# Machine Design Homework 3

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[1]:
```python
# Notebook Preamble
import sympy as sp
import matplotlib.pyplot as plt
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')
```

## Contents

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# 1 Problem 5-3

## 1.1 Given

A ductile AISI 1030 hot-rolled steel bar has a minimum yield strength in tension and compression of 37.5 $ksi$.

## 1.2 Find

Use the distortion energy and maximum shear stress theories to determine the factors of safety for the following plane stress states:

a. $\sigma_x = 25\ ksi$, $\sigma_y = 15\ ksi$
b. $\sigma_x = -12\ ksi$, $\sigma_y = 15\ ksi$, $\tau_{xy} = -9\ ksi$
c. $\sigma_x = -24\ ksi$, $\sigma_y = -24\ ksi$, $\tau_{xy} = -15\ ksi$

## 1.3 Solution

The relationship comes from Eq. 5-3 (maximum shear stress theory) and Eq. 5-19 (distortion energy theory),

$$\sigma_1 - \sigma_3 = \frac{S_y}{n}$$
$$\sigma' = \frac{S_y}{n}$$

### 1.3.1 Part A

[2]:
```
Sy = sp.S('37.5')

# Getting the principal stresses
sig_x, sig_y, sig_z, tau_xy, tau_zx, tau_yz = sp.symbols(r'\sigma_x \sigma_y␣
↪\sigma_z \tau_{xy} \tau_{zx} \tau_{yz}')
sig = sp.Symbol(r'\sigma')
sig1, sig2, sig3 = sp.symbols(r'\sigma_1 \sigma_2 \sigma_3')

poly = sig**3 - (sig_x + sig_y + sig_z)*sig**2 + (sig_x*sig_y + sig_x*sig_z +␣
↪sig_y*sig_z - tau_xy**2 - tau_yz**2 - tau_zx**2)*sig - (sig_x*sig_y*sig_z +␣
↪2*tau_xy*tau_yz*tau_zx - sig_x*tau_yz**2 - sig_y*tau_zx**2 - sig_z*tau_xy**2)
display(sp.Eq(poly.simplify(), 0))

def get_principal(sx, sy, sz, txy, tyz, tzx):
    poly_ = poly.subs([(sig_x, sx), (sig_y, sy), (sig_z, sz), (tau_xy, txy),␣
↪(tau_yz, tyz), (tau_zx, tzx)])
    roots = [sp.re(root.n()) for root in sp.roots(poly_, sig)]
    roots_ = sorted(list(roots), reverse=True)
```

```
    for i, j in zip((sig1, sig2, sig3), roots_):
        display(sp.Eq(i, j))
    return roots_

def von_mises(s1_, s2_, s3_):
    return (1/sp.sqrt(2)*sp.sqrt((s1_ - s2_)**2 + (s2_ - s3_)**2 + (s3_ -
 ↪s1_)**2)).n()

s1, s2, s3 = get_principal(25, 15, 0, 0, 0, 0)
```

$\sigma^3 - \sigma^2\left(\sigma_x + \sigma_y + \sigma_z\right) + \sigma\left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right) - \sigma_x\sigma_y\sigma_z + \sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2 - 2\tau_{xy}\tau_{yz}\tau_{zx} = 0$

$\sigma_1 = 25.0$

$\sigma_2 = 15.0$

$\sigma_3 = 0$

[3]:
```
# Maximum shear stress theory
Sy/(s1 - s3)
```

[3]: 1.5

[4]:
```
# Distortion energy method
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[4]: 1.72061800402921

### 1.3.2 Part D

[5]:
```
s1, s2, s3 = get_principal(-12, 15, 0, -9, 0, 0)

# Maximum shear stress
Sy/(s1 - s3)
```

$\sigma_1 = 17.724980739588$

$\sigma_2 = 0$

$\sigma_3 = -14.724980739588$

[5]: 1.15562540880256

[6]:
```
# Distortion
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[6]: 1.33250447722257

### 1.3.3 Part E

[7]:
```
s1, s2, s3 = get_principal(-24, -24, 0, -15, 0, 0)

# Maximum shear stress
Sy/(s1 - s3)
```

$\sigma_1 = 0$

$\sigma_2 = -9.0$

$\sigma_3 = -39.0$

[7]: 0.9615384615384462

[8]:
```
# Distortion
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[8]: 1.06023616209996

# 2 Problem 5-17

## 2.1 Given

An AISI 4142 steel Q&T at 800°$F$ exhibits $S_{yt} = 235\ ksi$, $S_{yc} = 285\ ksi$, and $\epsilon_f = 0.07$.

$\sigma_x = -80\ ksi$, $\sigma_y = -125\ ksi$, $\tau_{xy} = 50\ ksi$

## 2.2 Find

Determine the factor of safety.

## 2.3 Solution

The strain at failure is above 0.05, which means that the material is considered ductile. We can apply Eq. 5-22,

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = \frac{1}{n}$$

[9]:
```
St, Sc = sp.S(235), sp.S(285)
s1, _, s3 = get_principal(-80, -125, 0, 50, 0, 0)
1/(s1/St - s3/Sc)
```

$\sigma_1 = 0$

$\sigma_2 = -47.6707195013467$

$\sigma_3 = -157.329280498653$

[9]: 1.81148734105118

This answer lines up with the answer in tha back of the book.

# 3 Problem 5-19

## 3.1 Given

A brittle material has properties $S_{ut} = 30\ ksi$ and $S_{uc} = 90\ ksi$.

$\sigma_x = 20\ ksi$, $\tau_{xy} = -10\ ksi$

## 3.2 Find

Using only the modified-Mohr theories, determine the factor of safety.

## 3.3 Solution

Start by computing the principal stresses.

[10]:
```
Sut = 30
s1, _, _ = get_principal(20, 0, 0, -10, 0, 0)
```

$\sigma_1 = 24.142135623731$

$\sigma_2 = 0$

$\sigma_3 = -4.14213562373095$

For this case, we can use Eq. 5-32a on p. 264,

$$\sigma_1 = \frac{S_{ut}}{n}$$

[11]:
```
Sut/s1
```

[11]: 1.24264068711929

# 4 Problem 5-24

## 4.1 Given

ASTM 30 cast iron.

$\sigma_x = -10\ ksi$, $\sigma_y = -25\ ksi$, and $\tau_{xy} = -10\ ksi$

## 4.2 Find

Determine the factor of safety using the modified-Mohr method.

## 4.3 Solution

Start with getting the principal stresses.

[12]:
```
s1, s2, s3 = get_principal(-10, -25, 0, -10, 0, 0)
```

$\sigma_1 = 0$

$\sigma_2 = -5.0$

$\sigma_3 = -30.0$

Use Eq. 5-32c,

$$n = \frac{-S_{uc}}{\sigma_3}$$

[13]:
```
Suc = 109  # Table A-24
-Suc/s3
```

[13]: 3.6333333333333333

# 5 Problem 5-98

## 5.1 Given

A cylinder subjected to internal pressure $p_i$ has an outer diameter of 14 in and a 1-in wall thickness. For the cylinder material, $K_{IC} = 72\ ksi\sqrt{in}$, $S_y = 170\ ksi$, and $S_{ut} = 192\ ksi$.

## 5.2 Find

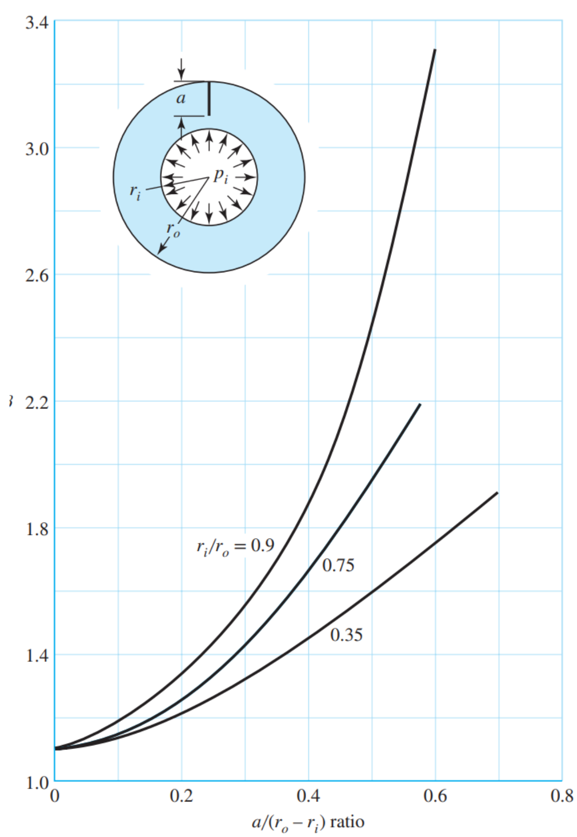
If the cylinder contains a radial crack in the longitudinal direction of depth 0.5 in, determine the pressure that will cause uncontrollable crack growth.

## 5.3 Solution

We use the following relationship,

$$K_I = \beta\sigma\sqrt{\pi a}$$

$\beta$ may be found from,



[14]:
```
a = sp.S('0.5')
ro, ri = sp.S(7), 6
a/(ro - ri)
```

[14]: 0.5

The correct line to use depends on $\frac{r_i}{r_o}$.

[15]:
```
ri/ro.n()
```

[15]: 0.857142857142857

[16]:
```
beta = sp.S(2.35) # from the above figure
K_IC = 72
sig = K_IC/(beta*sp.sqrt(sp.pi*a))
sig.n()  # ksi
```

[16]: 24.4458248416197

The relationship due to the internal stress is,

$$p = \sigma \frac{r_o^2 - r_i^2}{r_i^2 \left(1 + \frac{r_o^2}{r^2}\right)}$$

[17]:
```
r = ro - a
p = sig*(ro**2 - ri**2)/(ri**2*(1 + ro**2/r**2))
p.n()  # ksi
```

[17]: 4.08732703021602