FEA Homework 4

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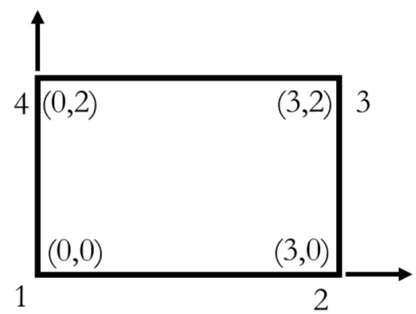
[1]:
```python
# Notebook Preamble
import sympy as sp
import numpy as np
from IPython.core.interactiveshell import InteractiveShell
import matplotlib.pyplot as plt

InteractiveShell.ast_node_interactivity = 'all'
plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1 Problem 1

1.1 Given



For a rectangular element, the displacements at four nodes are given by

$$u_1 = 0$$

$$v_1 = 0$$

$$u_2 = -0.5$$

$$v_2 = -0.5$$

$$u_3 = 0.75$$

$$v_3 = 1.25$$

$$u_4 = 0.5$$

$$v_4 = 1$$

1.2 Find

a. Calculate the displacement (u, v) at point $(x, y) = (0.7, 1.3)$.
b. Calculate the strain ϵ_{xx} at point $(x, y) = (0.7, 1.3)$

1.3 Solution

From Fig. 3.4-1 in the text, $a = 1.5$ and $b = 1$. These are the center to edge dimensions of the rectangle and are used in the shape functions/strain displacement matrix.

The analysis of the bilinear quadrilateral depends on the origin being at the center of the rectangle. Therefore, the point $(0.7, 1.3)$ relative to the origin at the center is $(0.7, 1.3) - (a, b) = (-0.8, 0.3)$.

1.3.1 Part A

The displacements u, v can be defined as the dot product between the shape function and the displacement vectors

$$u(x,y) = \vec{N} \cdot \vec{u}$$
$$v(x,y) = \vec{N} \cdot \vec{v}$$

where the shape function in the vector form is,

$$N = \begin{bmatrix} \frac{(a-x)(b-y)}{4ab} \\ \frac{(a+x)(b-y)}{4ab} \\ \frac{(a+x)(b+y)}{4ab} \\ \frac{(a-x)(b+y)}{4ab} \end{bmatrix}$$

and $\vec{u}$ and $\vec{v}$ are $\langle u_1, u_2, u_3, u_4 \rangle$ and $\langle v_1, v_2, v_3, v_4 \rangle$.

[2]:

```
# Define known parameters
# The underscore denotes a numerical value, while no underscore denotes a␣
↪symbol.
a_, b_ = 1.5, 1
x_, y_ = -0.8, 0.3
u_, v_ = [0, -0.5, 0.75, 0.5], [0, -0.5, 1.25, 1]
d_ = np.array(list(zip(u_, v_))).flatten()

# Define symbols
a, b, x, y = sp.symbols('a b x y')
u_vec, v_vec = sp.Matrix(u_), sp.Matrix(v_)
d_vec = sp.Matrix(d_)

# Shape function
N = 1/(4*a*b)*sp.Matrix([(a - x)*(b - y), (a + x)*(b - y), (a + x)*(b + y), (a␣
↪- x)*(b + y)])

sub = [(a, a_), (b, b_), (x, x_), (y, y_)]
N_ = N.subs(sub)

u = sp.DotProduct(N, u_vec)
v = sp.DotProduct(N, v_vec)
sp.Eq(u, N_.dot(u_vec), evaluate=False)
sp.Eq(v, N_.dot(v_vec), evaluate=False)
```

[2]:

$$DotProduct\left(\begin{bmatrix} \frac{(a-x)(b-y)}{4ab} \\ \frac{(a+x)(b-y)}{4ab} \\ \frac{(a+x)(b+y)}{4ab} \\ \frac{(a-x)(b+y)}{4ab} \end{bmatrix}, \begin{bmatrix} 0 \\ -0.5 \\ 0.75 \\ 0.5 \end{bmatrix}\right) = 0.322083333333333$$

[2]:

$$DotProduct\left(\begin{bmatrix}\frac{(a-x)(b-y)}{4ab}\\\frac{(a+x)(b-y)}{4ab}\\\frac{(a+x)(b+y)}{4ab}\\\frac{(a-x)(b+y)}{4ab}\end{bmatrix},\begin{bmatrix}0\\-0.5\\1.25\\1\end{bmatrix}\right)=0.647083333333333$$

Thus, the displacements u, v at $(-0.8, 0.3)$ are $u = 0.322$ and $v = 0.647$.

1.3.2 Part B

The strain displacement matrix is

[3]:
```python
# Getting B
B = 1/(4*a*b)*sp.Matrix([
    [-(b - y), 0, b - y, 0, b + y, 0, -(b + y), 0],
    [0, -(a - x), 0, -(a + x), 0, (a + x), 0, (a - x)],
    [-(a - x), -(b - y), -(a + x), b - y, a + x, b + y, a - x, -(b + y)]
])
sp.Eq(sp.Symbol('B'), B, evaluate=False)
```

[3]:

$$B=\begin{bmatrix}\frac{-b+y}{4ab}&0&\frac{b-y}{4ab}&0&\frac{b+y}{4ab}&0&\frac{-b-y}{4ab}&0\\0&\frac{-a+x}{4ab}&0&\frac{-a-x}{4ab}&0&\frac{a+x}{4ab}&0&\frac{a-x}{4ab}\\\frac{-a+x}{4ab}&\frac{-b+y}{4ab}&\frac{-a-x}{4ab}&\frac{b-y}{4ab}&\frac{a+x}{4ab}&\frac{b+y}{4ab}&\frac{a-x}{4ab}&\frac{-b-y}{4ab}\end{bmatrix}$$

The strain may be calculated using $\epsilon = Bd$.

[4]:
```python
Bd = sp.MatMul(B, d_vec)
Bd_doit = Bd.doit().subs(sub)
sp.Eq(Bd, Bd_doit, evaluate=False)
```

[4]:

$$\begin{bmatrix}\frac{-b+y}{4ab}&0&\frac{b-y}{4ab}&0&\frac{b+y}{4ab}&0&\frac{-b-y}{4ab}&0\\0&\frac{-a+x}{4ab}&0&\frac{-a-x}{4ab}&0&\frac{a+x}{4ab}&0&\frac{a-x}{4ab}\\\frac{-a+x}{4ab}&\frac{-b+y}{4ab}&\frac{-a-x}{4ab}&\frac{b-y}{4ab}&\frac{a+x}{4ab}&\frac{b+y}{4ab}&\frac{a-x}{4ab}&\frac{-b-y}{4ab}\end{bmatrix}\begin{bmatrix}0\\0\\-0.5\\-0.5\\0.75\\1.25\\0.5\\1.0\end{bmatrix}=\begin{bmatrix}-0.00416666666666666\\0.5875\\0.333333333333333\end{bmatrix}$$

Thus, $\epsilon_{xx} = -0.00417$.