

# Machine Design Homework 2

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```
[1]: # Notebook Preamble
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Markdown

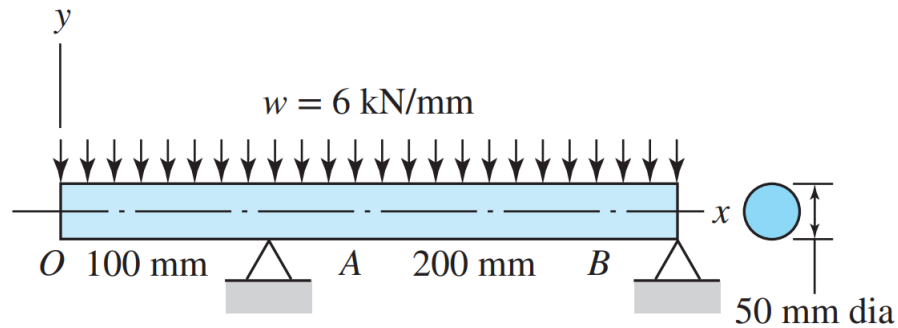
plt.style.use('maroon_ipynb.mplstyle')
```

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## 1 Problem 3-39

### 1.1 Given

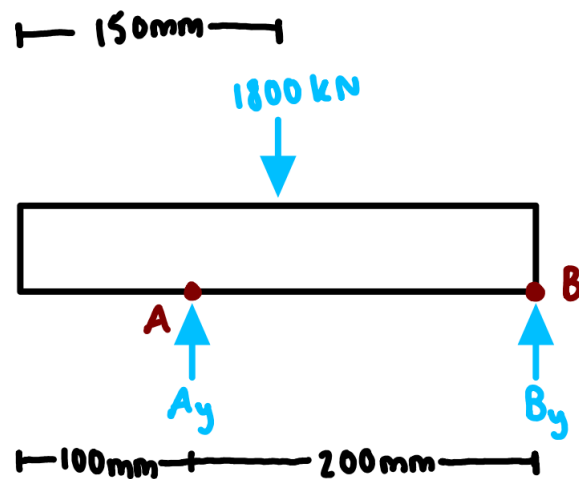


### 1.2 Find

For the beam above, find the maximum tensile stress due to  $M$  and the maximum shear stress due to  $V$ .

### 1.3 Solution

The free body diagram is,



```
[2]: # Getting reaction forces
Ay, By = sp.symbols('A_y B_y')
eq1 = sp.Eq(Ay + By, 1800)
eq2 = sp.Eq(200*Ay, 150*1800)

sol = sp.solve([eq1, eq2], dict=True)[0]
[display(eq) for eq in [eq1, eq2]]
```

```
display(Markdown('---'))

for key, value in sol.items():
    display(sp.Eq(key, value))
```

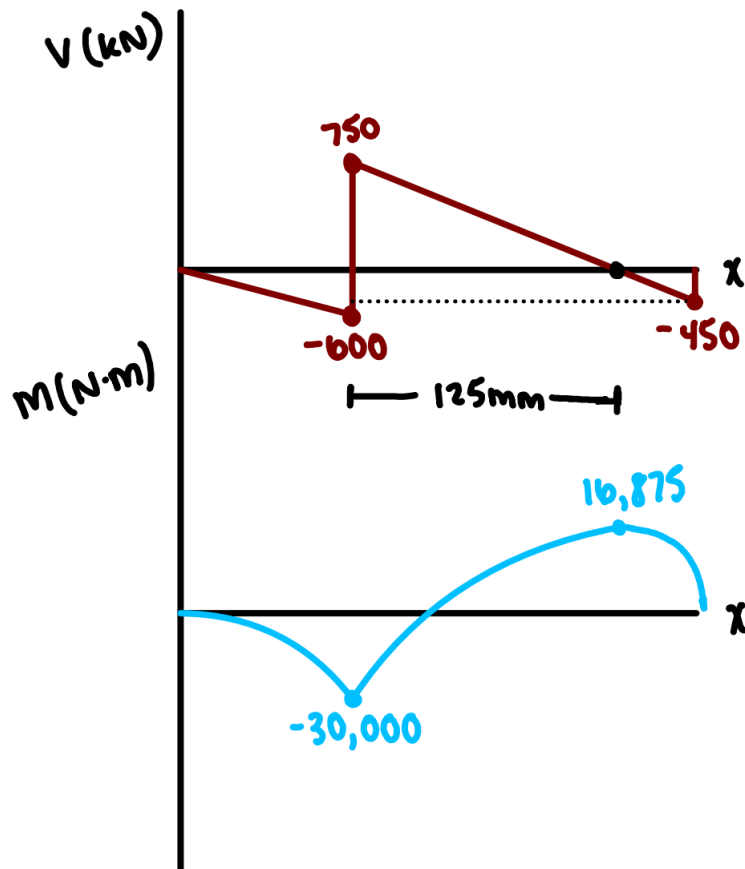
$$A_y + B_y = 1800$$

$$200A_y = 270000$$

$$A_y = 1350$$

$$B_y = 450$$

The shear and moment diagram is,



The maximum shear and tensile stress occur at  $x = 100 \text{ mm}$ .

```
[3]: # Calculating stress due to bending
M, c = 30_000, sp.S(0.025)
(M*c/(sp.pi/4*c**4)).n() # in Pa
```

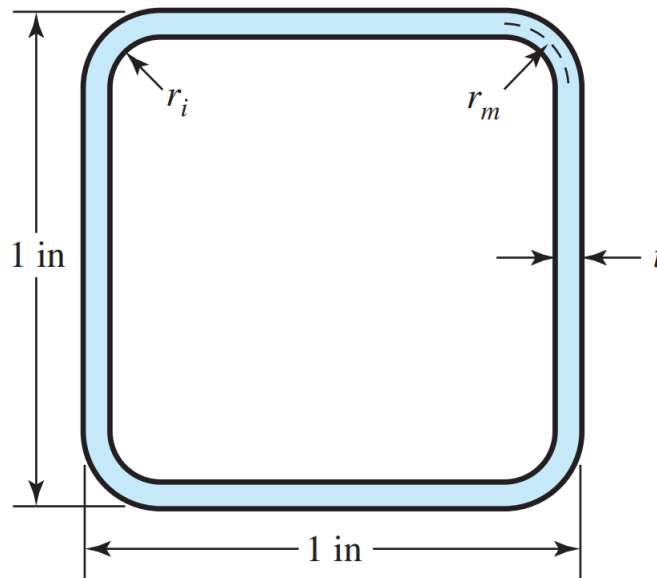
```
[3]: 2444619925.89151
```

```
[4]: # Calculating the maximum shear stress
V = 750_000
(sp.Rational(4, 3)*V/(sp.pi*c**2)).n() # in Pa
```

```
[4]: 509295817.894065
```

## 2 Problem 3-62 Part A

### 2.1 Given



The tube is 36 in long and  $r_i = r_m = 0$ . The thickness  $t$  is  $\frac{1}{16}$  in.

### 2.2 Find

The maximum torque that can be applied and the corresponding angle of twist of the tube.

### 2.3 Solution

For thin-walled tubes,

$$\tau = \frac{T}{2A_m t}$$
$$\theta_1 = \frac{TL_m}{4GA_m^2 t}$$

See p. 129 for additional details of the above formulas.

```
[5]: # Calculating the maximum torque
tau_max = 12_000 # lbf in
t = sp.Rational(1, 16) # thickness in inches
Am = (1 - t)**2
T = tau_max*2*Am*t
T.n() # lbf in
```

[5]: 1318.359375

From table A-5, the modulus of rigidity is 11.5 *Mpsi*.

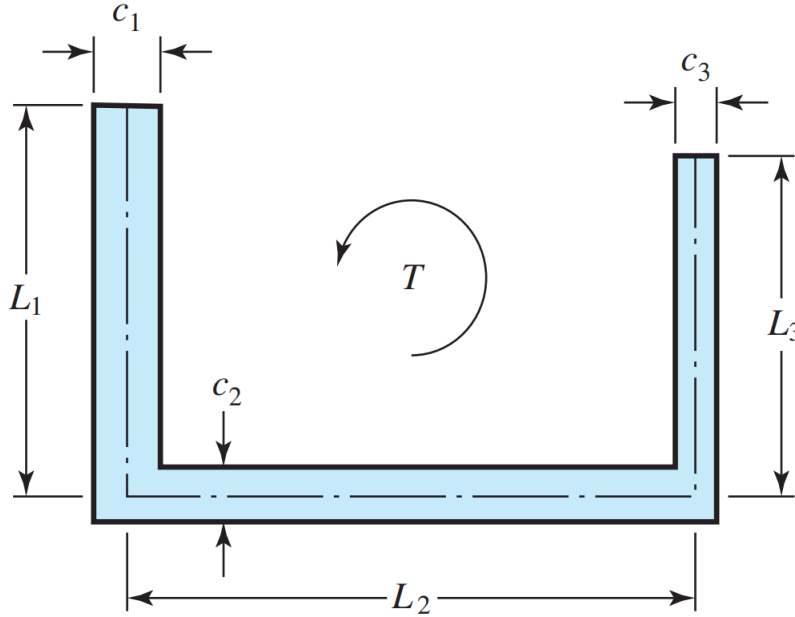
```
[6]: G = 11.5e6
      Lm = (1 - t)*4 # total length
      L = 36 # inches
      phi_1 = T*Lm/(4*G*Am**2*t)*L
      (phi_1*180/sp.pi).n() # in degrees
```

[6]: 4.59163394776145

The expression gets multiplied by  $L$  because  $\theta_1$  is the angle of twist per unit length.

### 3 Problem 3-64

#### 3.1 Given



$c_1 = 2 \text{ mm}$ ,  $L_1 = 20 \text{ mm}$ ,  $c_2 = 3 \text{ mm}$ ,  $L_2 = 30 \text{ mm}$ ,  $c_3 = 0 \text{ mm}$ , and  $L_3 = 0 \text{ mm}$ . The material is steel and the maximum shear is  $\tau_{allow} = 12 \text{ ksi}$ . The angle of twist is the same for each section.

#### 3.2 Find

- Determine the torque transmitted by each leg and the torque transmitted by the entire section.
- Determine the angle of twist per unit length.

#### 3.3 Solution

The relationship for open looped geometry is,

$$T_i = \frac{\theta_i G L_i c_i^3}{3}$$

$$\tau_{max} = G \theta_i c_{max}$$

From Table A-5,  $G_{steel} = 79.3 \text{ GPa}$ . I will find Part B first because it is required to answer Part A.



### 3.3.1 Part B

```
[7]: tau_max = sp.S(82.7371e6) # shear stress in Pa
G = sp.S(79.3e9) # modulus of rigidity in Pa
c = [sp.S(c_) for c_ in (0.002, 0.003, 0)] # in m
L = [sp.S(L_) for L_ in (0.02, 0.03, 0)] # in m
c_max = max(c)

phi_i = tau_max/(G*c_max)
phi_i # in rad per m
```

```
[7]: 0.347781000420345
```

### 3.3.2 Part A

```
[8]: T = []
for i in range(len(c)):
    T_i = phi_i*G*L[i]*c[i]**3/3
    display(sp.Eq(sp.Symbol(f'T_{i + 1}'), T_i))
    T.append(T_i)
T = sum(T)
display(sp.Eq(sp.Symbol('T'), T)) # torques in N m
```

$$T_1 = 1.47088177777778$$

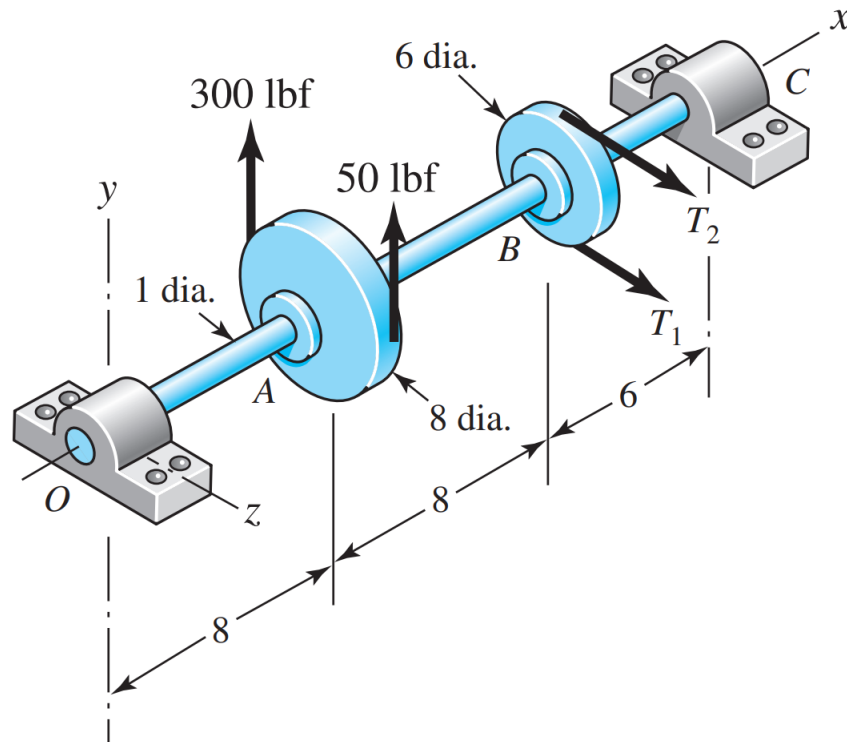
$$T_2 = 7.446339$$

$$T_3 = 0$$

$$T = 8.91722077777778$$

## 4 Problem 3-81

### 4.1 Given



A countershaft carrying two V-belt pulleys is shown in the figure. Pulley A receives power from a motor through a belt with the belt tensions shown. The power is transmitted through the shaft and delivered to the belt on pulley B. Assume the belt tension on the loose side at B is 15 percent of the tension on the tight side.

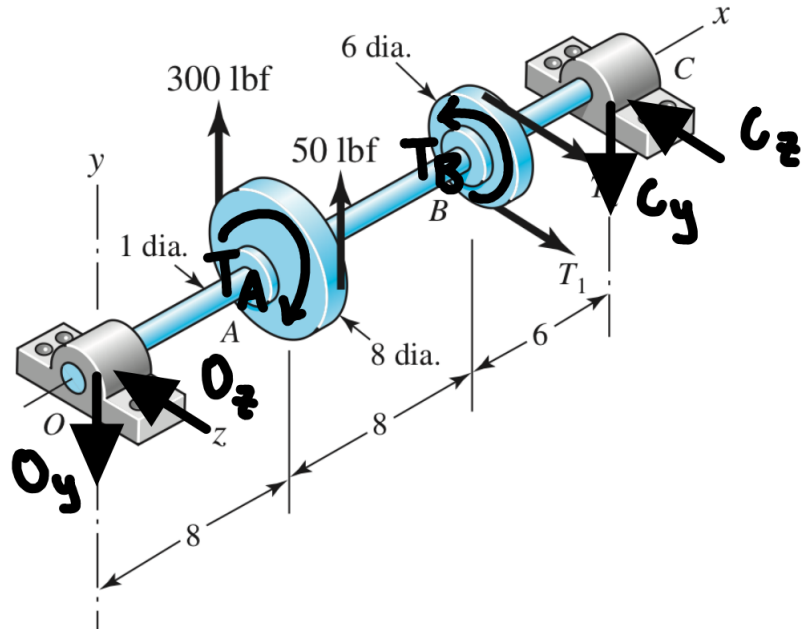
### 4.2 Find

- Determine the tensions in the belt on pulley B, assuming the shaft is running at a constant speed.
- Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.
- Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.
- At the point of maximum bending moment, determine the bending stress and the torsional shear stress.
- At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

## 4.3 Solution

### 4.3.1 Part A

The directions of the torques about  $A$  and  $B$  are,



Since the shaft has no angular acceleration,  $T_A = T_B$  (with directions shown above). It should also be noted that  $T_1$  must be greater than  $T_2$  because the torque shows that the pulley is more tensile at the bottom.

```
[9]: # Solving for T1 and T2
T1, T2 = sp.symbols('T_1 T_2')
T_A = 4*(sp.S(300) - 50)
eq1 = sp.Eq(3*(T1 - T2), T_A)
eq2 = sp.Eq(T2, sp.S(0.15)*T1)

[display(eq) for eq in [eq1, eq2, Markdown('---')]]

sol = sp.solve([eq1, eq2], dict=True)[0]
_ = [display(sp.Eq(key, value)) for key, value in sol.items()]
```

$$3T_1 - 3T_2 = 1000$$

$$T_2 = 0.15T_1$$

---

$$T_1 = 392.156862745098$$

$$T_2 = 58.8235294117647$$

### 4.3.2 Part B

```
[10]: # Solving for the reactions
Oy, Oz, Cy, Cz = sp.symbols('O_y O_z C_y C_z')

eq1 = sp.Eq(300 + 50 - Oy - Cy, 0) # Forces in y direction
eq2 = sp.Eq(sol[T1] + sol[T2] - Oz - Cz, 0) # Forces in z direction
eq3 = sp.Eq(8*sp.S(350) - Cy*22, 0) # Moments about z-axis
eq4 = sp.Eq(-16*(sol[T1] + sol[T2]) + Cz*22, 0) # Moments about the y-axis

sol2 = sp.solve([eq1, eq2, eq3, eq4], dict=True)[0]
[display(eq) for eq in [eq1, eq2, eq3, eq4, Markdown('---')]]
_ = [display(sp.Eq(key, value)) for key, value in sol2.items()]
```

$$-C_y - O_y + 350 = 0$$

$$-C_z - O_z + 450.980392156863 = 0$$

$$2800 - 22C_y = 0$$

$$22C_z - 7215.6862745098 = 0$$

---

$$C_y = 127.272727272727$$

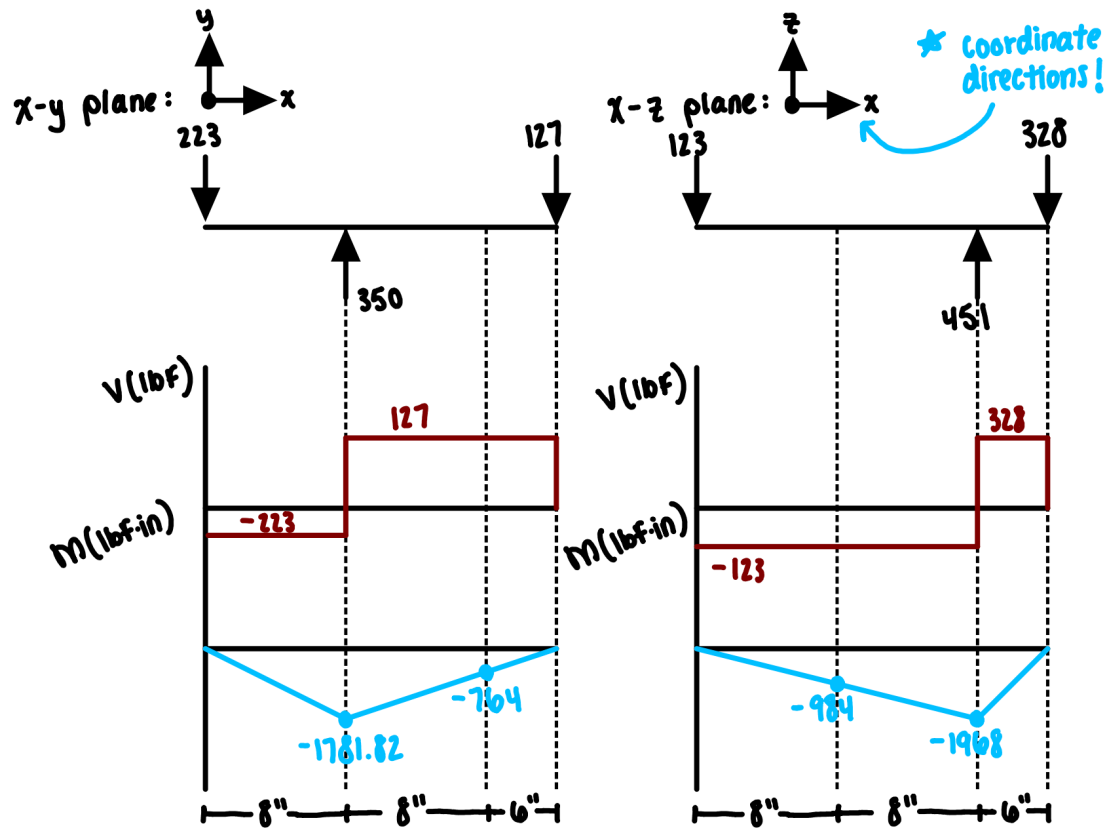
$$O_y = 222.727272727273$$

$$C_z = 327.985739750445$$

$$O_z = 122.994652406418$$

### 4.3.3 Part C

The shear and moment diagram for the two planes is,



#### 4.3.4 Part D

```
[11]: # Getting max bending moment
# At A,
M_A = sp.sqrt(1781.8181818181818**2 + 983.9572195**2)
M_B = sp.sqrt(763.6363636363636**2 + 1967.914439**2)
sp.Matrix([M_A, M_B])
```

```
[11]: [2035.44782366535]
      [2110.88316471859]
```

The maximum bending moment occurs at point B with a value of 2110.88316471859 *lb f in*.

```
[12]: # Getting the bending stress
c = sp.S(0.5)
sig_x = (M_B*c/(sp.pi/4*c**4)).n()
sig_x # psi
```

```
[12]: 21501.2793570833
```

```
[13]: # Getting the torsional stress
t_xz = (1000*c/(sp.pi/2*c**4)).n()
t_xz # in psi
```

```
[13]: 5092.95817894065
```

#### 4.3.5 Part E

**Center and Radius:**

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 10750.6396785417$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 11895.9857309209$$

---

**Principle Stresses:**

$$\sigma_1 = C + R = 22646.6254094625$$

$$\sigma_2 = C - R = -1145.3460523792$$

$$\tau_1 = R = 11895.9857309209$$

$$\tau_2 = -R = -11895.9857309209$$

---

**Angle of Occurrence:**

$$2\phi_p = \text{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 25.348569568567$$