Machine Design Homework 3

June 13, 2022

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[1]:
```
# Notebook Preamble
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1 Problem 5-3

1.1 Given

A ductile AISI 1030 hot-rolled steel bar has a minimum yield strength in tension and compression of 37.5 ksi.

1.2 Find

Use the distortion energy and maximum shear stress theories to determine the factors of safety for the following plane stress states:

a. $\sigma_x = 25\ ksi$, $\sigma_y = 15\ ksi$
b. $\sigma_x = -12\ ksi$, $\sigma_y = 15\ ksi$, $\tau_{xy} = -9\ ksi$
c. $\sigma_x = -24\ ksi$, $\sigma_y = -24\ ksi$, $\tau_{xy} = -15\ ksi$

1.3 Solution

The relationship comes from Eq. 5-3 (maximum shear stress theory) and Eq. 5-19 (distortion energy theory),

$$\sigma_1 - \sigma_3 = \frac{S_y}{n}$$

$$\sigma' = \frac{S_y}{n}$$

1.3.1 Part A

[2]:
```
Sy = sp.S('37.5')

# Getting the principal stresses
sig_x, sig_y, sig_z, tau_xy, tau_zx, tau_yz = sp.symbols(r'\sigma_x \sigma_y␣
↪\sigma_z \tau_{xy} \tau_{zx} \tau_{yz}')
sig = sp.Symbol(r'\sigma')
sig1, sig2, sig3 = sp.symbols(r'\sigma_1 \sigma_2 \sigma_3')

poly = sig**3 - (sig_x + sig_y + sig_z)*sig**2 + (sig_x*sig_y + sig_x*sig_z +␣
↪sig_y*sig_z - tau_xy**2 - tau_yz**2 - tau_zx**2)*sig - (sig_x*sig_y*sig_z +␣
↪2*tau_xy*tau_yz*tau_zx - sig_x*tau_yz**2 - sig_y*tau_zx**2 - sig_z*tau_xy**2)
display(sp.Eq(poly.simplify(), 0))

def get_principal(sx, sy, sz, txy, tyz, tzx):
    poly_ = poly.subs([(sig_x, sx), (sig_y, sy), (sig_z, sz), (tau_xy, txy),␣
↪(tau_yz, tyz), (tau_zx, tzx)])
    roots = [sp.re(root.n()) for root in sp.roots(poly_, sig)]
    roots_ = sorted(list(roots), reverse=True)
```

```
    for i, j in zip((sig1, sig2, sig3), roots_):
        display(sp.Eq(i, j))
    return roots_

def von_mises(s1_, s2_, s3_):
    return (1/sp.sqrt(2)*sp.sqrt((s1_ - s2_)**2 + (s2_ - s3_)**2 + (s3_ -
 ↪s1_)**2)).n()

s1, s2, s3 = get_principal(25, 15, 0, 0, 0, 0)
```

$\sigma^3 - \sigma^2 \left(\sigma_x + \sigma_y + \sigma_z\right) + \sigma \left(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2\right) - \sigma_x\sigma_y\sigma_z + \sigma_x\tau_{yz}^2 + \sigma_y\tau_{zx}^2 + \sigma_z\tau_{xy}^2 - 2\tau_{xy}\tau_{yz}\tau_{zx} = 0$

$\sigma_1 = 25.0$

$\sigma_2 = 15.0$

$\sigma_3 = 0$

[3]:
```
# Maximum shear stress theory
Sy/(s1 - s3)
```

[3]: 1.5

[4]:
```
# Distortion energy method
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[4]: 1.72061800402921

1.3.2 Part D

[5]:
```
s1, s2, s3 = get_principal(-12, 15, 0, -9, 0, 0)

# Maximum shear stress
Sy/(s1 - s3)
```

$\sigma_1 = 17.724980739588$

$\sigma_2 = 0$

$\sigma_3 = -14.724980739588$

[5]: 1.15562540880256

[6]:
```
# Distortion
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[6]: 1.33250447722257

1.3.3 Part E

[7]:
```
s1, s2, s3 = get_principal(-24, -24, 0, -15, 0, 0)

# Maximum shear stress
Sy/(s1 - s3)
```

$\sigma_1 = 0$

$\sigma_2 = -9.0$

$\sigma_3 = -39.0$

[7]: 0.961538461538462

[8]:
```
# Distortion
s_vm = von_mises(s1, s2, s3)
Sy/s_vm
```

[8]: 1.06023616209996