0, V_0, V_A, V_A, V_B, V_B, V_C]
x_bending = [O, A, B, C]
M = [M_0, M_A, M_B, M_C]

fig, shear = plt.subplots()
bend = shear.twinx()

shear.plot(x, V, label='Shear')

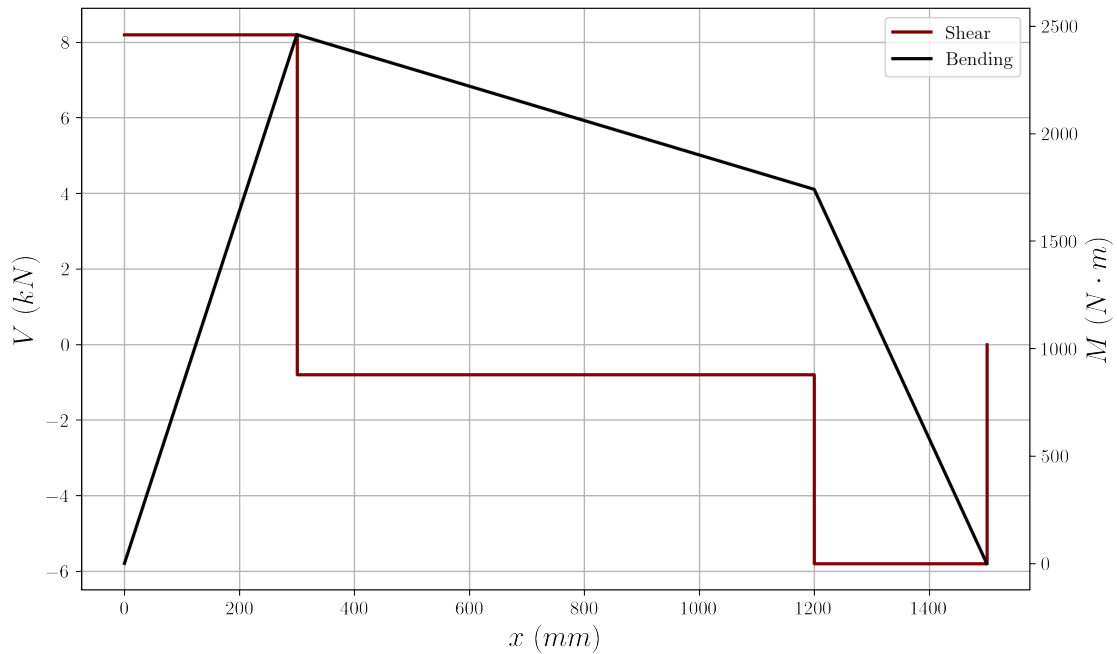
shear.set_xlabel('$x$ ($mm$)')
shear.set_ylabel('$V$ ($kN$)')

bend.plot(x_bending, M, label='Bending', color='black')
bend.set_ylabel(r'$M$ ($N\cdot m$)')
bend.grid(visible=False)
```



```
shear.legend(handles=[shear.lines[0], bend.lines[0]], labels=['Shear', 'Bending'])
```

```
plt.show()
```



Notice that the plot above has a dual y-axis.

The values at the points are (shortly after for the shear values),

- $V_O = 8.2 \text{ kN}$, $M_O = 0 \text{ N} \cdot \text{m}$
- $V_A = -0.8 \text{ kN}$, $M_A = 2460 \text{ N} \cdot \text{m}$
- $V_B = -5.8 \text{ kN}$, $M_B = 1740 \text{ N} \cdot \text{m}$
- $V_C = 0 \text{ kN}$, $M_C = 0 \text{ N} \cdot \text{m}$

3 Problem 15

3.1 Given

Plane stress values,

- a. $\sigma_x = 20 \text{ kpsi}$, $\sigma_y = -10 \text{ kpsi}$, and $\tau_{xy} = 8 \text{ kpsi cw}$
- b. $\sigma_x = -12 \text{ kpsi}$, $\sigma_y = 22 \text{ kpsi}$, and $\tau_{xy} = 12 \text{ kpsi cw}$

3.2 Find

Draw a Mohr's circle diagram. Find the principal normal and shear stresses, and determine the angle from the x-axis to σ_1 . Draw the stress elements as in Figure 3-11c and d.

3.3 Solution

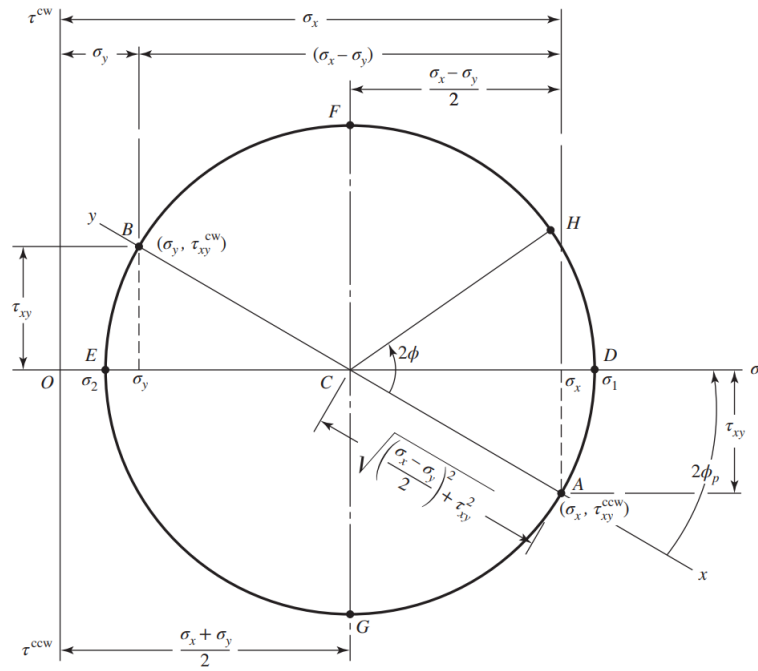
Mohr's circle is a circle that has the following characteristics:

- The x-axis is σ and the y-axis is τ .
- The center of the circle is along the x-axis at the average of σ_x and σ_y .
- The radius of the circle is $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$.

The principle stresses and max angles may be calculated using the following,

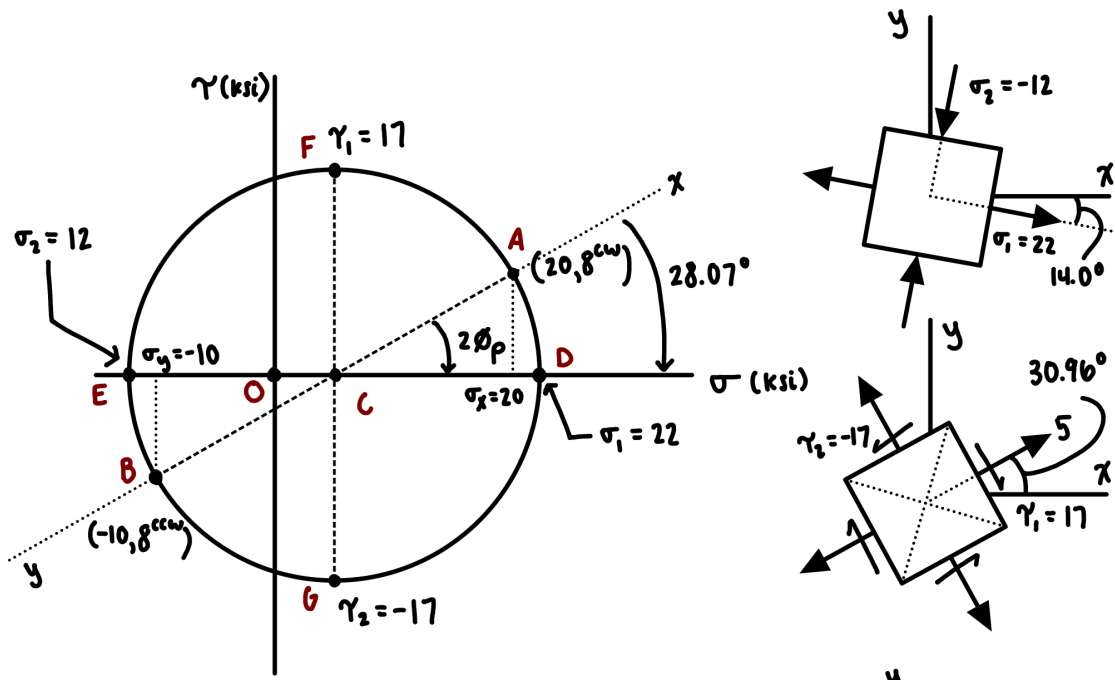
$$\begin{aligned}\sigma_1, \sigma_2 &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \tau_1, \tau_2 &= \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ \tan 2\phi_p &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \\ \tan 2\phi_s &= -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\end{aligned}$$

Here is a breakdown of Mohr's circle,



3.3.1 Part A

Using the information from the figure above,



Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 5$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 17.0$$

Principle Stresses:

$$\sigma_1 = C + R = 22.0$$

$$\sigma_2 = C - R = -12.0$$

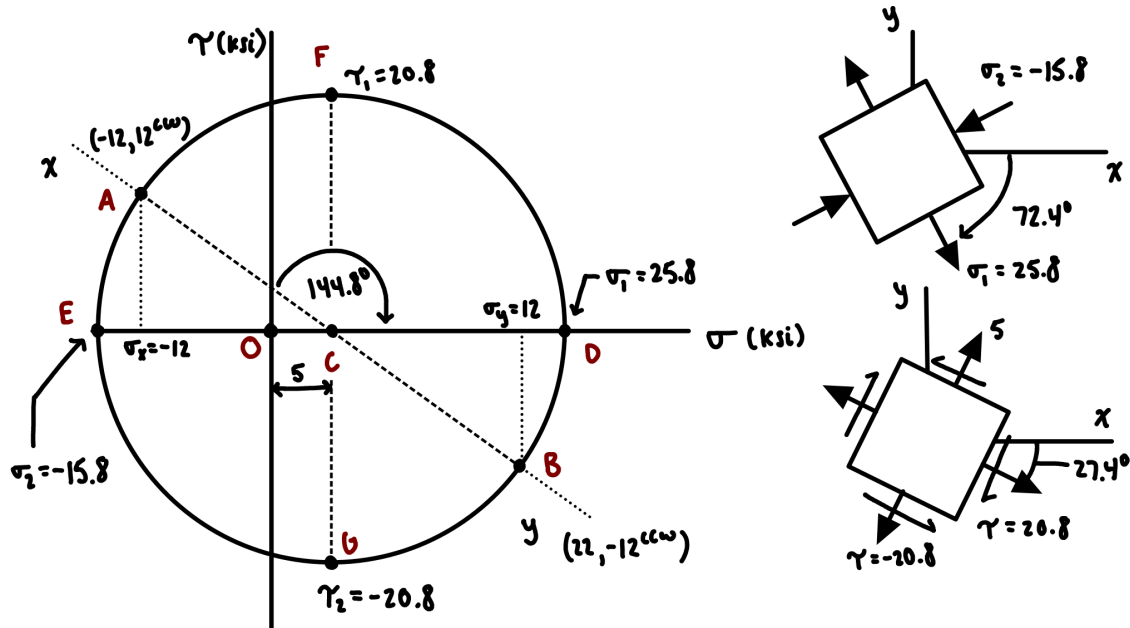
$$\tau_1 = R = 17.0$$

$$\tau_2 = -R = -17.0$$

Angle of Occurrence:

$$2\phi_p = \text{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 28.072486935853$$

3.3.2 Part D



Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 5.0$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 20.8086520466848$$

Principle Stresses:

$$\sigma_1 = C + R = 25.8086520466848$$

$$\sigma_2 = C - R = -15.8086520466848$$

$$\tau_1 = R = 20.8086520466848$$

$$\tau_2 = -R = -20.8086520466848$$

Angle of Occurrence:

$$2\phi_p = \text{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 144.782407031807$$

4 Problem 20

4.1 Given

Element stresses in *k*si:

- $\sigma_x = -6$
- $\sigma_y = 18$
- $\sigma_z = -12$
- $\tau_{xy} = 9$
- $\tau_{yz} = 6$
- $\tau_{zx} = -15$

4.2 Find

Determine the principal stresses and draw a complete Mohr's three-circle diagram, labeling all points of interest. Report the maximum shear stress for this case.

4.3 Solution

4.3.1 Principal Stresses and Max Shear Stress

The principal stresses may be found by finding the roots of the following expression,

$$\sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) - \sigma_x\sigma_y\sigma_z + \sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2 - 2\tau_{xy}\tau_{yz}\tau_{zx} = 0$$

```
[7]: sig_x, sig_y, sig_z, tau_xy, tau_zx, tau_yz = sp.symbols(r'\sigma_x \sigma_y \sigma_z \tau_{xy} \tau_{yz} \tau_{zx}')
sig = sp.Symbol(r'\sigma')
sig1, sig2, sig3 = sp.symbols(r'\sigma_1 \sigma_2 \sigma_3')

poly = sig**3 - (sig_x + sig_y + sig_z)*sig**2 + (sig_x*sig_y + sig_x*sig_z + sig_y*sig_z - tau_xy**2 - tau_yz**2 - tau_zx**2)*sig - (sig_x*sig_y*sig_z + 2*tau_xy*tau_yz*tau_zx - sig_x*tau_yz**2 - sig_y*tau_zx**2 - sig_z*tau_xy**2)

display(sp.Eq(poly.simplify(), 0))
poly_ = poly.subs([(sig_x, -6), (sig_y, 18), (sig_z, -12), (tau_xy, 9), (tau_yz, 6), (tau_zx, -15)])
roots = [sp.re(root.n()) for root in sp.roots(poly_, sig)]
roots_ = sorted(list(roots), reverse=True)
s1, s2, s3 = roots_
for i, j in zip((sig1, sig2, sig3), roots_):
    display(sp.Eq(i, j))
```

$$\sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) - \sigma_x\sigma_y\sigma_z + \sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2 - 2\tau_{xy}\tau_{yz}\tau_{zx} = 0$$

$$\sigma_1 = 21.0369191756785$$

$$\sigma_2 = 5.67061153118965$$

$$\sigma_3 = -26.7075307068682$$

The max shear stress is,

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

```
[8]: (s1 - s3)/2
```

```
[8]: 23.8722249412734
```

4.3.2 Von Mises Stress

The Von Mises Stress is,

```
[9]: (1/sp.sqrt(2)*sp.sqrt((s1 - s2)**2 + (s2 - s3)**2 + (s3 - s1)**2)).n()
```

```
[9]: 42.2137418384109
```

The Von Mises Stress may also be computed without using the principal stresses,

```
[10]: sig_x, sig_y, sig_z, tau_xy, tau_yz, tau_zx = -6, 18, -12, 9, 6, -15
      (1/sp.sqrt(2)*sp.sqrt((sig_x - sig_y)**2 + (sig_y - sig_z)**2 + (sig_z -
      ↪ sig_x)**2 + 6*(tau_xy**2 + tau_yz**2 + tau_zx**2))).n()
```

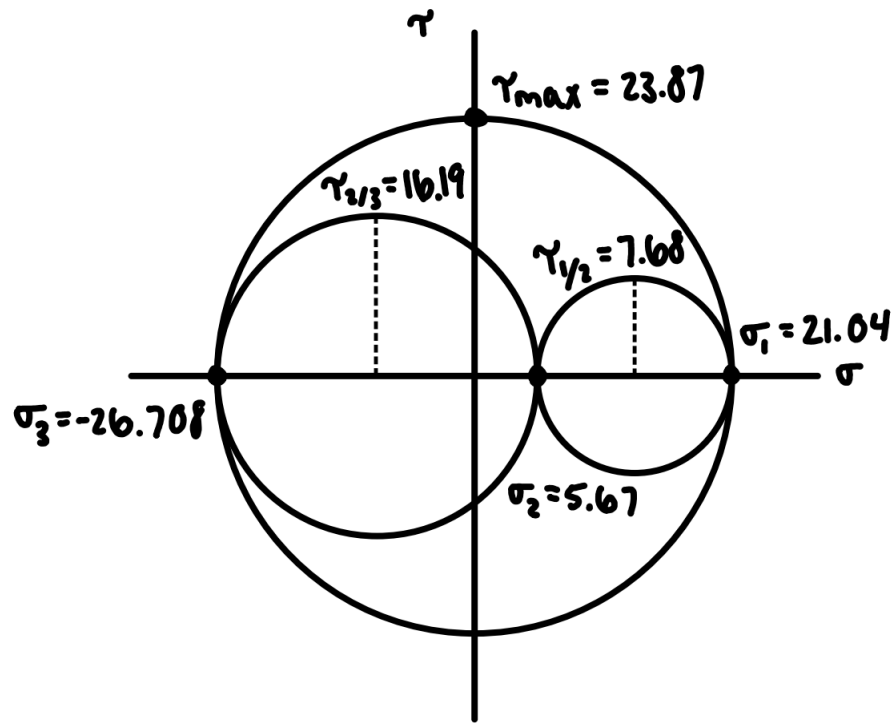
```
[10]: 42.2137418384109
```

4.3.3 Mohr's Circle Diagram

```
[11]: # Calculating additional shear stress values
      tau_1_2, tau_2_3 = sp.symbols(r'\tau_{1/2} \tau_{2/3}')
      display(sp.Eq(tau_1_2, (s1 - s2)/2))
      display(sp.Eq(tau_2_3, (s2 - s3)/2))
```

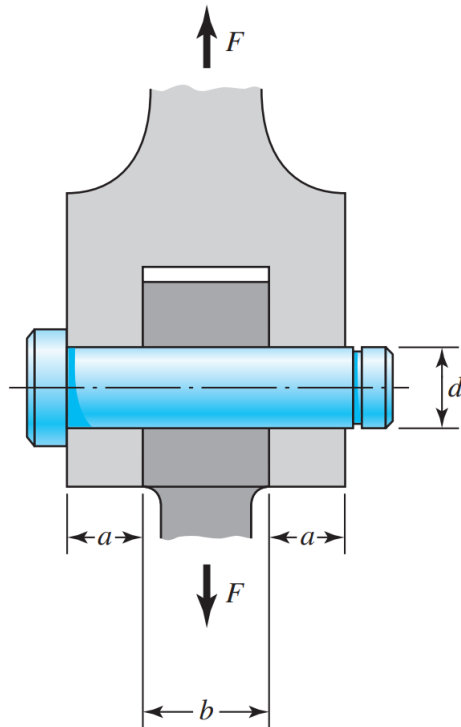
$$\tau_{1/2} = 7.68315382224444$$

$$\tau_{2/3} = 16.1890711190289$$



5 Problem 42

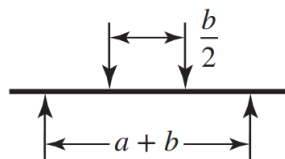
5.1 Given



$F = 4 \text{ kN}$, $a = 6 \text{ mm}$, $b = 18 \text{ mm}$, and $d = 12 \text{ mm}$

5.2 Find

Find the maximum bending and shear stress if modeled like so,



5.3 Solution

Start with a shear and moment diagram.

```
[12]: A, B, C, D = x = [0, 7.5, 16.5, 24]
x_shear = [A, B, B, C, C, D, D]
V = [2, 2, 0, 0, -2, -2, 0]
M = [0, 2*7.5, 2*7.5 - 0, 2*7.5 - 2*7.5]
```

```

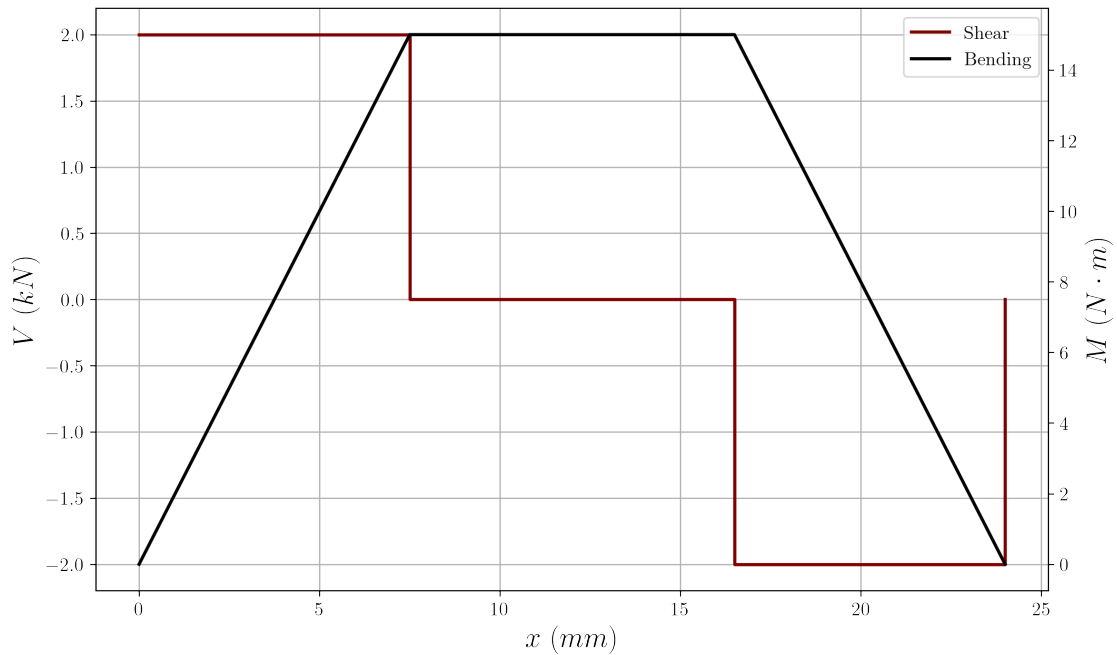
fig, ax1 = plt.subplots()
ax2 = ax1.twinx()

ax1.plot(x_shear, V, label='Shear')
ax1.set_xlabel('$x$ ($mm$)')
ax1.set_ylabel('$V$ ($kN$)')

ax2.plot(x, M, label='Bending', color='black')
ax2.set_ylabel(r'$M$ ($N\cdot m$)')
ax2.grid(visible=False)

ax1.legend(handles=[ax1.lines[0], ax2.lines[0]], labels=['Shear', 'Bending'])
plt.show()

```



The maximum bending moment is $15 \text{ N}\cdot\text{m}$, and the maximum shear stress is $\tau_{max} = 2000 \text{ N}$. The normal and shear stress due to bending is,

$$\sigma_{max} = \frac{Mc}{I}$$

$$I = \frac{1}{4}\pi r^4$$

and for shear, the maximum stress for circular cross-sections is,

$$\tau_{max} = \frac{4}{3} \frac{V}{A}$$

```
[13]: M, c = sp.S(15), sp.S(0.006)
      I = sp.S(0.25)*sp.pi*sp.S(0.006)**4
      sig_max = M*c/I
      sig_max.n()  # in Pa
```

```
[13]: 88419412.8288307
```

```
[14]: V, A = 2000, sp.pi*sp.S(0.006)**2
      tau_max = 4/3*V/A
      tau_max.n()  # in Pa
```

```
[14]: 23578510.0876882
```