

Machine Design Final Exam

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```
[1]: # Notebook Preamble
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')

def sub_return(expression, substitution, sym=None):
    """
    Displays a substituted expression and returns the value.

    :param expression: Symbolic expression
    :param substitution: A list of tuples to be passed into the .sub method
    :param sym: An optional symbol to display prior to operations
    :return: A simplified value of what is substituted.
    """
    with sp.evaluate(False):
        expr_sub = expression.subs(substitution)
        simp = expr_sub.doit().n()
        if sym is None:
            lev1 = sp.Eq(expression, expr_sub)
            lev2 = sp.Eq(lev1, simp, evaluate=False)
            display(lev2)
        else:
            lev1 = sp.Eq(sym, expression)
            lev2 = sp.Eq(lev1, expr_sub, evaluate=False)
            lev3 = sp.Eq(lev2, simp, evaluate=False)
            display(lev3)
    return simp

def get_principal(s_x, s_y, t_xy):
    """
    Shows the principal stress calculations for a 2D element.

    :param s_x: normal stress along the x-axis
    :param s_y: normal stress along the y-axis
```

```

:param t_xy: shear stress on face x in the direction of y
:return: principal stresses
"""
s_x_, s_y_, t_xy_ = sp.symbols(r'\sigma_x \sigma_y \tau_{xy}')
symbols_ = sp.symbols(r'\sigma_1 \sigma_2 \tau_1 \tau_2')
expr1_ = (s_x_ + s_y_)/2 + sp.sqrt(((s_x_ - s_y_)/2)**2 + t_xy_**2)
expr2_ = (s_x_ + s_y_)/2 - sp.sqrt(((s_x_ - s_y_)/2)**2 + t_xy_**2)
expr3_ = sp.sqrt(((s_x_ - s_y_)/2)**2 + t_xy_**2)
expr4_ = -sp.sqrt(((s_x_ - s_y_)/2)**2 + t_xy_**2)
sub_list = [(s_x_, s_x), (s_y_, s_y), (t_xy_, t_xy)]
ans = []
for sym_, expr_ in zip(symbols_, [expr1_, expr2_, expr3_, expr4_]):
    ans.append(sub_return(expr_, sub_list, sym=sym_))
return ans

def von_mises(s_x, s_y, s_z, t_xy, t_yz, t_zx):
    return 1/sp.sqrt(2)*sp.sqrt((s_x - s_y)**2 + (s_y - s_z)**2 + (s_z -
↪s_x)**2 + 6*(t_xy**2 + t_yz**2 + t_zx**2))

def log10(x_):
    return sp.log(x_)/sp.log(10)

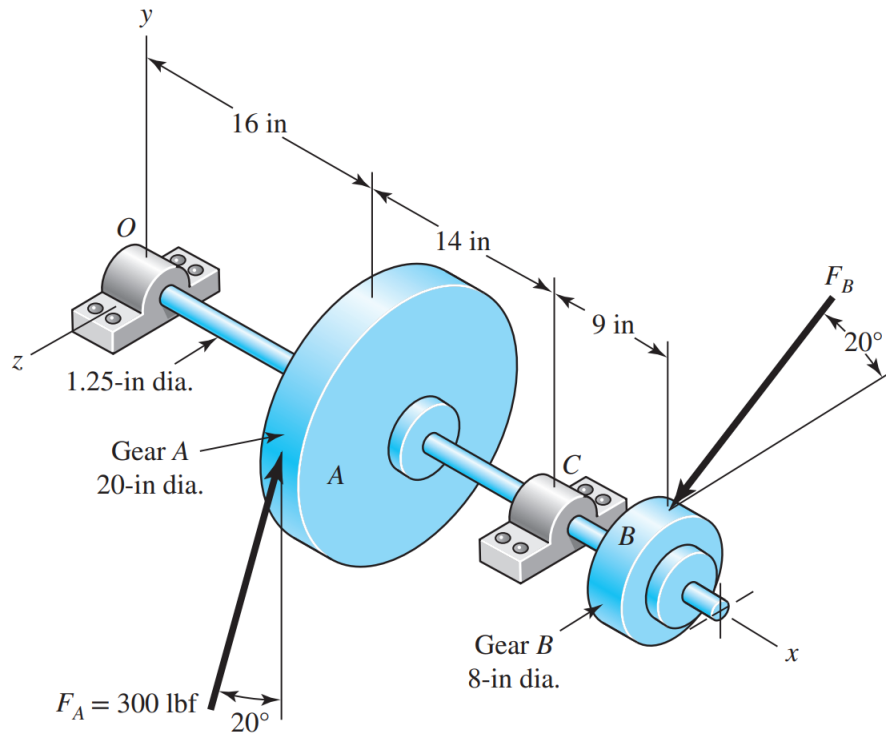
```

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1 Problem 3-83

1.1 Given



A gear reduction unit uses the countershaft shown in the figure above. Gear A receives power from another gear with a transmitted force F_A applied at the 20° pressure angle as shown. The power is transmitted through the shaft and delivered through gear B through a transmitted force F_B at the pressure angle shown.

1.2 Find

- Determine the force F_B , assuming the shaft is running at a constant speed.
- Find the bearing reaction forces, assuming the bearings act as simple supports.
- Draw a shear-force and bending moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.
- At the point of maximum bending moment, determine the bending stress and the torsional shear stress.
- At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

1.3 Solution

1.3.1 Part A and Part B

F_B and the reaction forces may be found by summing the forces and taking the moments about the point O .

```
[2]: # Using matrices takes less thinking and more letting python do the work.
Oy, Oz, Cy, Cz, F_B = sp.symbols('O_y O_z C_y C_z F_B')
zero = sp.ZeroMatrix(3, 1)
F_A_mat = sp.Matrix([0, 300*sp.cos(sp.rad(20)), -300*sp.sin(sp.rad(20))])
F_B_mat = sp.Matrix([0, -F_B*sp.sin(sp.rad(20)), F_B*sp.cos(sp.rad(20))])
O = sp.Matrix([0, Oy, Oz])
C = sp.Matrix([0, Cy, Cz])

# Sum forces
sys1 = sp.Eq(O + F_A_mat + F_B_mat + C, zero)

# Sum moments about O
rA = sp.Matrix([16, 0, 10])
rC = sp.Matrix([16 + 14, 0, 0])
rB = sp.Matrix([16 + 14 + 9, 4, 0])
sys2 = sp.Eq(rA.cross(F_A_mat) + rC.cross(C) + rB.cross(F_B_mat), zero)

sol = sp.solve([sys1, sys2], dict=True)[0]
for key, value in sol.items():
    display(sp.Eq(key, value.n())) # lbf
```

$$C_y = 183.118820416782$$

$$C_z = -861.477082334154$$

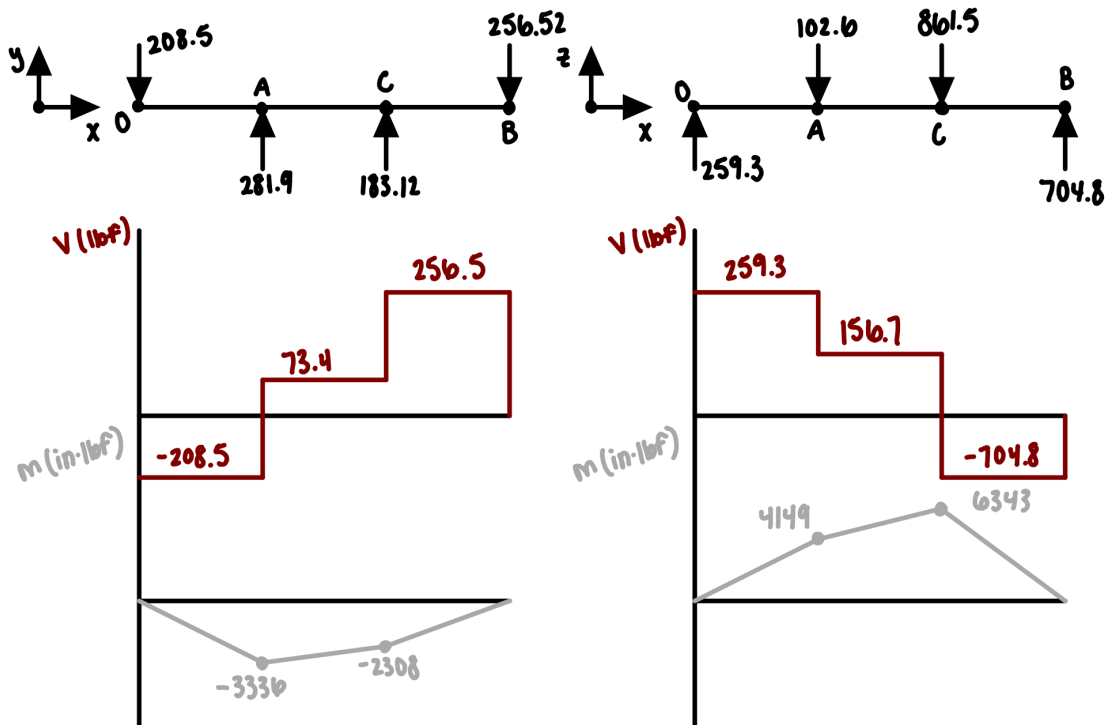
$$F_B = 750.0$$

$$O_y = -208.511499158303$$

$$O_z = 259.313659742423$$

The reaction forces above assume all positive directions.

1.3.2 Part C



1.3.3 Part D

The magnitudes of the moment at section A and section C must be compared.

```
[3]: M_A = sp.sqrt(sp.S('3336.18399')**2 + sp.S('4149.01856')**2)
M_C = sp.sqrt(sp.S('6342.92519')**2 + sp.S('2308.63597')**2)
display(M_A, M_C)
```

5323.95328927262

6750.00000058625

```
[4]: # M_C is the greater value
M_C = sp.S(6750) # in*lb
d = sp.S('1.25')
sig = M_C*32/(sp.pi*d**3) # psi
T = 300*sp.cos(sp.rad(20))*10 # static torque
tau = T*16/(sp.pi*d**3) # psi
display(sig.n(), tau.n())
```

35202.5269328378

7351.01217595663

1.3.4 Part E

$$\sigma_1 = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} + \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 35203.0 + \sqrt{7351.0^2 + \left(\left(-\frac{1}{2}\right) 0 + \frac{1}{2} \cdot 35203.0\right)^2} = 36675.875$$

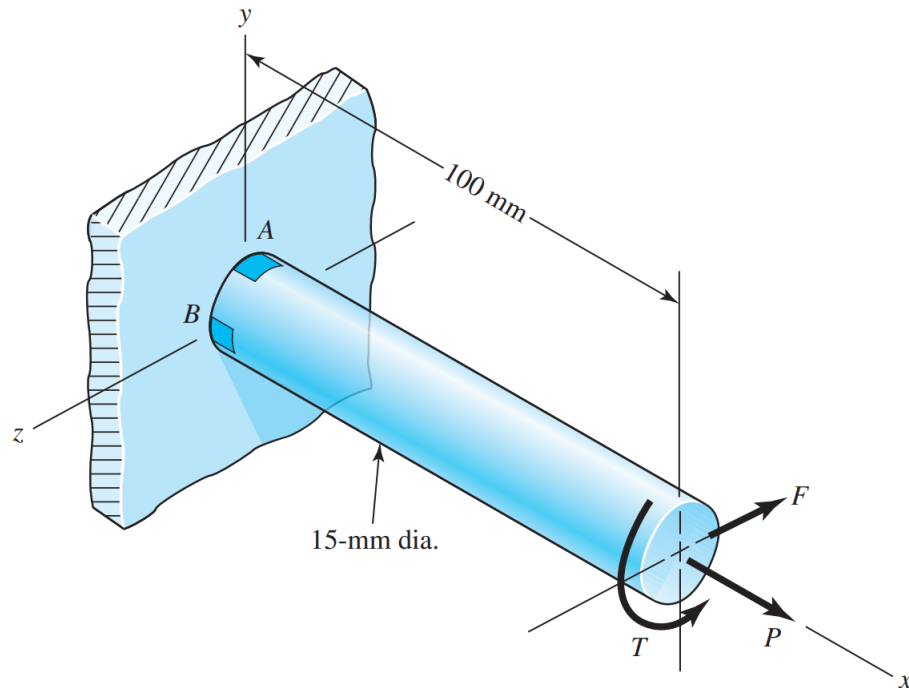
$$\sigma_2 = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} - \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = -\sqrt{7351.0^2 + \left(\left(-\frac{1}{2}\right) 0 + \frac{1}{2} \cdot 35203.0\right)^2} + \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 35203.0 = -1473.375$$

$$\tau_1 = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = \sqrt{7351.0^2 + \left(\left(-\frac{1}{2}\right) 0 + \frac{1}{2} \cdot 35203.0\right)^2} = 19074.625$$

$$\tau_2 = -\sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = -\sqrt{7351.0^2 + \left(\left(-\frac{1}{2}\right) 0 + \frac{1}{2} \cdot 35203.0\right)^2} = -19074.625$$

2 Problem 5-36

2.1 Given



The figure above illustrates that the factor of safety for a machine element depends on the particular point selected for analysis. The bar is made of AISI 1006 cold-drawn steel and is loaded by the forces $F = 0.55 \text{ kN}$, $P = 4.0 \text{ kN}$, and $T = 25 \text{ N} \cdot \text{m}$.

2.2 Find

Find the factor of safety for both elements using the distortion energy method.

2.3 Solution

The yield strength is $S_y = 280 \text{ MPa}$.

2.3.1 Element A

```
[6]: P = sp.S(4_000)
     Sy = sp.S(280e6)
     T = sp.S(25)
     d = sp.S('0.015')
     F = sp.S('0.55e3')
     l = sp.S('0.1')
```



```
sig_x = 4*P/(sp.pi*d**2)
tau_xy = 16*T/(sp.pi*d**3) - 4*sp.S('0.55e3')/(3*(sp.pi/4)*(sp.S('0.015')**2))
display(sig_x.n(), tau_xy.n()) # Pa
```

22635369.6841807

33575798.364868

```
[7]: sig_prime = von_mises(sig_x, 0, 0, tau_xy, 0, 0)
sig_prime.n() # Pa
```

[7]: 62404828.8857068

```
[8]: (Sy/sig_prime).n()
```

[8]: 4.48683226922735

2.3.2 Element B

```
[9]: sig_x = 32*F*1/(sp.pi*d**3) + 4*P/(sp.pi*d**2)
tau_xy = 16*T/(sp.pi*d**3)
display(sig_x.n(), tau_xy.n()) # Pa
```

188628080.701506

37725616.1403011

```
[10]: sig_prime = von_mises(sig_x, 0, 0, tau_xy, 0, 0)
sig_prime.n() # Pa
```

[10]: 199625196.727842

```
[11]: (Sy/sig_prime).n()
```

[11]: 1.40262854884865

3 Problem 6-62

3.1 Given

The material properties of a machine part are $S_{ut} = 85 \text{ ksi}$, $f = 0.86$, and a fully corrected endurance limit of $S_e = 45 \text{ ksi}$. The part is to be cycled at $\sigma_a = 35 \text{ ksi}$ and $\sigma_m = 30 \text{ ksi}$ for $12 \cdot 10^3$ cycles.

3.2 Find

Using the Goodman criterion, estimate the new endurance limit after cycling.

- Use Miner's method.
- Use Manson's method.

3.3 Solution

3.3.1 Part A

```
[12]: Se = sp.S(45)
      Sut = sp.S(85)
      sig_a = sp.S(35)
      sig_m = sp.S(30)
      N = sp.S(12_000)
      # f = sp.S('1.06') - sp.S('2.8e-3')*Sut + sp.S('6.9e-6')*Sut**2
      f = sp.S('0.86')

      sig_ar = sig_a/(1 - (sig_m/Sut)) # 6-59
      sig_ar.n() # ksi
```

```
[12]: 54.0909090909091
```

```
[13]: a = (f*Sut)**2/Se
      b = -sp.Rational(1, 3)*log10(f*Sut/Se)
      display(a, b.n())
```

```
118.746888888889
```

```
-0.0702349543941723
```

The total number of cycles needs to be calculated, then the remaining number of cycles may be found.

```
[14]: N1 = (sig_ar/a)**(1/b)
      N1.n()
```

```
[14]: 72815.2378713139
```

```
[15]: N_remaining = N1 - N  
      N_remaining.n()
```

```
[15]: 60815.2378713139
```

```
[16]: a_prime = sig_ar/(N_remaining**b)  
      Se_prime = a_prime*1e6**b  
      Se_prime.n()  # ksi
```

```
[16]: 44.4344130331323
```

3.3.2 Part B

See p. 355.

```
[17]: Sf = f*Sut  
      b_prime = log10(Sf/sig_ar)/log10(1e3/N_remaining)  
      a_prime = sig_ar/(N_remaining**b_prime)  
      Se_prime = a_prime*1e6**b_prime  
      Se_prime.n()  # ksi
```

```
[17]: 44.0529868187135
```

4 Problem 8-26

4.1 Given

A $\frac{3}{4}$ in-16 UNF series SAE grade 5 bolt has a $\frac{3}{4}$ -in ID steel tube 10 in long, clamped between washer faces of bolt and nut by turning the nut snug and adding one-third of a turn. The tube OD is the washer face diameter $d_w = 1.5d = 1.5(0.75) = 1.125$ in.

4.2 Find

- Determine the bolt stiffness, the tube stiffness, and the joint constant C .
- When the one-third turn of the nut is applied, what is the initial tension F_i in the bolt?

4.3 Solution

4.3.1 Part A

The bolt stiffness comes from Eq. 8-17.

```
[18]: l = 10 # inches
      H = sp.Rational(41, 64)
      L = l + H
      L.n()
```

```
[18]: 10.640625
```

```
[19]: d = sp.S('0.75')
      L = sp.S('10.75') # to the nearest quarter inch
      LT = 2*d + sp.S('0.5')
      ld = L - LT
      lt = l - ld
      Ad = sp.pi*d**2/4
      At = sp.S('0.373') # table 8-2
      E = sp.S(30e6) # psi (Table A-5)
      kb = Ad*At*E/(Ad*lt + At*ld)
      kb.n() # psi
```

```
[19]: 1295495.93286476
```

The stiffness the member may be calculated as a long bar (Eq. 4-4),

$$k = \frac{AE}{l}$$

```
[20]: km = sp.pi/4*(sp.S('1.125')**2 - sp.S('0.75')**2)*E/l
      km.n()
```

```
[20]: 1656699.25091649
```

On p. 448, the joint constant is

$$C = \frac{k_b}{k_b + k_m}$$

```
[21]: C = kb/(kb + km)
      C.n()
```

```
[21]: 0.43882462107585
```

4.3.2 Part B

The total displacement may be described as,

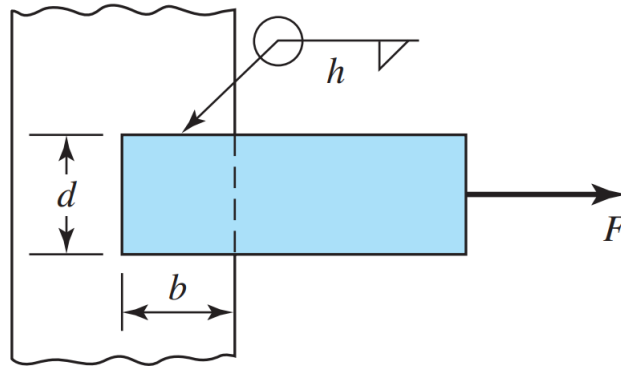
$$\delta = \delta_b + \delta_m = \frac{N_{turns}}{N_{threads}}$$

```
[22]: Fi, kb_, km_, Nt_, N = sp.symbols('F_i k_b k_m N_t N')
      db = Fi/kb_
      dm = Fi/km_
      eq = sp.Eq(db + dm, Nt_/N)
      sol = sp.solve(eq, Fi)[0]
      _ = sub_return(sol, [(km_, km.n(5)), (kb_, kb.n(5)), (Nt_, sp.Rational(1, 3)),
      ↪(N, 16)], sym=Fi) # lbf
```

$$F_i = \frac{N_t k_b k_m}{N (k_b + k_m)} = \frac{1}{3} \cdot 1.2955 \cdot 10^6 \cdot 1.6567 \cdot 10^6 \cdot \frac{1}{16} \cdot \frac{1}{1.2955 \cdot 10^6 + 1.6567 \cdot 10^6} = 15145.84375$$

5 Problem 9-13

5.1 Given



A steel bar of thickness h is welded to a vertical support as shown in the figure above.

$$b = 50 \text{ mm}$$

$$d = 50 \text{ mm}$$

$$h = 5 \text{ mm}$$

$$F = 100 \text{ kN}$$

5.2 Find

What is the shear stress in the throat of the welds due to the force F ?

5.3 Solution

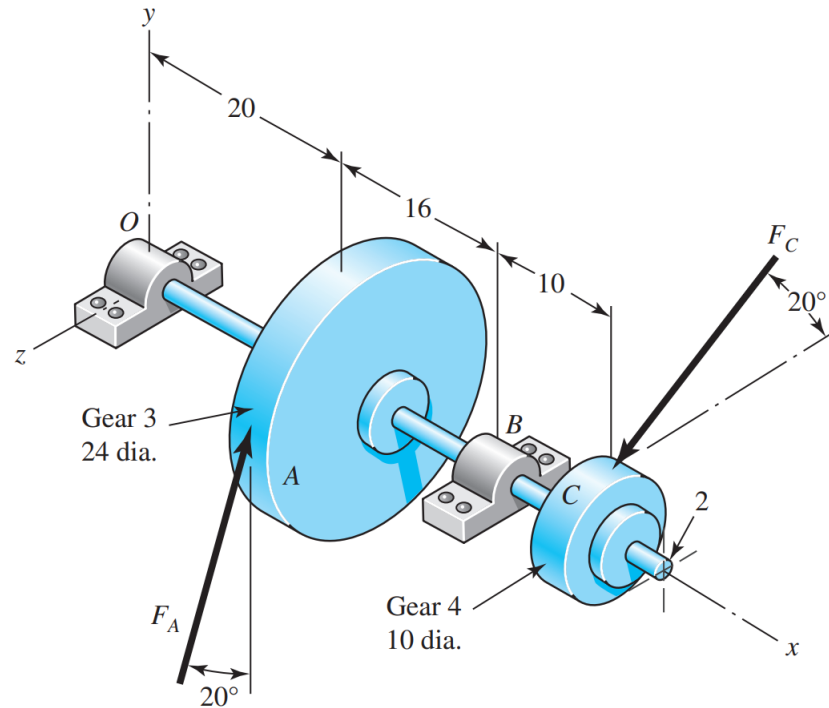
Use Eq. 9-3

```
[23]: F = sp.S(100_000)
      h = sp.S('0.005')
      l = sp.S('0.2')
      tau = sp.sqrt(2)*F/(h*l)
      tau.n() # Pa
```

```
[23]: 141421356.23731
```

6 Problem 11-34

6.1 Given



The figure above is a geared countershaft with an overhanging pinion at C . The force on gear A is $F_A = 600 \text{ lbf}$, and the shaft is to run at a speed of 420 rpm. The solution of the statics problem gives the force of the bearings against the shaft at O as $R_O = -387j + 467k \text{ lbf}$, and at B as $R_B = 316j - 1615k \text{ lbf}$.

6.2 Find

Select an angular contact ball bearing from Table 11-2 for mounting at O and an O2-series cylindrical roller bearing from Table 11-3 for mounting at B . Specify the bearings required, using an application factor of 1.2, a desired life of 40,000 hours, and a combined reliability goal of 0.95, assuming distribution data from manufacturer 2 in Table 11-6.

6.3 Solution

```
[24]: # Find xD
xD = 60*sp.S(40_000)*420/1e6
xD
```

[24]: 1008.0

6.3.1 Bearing O

```
[25]: x0, theta = sp.S('0.02'), sp.S('4.459')
      b = sp.S('1.483')
      RD = sp.sqrt(sp.S('0.95'))

      kN = sp.S('4.44822')/sp.S(1000)

      a = sp.S(3)
      af = sp.S('1.2')
      FD = sp.sqrt(sp.S(-387)**2 + sp.S(467)**2)
      C10 = af*FD*(xD/(x0 + (theta - x0)*(sp.log(1/RD))**(1/b))))**(1/a)
      (C10*kN).n() # kN
```

[25]: 44.2269113767268

For bearing O, select the 02-55 mm angular contact bearing with a rating of 46.2 kN.

6.3.2 Bearing B

```
[26]: a = sp.Rational(10, 3)
      af = sp.S('1.2')
      FD = sp.sqrt(sp.S(316)**2 + sp.S(-1615)**2)
      C10 = af*FD*(xD/(x0 + (theta - x0)*(sp.log(1/RD))**(1/b))))**(1/a)
      (C10*kN).n() # kN
```

[26]: 92.3909500897365

For bearing B, select the 02-75 mm with a rating of 93.1 kN.