Vibrations and Controls Homework 11

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[1]:

```python
# Notebook Preamble
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Latex, Markdown

plt.style.use('maroon_ipynb.mplstyle')
```

1 Problem 11.3

1.1 Given and Find

Sketch the root locus of the armature-controlled dc motor model in terms of the damping constant c, and evaluate the effect on the motor time constant. The characteristic equation is

$$L_a I s^2 + (R_a I + cL_a)s + cR_a + K_b K_T = 0$$

Use the following parameter values:

$$K_b = K_T = 0.1\, N \cdot m/A$$
$$R_a = 2\,\Omega$$
$$I = 12 \times 10^{-5}\, kg \cdot m^2$$
$$L_a = 3 \times 10^{-3}\, H$$

1.2 Solution

[2]:

```python
# Define symbols and substitute in for the characteristic equation
La, I, s, Ra, c, Kb, KT = sp.symbols(r'L_a I s R_a c K_b K_T')
Kb_ = KT_ = 0.1
I_ = 12e-5
Ra_ = 2
La_ = 3e-3
eq1 = sp.Eq(La*I*s**2 + (Ra*I + c*La)*s + c*Ra + Kb*KT, 0)
eq2 = eq1.subs([
    (Kb, Kb_),
    (KT, KT_),
    (Ra, Ra_),
    (I, I_),
    (La, La_)
])
poly = eq2.lhs
display(eq1, eq2)
```

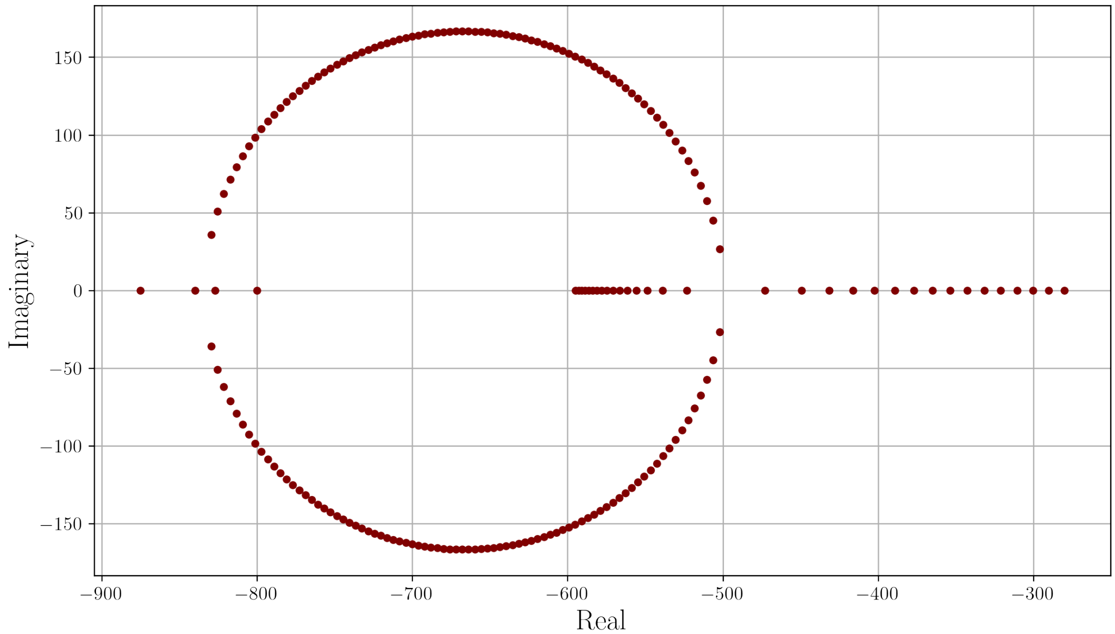
$IL_a s^2 + K_T K_b + R_a c + s\left(IR_a + L_a c\right) = 0$

$2c + 3.6 \cdot 10^{-7} s^2 + s\left(0.003c + 0.00024\right) + 0.01 = 0$

[3]:

```python
c_values = np.linspace(0.025, 0.121, 100)
fig, ax = plt.subplots()
ax.set_aspect('equal')
ax.set_xlabel('Real')
ax.set_ylabel('Imaginary')
```

```
for c_ in c_values:
    A, B, C = I_*La_, I_*Ra_ + La_*c_, KT_*Kb_ + Ra_*c_
    roots = np.roots([A, B, C])
    ax.scatter(np.real(roots), np.imag(roots), color='maroon', zorder=3,␣
 ↪marker='.')
plt.show()
```



The value of the time constant may be observed by testing increasing values of c.

[4]:
```
c_list = [0, 10, 100, 1000]
for c_ in c_list:
    roots = list(sp.roots(poly.subs(c, c_)))
    display(Latex(fr'$c={c_}\rightarrow{sp.latex(sp.Matrix(roots).
 ↪transpose())}$'))
```

$c = 0 \rightarrow \begin{bmatrix} -622.008467928146 & -44.6581987385205 \end{bmatrix}$

$c = 10 \rightarrow \begin{bmatrix} -83332.9973104621 & -667.002689537906 \end{bmatrix}$

$c = 100 \rightarrow \begin{bmatrix} -833333.299973311 & -666.700026689354 \end{bmatrix}$

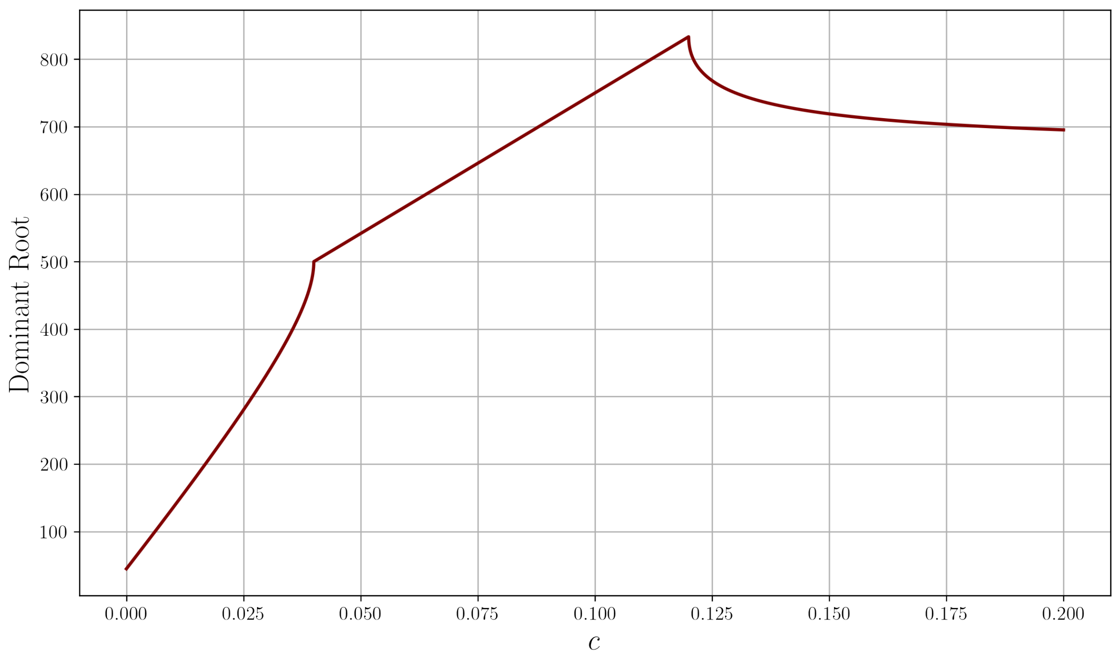
$c = 1000 \rightarrow \begin{bmatrix} -8333333.32999973 & -666.670000266689 \end{bmatrix}$

Since the dominant time constants corresponds to the smallest root value in magnitude, it may be observed that the root above converges at -666.7, meaning that the time constant is $-\frac{1}{r} = \frac{1}{666.7} = 0.0015\,s$ for high damping constants.

The smallest possible time constant may be computed by considering the dominant time constant across the range of c values.

[5]:
```
c_values = np.linspace(0, 0.2, 10_000)
dom_reals = []
for c_ in c_values:
    A, B, C = I_*La_, I_*Ra_ + La_*c_, KT_*Kb_ + Ra_*c_
    roots = np.roots([A, B, C])
    real = abs(np.real(roots))
    dom_reals.append(min(real))

fig2, ax2 = plt.subplots()
ax2.plot(c_values, dom_reals)
ax2.set_xlabel('$c$')
ax2.set_ylabel(r'Dominant Root')
plt.show()
```



The maximum value corresponds to the smallest time constant and was found to be $\frac{1}{833.300} = 0.00120\,s$.

2 Problem 11.5 Part C

2.1 Given

$$5s^3 + 3ps^2 + 5s + p = 0$$

2.2 Find

Identify the root locus plotting parameter K and its range in terms of the parameter p, where $p \geq 0$.

2.3 Solution

The relationship needs to be rearranged such that $D(s) + K \cdot N(s) = 0$.

[7]:
```
p = sp.Symbol('p')
# eq = sp.Eq(3*s**2 + (6 + p)*s + 5 + 2*p, 0)
eq = sp.Eq(5*s**3 + 3*p*s**2 + 5*s + p, 0)
A, *_ = sp.Poly(eq.lhs, s).coeffs()
eq1 = sp.Eq(sp.collect(eq.lhs/A, p).expand(), 0)
display(eq, eq1)
```

$3ps^2 + p + 5s^3 + 5s = 0$

$\frac{3ps^2}{5} + \frac{p}{5} + s^3 + s = 0$

Thus, $K = \frac{3p}{5}$, $N(s) = s^2 + \frac{1}{3}$, and $D(s) = s^3 + s$.