Fatigue Homework 6

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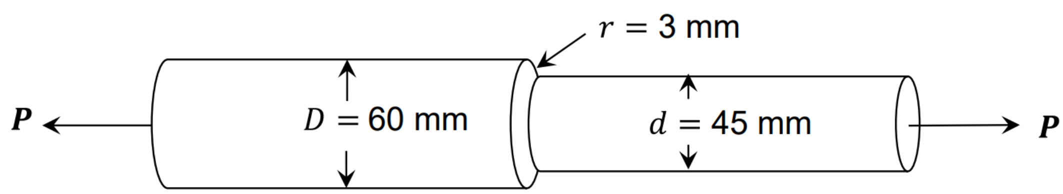
[1]:
```python
# Notebook Preamble
import matplotlib.pyplot as plt
import numpy as np

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1

1.1 Given

A stepped circular rod of 4340 steel (with $S_u = 1468$ MPa) with diameters of 60 and 45 mm has a root radius of 3 mm at the stepped section. The rod is to be subjected to axial cyclic loading.



The cyclic yield strength (S'_y) is estimated from:

$$S'_y = K'(0.002)^{n'}$$

where K' and n' are given in Table A.2

For the purpose of constructing Haigh diagram, exact value of σ_f is not needed, as the diagram is not very sensitive to its value. Since σ_f is not listed in Table A.2 for this material, we use Eq. 5.20 in the textbook to approximate it as

$$\sigma_f \approx S_u + 345\ (MPa)$$

1.2 Find

Using a Haigh diagram, determine the following for an approximate median fatigue life of 10^6 cycles:

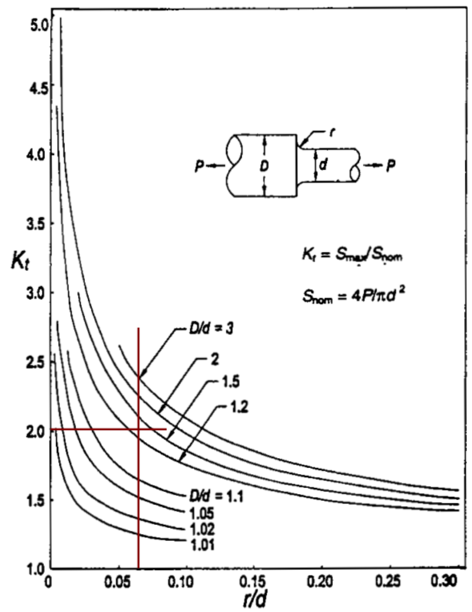
a. What fully reversed alternating force, P_a, can be applied?
b. What is the maximum value of P_a, if proper compressive residual stresses are present at the notch root? What is the magnitude of the compressive residual stress needed to obtain this maximum alternating stress?
c. What value of P_a can be applied if the residual stress calculated in (b) is tensile? What fully reversed alternating force, P_a, can be applied?

1.3 Solution

According to equation 4.3b, the endurance limit is 700 MPa for materials with an ultimate strength greater than 1400 MPa. With the size effect, the endurance limit becomes

$$S_f = 0.85(700) = 595\ MPa$$

1.3.1 Part A



[2]:
```
D, d, r = 60, 45, 3  # mm
Su = 1468  # MPa
size_effect = 0.85

if Su <= 1400:
    Sf = 0.5*Su*size_effect
else:
    Sf = 700*size_effect

D/d
```

[2]: 1.3333333333333333

[3]:
```
r/d
```

[3]: 0.06666666666666667

From above the stress concentration factor is $K_t = 2$. The fatigue notch factor for the fully reversed condition is

$$K_f = 1 + \frac{K_t - 1}{1 + a/r}$$

where $a = 0.0254\left(\frac{2070}{S_u}\right)^{1.8}$ with a in mm and S_u in MPa.

[4]:
```
Kt = 2
a = 0.0254*(2070/Su)**1.8
```

```
a  # mm
```

[4]: 0.047149103389883054

[5]:
```
Kf = 1 + (Kt - 1)/(1 + a/r)
Kf
```

[5]: 1.9845268144780213

Since the stress is maximized at the smaller diameter,

$$P_a = \frac{S_f}{K_f}\left(\frac{\pi}{4}\right)(d)^2$$

[6]:
```
Pa = Sf/Kf*np.pi/4*d**2
Pa  # N
```

[6]: 476842.4418454644

1.3.2 Part B

From Table A.2,

Property	Value
S_y	1371 MPa
S_y'	863 MPa
S_f	595 MPa
σ_f	1813 MPa
K_f	1.98
S_{cat}	70 MPa

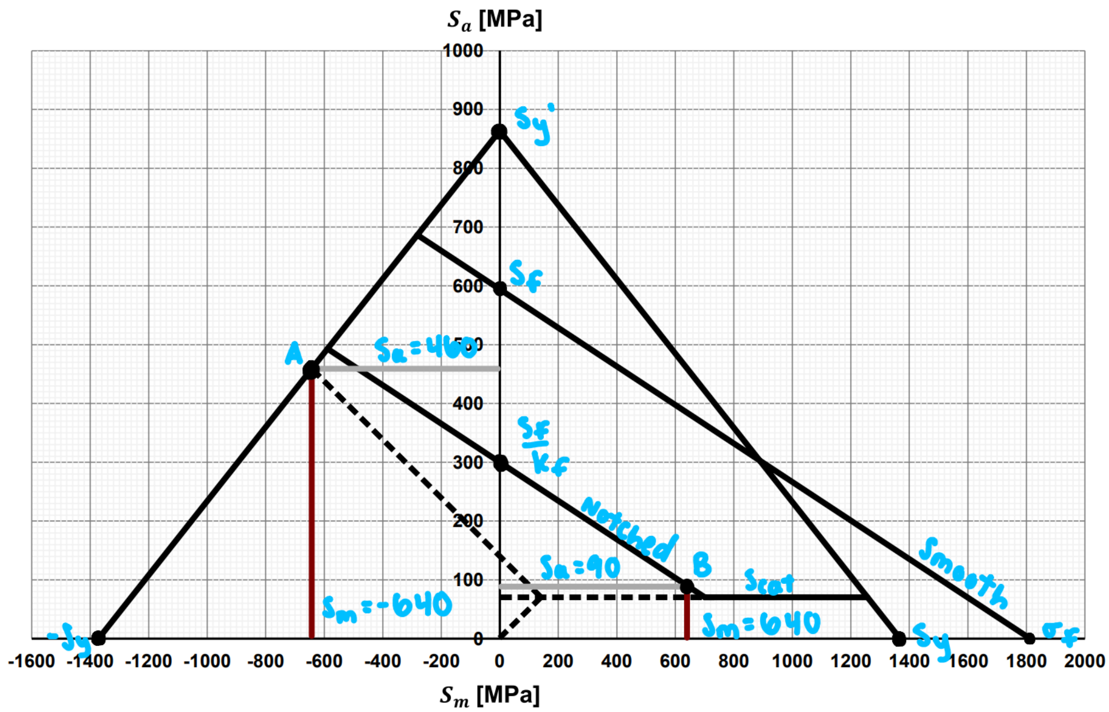
S_{cat} comes from the fact that this is a hard steel. The other calculations are shown below.

[7]:
```
sig_f = Su + 345
sig_f  # MPa
```

[7]: 1813

[8]:
```
K_prime, n_prime = 1996, 0.135
Sy_prime = K_prime*0.002**n_prime
Sy_prime  # MPa
```

[8]: 862.5804014077875




```
[9]: Sa = 460   # MPa
     Pa = Sa*np.pi/4*d**2
     Pa  # N
```

[9]: 731598.3892047232

1.3.3 Part C

From above, point B is when the mean stress is tensile.

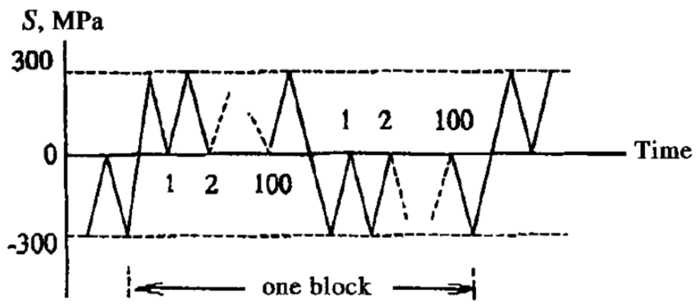
```
[10]: Sa = 90
      Pa = Sa*np.pi/4*d**2
      Pa  # N
```

[10]: 143138.81527918496

2

2.1 Given

An axially loaded member made of 2024-T3 aluminum is repeatedly subjected to the block of stress history shown below.



2.2 Find

a. Complete a summary of the loading block.
b. Using the Basquin equation, $S_{Nf} = \sigma'_f(2N_f)^b$, determine the fatigue strength S_{Nf}, the fatigue life N_f and the damage ratio n/N_f for each load segment and estimate the expected life if the member is smooth.
c. Estimate the expected life if the member has a notch with $K_t = 2$ and the notch root radius is 1 mm. For the notched member, assume that the given nominal stress block and K_t are based on net stress.

2.3 Solution

2.3.1 Part A

Load Segment	S_{min} (MPa)	S_{max} (MPa)	S_a (MPa)	S_m (MPa)	n
1	0	300	150	150	100
2	-300	300	300	0	1
3	-300	0	150	-150	100

2.3.2 Part B

Use the modified Goodman to find S_{Nf} with $S_u = 469$ MPa.

$$\frac{S_a}{S_{Nf}} + \frac{S_m}{S_u} = 1 \rightarrow S_{Nf} = \frac{S_a S_u}{S_u - S_m}$$

$$S_{Nf} = \sigma'_f(2N_f)^b \rightarrow N_f = \frac{1}{2}\left(\frac{S_{Nf}}{\sigma'_f}\right)^{1/b}$$

[11]:
```python
Su, sig_prime, b = 469, 1100, -0.124

SNf_lamb = lambda Sa_, Su_, Sm_: Sa_*Su_/(Su_ - Sm_)
Nf_lamb = lambda SNf_, sig_prime_, b_: 0.5*(SNf_/sig_prime_)**(1/b_)

# Load 1
SNf1 = SNf_lamb(150, Su, 150)
SNf1  # MPa
```

[11]: 220.53291536050156

[12]:
```python
Nf1 = Nf_lamb(SNf1, sig_prime, b)
Nf1
```

[12]: 212496.20843121517

[13]:
```python
d1 = 100/Nf1
d1
```

[13]: 0.0004705966319976477

[14]:
```python
# Load 2
SNf2 = SNf_lamb(300, Su, 0)
SNf2  # MPa
```

[14]: 300.0

[15]:
```python
Nf2 = Nf_lamb(SNf2, sig_prime, b)
Nf2
```

[15]: 17764.216450750755

[16]:
```python
d2 = 1/Nf2
d2
```

[16]: 5.629294164324021e-05

[17]:
```python
# Load 3
SNf3 = SNf_lamb(150, Su, -150)
SNf3  # MPa
```

[17]: 113.65105008077545

[18]:
```python
Nf3 = Nf_lamb(SNf3, sig_prime, b)
Nf3
```

[18]: 44578464.41972726

[19]:
```
d3 = 100/Nf3
d3
```

[19]: 2.24323563634792e-06

Now the expected life is calculated as the reciprocal of the summation of the damage ratios.

[20]:
```
1/sum([d1, d2, d3])  # Blocks
```

[20]: 1889.8846990152454

Load Segment	S_{Nf} (MPa)	N_f	n	n/N_f
1	221	212496	100	$4.706 \cdot 10^{-4}$
2	300	17764	1	$5.630 \cdot 10^{-5}$
3	114	44,578,464	100	$2.243 \cdot 10^{-6}$
Total	-	-	-	1890

2.3.3 Part C

Everything remains the same, but now the endurance limit is changed to S_f/K_f, and a new b is found by assuming that the endurance limit is reached at 10^6 cycles.

[21]:
```
a = 0.0254*(2070/Su)**1.8
a  # mm
```

[21]: 0.3676793350247542

[22]:
```
Kt, r = 2, 1
Kf = 1 + (Kt - 1)/(1 + a/r)
Kf
```

[22]: 1.7311655403361788

[23]:
```
Sf = sig_prime*2e6**b
Sf  # MPa
```

[23]: 181.9973086280446

[24]:
```
Sf_new = Sf/Kf
B = np.log(Sf_new/sig_prime)/np.log(2e6)
B
```

[24]: -0.16182533948270703

The S_{Nf} values stay the same, but the N_f values change.

[25]:
```
Nf1 = Nf_lamb(SNf1, sig_prime, B)
Nf1
```

[25]: 10274.56557852558

[26]:
```
d1 = 100/Nf1
d1
```

[26]: 0.009732771593672595

[27]:
```
Nf2 = Nf_lamb(SNf2, sig_prime, B)
Nf2
```

[27]: 1534.2156409133563

[28]:
```
d2 = 1/Nf2
d2
```

[28]: 0.0006517988562577001

[29]:
```
Nf3 = Nf_lamb(SNf3, sig_prime, B)
Nf3
```

[29]: 617791.4135432595

[30]:
```
d3 = 100/Nf3
d3
```

[30]: 0.00016186693082453747

[31]:
```
1/sum([d1, d2, d3])  # Blocks
```

[31]: 94.81874910904062

Load Segment	S_{Nf} (MPa)	N_f	n	n/N_f
1	221	10275	100	$9.732 \cdot 10^{-3}$
2	300	1534	1	$6.518 \cdot 10^{-4}$
3	114	617791	100	$1.619 \cdot 10^{-4}$
Total	-	-	-	95