

Vibrations and Controls Homework 6

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```
[1]: import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Latex

plt.style.use('maroon.mplstyle')

s, t = sp.symbols('s t')

display_latex = lambda text: display(Latex(text))
```

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1 Problem 9.11 Part A

1.1 Given

$$T(s) = \frac{Y(s)}{F(s)} = \frac{5}{(5s+1)(2s+1)}$$
$$f(t) = 10 \sin(0.2t)$$

1.2 Find

The steady state response $y_{ss}(t)$

1.3 Solution

$$y_{ss}(t) = |T(j\omega)|A \sin(\omega t + \angle T(j\omega))$$

```
[2]: T_s = 5/((5*s + 1)*(2*s + 1))  
T_s
```

```
[2]: 5  
(2s + 1)(5s + 1)
```

```
[3]: T_jw = T_s.subs(s, sp.I*0.2)  
T_jw
```

```
[3]: 2.1551724137931(1 - 1.0i)(1 - 0.4i)
```

```
[4]: M = sp.Abs(T_jw)  
phi = sp.arg(T_jw)  
  
M.n() # Magnitude
```

```
[4]: 3.28266082149306
```

```
[5]: phi # The angle
```

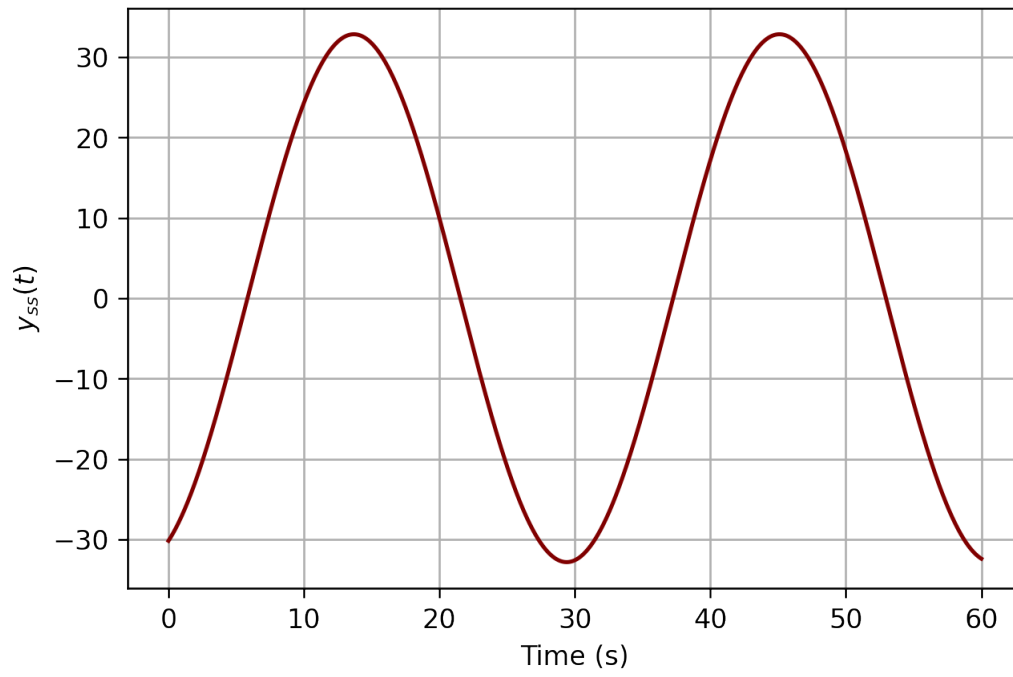
```
[5]: -1.16590454050981
```

```
[6]: y_ss = 10*M*sp.sin(0.2*t + phi)  
y_ss
```

```
[6]: 32.8266082149306 sin(0.2t - 1.16590454050981)
```

```
[7]: # Plotting it  
y_ss_lamb = sp.lambdify(t, y_ss)  
time = np.linspace(0, 60, 1000)  
  
plt.plot(time, y_ss_lamb(time))  
plt.xlabel('Time (s)')  
plt.ylabel('$y_{ss}(t)$')
```

```
plt.show()
```



2 Problem 9.11 Part A

2.1 Given

$$T(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^2 + 10s + 100}$$
$$f(t) = 16 \sin(5t)$$

2.2 Find

The steady state response $y_{ss}(t)$

2.3 Solution

$$y_{ss}(t) = |T(j\omega)|A \sin(\omega t + \angle T(j\omega))$$

```
[8]: T_s = 1/(s**2 + 10*s + 100)
     T_s
```

```
[8]: 1
     s^2 + 10s + 100
```

```
[9]: T_jw = T_s.subs(s, sp.I*5)
     T_jw
```

```
[9]: 75 - 50i
     8125
```

```
[10]: M = sp.Abs(T_jw)
      phi = sp.arg(T_jw)

      M.n() # The magnitude
```

```
[10]: 0.0110940039245046
```

```
[11]: phi.n() # The angle
```

```
[11]: -0.588002603547568
```

```
[12]: y_ss = 16*M.n()*sp.sin(5*t + phi.n())
      y_ss
```

```
[12]: 0.177504062792073 sin(5t - 0.588002603547568)
```

```
[13]: y_ss_lamb = sp.lambdify(t, y_ss)
      time = np.linspace(0, 5, 1000)

      plt.plot(time, y_ss_lamb(time))
      plt.xlabel('Time (s)')
      plt.ylabel('$y_{ss}(t)$')
```

```
plt.show()
```

