Machine Design Test 1

June 15, 2022

Gabe Morris

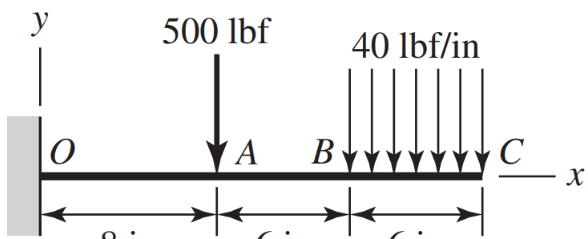
[1]:
```python
# Notebook Preamble
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Markdown

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1 Problem 3-6

1.1 Given

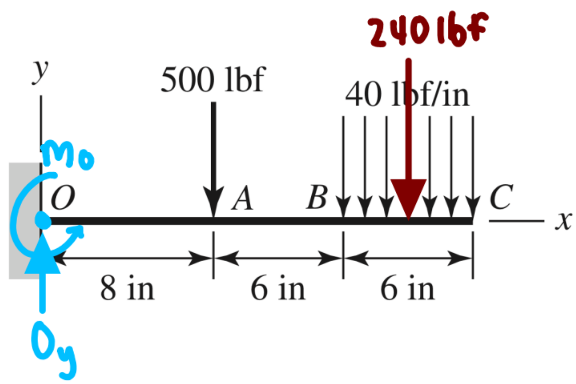


1.2 Find

Find the reaction forces and plot the shear and bending diagram.

1.3 Solution

1.3.1 Reaction Forces

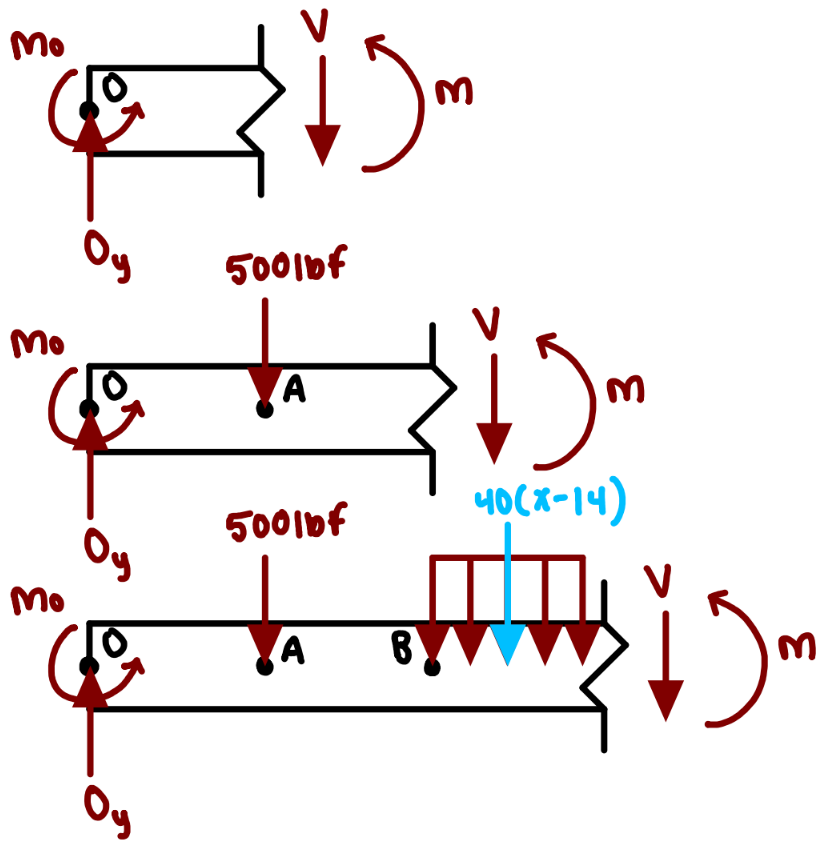


[2]:
```
# Getting the reaction forces
Oy_sym, Mo_sym = sp.symbols('O_y M_o')
Oy = 240 + 500
Mo = 500*8 + 240*17
display(sp.Eq(Oy_sym, Oy), sp.Eq(Mo_sym, Mo))  # lbf and lbf*in
```

$O_y = 740$

$M_o = 8080$

1.3.2 Bending and Moment Diagram



The equation may be described as the piecewise relationship coded below.

[3]:
```
V, M, x = sp.symbols('V M x')

# From O to A
V1 = Oy
M1 = -Mo + Oy*x

# From A to B
V2 = Oy - 500
M2 = -Mo + Oy*x - 500*(x - 8)

# From B to C
V3 = Oy - 500 - 40*(x - 14)
M3 = -Mo + Oy*x - 500*(x - 8) - 40*(x - 14)*(x - 14)/2

eq1 = sp.Eq(V, sp.Piecewise((V1, (x >= 0) & (x < 8)), (V2, (x >= 8) & (x <
    14)), (V3, (x >= 14) & (x <= 20))))
eq2 = sp.Eq(M, sp.Piecewise((M1, (x >= 0) & (x < 8)), (M2, (x >= 8) & (x <
    14)), (M3, (x >= 14) & (x <= 20))))
```

```python
display(eq1, eq2)
```

$$V = \begin{cases} 740 & \text{for}\ x \geq 0 \wedge x < 8 \\ 240 & \text{for}\ x \geq 8 \wedge x < 14 \\ 800 - 40x & \text{for}\ x \geq 14 \wedge x \leq 20 \end{cases}$$

$$M = \begin{cases} 740x - 8080 & \text{for}\ x \geq 0 \wedge x < 8 \\ 240x - 4080 & \text{for}\ x \geq 8 \wedge x < 14 \\ 240x - \frac{(x-14)(40x-560)}{2} - 4080 & \text{for}\ x \geq 14 \wedge x \leq 20 \end{cases}$$

The important key points for shear are shown in the piecewise function expression above. The key points for the bending moment are,

[4]:
```python
points = ['O', 'A', 'B', 'C']
values = [0, 8, 14, 20]
for p, v in zip(points, values):
    display(sp.Eq(sp.Symbol(f'M_{p}'), eq2.rhs.subs(x, v)))  # in lbf*in
```

$M_O = -8080$

$M_A = -2160$

$M_B = -720$

$M_C = 0$

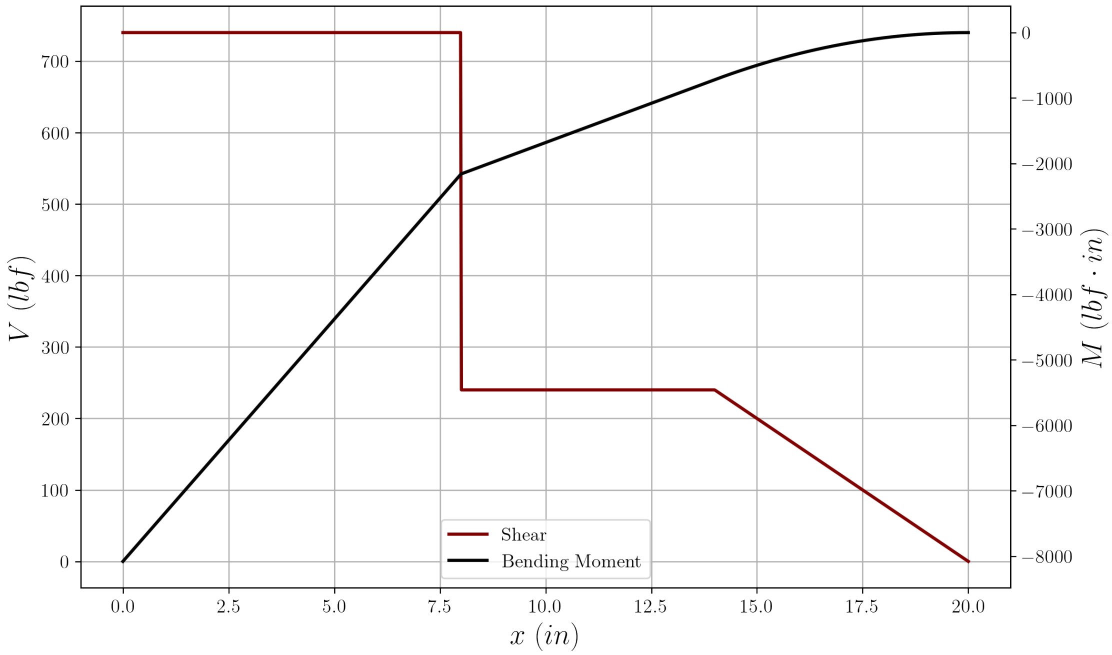
[5]:
```python
# Getting shear and bending diagram
x_ = np.linspace(0, 20, 1000)
V_ = sp.lambdify(x, eq1.rhs, modules='numpy')
M_ = sp.lambdify(x, eq2.rhs, modules='numpy')

fig, ax = plt.subplots()
ax2 = ax.twinx()

ax.plot(x_, V_(x_), label='Shear')
ax2.plot(x_, M_(x_), label='Bending Moment', color='black')

ax2.grid(visible=False)
ax.legend(handles=[ax.lines[0], ax2.lines[0]], loc='lower center')

ax.set_xlabel('$x$ ($in$)')
ax.set_ylabel('$V$ ($lbf$)')
ax2.set_ylabel(r'$M$ ($lbf\cdot in$)')
plt.show()
```



Notice that the graph has a duel y-axis.

2 Problem 3-17

2.1 Given

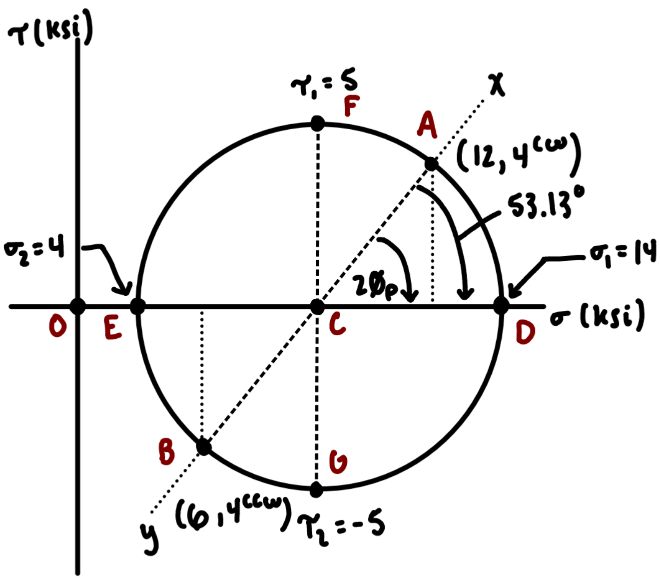
a. $\sigma_x = 12\ ksi$, $\sigma_y = 6\ ksi$, $\tau_{xy} = 4\ ksi\ cw$
b. $\sigma_x = 9\ ksi$, $\sigma_y = 19\ ksi$, $\tau_{xy} = 8\ ksi\ cw$

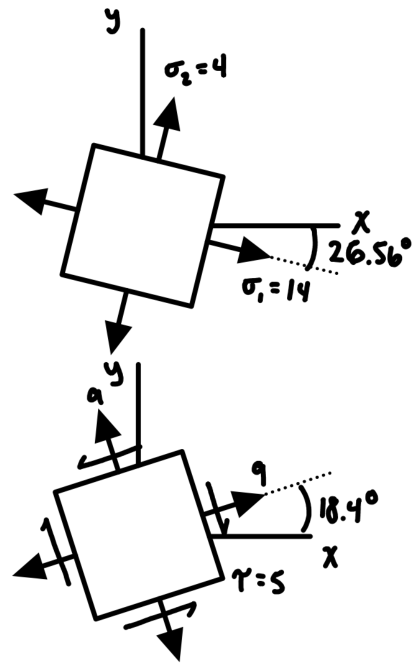
2.2 Find

Draw the plane stress element as seen in Figure 3-11c and d. Also draw Mohr's circle fully labeled.

2.3 Solution

2.3.1 Part A





Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 9.0$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 5.0$$

Principle Stresses:

$$\sigma_1 = C + R = 14.0$$

$$\sigma_2 = C - R = 4.0$$

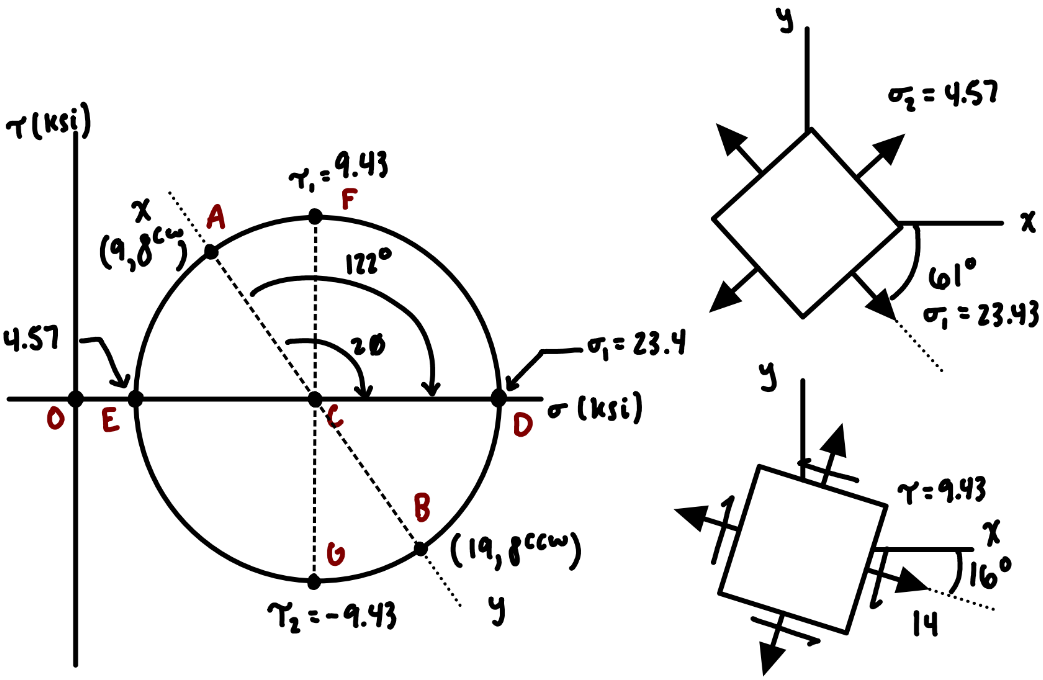
$$\tau_1 = R = 5.0$$

$\tau_2 = -R = -5.0$

Angle of Occurrence:

$$2\phi_p = \text{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 53.130102354156$$

2.3.2 Part D



Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 14.0$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 9.4339811320566$$

Principle Stresses:

$\sigma_1 = C + R = 23.4339811320566$

$\sigma_2 = C - R = 4.5660188679434$

$\tau_1 = R = 9.4339811320566$

$\tau_2 = -R = -9.4339811320566$

Angle of Occurrence:

$$2\phi_p = \operatorname{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 122.005383208084$$

3 Problem 3-72

3.1 Given

A 2-foot-long steel bar with a $\frac{3}{4}$ in diameter is to be used as a torsion spring. The torsional stress in the bar is not to exceed 30 ksi.

3.2 Find

What is the maximum angle of twist of the bar?

3.3 Solution

Use the following relationship to determine the torque,

$$\tau = \frac{Tc}{J}$$

The angle of twist is,

$$\phi = \frac{TL}{JG}$$

[8]:
```
# Find torque
c = sp.S('0.75')/2
J = sp.pi/2*c**4
tau = 30_000
T = tau*J/c
T.n()  # torque in lbf*in
```

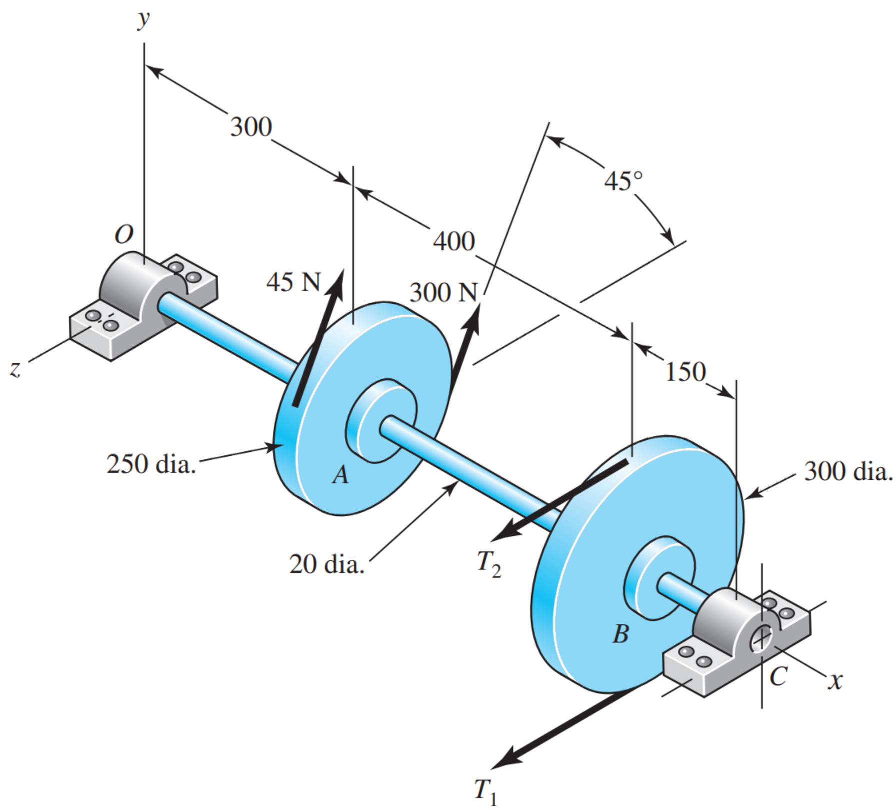
[8]: 2485.04887637474

[9]:
```
# Find angle of twist
G = sp.S('11.5e6')  # from Table A-5
L = 24
phi = (T*L/(J*G))
(phi*180/sp.pi).n()  # angle of twist in degrees
```

[9]: 9.56590405783635

4 Problem 3-82

4.1 Given



A counter shaft carrying two V-belt pulleys is shown in the figure. Pulley A receives power from a motor through a belt with the belt tensions shown. The power is transmitted through the shaft and delivered to the belt on pulley B. Assume the belt tension on the loose side at B is 15 percent of the tension on the tight side.

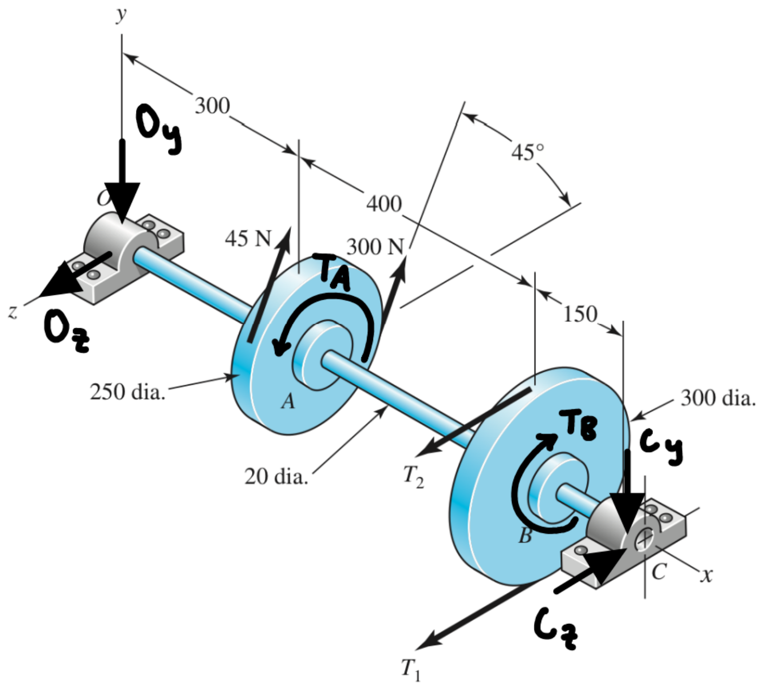
4.2 Find

a. Determine the tensions in the belt on pulley B, assuming the shaft is running at a constant speed.
b. Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.
c. Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.
d. At the point of maximum bending moment, determine the bending stress and the torsional shear stress.
e. At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.

4.3 Solution

4.3.1 Part A

The directions of the torques about A and B are,



Since the shaft has no angular acceleration, $T_A = T_B$ (with directions shown above). It should also be noted that T_1 must be greater than T_2 because the torque shows that the pulley is more tensile at the bottom.

[10]:
```
# Solving for T1 and T2
T1, T2 = sp.symbols('T_1 T_2')
T_A = sp.S('0.125')*(300 - 45)
eq1 = sp.Eq(sp.S('0.15')*(T1 - T2), T_A)
eq2 = sp.Eq(T2, sp.S(0.15)*T1)

[display(eq) for eq in [eq1, eq2, Markdown('---')]]

sol = sp.solve([eq1, eq2], dict=True)[0]
_ = [display(sp.Eq(key, value)) for key, value in sol.items()]
```

$0.15T_1 - 0.15T_2 = 31.875$

$T_2 = 0.15T_1$

$T_1 = 250.0$

$T_2 = 37.5$

4.3.2 Part B

[11]:
```
# Solving for the reactions
Oy, Oz, Cy, Cz = sp.symbols('O_y O_z C_y C_z')

eq1 = sp.Eq((300 + 45)*sp.sin(sp.pi/4) - Oy - Cy, 0)  # Forces in y direction
eq2 = sp.Eq(sol[T1] + sol[T2] + Oz - Cz - (45 + 300)*sp.cos(sp.pi/4), 0)  #␣
↪Forces in z direction
eq3 = sp.Eq(sp.S('0.3')*(45 + 300)*sp.sin(sp.pi/4) - Cy*sp.S('0.85'), 0)  #␣
↪Moments about z-axis
eq4 = sp.Eq(sp.S('0.3')*(45 + 300)*sp.cos(sp.pi/4) - sp.S('0.7')*(sol[T1] +␣
↪sol[T2]) + Cz*sp.S('0.85'), 0)  # Moments about the y-axis

sol2 = sp.solve([eq1, eq2, eq3, eq4], dict=True)[0]
[display(eq) for eq in [eq1, eq2, eq3, eq4, Markdown('---')]]
_ = [display(sp.Eq(key, value)) for key, value in sol2.items()]
```

$$-C_y - O_y + \frac{345\sqrt{2}}{2} = 0$$

$$-C_z + O_z - \frac{345\sqrt{2}}{2} + 287.5 = 0$$

$$-0.85C_y + 51.75\sqrt{2} = 0$$

$$0.85C_z - 201.25 + 51.75\sqrt{2} = 0$$

$$C_y = 86.1006492385973$$

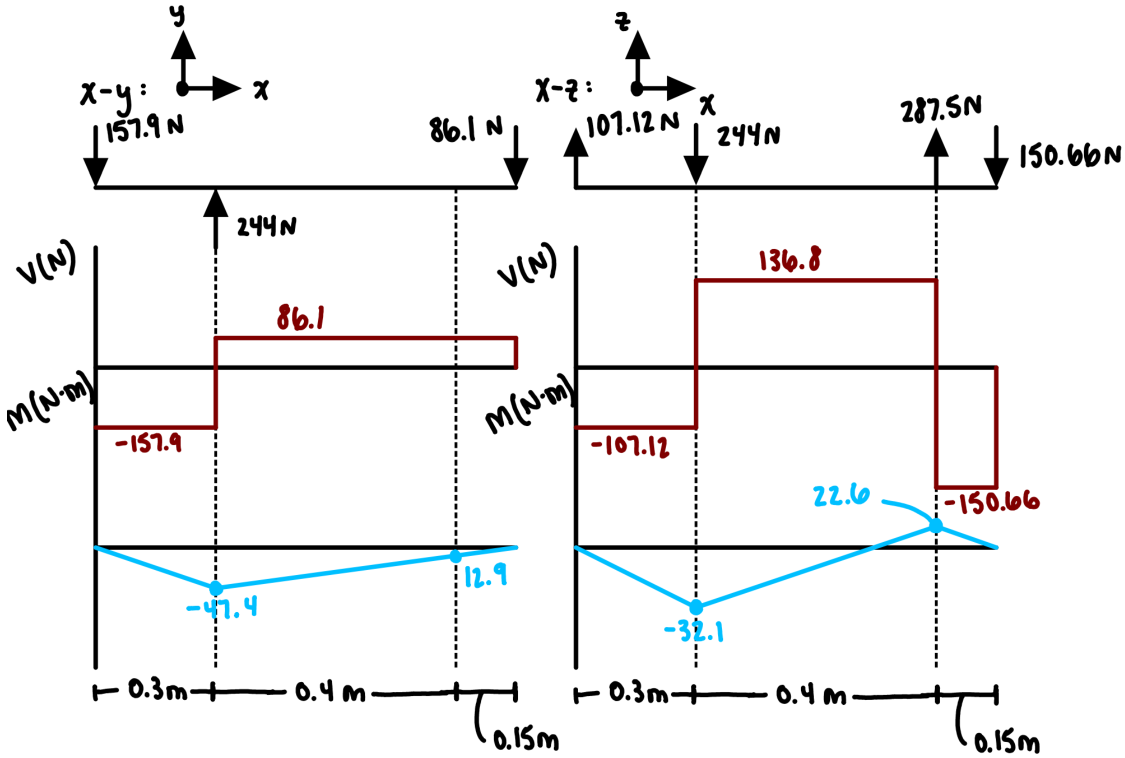
$$O_y = 157.851190270762$$

$$C_z = 150.664056643756$$

$$O_z = 107.115896153115$$

4.3.3 Part C

The shear and bending moment diagram for the two planes is,



4.3.4 Part D

[12]:
```python
# Getting max bending moment
M_A = sp.sqrt(47.35535708**2 + 32.13476885**2)
M_B = sp.sqrt(12.91509739**2 + 22.59960847**2)
sp.Matrix([M_A, M_B])
```

[12]:

$$\begin{bmatrix}57.2291290621938\\26.0296377921492\end{bmatrix}$$

The maximum bending moment occurs at point A.

[13]:
```python
# Getting the bending stress
c = sp.S('0.01')
sig_x = (M_A*c/(sp.pi/4*c**4)).n()
sig_x  # in Pa
```

[13]: 72866390.2327375

[14]:
```python
# Getting the torsional stress
t_xz = (31.875*c/(sp.pi/2*c**4)).n()
t_xz  # in Pa
```

[14]: 20292255.2442167

4.3.5 Part E

[15]:
```
mohr(sig_x, 0, t_xz)
```

Center and Radius:

$$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 36433195.1163688$$

$$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 41703157.3059383$$

Principle Stresses:

$\sigma_1 = C + R = 78136352.422307$

$\sigma_2 = C - R = -5269962.18956954$

$\tau_1 = R = 41703157.3059383$

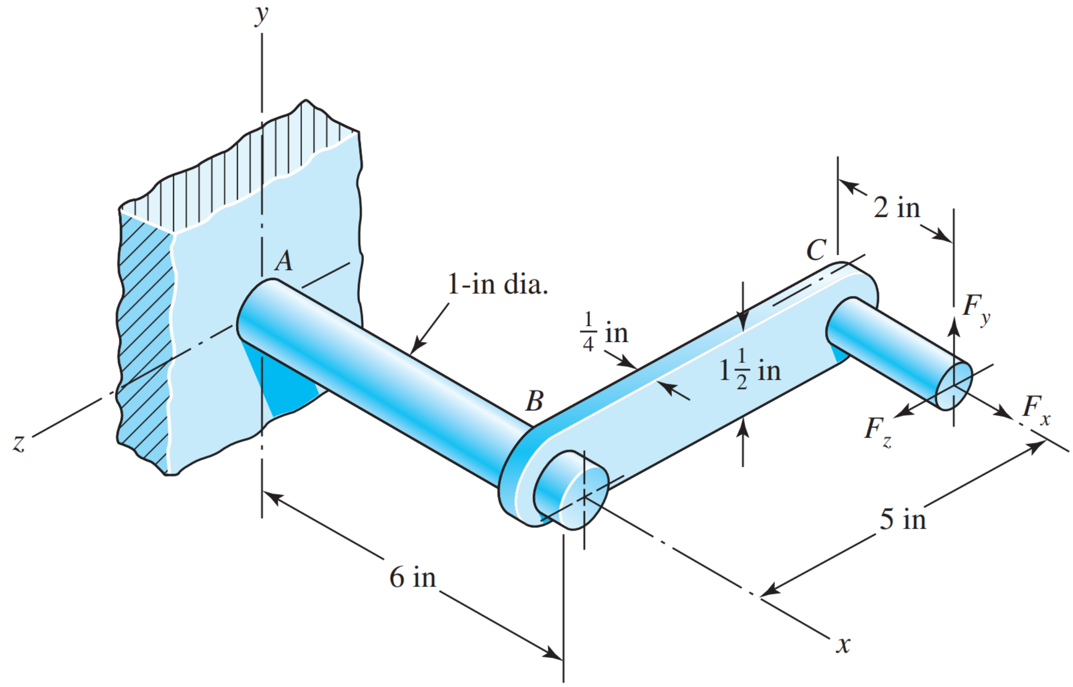
$\tau_2 = -R = -41703157.3059383$

Angle of Occurrence:

$$2\phi_p = \mathrm{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 29.1165652891492$$

5 Problem 3-91

5.1 Given



The cantilevered bar in the figure is made from a ductile material and is statically loaded with $F_y = 200\ lbf$ and $F_x = F_z = 0$.

5.2 Find

Analyze the stress situation on rod AB by obtaining the following:

a. Determine the precise location of the critical stress element.
b. Sketch the critical stress element and determine magnitudes and directions for all stresses acting on it. (Transverse shear may only be neglected if you can justify this decision.)
c. For the critical stress element, determine the principal stresses and the maximum shear stress.

5.3 Solution

5.3.1 Part A

The critical stress element will be at the top or bottom ($y = \pm 0.5\ in$) because both the bending stress and shear stress are maximized at the farthest distance away from the neutral axis.

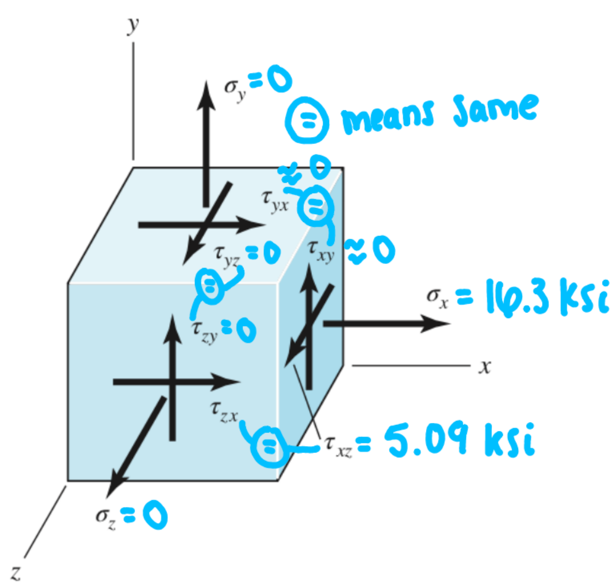
5.3.2 Part B

[16]:
```
# Acquiring shear stress
T = 5*200
c = sp.S('0.5')
J = sp.pi/2*c**4
t_xz = (T*c/J).n()
t_xz  # in psi
```

[16]: 5092.95817894065

[17]:
```
# Acquiring the bending stress
M = 8*200
I = sp.pi/4*c**4
sig_x = (M*c/I).n()
sig_x  # in psi
```

[17]: 16297.4661726101



The transverse shear, τ_{xy}, is being neglected because the rod is a magnitude longer than its diameter.

5.3.3 Part C

Center and Radius:

$C = \frac{\sigma_x}{2} + \frac{\sigma_y}{2} = 8148.73308630504$

$R = \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x}{2} - \frac{\sigma_y}{2}\right)^2} = 9609.37427329589$

Principle Stresses:

$\sigma_1 = C + R = 17758.1073596009$

$\sigma_2 = C - R = -1460.64118699085$

$\tau_1 = R = 9609.37427329589$

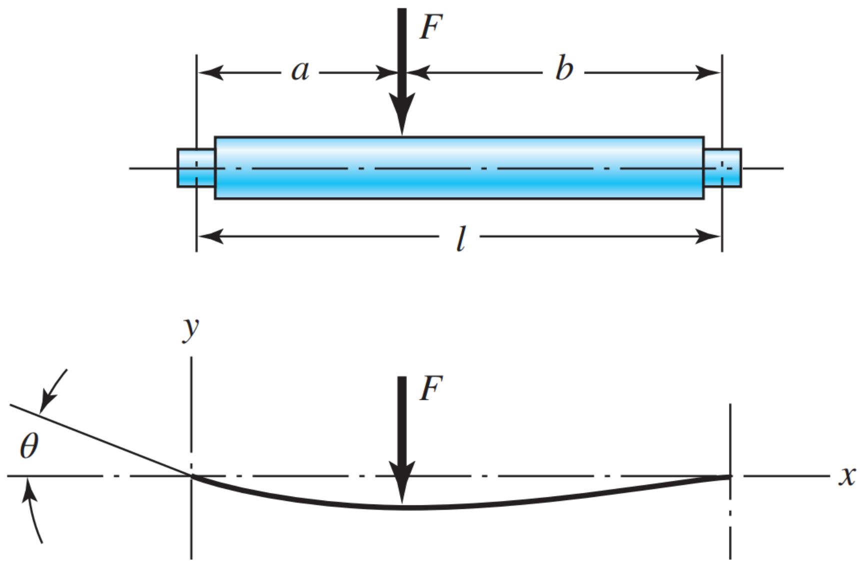
$\tau_2 = -R = -9609.37427329589$

Angle of Occurrence:

$2\phi_p = \operatorname{atan}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) = 32.0053832080835$

6 Problem 4-46

6.1 Given



The diameter is uniform with $l = 300\ mm$, $a = 100\ mm$, and $F = 3\ kN$. The allowable slope ath the bearings is 0.001 mm/mm and the design factor is 1.28. The shaft is steel with $E = 207\ GPa$. The relationship for the diameter is,

$$d = \left|\frac{32Fb(l^2 - b^2)}{3\pi El\xi}\right|^{1/4}$$

6.2 Find

What uniform diameter will the shaft support? Determine the maximum deflection of the shaft.

6.3 Solution

6.3.1 Uniform Diameter

We can use the given relationship to determine the diameter.

[19]:
```
E = sp.S('207e9')
l, a = sp.S('0.3'), sp.S('0.1')
xi = sp.S('0.001')
b = l - a
n = sp.S('1.28')
F = n*3_000

d = (sp.Abs(32*F*b*(l**2 - b**2)/(3*sp.pi*E*l*xi))**sp.S('0.25')).n()
```

```
d  # in meters
```

[19]:

0.0380653317176321

6.3.2 Maximum Deflection

The maximum deflection equation may be found from Table A-9. The deflection may be graphed like so,

[20]:

```
x = sp.Symbol('x')
I = sp.pi/4*(d/2)**4
F = 3_000
y_AB = F*b*x/(6*E*I*l)*(x**2 + b**2 - l**2)
y_BC = F*a*(l - x)/(6*E*I*l)*(x**2 + a**2 - 2*l*x)

y = sp.Piecewise((y_AB, (x >= 0) & (x < a)), (y_BC, (x >= a) & (x <= l)))
y
```

[20]:

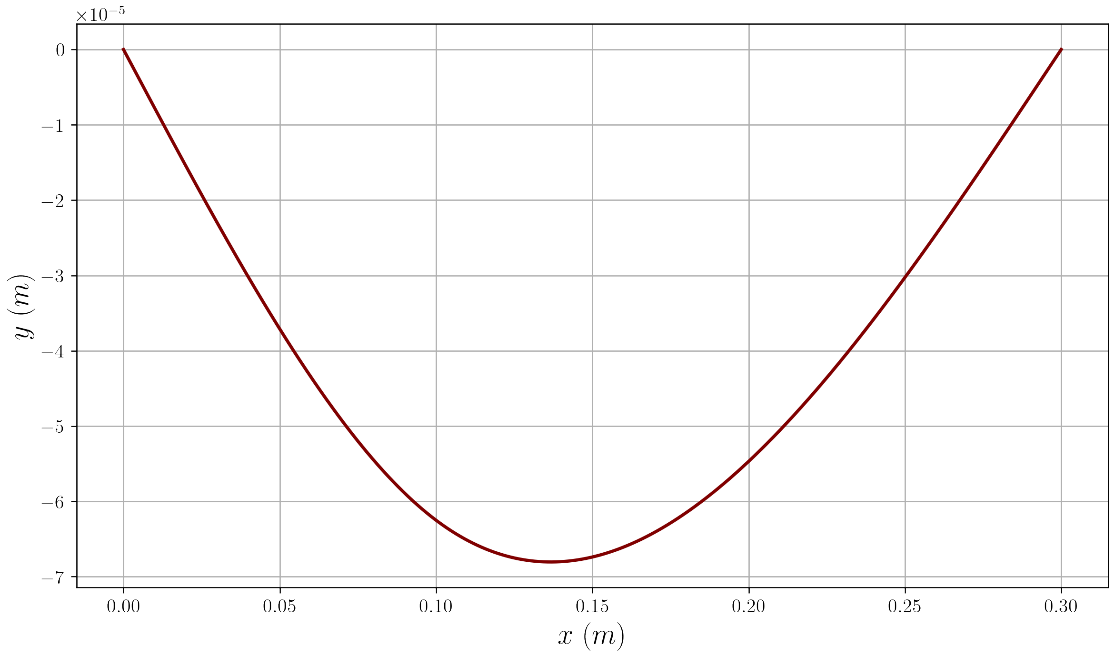
$$\begin{cases} \frac{0.0490873852123405x(x^2-0.05)}{\pi} & \text{for } x \geq 0 \wedge x < 0.1 \\ \frac{8.18123086872342 \cdot 10^{-5} \cdot (90.0-300.0x)(x^2-0.6x+0.01)}{\pi} & \text{for } x \geq 0.1 \wedge x \leq 0.3 \end{cases}$$

[21]:

```
x_ = np.linspace(0, 0.3, 10_000)
y_lamb = sp.lambdify(x, y, modules='numpy')

fig, ax = plt.subplots()
ax.plot(x_, y_lamb(x_))
ax.set_xlabel('$x$ ($m$)')
ax.set_ylabel('$y$ ($m$)')

plt.show()
```



With access to numerical tools, the maximum deflection may be obtained by simply calculating the maximum value from a discritized data set rather than taking the derivative symbolically.

[22]:
```
# Getting the maximum magnitude
y_max = np.max(np.abs(y_lamb(x_)))
y_max*1_000  # in mm
```

[22]:
```
0.06804138155566049
```

This answer is not rounded, but the one in the back of the book is rounded.

7 Problem 5-1

7.1 Given

A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa.

$\sigma_x = -50\ MPa$, $\sigma_y = -75\ MPa$, and $\tau_{xy} = -50\ MPa$

7.2 Find

Use the distortion-energy and maximum shear stress methods to determine the factor of safety.

7.3 Solution

Begin by getting the principal stresses.

[23]:
```
sig_x, sig_y, sig_z, tau_xy, tau_zx, tau_yz = sp.symbols(r'\sigma_x \sigma_y
 ↪\sigma_z \tau_{xy} \tau_{zx} \tau_{yz}')
sig = sp.Symbol(r'\sigma')
sig1, sig2, sig3 = sp.symbols(r'\sigma_1 \sigma_2 \sigma_3')

poly = sig**3 - (sig_x + sig_y + sig_z)*sig**2 + (sig_x*sig_y + sig_x*sig_z +
 ↪sig_y*sig_z - tau_xy**2 - tau_yz**2 - tau_zx**2)*sig - (sig_x*sig_y*sig_z +
 ↪2*tau_xy*tau_yz*tau_zx - sig_x*tau_yz**2 - sig_y*tau_zx**2 - sig_z*tau_xy**2)
display(sp.Eq(poly.simplify(), 0))

def get_principal(sx, sy, sz, txy, tyz, tzx):
    poly_ = poly.subs([(sig_x, sx), (sig_y, sy), (sig_z, sz), (tau_xy, txy),
 ↪(tau_yz, tyz), (tau_zx, tzx)])
    roots = [sp.re(root.n()) for root in sp.roots(poly_, sig)]
    roots_ = sorted(list(roots), reverse=True)
    for i, j in zip((sig1, sig2, sig3), roots_):
        display(sp.Eq(i, j))
    return roots_

def von_mises(s1_, s2_, s3_):
    return (1/sp.sqrt(2)*sp.sqrt((s1_ - s2_)**2 + (s2_ - s3_)**2 + (s3_ -
 ↪s1_)**2)).n()

s1, s2, s3 = get_principal(-50, -75, 0, -50, 0, 0)
```

$\sigma^3 - \sigma^2(\sigma_x + \sigma_y + \sigma_z) + \sigma(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2) - \sigma_x\sigma_y\sigma_z + \sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2 - 2\tau_{xy}\tau_{yz}\tau_{zx} = 0$

$\sigma_1 = 0$

$\sigma_2 = -10.9611796797792$

$\sigma_3 = -114.038820320221$

7.3.1 Maximum Shear Stress Method

[24]:
```
Sy = 350
Sy/(s1 - s3)
```

[24]:

3.06913031033819

7.3.2 Distortion Energy Method

[25]:
```
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[25]:

3.21182027418786

8 Problem 5-12

8.1 Given

A ductile material has the properties $S_{yt} = 60\ ksi$ and $S_{yc} = 75\ ksi$.

8.2 Find

Using the ductile Coulomb-Mohr theory, determine the factor of safety for the states of plane stresses,

a. $\sigma_x = 25\ ksi$, $\sigma_y = 15\ ksi$
b. $\sigma_x = 15\ ksi$, $\sigma_y = -15\ ksi$
c. $\sigma_x = 20\ ksi$, $\tau_{xy} = -10\ ksi$
d. $\sigma_x = -12\ ksi$, $\sigma_y = 15\ ksi$, $\tau_{xy} = -9\ ksi$
e. $\sigma_x = -24\ ksi$, $\sigma_y = -24\ ksi$, $\tau_{xy} = -15\ ksi$

8.3 Solution

Using Eq. 5-26, the block below will execute all parameters in a for loop.

[26]:
```python
sig_values = [[25, 15, 0],
              [15, -15, 0],
              [20, 0, -10],
              [-12, 15, -9],
              [-24, -24, -15]]
a = 'ABCDE'
St, Sc = 60, 75

for l, row in zip(a, sig_values):
    s_x_, s_y_, t_xy_ = row
    display(Markdown(f'### Part {l}'))
    s1, s2, s3 = get_principal(s_x_, s_y_, 0, t_xy_, 0, 0)
    n = 1/(s1/St - s3/Sc)
    display(Markdown('---'))
    display(sp.Eq(sp.Symbol("n"), n.n()))
```

8.3.1 Part A

$\sigma_1 = 25.0$

$\sigma_2 = 15.0$

$\sigma_3 = 0$

$n = 2.4$

8.3.2 Part B

$\sigma_1 = 15.0$

$\sigma_2 = 0$

$\sigma_3 = -15.0$

$n = 2.22222222222222$

8.3.3 Part C

$\sigma_1 = 24.142135623731$

$\sigma_2 = 0$

$\sigma_3 = -4.14213562373095$

$n = 2.18532709217848$

8.3.4 Part D

$\sigma_1 = 17.724980739588$

$\sigma_2 = 0$

$\sigma_3 = -14.724980739588$

$n = 2.03355602443072$

8.3.5 Part E

$\sigma_1 = 0$

$\sigma_2 = -9.0$

$\sigma_3 = -39.0$

$n = 1.92307692307692$