Vibrations and Controls Homework 6

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```python
[1]: import sympy as sp
     import numpy as np
     import matplotlib.pyplot as plt
     from IPython.display import display, Latex

     plt.style.use('maroon.mplstyle')

     s, t = sp.symbols('s t')

     display_latex = lambda text: display(Latex(text))
```

Contents

1   Problem 9.11 Part A

1.1   Given

$T(s) = \frac{Y(s)}{F(s)} = \frac{5}{(5s+1)(2s+1)}$
$f(t) = 10\sin(0.2t)$

1.2   Find

The steady state response $y_{ss}(t)$

1.3   Solution

$y_{ss}(t) = |T(j\omega)|A\sin(\omega t + \angle T(j\omega))$

```
[2]: T_s = 5/((5*s + 1)*(2*s + 1))
     T_s
```

[2]:
$$\frac{5}{(2s+1)(5s+1)}$$

```
[3]: T_jw = T_s.subs(s, sp.I*0.2)
     T_jw
```

[3]:
$$2.1551724137931 \left(1 - 1.0i\right)\left(1 - 0.4i\right)$$

```
[4]: M = sp.Abs(T_jw)
     phi = sp.arg(T_jw)

     M.n()   # Magnitude
```
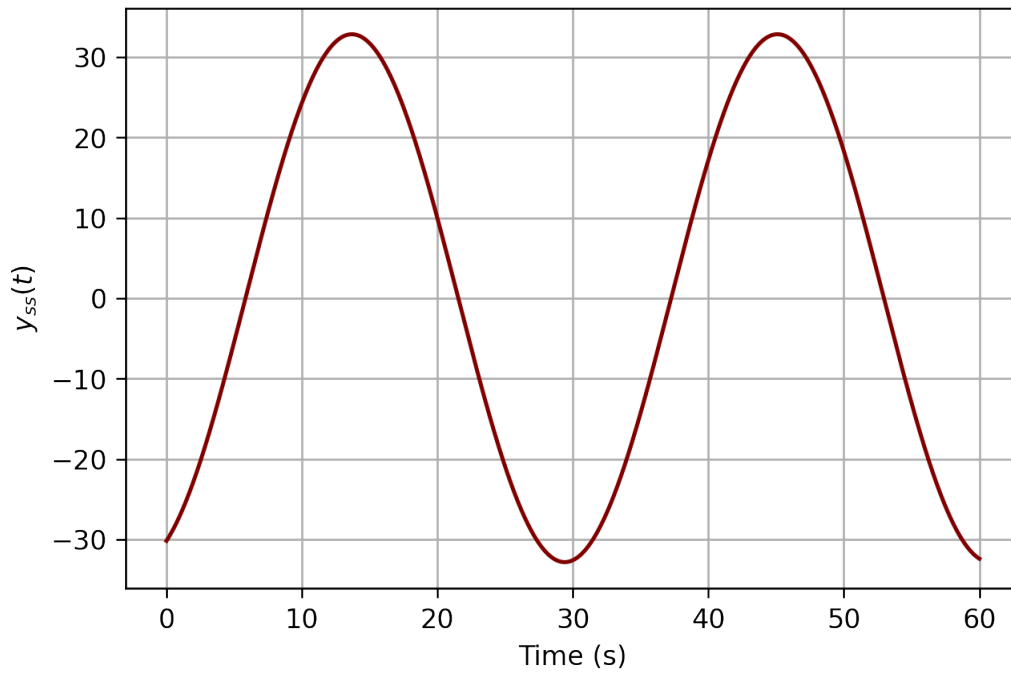
[4]:
3.28266082149306

```
[5]: phi   # The angle
```

[5]:
-1.16590454050981

```
[6]: y_ss = 10*M*sp.sin(0.2*t + phi)
     y_ss
```

[6]:
$32.8266082149306\sin\left(0.2t - 1.16590454050981\right)$

```
[7]: # Plotting it
     y_ss_lamb = sp.lambdify(t, y_ss)
     time = np.linspace(0, 60, 1000)

     plt.plot(time, y_ss_lamb(time))
     plt.xlabel('Time (s)')
     plt.ylabel('$y_{ss}(t)$')
```
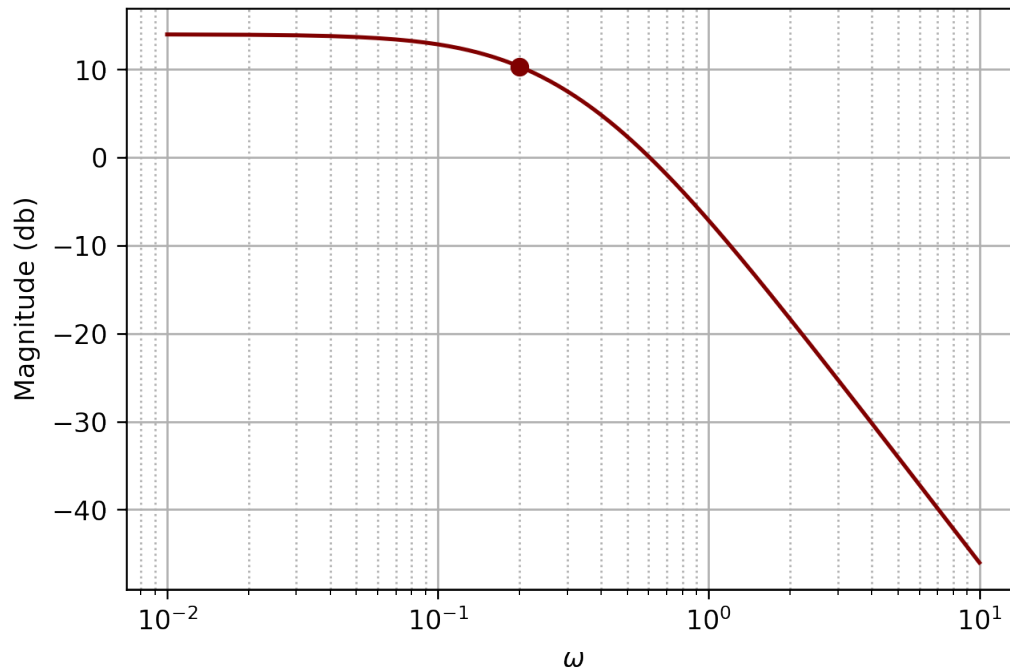
```
plt.show()
```


1.3.1 Frequency Response

```
[8]: # Getting a plot of the magnitude response
     omega_ = np.linspace(0.01, 10, 1000)
     c_nums = 5/((5*1j*omega_ + 1)*(2*1j*omega_ + 1))

     M = np.abs(c_nums)
     plt.xscale('log')
     plt.plot(omega_, 20*np.log10(M))
     plt.scatter(0.2, 20*np.log10(3.283), zorder=2)
     plt.xlabel(r'$\omega$')
     plt.ylabel('Magnitude (db)')
     plt.grid(which='minor', ls=':')
     plt.show()
```

```
[9]:  # Getting a plot of the phase response
      phi = np.angle(c_nums)
      plt.xscale('log')
      plt.plot(omega_, phi)
      plt.scatter(0.2, -1.166, zorder=2)
      plt.xlabel(r'$\omega$')
      plt.ylabel(r'$\phi$')
      plt.grid(which='minor', ls=':')
      plt.show()
```
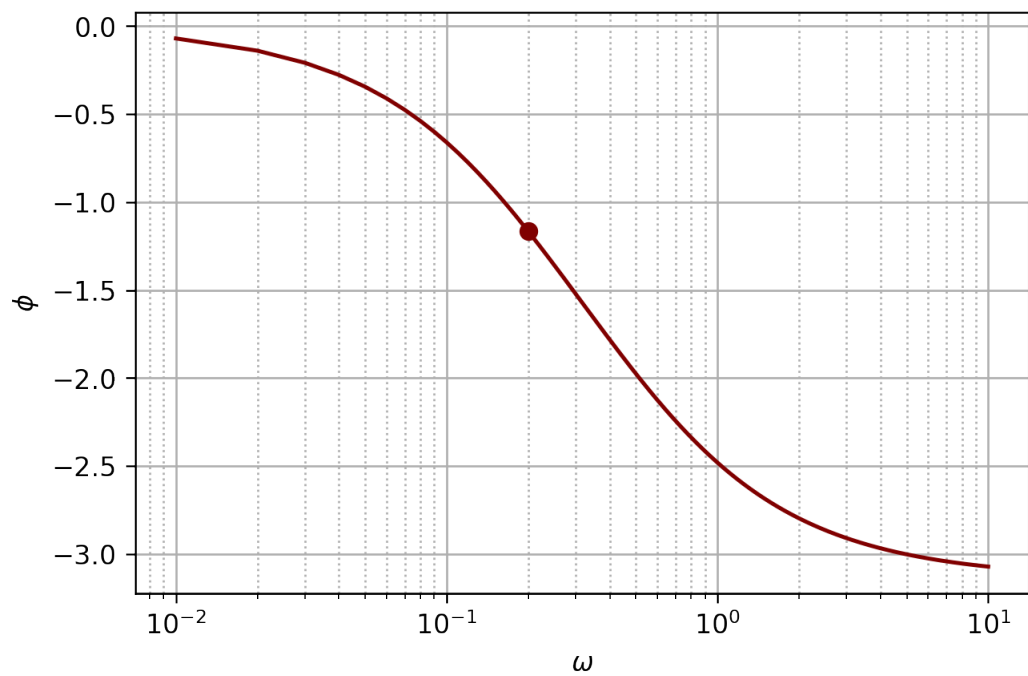
2   Problem 9.11 Part B

2.1   Given

$T(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^2 + 10s + 100}$
$f(t) = 16\sin(5t)$

2.2   Find

The steady state response $y_{ss}(t)$

2.3   Solution

$y_{ss}(t) = |T(j\omega)|A\sin(\omega t + \angle T(j\omega))$

```
[10]:  T_s = 1/(s**2 + 10*s + 100)
       T_s
```

[10]:
$$\frac{1}{s^2 + 10s + 100}$$

```
[11]:  T_jw = T_s.subs(s, sp.I*5)
       T_jw
```

[11]: $\frac{75 - 50i}{8125}$

```
[12]:  M = sp.Abs(T_jw)
       phi = sp.arg(T_jw)

       M.n()   # The magnitude
```

[12]:
0.0110940039245046

```
[13]:  phi.n()   # The angle
```

[13]:
-0.588002603547568
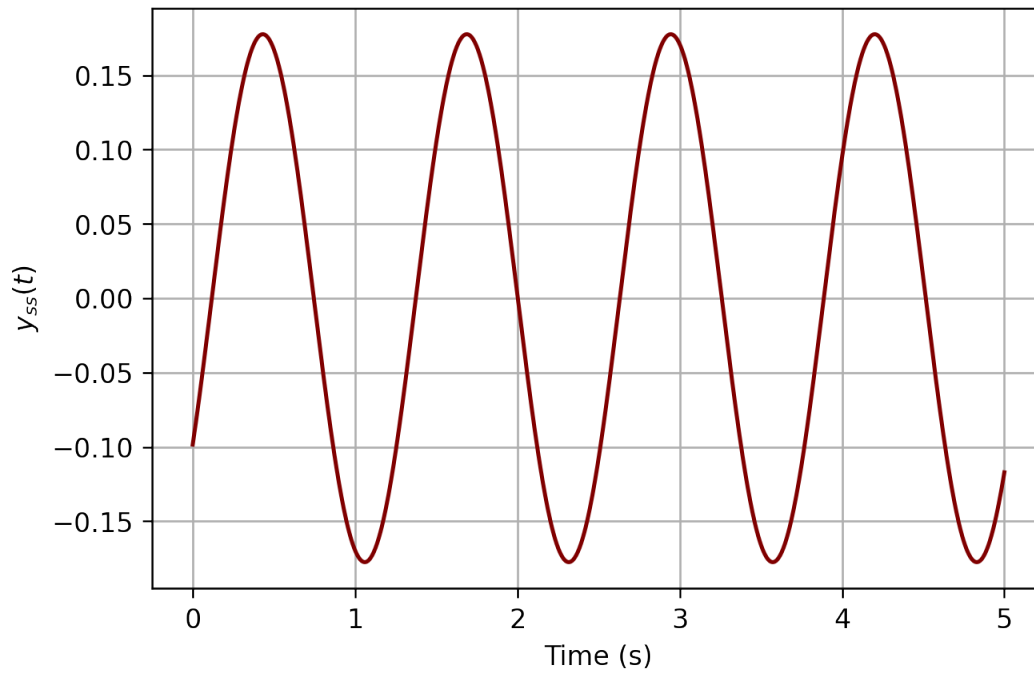
```
[14]:  y_ss = 16*M.n()*sp.sin(5*t + phi.n())
       y_ss
```

[14]:
$0.177504062792073\sin(5t - 0.588002603547568)$

```
[15]:  y_ss_lamb = sp.lambdify(t, y_ss)
       time = np.linspace(0, 5, 1000)

       plt.plot(time, y_ss_lamb(time))
       plt.xlabel('Time (s)')
       plt.ylabel('$y_{ss}(t)$')
```
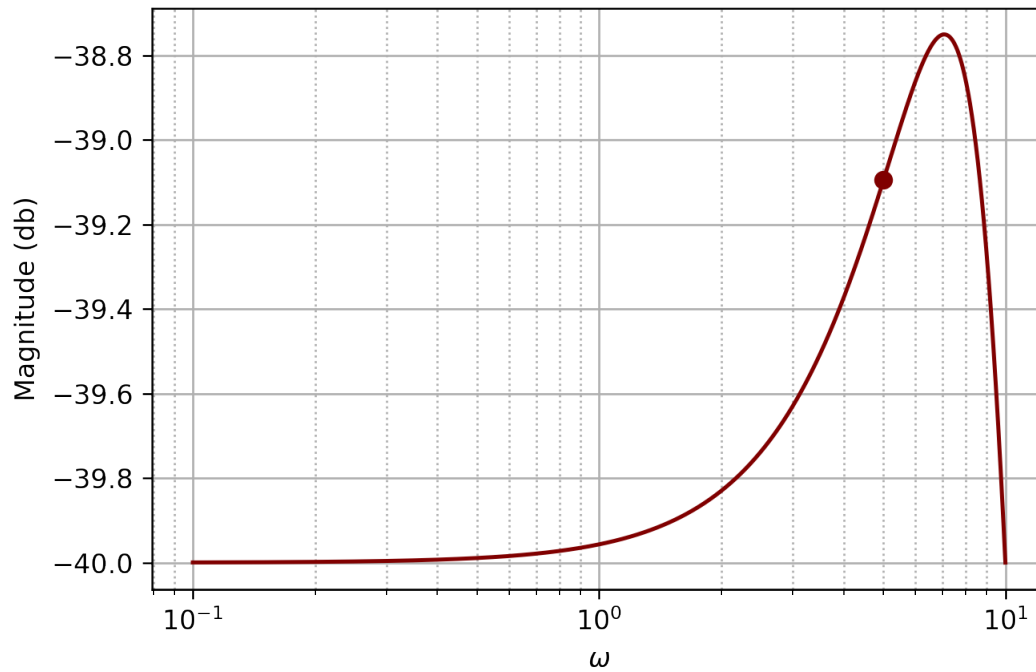
```
plt.show()
```


2.3.1 Frequency Response

```
[16]: # Plotting the magnitude response
      omega_ = np.linspace(0.1, 10, 1000)
      c_nums = 1/((1j*omega_)**2 + 10*1j*omega_ + 100)

      M = np.abs(c_nums)
      plt.xscale('log')
      plt.plot(omega_, 20*np.log10(M))
      plt.scatter(5, 20*np.log10(0.0111), zorder=2)
      plt.xlabel(r'$\omega$')
      plt.ylabel(r'Magnitude (db)')
      plt.grid(which='minor', ls=':')
      plt.show()
```

```
[17]:  # Plotting the phase response
       plt.xscale('log')
       plt.plot(omega_, np.angle(c_nums))
       plt.scatter(5, -0.588, zorder=2)
       plt.xlabel(r'$\omega$')
       plt.ylabel(r'$\phi$')
       plt.grid(which='minor', ls=':')
       plt.show()
```