Vibrations and Controls Homework 6

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[1]:
```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Latex

plt.style.use('maroon.mplstyle')

s, t = sp.symbols('s t')

display_latex = lambda text: display(Latex(text))
```

Contents

1 Problem 9.11 Part A

1.1 Given

$T(s) = \frac{Y(s)}{F(s)} = \frac{5}{(5s+1)(2s+1)}$
$f(t) = 10\sin(0.2t)$

1.2 Find

The steady state response $y_{ss}(t)$

1.3 Solution

$y_{ss}(t) = |T(j\omega)|A\sin(\omega t + \angle T(j\omega))$

[2]:
```
T_s = 5/((5*s + 1)*(2*s + 1))
T_s
```

[2]:
$$\frac{5}{(2s+1)(5s+1)}$$

[3]:
```
T_jw = T_s.subs(s, sp.I*0.2)
T_jw
```

[3]: $2.1551724137931\,(1-1.0i)\,(1-0.4i)$

[4]:
```
M = sp.Abs(T_jw)
phi = sp.arg(T_jw)

M.n()  # Magnitude
```

[4]: 3.28266082149306

[5]:
```
phi  # The angle
```

[5]: -1.16590454050981

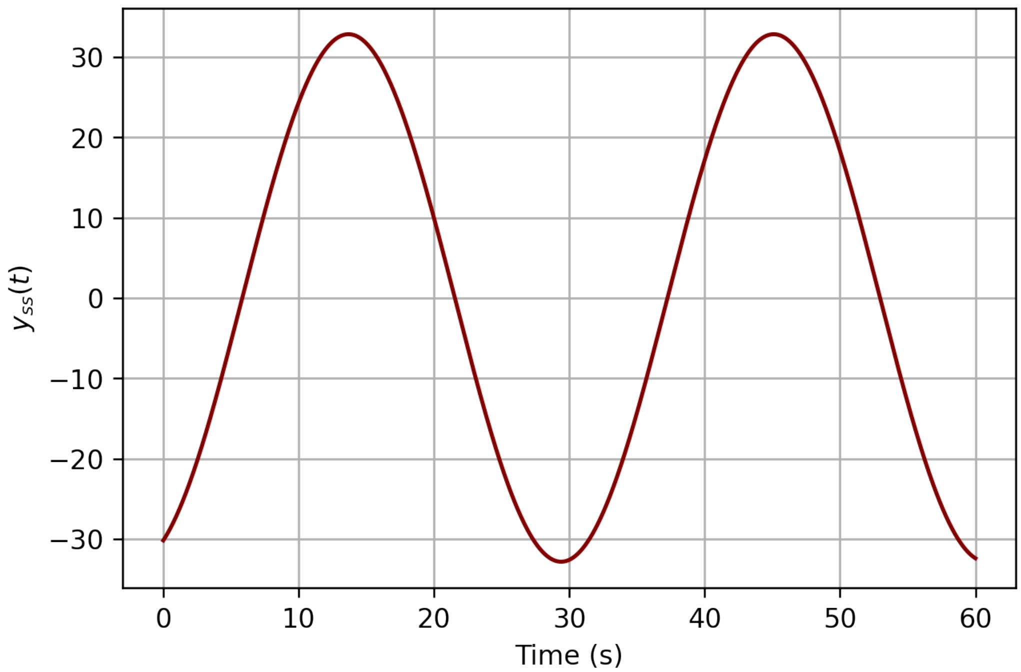
[6]:
```
y_ss = 10*M*sp.sin(0.2*t + phi)
y_ss
```

[6]: $32.8266082149306\sin(0.2t - 1.16590454050981)$

[7]:
```
# Plotting it
y_ss_lamb = sp.lambdify(t, y_ss)
time = np.linspace(0, 60, 1000)

plt.plot(time, y_ss_lamb(time))
plt.xlabel('Time (s)')
plt.ylabel('$y_{ss}(t)$')
```

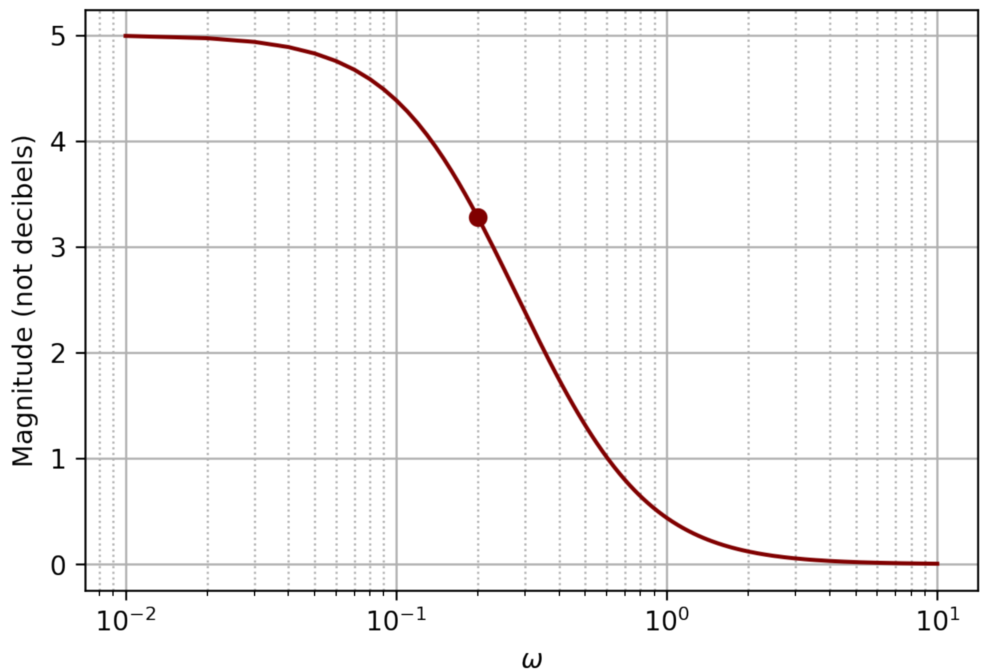
```
plt.show()
```



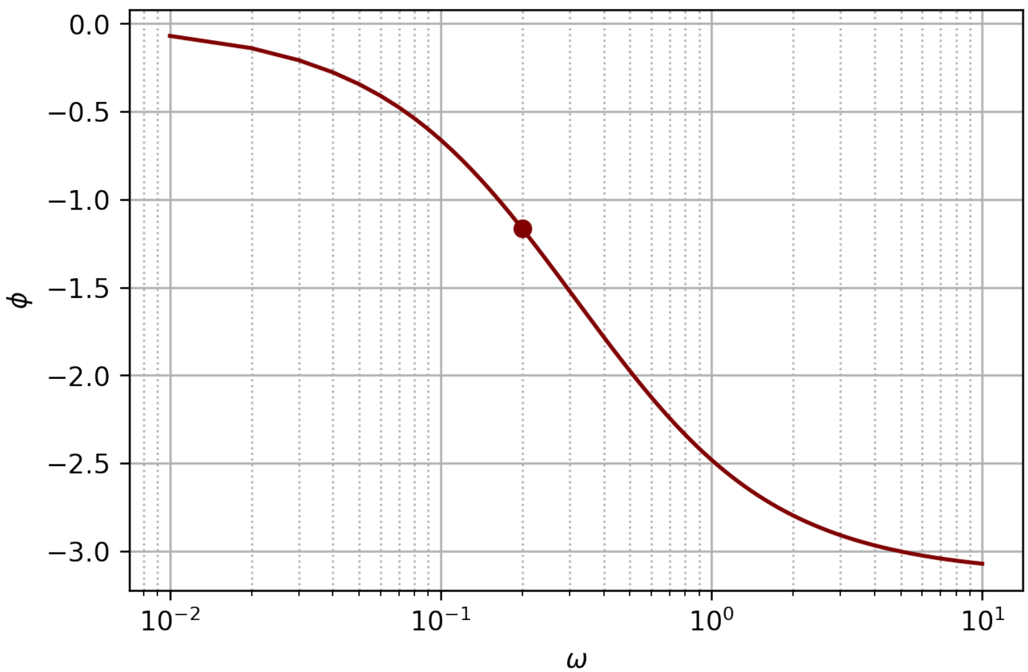
1.3.1 Frequency Response

[8]:
```
# Getting a plot of the magnitude response
omega_ = np.linspace(0.01, 10, 1000)
c_nums = 5/((5*1j*omega_ + 1)*(2*1j*omega_ + 1))

M = np.abs(c_nums)
plt.xscale('log')
plt.plot(omega_, M)
plt.scatter(0.2, 3.283, zorder=2)
plt.xlabel(r'$\omega$')
plt.ylabel('Magnitude (not decibels)')
plt.grid(which='minor', ls=':')
plt.show()
```



[9]:
```
# Getting a plot of the phase response
phi = np.angle(c_nums)
plt.xscale('log')
plt.plot(omega_, phi)
plt.scatter(0.2, -1.166, zorder=2)
plt.xlabel(r'$\omega$')
plt.ylabel(r'$\phi$')
plt.grid(which='minor', ls=':')
plt.show()
```


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
−3.0
ϕ
10^{-2}
10^{-1}
10^{0}
10^{1}
ω

2 Problem 9.11 Part B

2.1 Given

$T(s) = \frac{Y(s)}{F(s)} = \frac{1}{s^2+10s+100}$
$f(t) = 16\sin(5t)$

2.2 Find

The steady state response $y_{ss}(t)$

2.3 Solution

$y_{ss}(t) = |T(j\omega)|A\sin(\omega t + \angle T(j\omega))$

[10]:
```
T_s = 1/(s**2 + 10*s + 100)
T_s
```

[10]: $\frac{1}{s^2 + 10s + 100}$

[11]:
```
T_jw = T_s.subs(s, sp.I*5)
T_jw
```

[11]: $\frac{75 - 50i}{8125}$

[12]:
```
M = sp.Abs(T_jw)
phi = sp.arg(T_jw)

M.n()   # The magnitude
```

[12]: 0.0110940039245046

[13]:
```
phi.n()   # The angle
```

[13]: -0.588002603547568

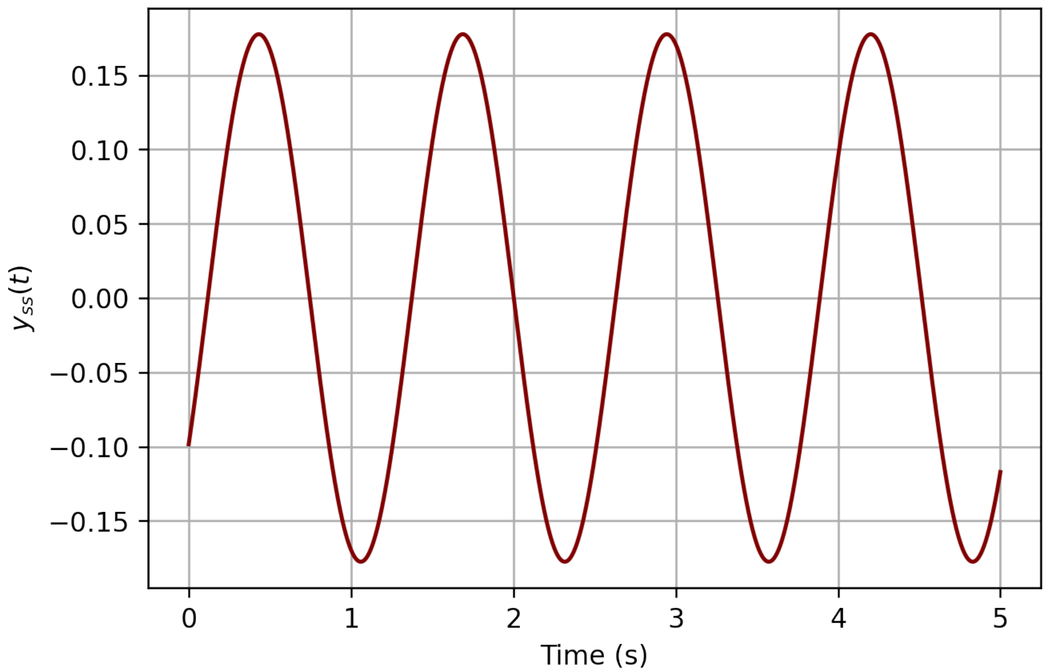
[14]:
```
y_ss = 16*M.n()*sp.sin(5*t + phi.n())
y_ss
```

[14]: $0.177504062792073\sin(5t - 0.588002603547568)$

[15]:
```
y_ss_lamb = sp.lambdify(t, y_ss)
time = np.linspace(0, 5, 1000)

plt.plot(time, y_ss_lamb(time))
plt.xlabel('Time (s)')
plt.ylabel('$y_{ss}(t)$')
```

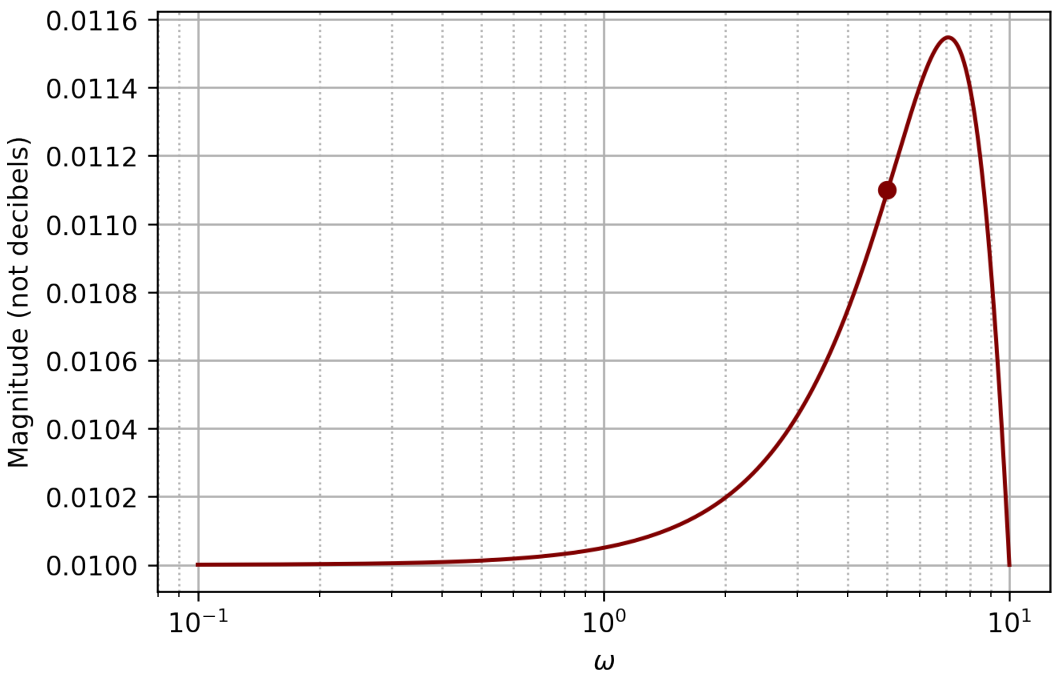
```
plt.show()
```



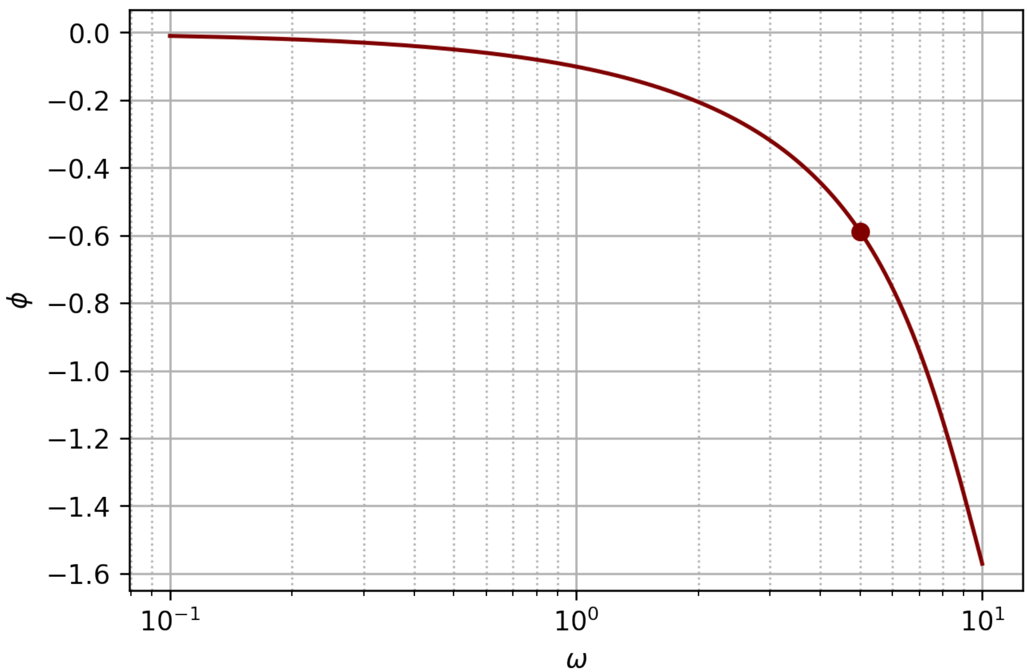
2.3.1 Frequency Response

[16]:
```
# Plotting the magnitude response
omega_ = np.linspace(0.1, 10, 1000)
c_nums = 1/((1j*omega_)**2 + 10*1j*omega_ + 100)

plt.xscale('log')
plt.plot(omega_, np.abs(c_nums))
plt.scatter(5, 0.0111, zorder=2)
plt.xlabel(r'$\omega$')
plt.ylabel(r'Magnitude (not decibels)')
plt.grid(which='minor', ls=':')
plt.show()
```



[17]:
```
# Plotting the phase response
plt.xscale('log')
plt.plot(omega_, np.angle(c_nums))
plt.scatter(5, -0.588, zorder=2)
plt.xlabel(r'$\omega$')
plt.ylabel(r'$\phi$')
plt.grid(which='minor', ls=':')
plt.show()
```



0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
−1.6
ϕ
10⁻¹
10⁰
10¹
ω