# Vibrations and Controls Homework 9

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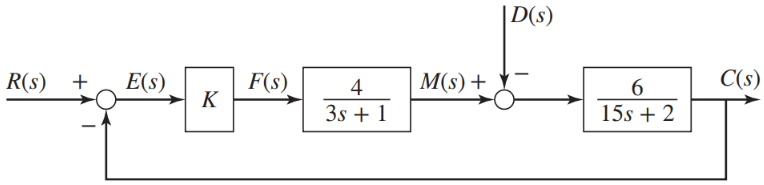
[1]:
```
# Notebook Preamble
%config ZMQInteractiveShell.ast_node_interactivity = 'all'
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

t, s = sp.symbols('t s')

plt.style.use('maroon_ipynb.mplstyle')
```

## Contents

# 1 Problem 10.4

## 1.1 Given




## 1.2 Find

Derive the output $C(s)$, error $E(s)$, and actuator $M(s)$ equations for the diagram above and obtain the characteristic polynomial.

## 1.3 Solution

[2]:

```python
# Use algebra to determine the functions
K = sp.Symbol('K')
R, E, F, M, D, C, T = [sp.Function(i)(s) for i in ['R', 'E', 'F', 'M', 'D',␣
↪'C', 'T']]
eq1 = sp.Eq(E, R - C)
eq1
eq2 = sp.Eq(F, E*K)
eq2
eq3 = sp.Eq(M, F*4/(3*s + 1))
eq3
eq4 = sp.Eq(C, (M - D)*6/(15*s + 2))
eq4
```

[2]:

$$E(s) = -C(s) + R(s)$$

[2]:

$$F(s) = KE(s)$$

[2]:

$$M(s) = \frac{4F(s)}{3s+1}$$

[2]:

$$C(s) = \frac{-6D(s) + 6M(s)}{15s+2}$$

The characteristic polynomial may be determined by solving for $\frac{C(s)}{R(s)}$.

[3]:

```python
subs = eq4.subs([
    (M, eq3.rhs),
    (F, eq2.rhs),
    (E, eq1.rhs),
    (C, T*R)
])
```

```python
sol = sp.solve(subs, T)[0]
sp.Eq(T, sol)
```

[3]:
$$T(s) = \frac{6 \cdot (4KR(s) - 3sD(s) - D(s))}{(24K + 45s^2 + 21s + 2)\, R(s)}$$

$\frac{C(s)}{R(s)}$ may be obtained by setting $D(s)$ equal to 0.

[4]:
```python
sp.Eq(C/R, sol.subs(D, 0))
```

[4]:
$$\frac{C(s)}{R(s)} = \frac{24K}{24K + 45s^2 + 21s + 2}$$

It is not necessary to try and rewrite $E(s)$, $M(s)$, and $F(s)$ in any other forms. The denominator of the $\frac{C(s)}{R(s)}$ function above is the characteristic polynomial,

$$45s^2 + 21s + 2 + 24K$$