ME 8253 Spring 2023
Homework #5
Due Date: Thursday April 13th, 2023

Please submit your homework through CANVAS as a PDF file. For all problems with calculation, present all calculation details.

Questions (10 points)

Answer the following questions:

(a) What are the expressions of the plastic zone size for plane stress and plane strain?
(b) What are the restrictions on the use of LEFM?
(c) What are the restrictions for the plane strain fracture toughness K_{Ic} value to be considered valid?

Problem 1 (*40 points*)

A gas turbine component is made of recrystallized, annealed Ti-6A1-4V with $K_{Ic} = 85\ \text{MPa}\sqrt{\text{m}}$ and $S_y = 815$ MPa. A surface semicircular crack ($a/c = 1$) similar to that in Figure 1

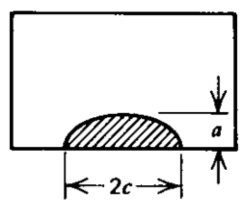

Figure 1 – Surface semi-elliptical crack.

is found during a routine maintenance inspection. If the component thickness is 25 mm, comment on the stress state (i.e., plane stress or plane strain).

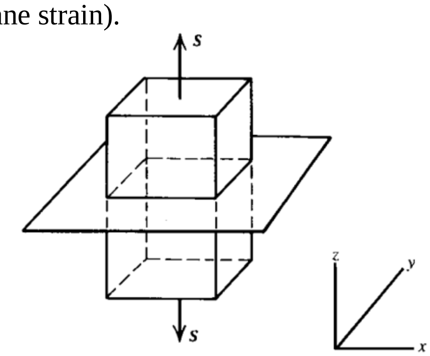

Figure 2 – Tensile loading and crack plane. Cracks are shown in the x–y plane.

a/c	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Φ	1.0	1.016	1.051	1.097	1.151	1.211	1.277	1.345	1.418	1.493	1.571

Figure 3 – Values of Φ.

(a) If a stress is applied normal to the crack plane like that in Figure 2, what maximum stress is required to cause fracture if $a = 8$ mm and $K_c = 105\ \text{MPa}\sqrt{\text{m}}$?
(b) If the thickness were doubled, what maximum stress would cause fracture?
(c) Comment on the conditions required for fracture at each thickness and whether LEFM is valid for each case.

Problem 2 (*50 points*)

A uniaxially loaded very wide sheet of medium-strength steel is subjected to constant amplitude loading at $R = 0$ with $S_{\max}$ =110 MPa. Let $K_c = 95$ MPa$\sqrt{\text{m}}$ and $S_y = 440$ MPa. The material displays the following region II Paris relationship for long crack behavior at $R = 0$:

$$\frac{da}{dN} = 2.4 \times 10^{-11}(\Delta K)^{2.75}$$

where da/dN is in m/cycle and ΔK is in MPa$\sqrt{\text{m}}$. Data on physically small cracks were generated for the same material at $R = 0$, and fitting a power law expression to the data yielded the following relationship:

$$\frac{da}{dN} = 1.8 \times 10^{-10}(\Delta K)^{1.75}$$

(a) Plot the equation for these two relationships (use **Figure 3** next page), the long crack equation between

$$10^{-8} \text{ m/cycle} < \frac{da}{dN} < 10^{-5} \text{ m/cycle}$$

and the small crack equation between 1 MPa$\sqrt{\text{m}}$ and where it merges with the long crack Paris equation. If $\Delta K_{\text{th}} = 5$ MPa$\sqrt{\text{m}}$ for the long crack data, also sketch the approximate sigmoidal portion of the long crack growth curve in region I. Complete the approximate sigmoidal long crack growth curve with K_c.

(b) Based on your plot, will extrapolation of the Paris equation to region I predict conservative or nonconservative fatigue life if a physically small crack exists in a component made from this material?

Laboratory experiments have shown that for this material, physically small crack growth occurs up to a length of 1 mm. If the wide sheet contains an initial edge crack with $a_i = 0.3$ mm:

(c) Calculate the fatigue life of the sheet using only the long crack.
(d) Calculate the fatigue life, taking into consideration the small crack and the long crack.
(e) Comment on your results and the use of this life prediction technique.

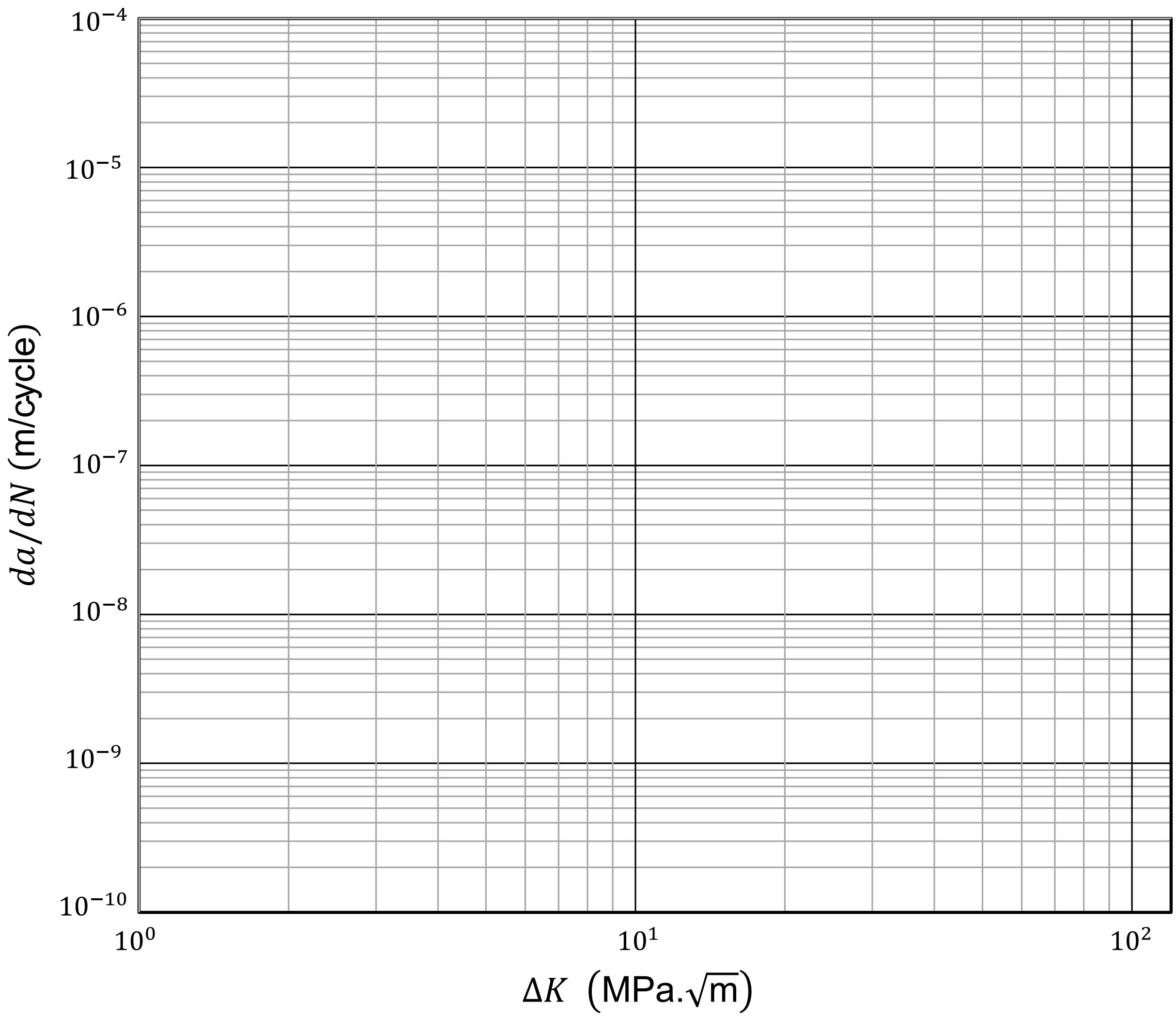

Figure 3 – Long crack and small crack regions.