Smart Material Chapter 4 Solutions

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[1]:
```
# toc
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

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Problem 4.1

Compute the stress required to produce 100 microstrain in APC 856 when the applied electric field is held constant at zero Compute the stress required to produce 100 microstrain when the electric displacement is held equal to zero.

Property	Unit	Symbol	APC 856	PZT-5H	PVDF
Relative dielectric constant	unitless	ε_r	4100	3800	12–13
Curie temperature	°C	T_c	150	250	
Coupling coefficient	unitless	k_{33}	0.73	0.75	
		k_{31}	0.36		0.12
		k_{15}	0.65		
Strain coefficient	10^{-12} C/N or m/V	d_{33}	620	650	−33
		$-d_{31}$	260	320	−23
		d_{15}	710		
Elastic compliance	10^{-12} m^2/N	s_{11}^E	15	16.1	250–500
		s_{33}^E	17	20	
Density	g/cm^3	ρ	7.5	7.8	1.78

Solution

We can use the constitutive equations.

$$S = sT + dE$$

$$D = dT + \epsilon E$$

[2]:
```
S = 100e-6
s = 17e-12  # m^2/N
T = S/s
T  # Pa
```

[2]: 5882352.941176471

For finding the open circuit stress, we need the open circuit compliance.

$$s^D = s^E(1 - k^2)$$

[3]:
```
SD = s*(1 - 0.73**2)
T = S/SD
T  # Pa
```

[3]: 12593348.193484202

Problem 4.2

A new composition of piezoelectric material is found to have a compliance at zero electric field of $18.2\mu m^2/N$, a piezoelectric strain coefficient of $330pm/V$, and a relative permittivity of 1500.

a. Write the one-dimensional constitutive relationship for the material with strain and electric displacement as the dependent variables.
b. Write the one-dimensional constitutive relationship with stress and electric field as the dependent variables.

Solution

Part A

[4]:
```
S, D, s, d, eps, T, E = sp.symbols(r'S D s d \epsilon T E')
A = sp.Matrix([[s, d], [d, eps]])
x = sp.Matrix([T, E])
b = sp.Matrix([S, D])
eq = sp.Eq(b, sp.MatMul(A, x))
eq
```

[4]:
$$\begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} s & d \\ d & \epsilon \end{bmatrix} \begin{bmatrix} T \\ E \end{bmatrix}$$

$\epsilon = \epsilon_r \cdot \epsilon_o$

[5]:
```
subs = {s: sp.S('18.2e-6'), d: sp.S('330e-12'), eps: 1500*sp.S('8.854e-12')}
eq.subs(subs)
```

[5]:
$$\begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} 1.82 \cdot 10^{-5} & 3.3 \cdot 10^{-10} \\ 3.3 \cdot 10^{-10} & 1.3281 \cdot 10^{-8} \end{bmatrix} \begin{bmatrix} T \\ E \end{bmatrix}$$

Part B

[6]:
```
eq = sp.Eq(x, sp.MatMul(A.inv(), b))
eq
```

[6]:
$$\begin{bmatrix} T \\ E \end{bmatrix} = \begin{bmatrix} \frac{\epsilon}{\epsilon s - d^2} & -\frac{d}{\epsilon s - d^2} \\ -\frac{d}{\epsilon s - d^2} & \frac{s}{\epsilon s - d^2} \end{bmatrix} \begin{bmatrix} S \\ D \end{bmatrix}$$

[7]:
```
eq.subs(subs)
```

[7]:
$$\begin{bmatrix} T \\ E \end{bmatrix} = \begin{bmatrix} 54945.0796995757 & -1365.24932616971 \\ -1365.24932616971 & 75295568.8978449 \end{bmatrix} \begin{bmatrix} S \\ D \end{bmatrix}$$

Problem 4.4

The short-circuit mechanical compliance of a piezoelectric material has been measured to be 20 $\mu m^2/N$ and the open-circuit mechanical compliance has been measured to be 16.2 $\mu m^2/N$. If the stress-free relative permittivity is equal to 2800, compute the relative permittivity of the material when the strain is constrained to be zero.

Solution

We can use the equation for the open circuit compliance to find the piezoelectric coupling coefficient, and apply that to the no strain permittivity relationship.

$$s^D = s^E(1 - k^2)$$
$$\epsilon^S = \epsilon^T(1 - k^2)$$

[8]:
```
sD, sE, epsT = 16.2e-6, 20e-6, 2800
k = np.sqrt(1 - sD/sE)
k
```

[8]: 0.43588989435406744

[9]:
```
epsS = epsT*(1 - k**2)
epsS
```

[9]: 2268.0