Vibrations and Controls Homework 5

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[1]:
```
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display, Latex

plt.style.use('maroon.mplstyle')

s, t = sp.symbols('s t')

display_latex = lambda text: display(Latex(text))
```

Contents

1 Problem 9.5 Part A

1.1 Given

$T(s) = \frac{Y(s)}{F(s)} = \frac{75}{14s+18}$
$f(t) = 10\sin(1.5t)$

1.2 Find

The steady state response $y_{ss}(t)$

1.3 Solution

$y_{ss}(t) = 10|T(j\omega)|\sin(1.5t + \angle T(j\omega))$

[2]:
```
T_s = 75/(14*s + 18)
T_s
```

[2]:

$$\frac{75}{14s + 18}$$

[3]:
```
T_jw = T_s.subs(s, sp.I*1.5)
T_jw
```

[3]:

$$0.0980392156862745\,(18 - 21.0i)$$

[4]:
```
mag = sp.Abs(T_jw)
mag
```

[4]:

$$2.7116307227332$$

[5]:
```
angle = sp.arg(T_jw)
angle
```

[5]:

$$-0.862170054667226$$

[6]:
```
y_t = 10*mag*sp.sin(1.5*t + angle)
y_t
```

[6]:

$$27.116307227332\sin(1.5t - 0.862170054667226)$$

1.4 Answer

$y_{ss}(t) = 27.1\sin(1.5t - 0.862)$

2 Problem 9.5 Part C

2.1 Given

$T(s) = \frac{Y(s)}{F(s)} = \frac{s+50}{s+150}$
$f(t) = 15\sin(100t)$

2.2 Find

The steady state response $y_{ss}(t)$

2.3 Solution

Do the same procedure as above. No need for tables in the book. Python makes this very easy.

[7]:
```
T_s = (s + 50)/(s + 150)
T_s
```

[7]:
$$\frac{s+50}{s+150}$$

[8]:
```
T_jw = T_s.subs(s, sp.I*100)
T_jw
```

[8]:
$$\frac{(50+100i)(150-100i)}{32500}$$

[9]:
```
mag = sp.Abs(T_jw)
mag.n()
```

[9]: 0.620173672946042

[10]:
```
angle = sp.arg(T_jw)
angle.n()
```

[10]: 0.519146114246523

[11]:
```
y_t = 15*mag*sp.sin(100*t + angle.n())
y_t.n()
```

[11]: $9.30260509419063\sin(100t + 0.519146114246523)$

2.4 Answer

$y_{ss}(t) = 9.3\sin(100t + 0.519)$