Machine Design Test 2

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[1]:
```python
# Notebook Preamble
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

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1 Problem 6-4

1.1 Given

A steel rotating-beam test specimen has an ultimate strength of 1600 MPa.

1.2 Find

Estimate the life of the specimen if it is tested at a completely reversed stress amplitude of 900 MPa.

1.3 Solution

The first step is to find S_e'.

[2]:
```
sig_ar, S_ut = sp.S(900), sp.S(1600)

# Eq. 6-10
S_e_prime = sp.S(700)
S_e_prime  # MPa
```

[2]: 700

The S_e' value will be used in place of S_e from Figure 6-23 description. We can use the following relationships to determine N.

$$N = \left(\frac{\sigma_{ar}}{a}\right)^{1/b}$$
$$a = \frac{(fS_{ut})^2}{S_e}$$
$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{Se}\right)$$

The value of f is 0.77 from Figure 6-23 (estimated even though it is off the graph).

[3]:
```
def log10(x_):
    return sp.log(x_)/sp.log(10)

f = sp.S('0.77')
a = (f*S_ut)**2/S_e_prime
b = -sp.Rational(1, 3)*log10(f*S_ut/S_e_prime)

display(sp.Eq(sp.Symbol('a'), a.n()),
        sp.Eq(sp.Symbol('b'), b.n()))

N = ((sig_ar/a)**(1/b)).n()
N  # cycles
```

$a = 2168.32$

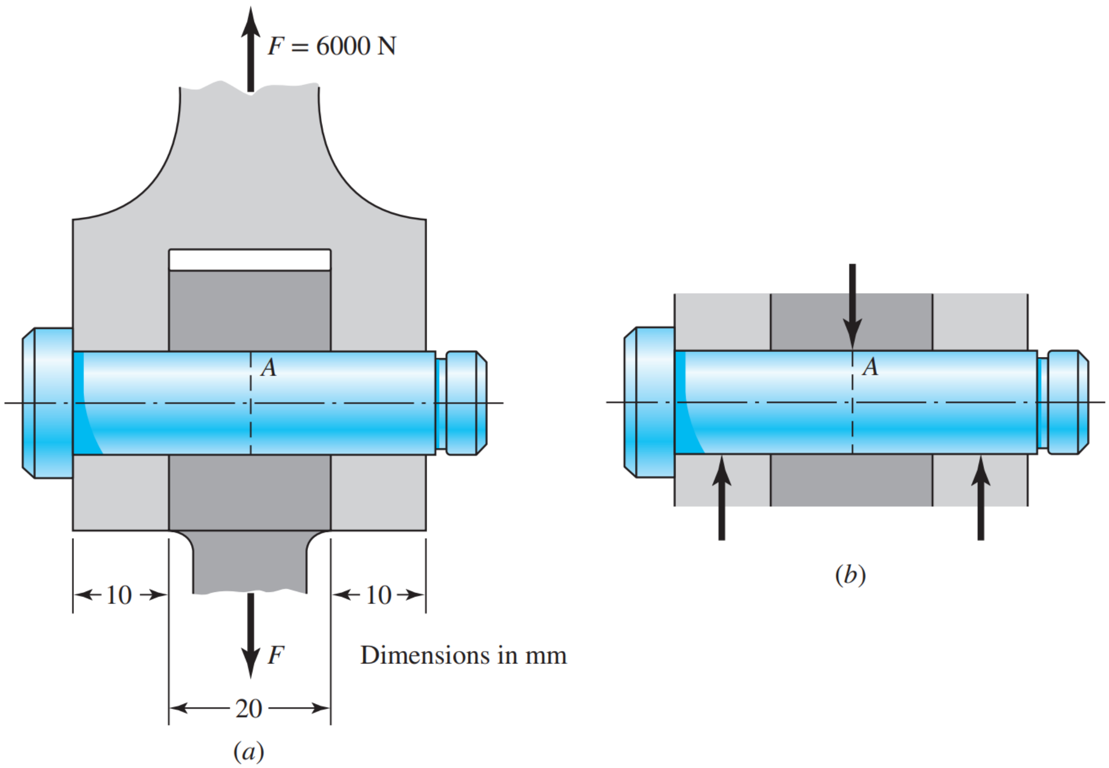
$b = -0.0818375559380499$

[3]: 46379.6905856764

The life of the specimen is 46400 cycles.

2 Problem 6-11

2.1 Given



A pin in a knuckle joint is shown in part (a) of the figure above. The joint is subject to a repeatedly applied and released load of 6000 N in tension. Assume the loading on the pin is modeled as concentrated forces as shown in part (b) of the figure. The shaft is made from AISI 1018 hot-rolled steel that has been machined to its final diameter.

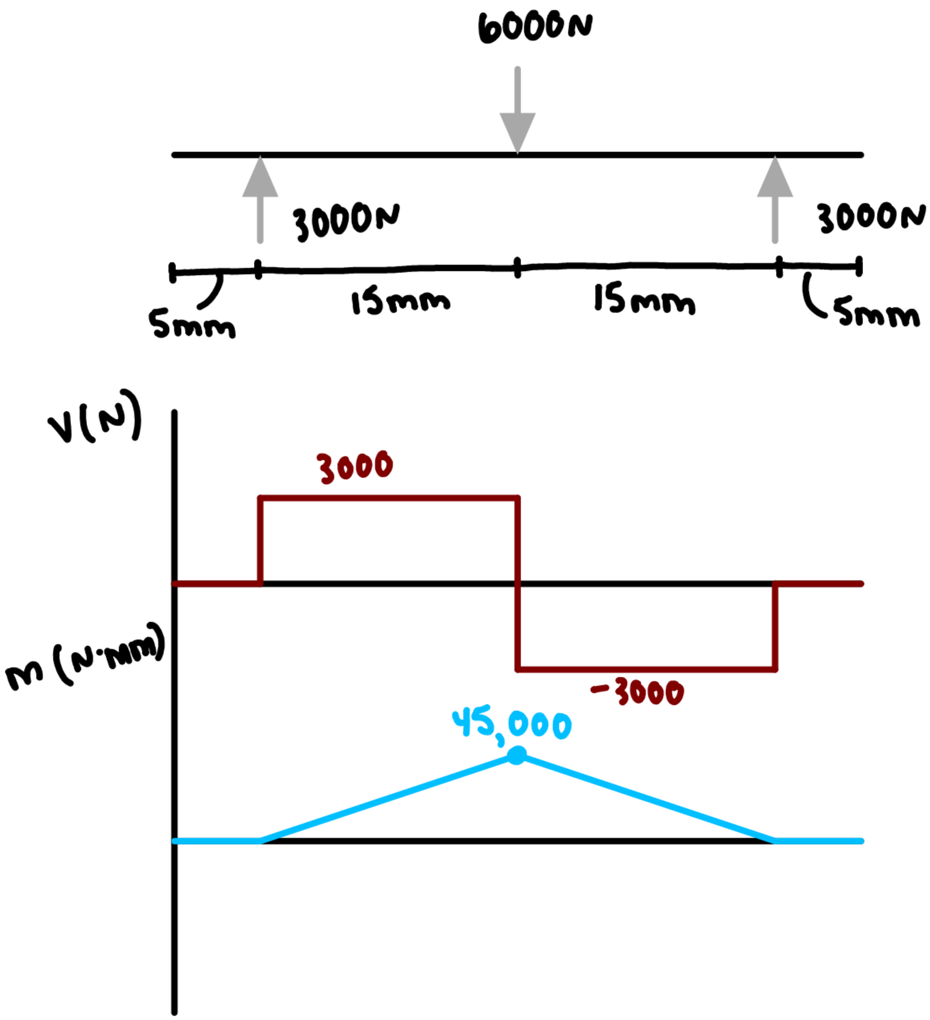
[4]:
```
# Table A-20
Sut, Sy = sp.S(400), sp.S(220)  # MPa
```

2.2 Find

Based on a stress element on the outer surface at the cross-section A, determine a suitable diameter of the pin, rounded up to the next mm increment, to provide at least a factor of safety of 1.5 for both infinite fatigue life and for yielding.

2.3 Solution

2.3.1 Yielding



The equation for the maximum stress may be found using the bending moment diagram above.

[5]:
```python
d = sp.Symbol('d', real=True, positive=True)
M = 45_000  # N*mm^2
c = d/2
I = sp.pi/64*d**4
sig_max = (M*c/I).n()
sig_max
```

[5]: $\frac{458366.236104659}{d^3}$

For yielding,

$$n = \frac{S_y}{\sigma_{max}}$$

[6]:
```
eq1 = sp.Eq(sp.S('1.5'), Sy/sig_max)
display(eq1)
d_ = sp.solve(eq1)[0]
display(sp.Eq(d, d_))
```

$1.5 = 0.000479965544298441d^3$

$d = 14.6204385310094$

So for yielding, a suitable diameter would be $d = 15\,mm$.

2.3.2 Infinite Life

The Goodman method will be used to determine the diameter that results in an infinite life with a factor of safety of 1.5. The relationship for this is,

$$\frac{1}{n_f} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}$$

The load is not reversible, but is fluctuating on and off, so the mean stress is positive.

[7]:
```
# Getting Se
# Equations come from the road map starting on page 359

S_e_prime = Sut/2

a, b = sp.S('3.04'), sp.S('-0.217')  # Table 6-2 (Machined)
k_a = (a*Sut**b).n()
k_b = sp.S('1.24')*d**sp.S('-0.107')

Se = k_a*k_b*S_e_prime
Se
```

[7]:

$$\frac{205.436802219618}{d^{0.107}}$$

[8]:
```
# Applying the Goodman relationship
sig_a = sig_m = sig_max/2  # minimum stress is 0
eq1 = sp.Eq(sig_a/Se + sig_m/Sut, 1/sp.S('1.5'))
display(eq1)
d_inf = sp.nsolve(eq1, d, 10)  # Numerical solver
display(sp.Eq(d, d_inf))
```

$$\frac{1115.58939574676}{d^{2.893}} + \frac{572.957795130823}{d^3} = 0.666666666666667$$

$d = 14.5624553895619$

The above equation is complex and the diameter can only be solved using the numerical solver in `sympy`. The results show that the $15\,mm$ diameter should be used for both yielding and infinite life.

3 Problem 3-61

3.1 Given

A machine part will be cycled at $\pm 350\,MPa$ for $5 \cdot 10^3$ cycles. Then loading will be changed to $\pm 260\,MPa$ for $5 \cdot 10^4$ cycles. Finally, the load will be changed to $\pm 225\,MPa$.

$$S_{ut} = 350\,MPA$$
$$f = 0.9$$
$$S_e = 210\,MPa$$

[9]:
```
Sut, f, Se = sp.S(530), sp.S('0.9'), sp.S(210)
```

3.2 Find

How many cycles of operation can be expected at this stress level using Miner's method?

3.3 Solution

Minor's method is,

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = 1$$

[10]:
```
def get_N(s_max, s_min, f_value, Sut_value, Se_value):
    s_m = (s_max + s_min)/2
    s_a = (s_max - s_min)/2
    s_ar = s_a/(1 - s_m/Sut_value)
    a_ = (f_value*Sut_value)**2/Se_value
    b_ = -sp.Rational(1, 3)*log10(f_value*Sut_value/Se_value)
    N_ = (s_ar/a_)**(1/b_)
    return N_.n()

n1, n2 = sp.S(5e3), sp.S(5e4)
N1 = get_N(sp.S(350), -sp.S(350), f, Sut, Se)
N2 = get_N(sp.S(260), -sp.S(260), f, Sut, Se)
N3 = get_N(sp.S(225), -sp.S(225), f, Sut, Se)

n3 = sp.Symbol('n_3')
eq1 = sp.Eq(n1/N1 + n2/N2 + n3/N3, 1)
display(eq1)
n3_ = sp.solve(eq1)[0]
display(sp.Eq(n3, n3_))
```

$1.78766904825898 \cdot 10^{-6} n_3 + 0.670863205354351 = 1$

$n_3 = 184115.060316224$

4 Problem 8-12

4.1 Given

An $M14 \times 2$ hex head bolt with a nut is used to clamp together two 15 mm steel plates. There is a 14R metric plain washer under the nut.

4.2 Find

a. Determine a suitable length for the bolt, rounded up to the nearest 5 mm.
b. Determine the bolt stiffness.
c. Determine the stiffness of the members.

4.3 Solution

4.3.1 Part A

The bolt dimensions are found in Table A-31.

[11]:
```
l, H, t = sp.S(30), sp.S('12.8'), sp.S('3.5')
L = l + H + t
L  # mm
```

[11]:

46.3

The bolt length is $50\,mm$.

4.3.2 Part B

The fastener stiffness along with more relationships come from Table 8-7.

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

[12]:
```
d = sp.S(14)  # mm
E = sp.S('207')  # GPa
L = 50  # mm

Ad = sp.pi*d**2/4
At = sp.S(115)
Lt = 2*d + 6
ld = L - Lt
lt = l + t - ld
kb = Ad*At*E/(Ad*lt + At*ld)
kb.n()  # MN/m
```

[12]:

808.240665447356

4.3.3 Part C

[13]:
```
# Eq. 8-22
x = sp.S('0.5774')
km = (x*sp.pi*E*d)/(2*sp.log(5*(x*(l + t) + sp.S('0.5')*d)/(x*(l + t) + sp.S('2.
↪5')*d)))
km.n()  # MN/m
```

[13]: 2968.8853220629

5 Problem 8-25

5.1 Given

An $M14 \times 2$ hex head bolt with a nut is used to clamp together two 20 mm steel plates.

5.2 Find

Compare the results of finding the overall member stiffness by use of Equations 8-20, 8-22, and 8-23.

5.3 Solution

[14]:
```
def get_k(E_, dia, t_, D_):
    return ((sp.S('0.5774')*sp.pi*E_*dia)/sp.log(((sp.S('1.155')*t_ + D_ -
 ↪dia)*(D_ + dia))/((sp.S('1.155')*t_ + D_ + dia)*(D_ - dia)))).n()

E = sp.S(207)  # GPa
```

5.3.1 Equation 8-20

The relationship is,

$$k = \frac{0.5774\pi E d}{\ln \frac{(1.155t+D-d)(D+d)}{(1.155t+D+d)(D-d)}}$$

[15]:
```
# The width of the bolt is 21 mm
d = sp.S(14)
w = sp.S(21)
k1 = get_k(E, d, 20, w)
k2 = get_k(E, d, 20, w)

km1 = (1/k1 + 1/k2)**-1
km1  # MN/m
```

[15]: 2761.53476109712

5.3.2 Equation 8-22

The relationship should result in an identical answer because this equation is for two sheets in series with same properties/geometry.

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\frac{0.5774l+0.5d}{0.5774l+2.5d}\right)}$$

[16]:
```python
l = 40
x = sp.S('0.5774')
km2 = (x*sp.pi*E*d)/(2*sp.log(5*(x*l + sp.S('0.5')*d)/(x*l + sp.S('2.5')*d)))
km2.n()  # MN/m
```

[16]: 2761.72060761183

The slight error is because of the rounded numbers in the expression.

5.3.3 Equation 8-23

This relationship is,

$$\frac{k_m}{Ed} = A\exp(Bd/l)$$

The values of A and B come from Table 8-8.

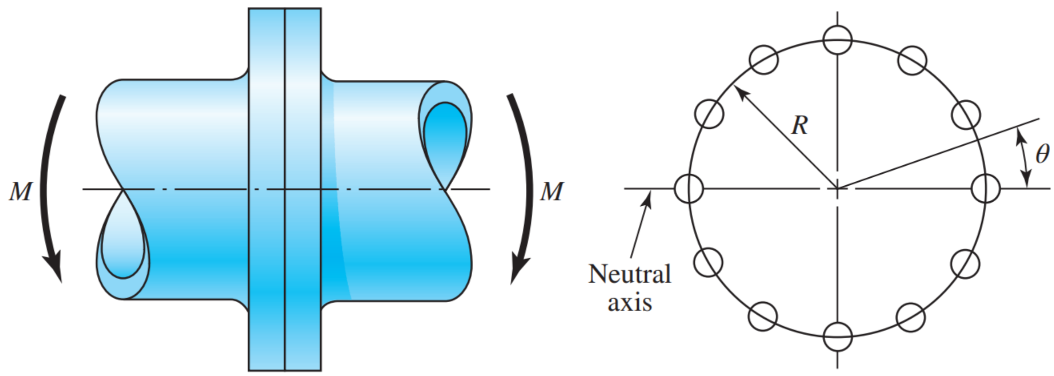
[17]:
```python
A, B = sp.S('0.78715'), sp.S('0.62873')
km3 = E*d*A*sp.exp(B*d/l)
km3.n()  # MN/m
```

[17]: 2842.65903019863

This answer is more conservative than the other two, but is based off a finite element study.

6 Problem 8-45

6.1 Given



Bolts distributed about a bolt circle are often called upon to resist an external bending moment as shown in the figure. The external moment is $12\,kip \cdot in$ and the bolt circle has a diameter of 8 in. The neutral axis for bending is a diameter of the bolt circle. What needs to be determined is the most severe external load seen by a bolt in the assembly.

6.2 Find

a. View the effect of the bolts as placing a line load around the bolt circle whose intensity F'_b, in pounds per inch, varies linearly with the distance from the neutral axis according to the relation $F'_b = F'_{b,max} R \sin\theta$. The load on any particular bolt can be viewed as the effect of the line load over the arc associated with the bolt. For example, there are 12 bolts shown in the figure. Thus, each bolt load is assumed to be distributed on a 30° arc of the bolt circle. Under these conditions, what is the largest bolt load?
b. View the largest load as the intensity $F'_{b,max}$ multiplied by the arc length associated with each bolt and find the largest bolt load.
c. Express the load on any bolt as $F = F_{max}\sin\theta$. Sum the moments due to all the bolts, and estimate the largest bolt load. Compare the results of these three approaches to decide how to attach such problems in the future.

6.3 Solution

6.3.1 Part A

Each bolt carries come fraction of the load, F'_b. The moment arm associated with each fraction is $R\sin\theta$. Adding up each differential,

$$M = \int_0^{2\pi} F'_b R \sin\theta d\theta$$

The maximum load is the value of the hypotenuse, $F_b' = F_{b,max}' R \sin\theta$. The next couple cells can execute this integral symbolically.

[18]:
```
# Show the original to make sure everything looks good
th, Fb, Fb_max, M, R = sp.symbols(r'\theta F_b F_{b\,max} M R')
integral = sp.Integral(Fb*R*sp.sin(th), (th, 0, 2*sp.pi))
sp.Eq(M, integral)
```

[18]:

$$M = \int_0^{2\pi} F_b R \sin(\theta)\, d\theta$$

[19]:
```
integral_max = integral.subs(Fb, Fb_max*R*sp.sin(th))
sp.Eq(M, integral_max)
```

[19]:

$$M = \int_0^{2\pi} F_{b,max} R^2 \sin^2(\theta)\, d\theta$$

The integral may now be evaluated.

[20]:
```
eq1 = integral_max.doit()
sp.Eq(M, eq1)
```

[20]:

$$M = \pi F_{b,max} R^2$$

Using $F_{b,max}$ as the load,

[21]:
```
Fb_max_ = M/(sp.pi*R**2)
new_int = Fb_max_*sp.Integral(R*sp.sin(th), (th, sp.rad(75), sp.rad(105)))
new_int_doit = new_int.doit()
sp.Eq(new_int, new_int_doit.simplify())
```

[21]:

$$\frac{M \int_{\frac{5\pi}{12}}^{\frac{7\pi}{12}} R \sin(\theta)\, d\theta}{\pi R^2} = \frac{M\left(-\sqrt{2} + \sqrt{6}\right)}{2\pi R}$$

[22]:
```
# Substitute
F_max1 = new_int_doit.subs([
    (M, 12_000),
    (R, 4)
])
F_max1.n()  # lbf
```

[22]: 494.307964732685

6.3.2 Part B

[23]:
```
F_max2 = Fb_max_*R*sp.rad(30)
F_max2
```

[23]:

$\frac{M}{6R}$

[24]:
```python
F_max2.subs([
    (M, 12_000),
    (R, 4)
])  # lbf
```

[24]: 500

6.3.3 Part C

[25]:
```python
arms = [sp.sin(sp.rad(i))**2 for i in range(0, 390, 30)]
F_max = sp.Symbol('F_{max}')
eq = sp.Eq(M, F_max*R*sum(arms))
eq
```

[25]: $M = 6F_{max}R$

[26]:
```python
F_max3 = sp.solve(eq, F_max)[0].subs([
    (M, 12_000),
    (R, 4)
])
F_max3  # lbf
```

[26]: 500

In general, the maximum force within a bolt would be $F_{max} = \frac{M}{6R}$.