Fatigue Homework 6

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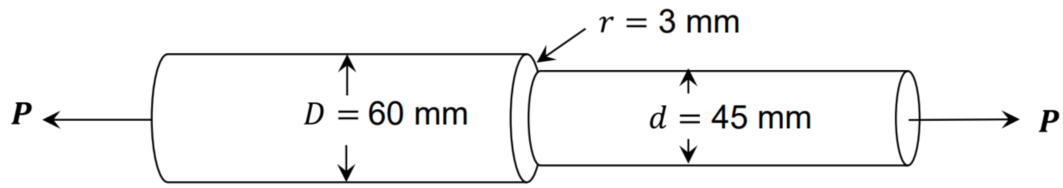
[1]:
```python
# Notebook Preamble
import matplotlib.pyplot as plt
import numpy as np

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1

1.1 Given

A stepped circular rod of 4340 steel (with $S_u = 1468$ MPa) with diameters of 60 and 45 mm has a root radius of 3 mm at the stepped section. The rod is to be subjected to axial cyclic loading.



The cyclic yield strength (S'_y) is estimated from:

$$S'_y = K'(0.002)^{n'}$$

where K' and n' are given in Table A.2

For the purpose of constructing Haigh diagram, exact value of σ_f is not needed, as the diagram is not very sensitive to its value. Since σ_f is not listed in Table A.2 for this material, we use Eq. 5.20 in the textbook to approximate it as

$$\sigma_f \approx S_u + 345 \ (MPa)$$

1.2 Find

Using a Haigh diagram, determine the following for an approximate median fatigue life of 10^6 cycles:

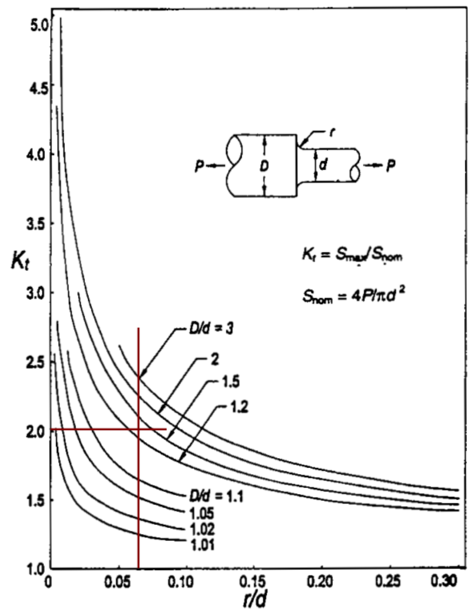
a. What fully reversed alternating force, P_a, can be applied?
b. What is the maximum value of P_a, if proper compressive residual stresses are present at the notch root? What is the magnitude of the compressive residual stress needed to obtain this maximum alternating stress?
c. What value of P_a can be applied if the residual stress calculated in (b) is tensile? What fully reversed alternating force, P_a, can be applied?

1.3 Solution

According to equation 4.3b, the endurance limit is 700 MPa for materials with an ultimate strength greater than 1400 MPa. With the size effect, the endurance limit becomes

$$S_f = 0.85(700) = 595 \ MPa$$

1.3.1 Part A



[2]:
```python
D, d, r = 60, 45, 3  # mm
Su = 1468   # MPa
size_effect = 0.85

if Su <= 1400:
    Sf = 0.5*Su*size_effect
else:
    Sf = 700*size_effect

D/d
```

[2]: 1.3333333333333333

[3]:
```python
r/d
```

[3]: 0.06666666666666667

From above the stress concentration factor is $K_t = 2$. The fatigue notch factor for the fully reversed condition is

$$K_f = 1 + \frac{K_t - 1}{1 + a/r}$$

where $a = 0.0254 \left(\frac{2070}{S_u}\right)^{1.8}$ with a in mm and S_u in MPa.

[4]:
```python
Kt = 2
a = 0.0254*(2070/Su)**1.8
```

```
a   # mm
```

[4]: 0.047149103389883054

[5]:
```
Kf = 1 + (Kt - 1)/(1 + a/r)
Kf
```

[5]: 1.9845268144780213

Since the stress is maximized at the smaller diameter,

$$P_a = \frac{S_f}{K_f}\left(\frac{\pi}{4}\right)(d)^2$$

[6]:
```
Pa = Sf/Kf*np.pi/4*d**2
Pa   # N
```

[6]: 476842.4418454644

1.3.2 Part B

From Table A.2,

Property	Value
S_y	1371 MPa
S_y'	863 MPa
S_f	595 MPa
σ_f	1813 MPa
K_f	1.98
S_{cat}	70 MPa

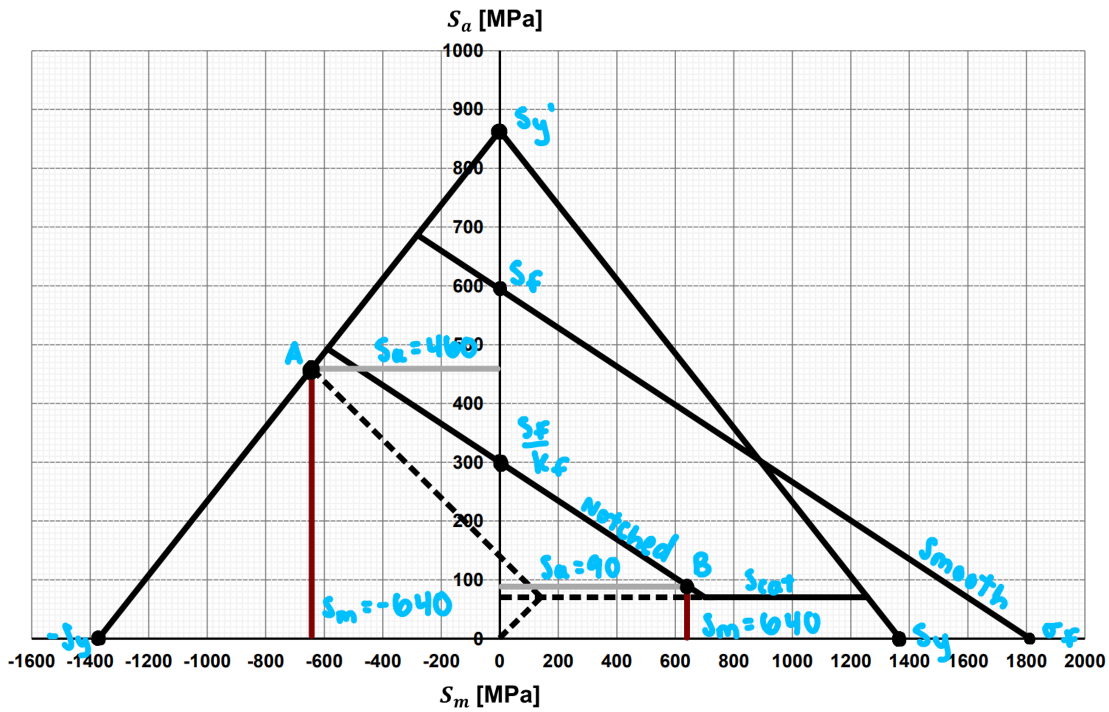
S_{cat} comes from the fact that this is a hard steel. The other calculations are shown below.

[7]:
```
sig_f = Su + 345
sig_f   # MPa
```

[7]: 1813

[8]:
```
K_prime, n_prime = 1996, 0.135
Sy_prime = K_prime*0.002**n_prime
Sy_prime   # MPa
```

[8]: 862.5804014077875



[9]:
```python
Sa = 460   # MPa
Pa = Sa*np.pi/4*d**2
Pa  # N
```

[9]: 731598.3892047232

1.3.3 Part C

From above, point B is when the mean stress is tensile.

[10]:
```python
Sa = 90
Pa = Sa*np.pi/4*d**2
Pa  # N
```

[10]: 143138.81527918496