Machine Design Test 1

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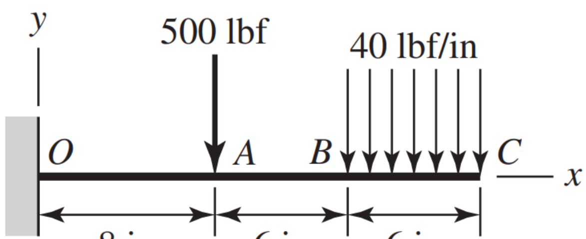
[1]:
```python
# Notebook Preamble
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Markdown

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1 Problem 3-6

1.1 Given

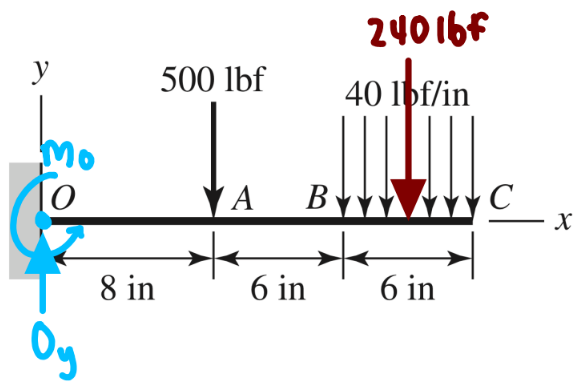


1.2 Find

Find the reaction forces and plot the shear and bending diagram.

1.3 Solution

1.3.1 Reaction Forces

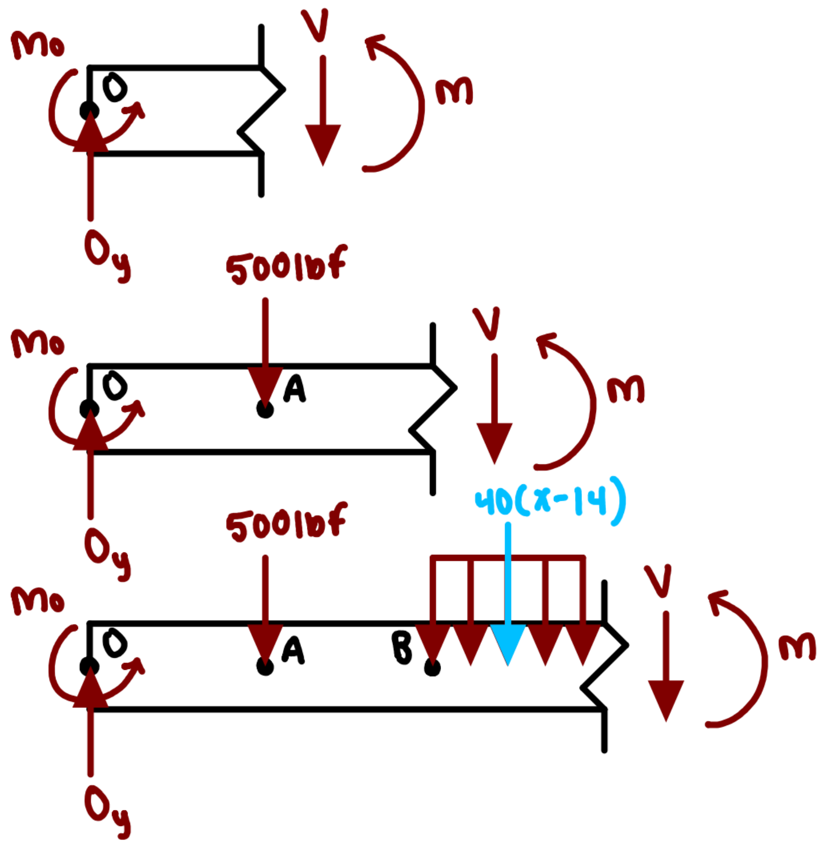


[2]:
```
# Getting the reaction forces
Oy_sym, Mo_sym = sp.symbols('O_y M_o')
Oy = 240 + 500
Mo = 500*8 + 240*17
display(sp.Eq(Oy_sym, Oy), sp.Eq(Mo_sym, Mo))  # lbf and lbf*in
```

$O_y = 740$

$M_o = 8080$

1.3.2 Bending and Moment Diagram



The equation may be described as the piecewise relationship coded below.

[3]:
```python
V, M, x = sp.symbols('V M x')

# From O to A
V1 = Oy
M1 = -Mo + Oy*x

# From A to B
V2 = Oy - 500
M2 = -Mo + Oy*x - 500*(x - 8)

# From B to C
V3 = Oy - 500 - 40*(x - 14)
M3 = -Mo + Oy*x - 500*(x - 8) - 40*(x - 14)*(x - 14)/2

eq1 = sp.Eq(V, sp.Piecewise((V1, (x >= 0) & (x < 8)), (V2, (x >= 8) & (x < 14)), (V3, (x >= 14) & (x <= 20))))
eq2 = sp.Eq(M, sp.Piecewise((M1, (x >= 0) & (x < 8)), (M2, (x >= 8) & (x < 14)), (M3, (x >= 14) & (x <= 20))))
```

```python
display(eq1, eq2)
```

$$V = \begin{cases} 740 & \text{for } x \geq 0 \wedge x < 8 \\ 240 & \text{for } x \geq 8 \wedge x < 14 \\ 800 - 40x & \text{for } x \geq 14 \wedge x \leq 20 \end{cases}$$

$$M = \begin{cases} 740x - 8080 & \text{for } x \geq 0 \wedge x < 8 \\ 240x - 4080 & \text{for } x \geq 8 \wedge x < 14 \\ 240x - \frac{(x-14)(40x-560)}{2} - 4080 & \text{for } x \geq 14 \wedge x \leq 20 \end{cases}$$

The important key points for shear are shown in the piecewise function expression above. The key points for the bending moment are,

[4]:
```python
points = ['O', 'A', 'B', 'C']
values = [0, 8, 14, 20]
for p, v in zip(points, values):
    display(sp.Eq(sp.Symbol(f'M_{p}'), eq2.rhs.subs(x, v)))  # in lbf*in
```

$M_O = -8080$

$M_A = -2160$

$M_B = -720$

$M_C = 0$

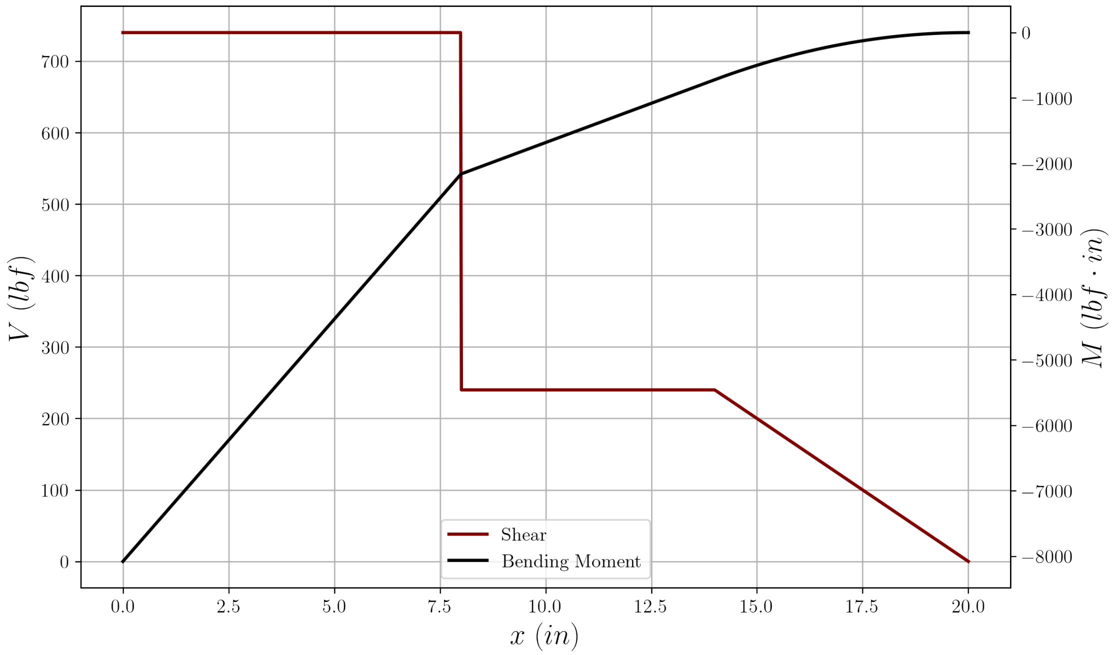
[5]:
```python
# Getting shear and bending diagram
x_ = np.linspace(0, 20, 1000)
V_ = sp.lambdify(x, eq1.rhs, modules='numpy')
M_ = sp.lambdify(x, eq2.rhs, modules='numpy')

fig, ax = plt.subplots()
ax2 = ax.twinx()

ax.plot(x_, V_(x_), label='Shear')
ax2.plot(x_, M_(x_), label='Bending Moment', color='black')

ax2.grid(visible=False)
ax.legend(handles=[ax.lines[0], ax2.lines[0]], loc='lower center')

ax.set_xlabel('$x$ ($in$)')
ax.set_ylabel('$V$ ($lbf$)')
ax2.set_ylabel(r'$M$ ($lbf\cdot in$)')
plt.show()
```



Notice that the graph has a duel y-axis.

2 Problem 3-17

2.1 Given

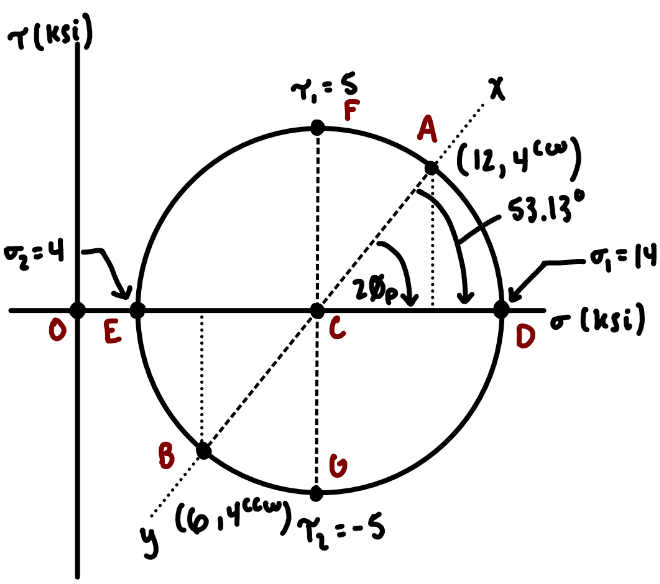
a. $\sigma_x = 12\ ksi$, $\sigma_y = 6\ ksi$, $\tau_{xy} = 4\ ksi\ cw$
b. $\sigma_x = 9\ ksi$, $\sigma_y = 19\ ksi$, $\tau_{xy} = 8\ ksi\ cw$

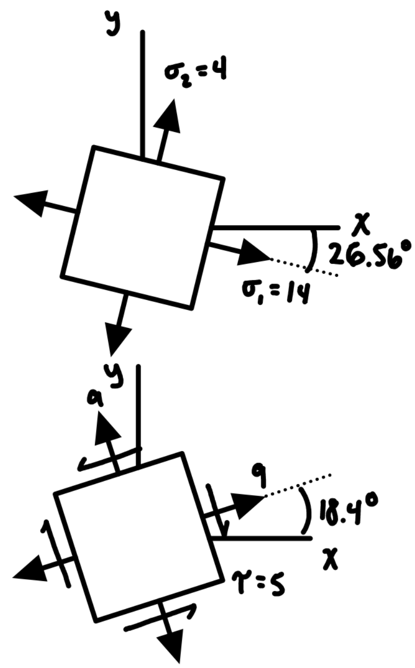
2.2 Find

Draw the plane stress element as seen in Figure 3-11c and d. Also draw Mohr's circle fully labeled.

2.3 Solution

2.3.1 Part A





Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 9.0$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 5.0$$

Principle Stresses:

$$\sigma_1 = C + R = 14.0$$

$$\sigma_2 = C - R = 4.0$$

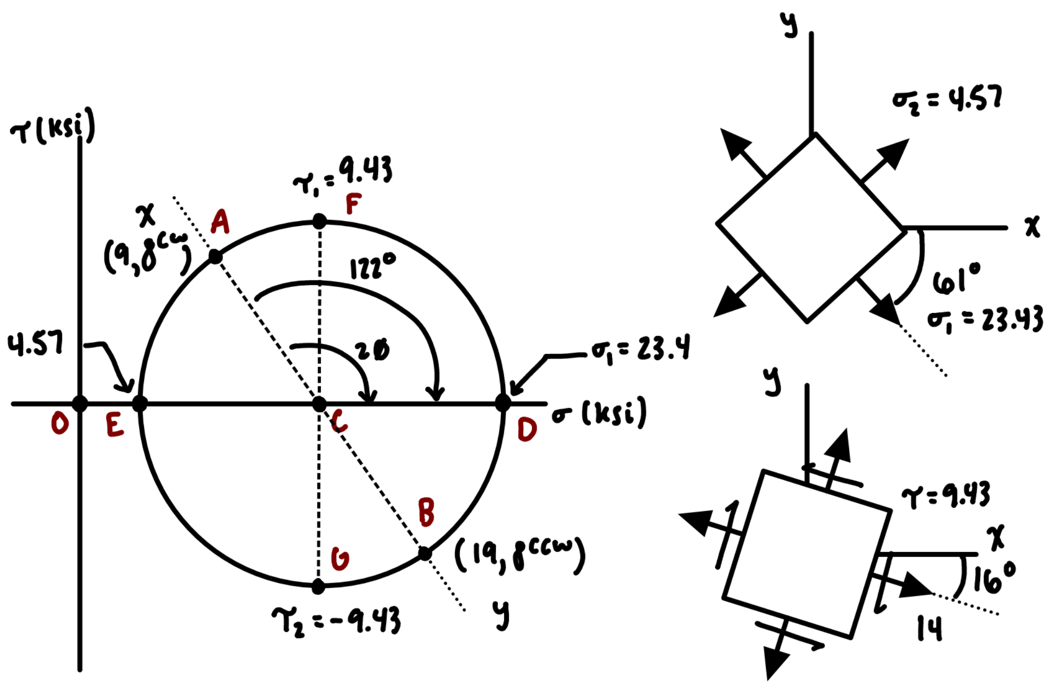
$$\tau_1 = R = 5.0$$

$\tau_2 = -R = -5.0$

Angle of Occurrence:

$$2\phi_p = \operatorname{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 53.130102354156$$

2.3.2 Part D



Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 14.0$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 9.4339811320566$$

Principle Stresses:

$\sigma_1 = C + R = 23.4339811320566$

$\sigma_2 = C - R = 4.5660188679434$

$\tau_1 = R = 9.4339811320566$

$\tau_2 = -R = -9.4339811320566$

Angle of Occurrence:

$$2\phi_p = \operatorname{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 122.005383208084$$