# Machine Design Test 2

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[1]:
```python
# Notebook Preamble
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')
```

# Contents

# 1 Problem 6-4

## 1.1 Given

A steel rotating-beam test specimen has an ultimate strength of 1600 MPa.

## 1.2 Find

Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 900 MPa.

## 1.3 Solution

The first step is to find $S_e'$.

[2]:
```
sig_ar, S_ut = sp.S(900), sp.S(1600)

# Eq. 6-10
S_e_prime = sp.S(700)
S_e_prime  # MPa
```

[2]: 700

The $S_e'$ value will be used in place of $S_e$ from Figure 6-23 description. We can use the following relationships to determine $N$.

$$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b}$$
$$a = \frac{(fS_{ut})^2}{S_e}$$
$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{Se}\right)$$

The value of $f$ is 0.77 from Figure 6-23 (estimated even though it is off the graph).

[3]:
```
def log10(x_):
    return sp.log(x_)/sp.log(10)

f = sp.S('0.77')
a = (f*S_ut)**2/S_e_prime
b = -sp.Rational(1, 3)*log10(f*S_ut/S_e_prime)

display(sp.Eq(sp.Symbol('a'), a.n()),
        sp.Eq(sp.Symbol('b'), b.n()))

N = ((sig_ar/a)**(1/b)).n()
N  # cycles
```

$a = 2168.32$

$b = -0.0818375559380499$

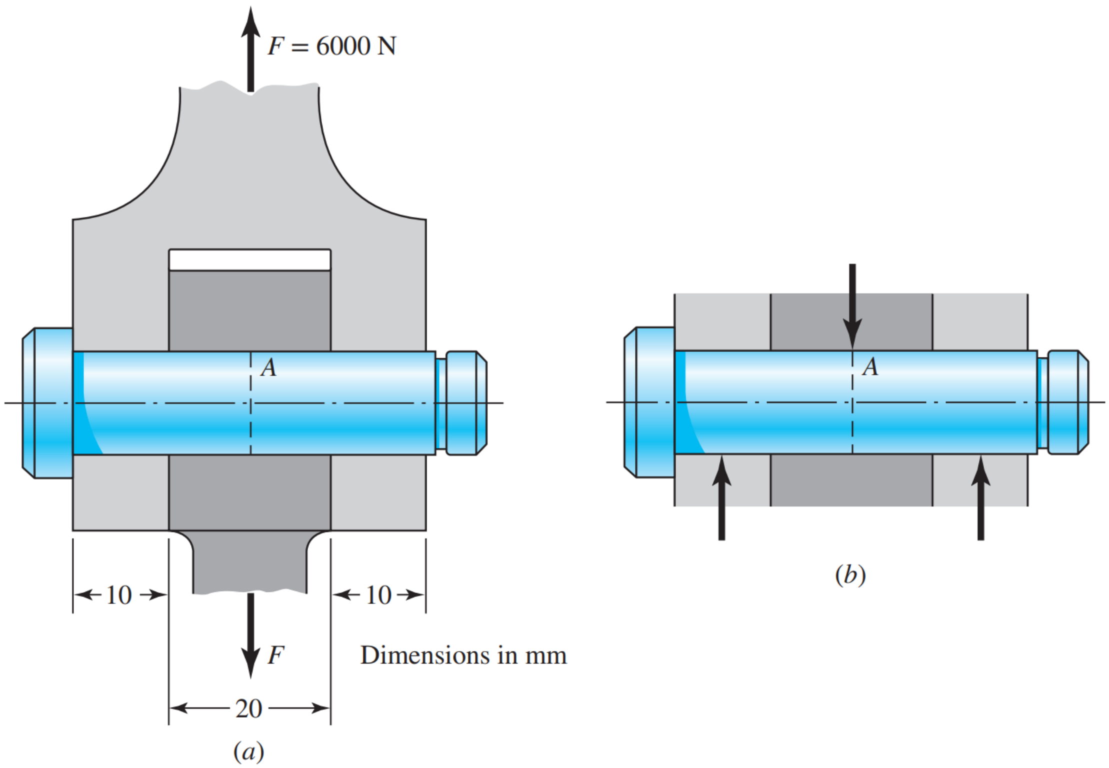
[3]:

46379.6905856764

The life of the specimen is 46400 cycles.

# 2 Problem 6-11

## 2.1 Given




A pin in a knuckle joint is shown in part (a) of the figure above. The joint is subject to a repeatedly applied and released load of 6000 N in tension. Assume the loading on the pin is modeled as concentrated forces as shown in part (b) of the figure. The shaft is made from AISI 1018 hot-rolled steel that has been machined to its final diameter.

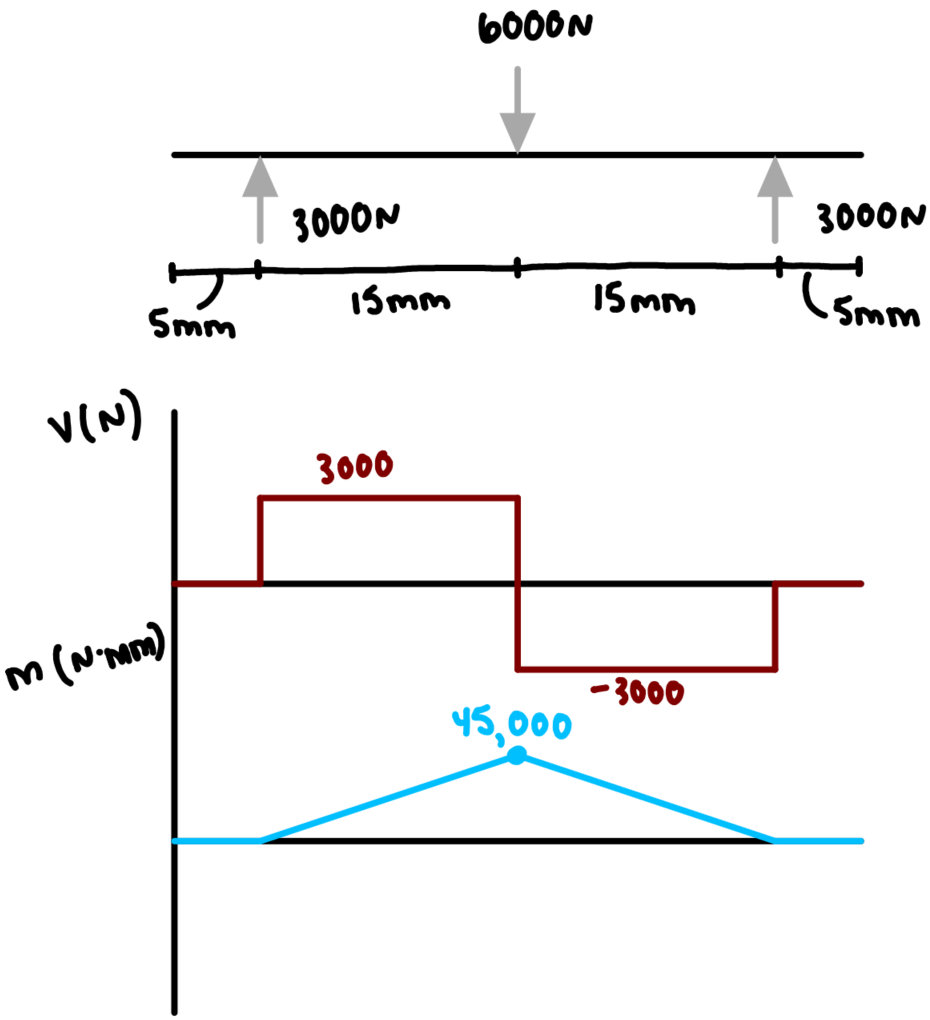
[4]:
```
# Table A-20
Sut, Sy = sp.S(400), sp.S(220)  # MPa
```

## 2.2 Find

Based on a stress element on the outer surface at the cross-section A, determine a suitable diameter of the pin, rounded up to the next mm increment, to provide at least a factor of safety of 1.5 for both infinite fatigue life and for yielding.

### 2.3 Solution

#### 2.3.1 Yielding




The equation for the maximum stress may be found using the bending moment diagram above.

[5]:
```
d = sp.Symbol('d', real=True, positive=True)
M = 45_000  # N*mm^2
c = d/2
I = sp.pi/64*d**4
sig_max = (M*c/I).n()
sig_max
```

[5]: $\frac{458366.236104659}{d^3}$

For yielding,

$$n = \frac{S_y}{\sigma_{max}}$$

[6]:
```
eq1 = sp.Eq(sp.S('1.5'), Sy/sig_max)
display(eq1)
d_ = sp.solve(eq1)[0]
display(sp.Eq(d, d_))
```

$1.5 = 0.000479965544298441d^3$

$d = 14.6204385310094$

So for yielding, a suitable diameter would be $d = 15\,mm$.

### 2.3.2 Infinite Life

The Goodman method will be used to determine the diameter that results in an infinite life with a factor of safety of 1.5. The relationship for this is,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

The load is not reversible, but is fluctuating on and off, so the mean stress is positive.

[7]:
```
# Getting Se
# Equations come from the road map starting on page 359

S_e_prime = Sut/2

a, b = sp.S('3.04'), sp.S('-0.217')  # Table 6-2 (Machined)
k_a = (a*Sut**b).n()
k_b = sp.S('1.24')*d**sp.S('-0.107')

Se = k_a*k_b*S_e_prime
Se
```

[7]: $\frac{205.436802219618}{d^{0.107}}$

[8]:
```
# Applying the Goodman relationship
sig_a = sig_m = sig_max/2  # minimum stress is 0
eq1 = sp.Eq(sig_a/Se + sig_m/Sut, 1/sp.S('1.5'))
display(eq1)
d_inf = sp.nsolve(eq1, d, 10)  # Numerical solver
display(sp.Eq(d, d_inf))
```

$\frac{1115.58939574676}{d^{2.893}} + \frac{572.957795130823}{d^3} = 0.666666666666667$

$d = 14.5624553895619$

The above equation is complex and the diameter can only be solved using the numerical solver in `sympy`. The results show that the 15 $mm$ diameter should be used for both yielding and infinite life.

# 3 Problem 3-61

## 3.1 Given

A machine part will be cycled at $\pm 350\, MPa$ for $5 \cdot 10^3$ cycles. Then loading will be changed to $\pm 260\, MPa$ for $5 \cdot 10^4$ cycles. Finally, the load will be changed to $\pm 225\, MPa$.

$$S_{ut} = 350\, MPA$$
$$f = 0.9$$
$$S_e = 210\, MPa$$

[9]:
```
Sut, f, Se = sp.S(530), sp.S('0.9'), sp.S(210)
```

## 3.2 Find

How many cycles of operation can be expected at this stress level using Miner's method?

## 3.3 Solution

Minor's method is,

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

[10]:
```
def get_N(s_max, s_min, f_value, Sut_value, Se_value):
    s_m = (s_max + s_min)/2
    s_a = (s_max - s_min)/2
    s_ar = s_a/(1 - s_m/Sut_value)
    a_ = (f_value*Sut_value)**2/Se_value
    b_ = -sp.Rational(1, 3)*log10(f_value*Sut_value/Se_value)
    N_ = (s_ar/a_)**(1/b_)
    return N_.n()

n1, n2 = sp.S(5e3), sp.S(5e4)
N1 = get_N(sp.S(350), -sp.S(350), f, Sut, Se)
N2 = get_N(sp.S(260), -sp.S(260), f, Sut, Se)
N3 = get_N(sp.S(225), -sp.S(225), f, Sut, Se)

n3 = sp.Symbol('n_3')
eq1 = sp.Eq(n1/N1 + n2/N2 + n3/N3, 1)
display(eq1)
n3_ = sp.solve(eq1)[0]
display(sp.Eq(n3, n3_))
```

$1.78766904825898 \cdot 10^{-6} n_3 + 0.670863205354351 = 1$

$n_3 = 184115.060316224$

# 4 Problem 8-12

## 4.1 Given

An $M14 \times 2$ hex head bolt with a nut is used to clamp together two 15 mm steel plates. There is a 14R metric plain washer under the nut.

## 4.2 Find

a. Determine a suitable length for the bolt, rounded up to the nearest 5 mm.
b. Determine the bolt stiffness.
c. Determine the stiffness of the members.

## 4.3 Solution

### 4.3.1 Part A

The bolt dimensions are found in Table A-31.

[11]:
```
l, H, t = sp.S(30), sp.S('12.8'), sp.S('3.5')
L = l + H + t
L  # mm
```

[11]:

46.3

The bolt length is $50\,mm$.

### 4.3.2 Part B

The fastener stiffness along with more relationships come from Table 8-7.

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

[12]:
```
d = sp.S(14)  # mm
E = sp.S('207')  # GPa
L = 50  # mm

Ad = sp.pi*d**2/4
At = sp.S(115)
Lt = 2*d + 6
ld = L - Lt
lt = l + t - ld
kb = Ad*At*E/(Ad*lt + At*ld)
kb.n()  # MN/m
```

[12]:

808.240665447356

### 4.3.3 Part C

[13]:
```
# Eq. 8-22
x = sp.S('0.5774')
km = (x*sp.pi*E*d)/(2*sp.log(5*(x*(l + t) + sp.S('0.5')*d)/(x*(l + t) + sp.S('2.
↪5')*d)))
km.n()  # MN/m
```

[13]:

2968.8853220629

# 5 Problem 8-25

## 5.1 Given

An $M14 \times 2$ hex head bolt with a nut is used to clamp together two 20 mm steel plates.

## 5.2 Find

Compare the results of finding the overall member stiffness by use of Equations 8-20, 8-22, and 8-23.

## 5.3 Solution

[14]:
```
def get_k(E_, dia, t_, D_):
    return ((sp.S('0.5774')*sp.pi*E_*dia)/sp.log(((sp.S('1.155')*t_ + D_ -
 ↪dia)*(D_ + dia))/((sp.S('1.155')*t_ + D_ + dia)*(D_ - dia)))).n()

E = sp.S(207)  # GPa
```

### 5.3.1 Equation 8-20

The relationship is,

$$k = \frac{0.5774\pi E d}{\ln \frac{(1.155t+D-d)(D+d)}{(1.155t+D+d)(D-d)}}$$

[15]:
```
# The width of the bolt is 21 mm
d = sp.S(14)
w = sp.S(21)
k1 = get_k(E, d, 20, w)
k2 = get_k(E, d, 20, w)

km1 = (1/k1 + 1/k2)**-1
km1  # MN/m
```

[15]: 2761.53476109712

### 5.3.2 Equation 8-22

The relationship should result in an identical answer because this equation is for two sheets in series with same properties/geometry.

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\frac{0.5774l+0.5d}{0.5774l+2.5d}\right)}$$

[16]:
```
l = 40
x = sp.S('0.5774')
km2 = (x*sp.pi*E*d)/(2*sp.log(5*(x*l + sp.S('0.5')*d)/(x*l + sp.S('2.5')*d)))
km2.n()  # MN/m
```

[16]: 2761.72060761183

The slight error is because of the rounded numbers in the expression.

### 5.3.3 Equation 8-23

This relationship is,

$$\frac{k_m}{Ed} = A\exp(Bd/l)$$

The values of $A$ and $B$ come from Table 8-8.

[17]:
```
A, B = sp.S('0.78715'), sp.S('0.62873')
km3 = E*d*A*sp.exp(B*d/l)
km3.n()  # MN/m
```

[17]: 2842.65903019863

This answer is more conservative than the other two, but is based of a finite element study.