

Machine Design Test 2

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```
[1]: # Notebook Preamble
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')
```

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1 Problem 6-4

1.1 Given

A steel rotating-beam test specimen has an ultimate strength of 1600 MPa.

1.2 Find

Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 900 MPa.

1.3 Solution

The first step is to find S'_e .

```
[2]: sig_ar, S_ut = sp.S(900), sp.S(1600)

# Eq. 6-10
S_e_prime = sp.S(700)
S_e_prime # MPa
```

```
[2]: 700
```

The S'_e value will be used in place of S_e from Figure 6-23 description. We can use the following relationships to determine N .

$$N = \left(\frac{\sigma_{ar}}{a} \right)^{1/b}$$
$$a = \frac{(fS_{ut})^2}{S_e}$$
$$b = -\frac{1}{3} \log \left(\frac{fS_{ut}}{S_e} \right)$$

The value of f is 0.77 from Figure 6-23 (estimated even though it is off the graph).

```
[3]: def log10(x_):
      return sp.log(x_)/sp.log(10)

f = sp.S('0.77')
a = (f*S_ut)**2/S_e_prime
b = -sp.Rational(1, 3)*log10(f*S_ut/S_e_prime)

display(sp.Eq(sp.Symbol('a'), a.n()),
        sp.Eq(sp.Symbol('b'), b.n()))

N = ((sig_ar/a)**(1/b)).n()
N # cycles
```

$$a = 2168.32$$

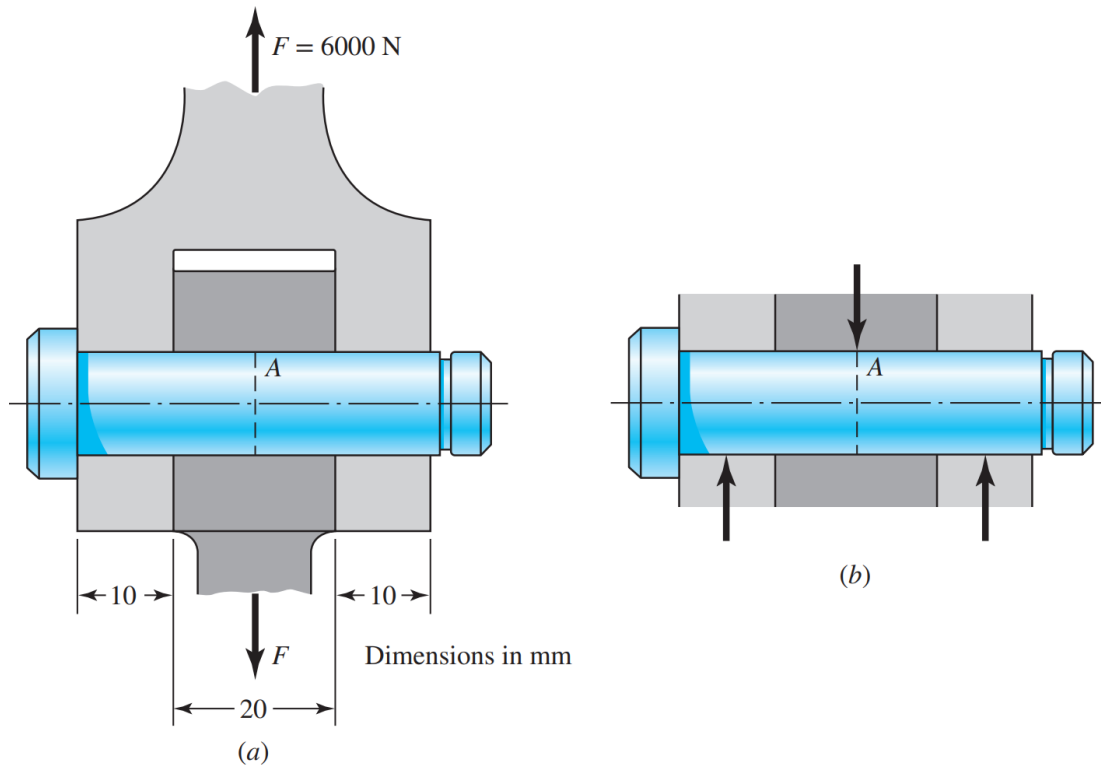
$$b = -0.0818375559380499$$

$$\text{[3]} : 46379.6905856764$$

The life of the specimen is 46400 cycles.

2 Problem 6-11

2.1 Given



A pin in a knuckle joint is shown in part (a) of the figure above. The joint is subject to a repeatedly applied and released load of 6000 N in tension. Assume the loading on the pin is modeled as concentrated forces as shown in part (b) of the figure. The shaft is made from AISI 1018 hot-rolled steel that has been machined to its final diameter.

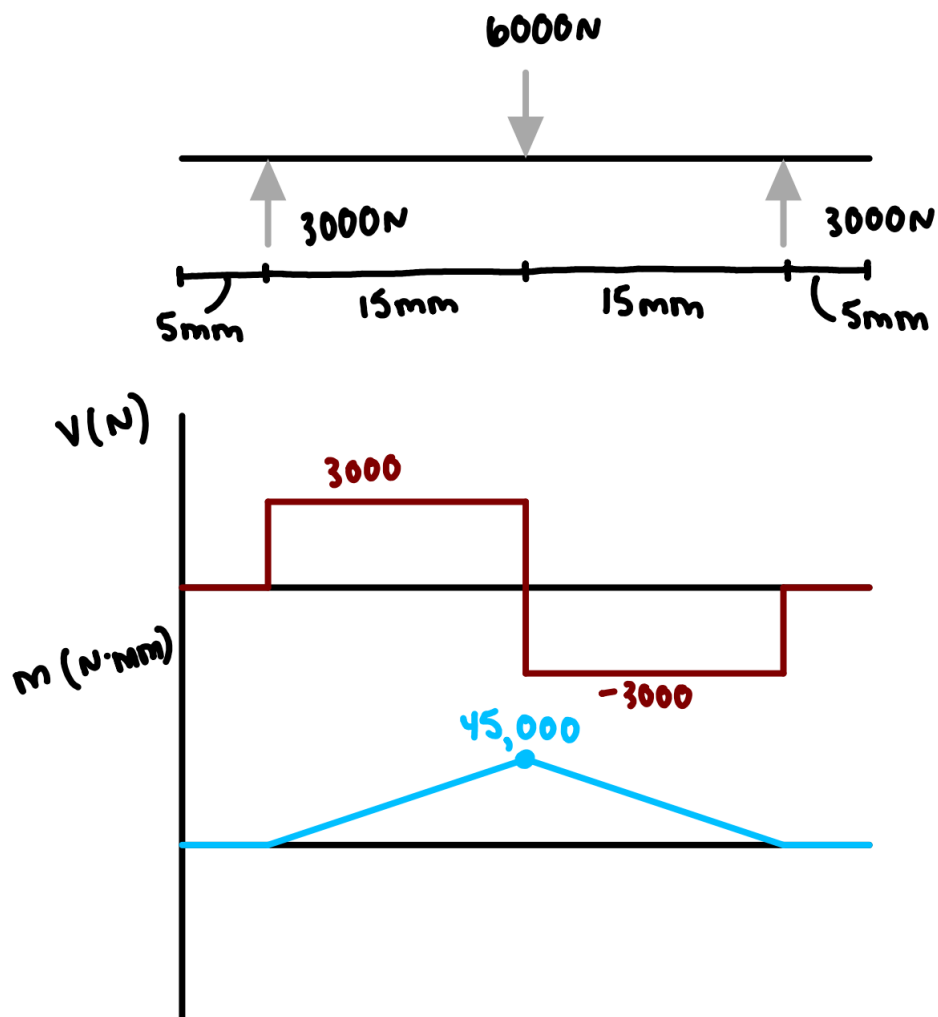
[4]: # Table A-20
Sut, Sy = sp.S(400), sp.S(220) # MPa

2.2 Find

Based on a stress element on the outer surface at the cross-section A, determine a suitable diameter of the pin, rounded up to the next mm increment, to provide at least a factor of safety of 1.5 for both infinite fatigue life and for yielding.

2.3 Solution

2.3.1 Yielding



The equation for the maximum stress may be found using the bending moment diagram above.

```
[5]: d = sp.Symbol('d', real=True, positive=True)
M = 45_000 # N*mm^2
c = d/2
I = sp.pi/64*d**4
sig_max = (M*c/I).n()
sig_max
```

```
[5]: 458366.236104659
      d^3
```

For yielding,

$$n = \frac{S_y}{\sigma_{max}}$$

```
[6]: eq1 = sp.Eq(sp.S('1.5'), Sy/sig_max)
      display(eq1)
      d_ = sp.solve(eq1)[0]
      display(sp.Eq(d, d_))
```

$$1.5 = 0.000479965544298441d^3$$

$$d = 14.6204385310094$$

So for yielding, a suitable diameter would be $d = 15 \text{ mm}$.

2.3.2 Infinite Life

The Goodman method will be used to determine the diameter that results in an infinite life with a factor of safety of 1.5. The relationship for this is,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

The load is not reversible, but is fluctuating on and off, so the mean stress is positive.

```
[7]: # Getting Se
      # Equations come from the road map starting on page 359

      S_e_prime = Sut/2

      a, b = sp.S('3.04'), sp.S('-0.217') # Table 6-2 (Machined)
      k_a = (a*Sut**b).n()
      k_b = sp.S('1.24')*d**sp.S('-0.107')

      Se = k_a*k_b*S_e_prime
      Se
```

```
[7]: 205.436802219618
      -----
      d0.107
```

```
[8]: # Applying the Goodman relationship
      sig_a = sig_m = sig_max/2 # minimum stress is 0
      eq1 = sp.Eq(sig_a/Se + sig_m/Sut, 1/sp.S('1.5'))
      display(eq1)
      d_inf = sp.nsolve(eq1, d, 10) # Numerical solver
      display(sp.Eq(d, d_inf))
```

$$\frac{1115.58939574676}{d^{2.893}} + \frac{572.957795130823}{d^3} = 0.6666666666666667$$

$$d = 14.5624553895619$$

The above equation is complex and the diameter can only be solved using the numerical solver in `sympy`. The results show that the 15 mm diameter should be used for both yielding and infinite life.

3 Problem 3-61

3.1 Given

A machine part will be cycled at $\pm 350 \text{ MPa}$ for $5 \cdot 10^3$ cycles. Then loading will be changed to $\pm 260 \text{ MPa}$ for $5 \cdot 10^4$ cycles. Finally, the load will be changed to $\pm 225 \text{ MPa}$.

$$S_{ut} = 350 \text{ MPa}$$

$$f = 0.9$$

$$S_e = 210 \text{ MPa}$$

```
[9]: Sut, f, Se = sp.S(530), sp.S('0.9'), sp.S(210)
```

3.2 Find

How many cycles of operation can be expected at this stress level using Miner's method?

3.3 Solution

Minor's method is,

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

```
[10]: def get_N(s_max, s_min, f_value, Sut_value, Se_value):
    s_m = (s_max + s_min)/2
    s_a = (s_max - s_min)/2
    s_ar = s_a/(1 - s_m/Sut_value)
    a_ = (f_value*Sut_value)**2/Se_value
    b_ = -sp.Rational(1, 3)*log10(f_value*Sut_value/Se_value)
    N_ = (s_ar/a_)**(1/b_)
    return N_.n()

n1, n2 = sp.S(5e3), sp.S(5e4)
N1 = get_N(sp.S(350), -sp.S(350), f, Sut, Se)
N2 = get_N(sp.S(260), -sp.S(260), f, Sut, Se)
N3 = get_N(sp.S(225), -sp.S(225), f, Sut, Se)

n3 = sp.Symbol('n_3')
eq1 = sp.Eq(n1/N1 + n2/N2 + n3/N3, 1)
display(eq1)
n3_ = sp.solve(eq1)[0]
display(sp.Eq(n3, n3_))
```


$$1.78766904825898 \cdot 10^{-6} n_3 + 0.670863205354351 = 1$$

$$n_3 = 184115.060316224$$