

# Machine Design Homework 4

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```
[1]: # Notebook Preamble
      %matplotlib inline
      import matplotlib.pyplot as plt
      import sympy as sp
      import numpy as np
      from IPython.display import display, Markdown

      plt.style.use('maroon_ipynb.mplstyle')
```

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## 1 Problem 6-1

### 1.1 Given

A 10-mm steel drill rod was heat treated and ground. The measured hardness was found to be 300 Brinell.

### 1.2 Find

Estimate the endurance strength in *MPa* if the rod is used in rotating bending.

### 1.3 Solution

Eq. 6-10 on p. 305,

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \leq 200 \text{ ksi (1400 MPa)} \\ 100 & S_{ut} > 200 \text{ ksi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

The ultimate strength of steel comes from Eq. 2-36,

$$S_{ut} = 3.4H_B$$

```
[2]: H_B = 300
      S_ut = sp.S('3.4')*H_B

      if S_ut <= 1400:
          S_e_prime = 0.5*S_ut
      else:
          S_e_prime = sp.S(700)

      S_e_prime # ksi
```

```
[2]: 510.0
```

This value is not the final value. The relationship for the refined value is,

$$S_e = k_a k_b k_c k_d k_e S'_e$$

The only necessary  $k$  values used for this analysis is  $k_a$  and  $k_b$ , whose equations are at 6-18 and 6-19 respectfully.

```
[3]: # See Table 6-2
      k_a = sp.S('1.38')*S_ut**-(sp.S('0.067'))
      d = 10
```

```
k_b = sp.S('1.24')*d**-(sp.S('0.107'))  
# display(k_a, k_b)  
S_e = k_a*k_b*S_e_prime  
S_e # MPa
```

[3]: 428.839455736079

## 2 Problem 6-3

### 2.1 Given

A steel rotating beam test specimen has an ultimate strength of 120 *ksi*.

### 2.2 Find

Estimate the life of the specimen if it is tested at completely reversed stress amplitude of 70 *ksi*.

### 2.3 Solution

Find  $S_e$  first.

```
[4]: S_ut = sp.S(120) # ksi

if S_ut <= 200:
    S_e_prime = 0.5*S_ut
else:
    S_e_prime = sp.S(100)

S_e_prime # ksi
```

```
[4]: 60.0
```

The  $S'_e$  value will be used in place of  $S_e$  from Figure 6-23 description. We can use the following relationships to determine  $N$ .

$$N = \left( \frac{\sigma_{ar}}{a} \right)^{1/b}$$
$$a = \frac{(fS_{ut})^2}{S_e}$$
$$b = -\frac{1}{3} \log \left( \frac{fS_{ut}}{S_e} \right)$$

The value of  $f$  is 0.82 from Figure 6-23. The  $S_{ut}$  value is  $2(S_e) = 120$  *ksi*.

```
[5]: def log10(x_):
    return sp.log(x_)/sp.log(10)

f = sp.S('0.82')
a = (f*S_ut)**2/S_e_prime
b = -sp.Rational(1, 3)*log10(f*S_ut/S_e_prime)

display(sp.Eq(sp.Symbol('a'), a.n()),
        sp.Eq(sp.Symbol('b'), b.n()))
```

```
sig_ar = 70  
N = ((sig_ar/a)**(1/b)).n()  
N # cycles
```

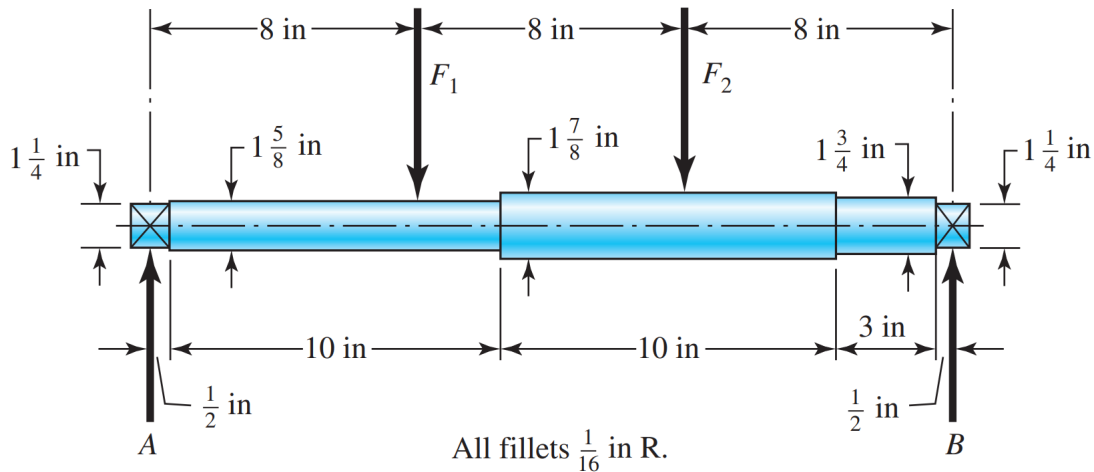
$a = 161.376$

$b = -0.0716146160158993$

[5]: 116192.956004683

### 3 Problem 6-17

#### 3.1 Given



The shaft shown in the figure above is machined from AISI 1040 CD steel. The shaft rotates at 1600 rpm and is supported in roller bearings at *A* and *B*. The applied forces are  $F_1 = 2500 \text{ lbf}$  and  $F_2 = 1000 \text{ lbf}$ .

#### 3.2 Find

Determine the minimum fatigue factor of safety based on achieving infinite life. If infinite life is not predicted, estimate the number of cycles to failure. Also check for yielding.

#### 3.3 Solution

The reaction forces need to be solved first.

```
[6]: A, B = sp.symbols('A B')
F1, F2 = 2500, 1000

eq1 = sp.Eq(A + B, F1 + F2)
eq2 = sp.Eq(B*24 - F1*8 - F2*16, 0)

sol = sp.solve([eq1, eq2], dict=True)[0]

display(eq1, eq2, Markdown('---'))
for key, value in sol.items():
    display(sp.Eq(key, value))
```

$$A + B = 3500$$

$$24B - 36000 = 0$$

$$A = 2000$$

$$B = 1500$$

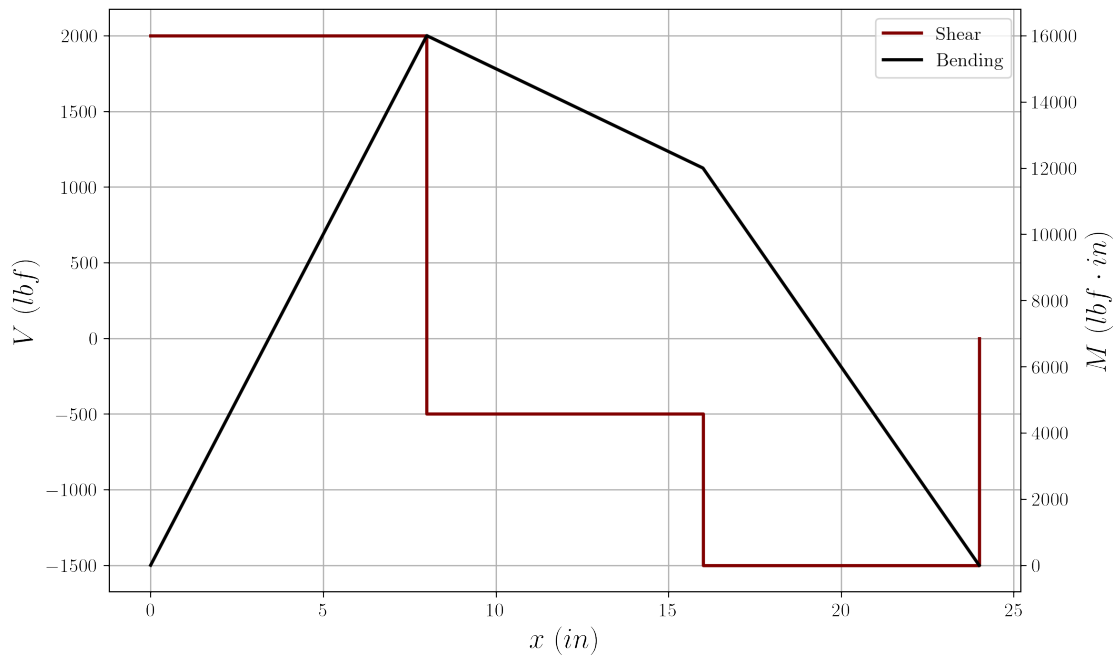
```
[7]: # Plotting Shear and Bending Moment Diagram
x = [0, 8, 16, 24]
x_shear = [0, 8, 8, 16, 16, 24, 24]
V1, V2, V3, V4 = [sol[A], sol[A] - F1, sol[A] - F1 - F2, sol[A] - F1 - F2 +
↳sol[B]]
V = [V1, V1, V2, V2, V3, V3, V4]
M = M1, M2, M3, M4 = [0, V1*8, V1*8 + V2*8, V1*8 + V2*8 + V3*8]

fig, ax = plt.subplots()
ax2 = ax.twinx()

ax.plot(x_shear, V, label='Shear')
ax2.plot(x, M, color='black', label='Bending')
ax2.grid(visible=False)

ax.legend(handles=[ax.lines[0], ax2.lines[0]])
ax.set_xlabel('$x$ (in$)')
ax.set_ylabel('$V$ (lbf$)')
ax2.set_ylabel(r'$M$ (lbf\cdot in$)')

plt.show()
```





We are interested in the stress at the fillet radius in which the smaller diameter is used.

```
[8]: M_mid = (M3 - M2)/8*(sp.S('10.5') - 8) + M2
      M_mid # in lbf*in
```

```
[8]: 14750.0
```

```
[9]: c = sp.S('1.625')/2
      I = sp.pi.n()/4*c**4
      sig = M_mid*c/I
      sig # in psi
```

```
[9]: 35013.218176932
```

The yield strength is 71 *ksi*, and this stress is far below this value. The ultimate strength is  $S_{ut} = 0.5(H_B) = 0.5(170) = 85$  *ksi*. To determine whether infinite life can be reached,

$$n_f = \frac{S_e}{K_f \sigma}$$

This is a variation of Eq. 6-42, but we are multiplying by the fatigue concentration factor to obtain the maximum stress value from the fillet geometry. From Eq. 6-32,

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

A maybe calculated using Eq. 6-35.  $K_t$  comes from Figure A-15-9.

```
[10]: r = sp.S('0.0625')
      K_t = sp.S('1.95')
      S_ut = sp.S(85)
      a = (sp.S('0.246') - sp.S('3.08e-3')*S_ut + sp.S('1.51e-5')*S_ut**2 - sp.S('2.
      ↪67e-8')*S_ut**3)**2
      Kf = 1 + (K_t - 1)/(1 + sp.sqrt(a/r))
      Kf
```

```
[10]: 1.72652106649163
```

$S_e$  maybe calculated using the same procedure as before.

```
[11]: if S_ut <= 200:
      S_e_prime = 0.5*S_ut
      else:
      S_e_prime = sp.S(100)

      S_e_prime # ksi
```

```
[11]: 42.5
```

```
[12]: a_factor, b_exponent = 2, sp.S('-0.217') # Table 6-2
      k_a = a_factor*S_ut**b_exponent
      d = sp.S('1.625')
      k_b = sp.S('0.879')*d**sp.S('-0.107')
      S_e = k_a*k_b*S_e_prime
      S_e # ksi
```

```
[12]: 27.0497081578753
```

```
[13]: # Getting nf
      S_e/(Kf*sig/1000)
```

```
[13]: 0.447464588712579
```

Because the factor of safety is less than one, infinite fatigue cannot be reached. There must be some finite number of cycles,  $N$ .

```
[14]: f = sp.S('0.867')

      a = (f*S_ut)**2/S_e
      b = -sp.Rational(1, 3)*log10(f*S_ut/S_e)

      display(sp.Eq(sp.Symbol('a'), a.n()),
              sp.Eq(sp.Symbol('b'), b.n()))

      N = ((sig/1000*Kf/a)**(1/b)).n()
      N # cycles
```

```
a = 200.776769690168
```

```
b = -0.145091813123711
```

```
[14]: 3917.08718671478
```

**Important:** The answer in the back of the book uses rounded values. For instance,

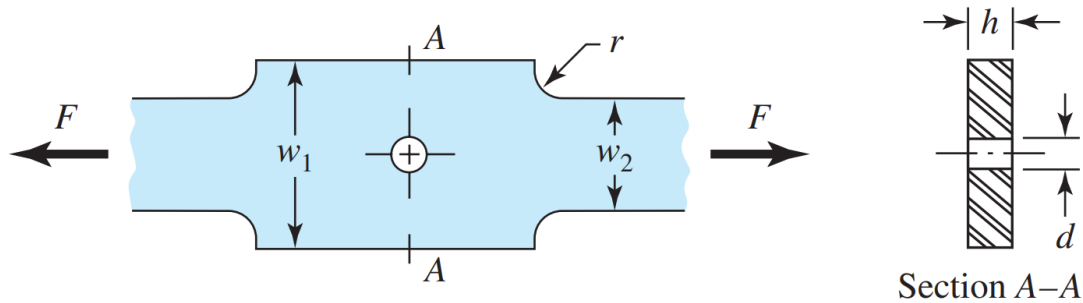
```
[15]: Kf = sp.S('1.72')
      a, b = sp.S('200.78'), sp.S('-0.145')
      sig = 35
      (sig*Kf/a)**(1/b)
```

```
[15]: 4052.76515886349
```

This relationship is very sensitive.

## 4 Problem 6-30

### 4.1 Given



The figure above shows the free body diagram of a connecting link portion having stress concentration at three sections. The dimensions are  $r = 0.25 \text{ in}$ ,  $d = 0.40 \text{ in}$ ,  $h = 0.50 \text{ in}$ ,  $w_1 = 3.50 \text{ in}$ , and  $w_2 = 3.0 \text{ in}$ . The forces  $F$  fluctuate between a tension of 5 kips and a compression of 16 kips. Neglect column action.

### 4.2 Find

Find the least factor of safety if the material is cold drawn AISI 1018 steel.

```
[16]: Sy = 54 # ksi
      Sut = 64 # ksi
      r, d, h, w1, w2 = sp.S('0.25'), sp.S('0.4'), sp.S('0.5'), sp.S('3.5'), sp.S(3)
```

### 4.3 Solution

The least factor of safety comes from the stress concentration due to the diameter because its  $K_t$  value is greater than the  $K_t$  value from the fillets by observing the figures from Table A-15.

```
[17]: d/w1
```

```
[17]: 0.114285714285714
```

$$K_t = 2.7$$

```
[18]: # Find notch sensitivity
      Kt = sp.S('2.7')
      a = (sp.S('0.246') - sp.S('3.08e-3')*Sut + sp.S('1.51e-5')*Sut**2 - sp.S('2.
      ↪67e-8')*Sut**3)**2
      q = 1/(1 + sp.sqrt(a/(d/2))) # notch radius refers to hole radius
      # q = sp.S('0.85') # Solution's approximation
      q
```

```
[18]:
```

0.811722489977041

```
[19]: # Find Kf
Kf = 1 + q*(Kt - 1)
Kf
```

```
[19]: 2.37992823296097
```

```
[20]: # Find Se prime
if Sut <= 200:
    S_e_prime = 0.5*sp.S(Sut)
else:
    S_e_prime = sp.S(100)

S_e_prime # ksi
```

```
[20]: 32.0
```

```
[21]: a_factor, b_exponent = 2, sp.S('-0.217') # Table 6-2
k_a = a_factor*Sut**b_exponent
k_b = 1 # Eq. 6-20
k_c = sp.S('0.85') # Eq. 6-25
S_e = k_a*k_b*k_c*S_e_prime
S_e # ksi
```

```
[21]: 22.0626586316956
```

Use the following to obtain the factor of safety,

$$n_f = \frac{S_e}{K_f \sigma_a}$$

```
[22]: sig_max = 5/(h*(w1 - d))
sig_min = -16/(h*(w1 - d))
sig_a = (sig_max - sig_min)/2
n_f = S_e/(Kf*sig_a)
n_f
```

```
[22]: 1.36847347329589
```

The answer in the back of the book is 1.33, but this comes from approximating that  $q = 0.85$ , which looks pretty conservative (see Figure 6-26).

## 5 Problem 6-35

### 5.1 Given

A steel part is loaded with a combination of bending, axial, and torsion such that the following stresses are created at a particular location:

- Bending: Completely reversed, with a maximum stress of 60 MPa
- Axial: Constant stress of 20 MPa
- Torsion: Repeated load, varying from 0 MPa to 70 MPa

Assume the varying stresses are in phase with each other. The part contains a notch such that  $K_{f,bending} = 1.4$ ,  $K_{f,axial} = 1.1$ , and  $K_{f,torsion} = 2.0$ . The material properties are  $S_y = 300 \text{ MPa}$  and  $S_u = 400 \text{ MPa}$ . The completely adjusted endurance limit is found to be  $S_e = 160 \text{ MPa}$ .

```
[23]: Sy, Su = sp.S(300), sp.S(400)
K_bend, K_axial, K_tors = sp.S('1.4'), sp.S('1.1'), sp.S(2)
Se = sp.S(160)

sig_bend_a, sig_bend_m = sp.S(60), 0
sig_axial_a, sig_axial_m = 0, 20
tau_tors_a, tau_tors_m = 35, 35
```

### 5.2 Find

Find the factor of safety for fatigue based on infinite life, using the Goodman criterion. If the life is not infinite, estimate the number of cycles, using the Walker criterion to find the equivalent completely reversed stress. Be sure to check for yielding.

### 5.3 Solution

Use Eq. 6-66 and 6-67,

$$\sigma'_a = \left\{ \left[ (K_f)_{\text{bending}} (\sigma_{a0})_{\text{bending}} + (K_f)_{\text{axial}} (\sigma_{a0})_{\text{axial}} \right]^2 + 3 \left[ (K_{fs})_{\text{torsion}} (\tau_{a0})_{\text{torsion}} \right]^2 \right\}^{1/2}$$

$$\sigma'_m = \left\{ \left[ (K_f)_{\text{bending}} (\sigma_{m0})_{\text{bending}} + (K_f)_{\text{axial}} (\sigma_{m0})_{\text{axial}} \right]^2 + 3 \left[ (K_{fs})_{\text{torsion}} (\tau_{m0})_{\text{torsion}} \right]^2 \right\}^{1/2}$$

```
[24]: sig_a = sp.sqrt((K_bend*sig_bend_a + K_axial*sig_axial_a)**2 +
↪ 3*(K_tors*tau_tors_a)**2)
sig_a
```

```
[24]: 147.499152539938
```

```
[25]: sig_m = sp.sqrt((K_bend*sig_bend_m + K_axial*sig_axial_m)**2 +
    ↪ 3*(K_tors*tau_tors_m)**2)
sig_m
```

```
[25]: 123.223374405995
```

Check for yielding using Eq. 6-43,

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m}$$

```
[26]: Sy/(sig_a + sig_m)
```

```
[26]: 1.10814568475092
```

It is greater than 1, meaning stress will stay under the yield stress.

Calculate the factor of safety using the Goodman equation (Eq. 6-41).

```
[27]: nf = 1/(sig_a/Se + sig_m/Su)
nf
```

```
[27]: 0.813055631442246
```

The life is not infinite because the value is less than 1. For the Walker criterion, use Eq. 6-57 and Eq. 6-62.

```
[28]: # Find gamma
gamma = -sp.S('0.0002')*Su + sp.S('0.8818')
gamma
```

```
[28]: 0.8018
```

```
[29]: # Get completely reversed stress
sig_reversed = (sig_m + sig_a)**(1 - gamma)*sig_a**gamma
sig_reversed
```

```
[29]: 166.364927970006
```

The number of cycles is (Eq. 6-15),

$$N = \left( \frac{\sigma_{ar}}{a} \right)^{1/b}$$

```
[30]: f = sp.S('0.9') # Figure 6-23
a = ((f*Su)**2/Se)
b = (-sp.Rational(1, 3)*log10(f*Su/Se))
N = (sig_reversed/a)**(1/b)
N.n()
```

```
[30]: 717273.099133359
```

The answer in the back of the book is heavily rounded.