Baja Project

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[1]:
```python
# Notebook Preamble
%config ZMQInteractiveShell.ast_node_interactivity = 'all'
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from IPython.display import display, Latex

plt.style.use('maroon_ipynb.mplstyle')
```

Contents

1 Parameters

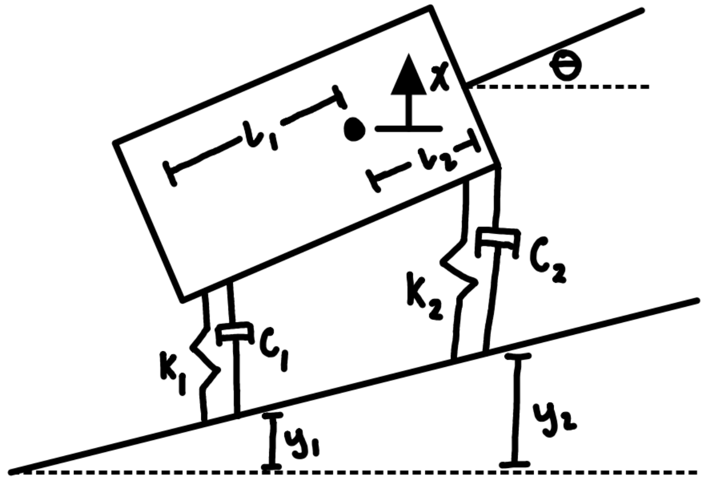


Figure 1: Baja Model

Property	Value
W_1	?
W_2	?
k_{e1}	?
k_{e2}	?
L_1	?
L_2	?
I_G	?

The values W_1 and W_2 were estimated to be ? and ? respectfully. This information was used to calculate the equivalent stiffness in the front and rear suspensions,

$$k_{e1} = \frac{W_1}{\Delta x}$$

$$k_{e2} = \frac{W_2}{\Delta x}$$

1.1 Calculating I_G

The pitch natural frequency ω_{n2}^2 (refers to the motion caused by a changing θ) was calculated by obtaining data from an accelerometer below. This data was obtained by producing a rocking motion with the accelerometer at the end of the vehicle. Similarly, the bounce natural frequency ω_{n1}^2 (refers to motion caused by vertical bouncing - changing x) was obtained from the accelerometer data below. The accelerometer was placed close to the center of gravity and vertical, bouncing motion was produced on the vehicle.

Show plots here and calculations

The mass moment of inertia may be calculated by obtaining the moment equation,

$$I_G\ddot{\theta} = -k_{e1}(y_1 - x + L_1\theta)L_1 + k_{e2}(y_2 - x - L_2\theta)L_2$$

Since the data was obtained on a level surface and x was very small, the equation simplifies to

$$I_G\ddot{\theta} = -k_{e1}L_1^2\theta - k_{e2}L_2^2\theta$$
$$I_G s^2\theta = -k_{e1}L_1^2\theta - k_{e2}L_2^2\theta$$

Plugging in the pitch natural frequency in for s and solving for I_G (only interested in magnitude),

$$I_G = \frac{k_{e1}L_1^2 + k_{e2}L_2^2}{\omega_{n2}^2}$$

Calculate the moment of inertia

2 Equations of Motion

Summing the moments and forces in the vertical direction,

$$\begin{cases} m\ddot{x} = k_{e1}(y_1 - x + L_1\theta) + k_{e2}(y_2 - x - L_2\theta) \\ I_G\ddot{\theta} = -k_{e1}(y_1 - x + L_1\theta)L_1 + k_{e2}(y_2 - x - L_2\theta)L_2 \end{cases}$$

In the laplace space,

$$\begin{cases} ms^2X(s) = k_{e1}(Y_1(s) - X(s) + L_1\theta(s)) + k_{e2}(Y_2(s) - X(s) - L_2\theta(s)) \\ I_G s^2\theta(s) = -k_{e1}(Y_1(s) - X(s) + L_1\theta(s))L_1 + k_{e2}(Y_2(s) - X(s) - L_2\theta(s))L_2 \end{cases}$$

2.1 Natural Frequencies

The system may be placed in the matrix form with y_1 and y_2 being set equal to zero because the free response is the main interest in determining the natural frequencies.

[2]:
```
# Input all measured quantities here
# For now, these are the quantities from example in the book
k1 = 1.6e4
k2 = 2.5e4
L1 = 1.5
L2 = 1.1
m = 730
IG = 1350

# Putting the system in the matrix form
# Defining symbols (underscore denotes symbolic variable)
L1_, L2_ = sp.symbols('L_1 L_2')
x_, k1_, k2_, IG_, s_, m_ = sp.symbols('x k_1 k_2 I_G s m')
X_, theta_ = sp.symbols(r'X \theta')
```

```python
eq1 = sp.Eq(m_*s_**2*X_, k1_*(-X_ + L1_*theta_) + k2_*(-X_ - L2_*theta_))
eq1
eq2 = sp.Eq(IG_*s_**2*theta_, -k1_*(-X_ + L1_*theta_)*L1_ + k2_*(-X_ -
    L2_*theta_)*L2_)
eq2

M_ = sp.linear_eq_to_matrix([eq1, eq2], (X_, theta_))[0]
X_matrix = sp.Matrix([X_, theta_])
sp.Eq(sp.MatMul(M_, X_matrix), sp.Matrix([0, 0]))
```

[2]: $Xms^2 = k_1 (L_1\theta - X) + k_2 (-L_2\theta - X)$

[2]: $I_G\theta s^2 = -L_1k_1 (L_1\theta - X) + L_2k_2 (-L_2\theta - X)$

[2]: $\begin{bmatrix} k_1 + k_2 + ms^2 & -L_1k_1 + L_2k_2 \\ -L_1k_1 + L_2k_2 & I_Gs^2 + L_1^2k_1 + L_2^2k_2 \end{bmatrix} \begin{bmatrix} X \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The expanded matrix is,

[3]:
```python
# Expanding the matrix symbolically
poly_ = M_.det()
poly_.collect(s_)

# Getting a numerical polynomial
poly = poly_.subs([
    (L1_, L1),
    (IG_, IG),
    (m_, m),
    (L2_, L2),
    (k1_, k1),
    (k2_, k2)
])
poly

display(Latex('Roots/Natural Frequencies:'))
roots = sp.roots(poly)
roots_keys = list(roots.keys())
nat_freq = [abs(sp.im(roots_keys[0])), abs(sp.im(roots_keys[-1]))]
roots_numpy = np.float64(nat_freq)
for root in roots:
    display(root)
```

[3]: $I_Gms^4 + L_1^2k_1k_2 + 2L_1L_2k_1k_2 + L_2^2k_1k_2 + s^2 (I_Gk_1 + I_Gk_2 + L_1^2k_1m + L_2^2k_2m)$

[3]: $985500s^4 + 103712500.0s^2 + 2704000000.0$

Roots/Natural Frequencies:

$-6.90068060114689i$

$6.90068060114689i$

$-7.59072228952376i$

$7.59072228952376i$

2.2 Mode Ratio $\frac{X}{\theta}$

The mode ratio $\frac{X}{\theta}$ may be found using the bounce equation (first equation in the systems above),

[4]:
```
# Finding the mode ratio
T_ = sp.Symbol('T')
w_ = sp.Symbol(r'\omega')

sol = sp.solve(eq1.subs(X_, T_*theta_), T_)[0]
sp.Eq(X_/theta_, sol)

sol_num = sol.subs([(L1_, L1), (L2_, L2), (k1_, k1), (k2_, k2), (m_, m)])
sp.Eq(X_/theta_, sol_num)

sol_omega = sol_num.subs(s_, w_*sp.I)
sp.Eq(X_/theta_, sol_omega)
```

[4]: $$\frac{X}{\theta} = \frac{L_1k_1 - L_2k_2}{k_1 + k_2 + ms^2}$$

[4]: $$\frac{X}{\theta} = -\frac{3500.0}{730s^2 + 41000.0}$$

[4]: $$\frac{X}{\theta} = -\frac{3500.0}{41000.0 - 730\omega^2}$$

Now we can calculate the mode shape based on the system's natural frequencies and plot over a range of frequencies.

[5]:
```
%config ZMQInteractiveShell.ast_node_interactivity = 'last_expr'
# Getting the mode shapes
# Units are length
mode1 = sol_omega.subs(w_, nat_freq[0]).n()
mode1
mode2 = sol_omega.subs(w_, nat_freq[1]).n()
mode2
```

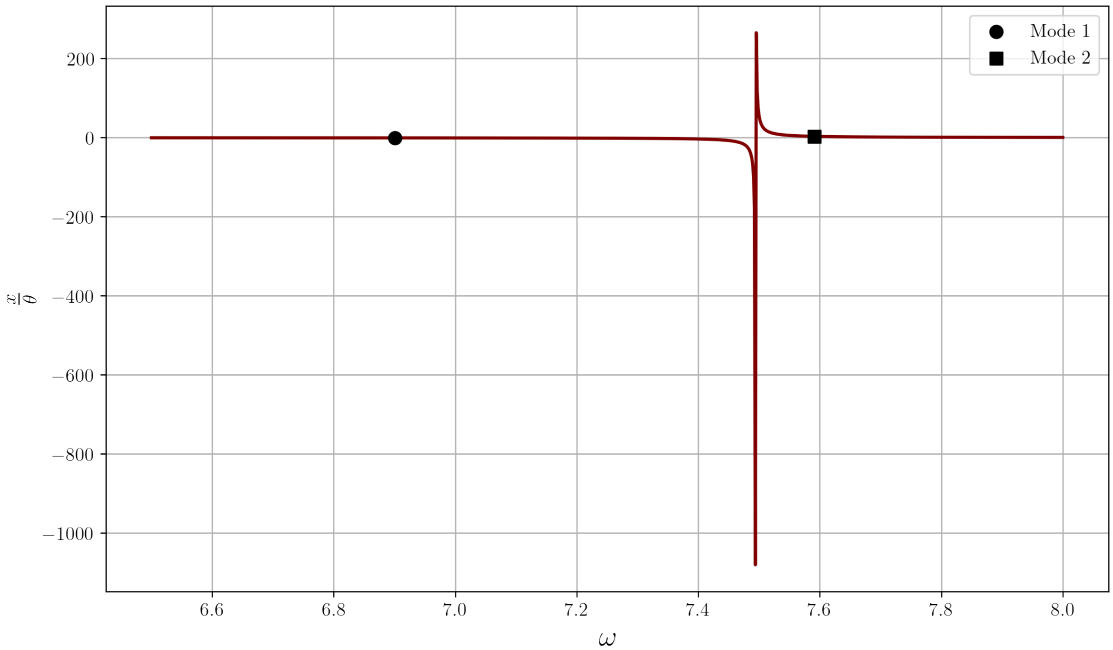
[5]: -0.561091364368354

[5]: 3.29592502385955

[6]:
```
# Getting a plot
nat1, nat2 = roots_numpy
mode_lamb = sp.lambdify(w_, sol_omega, modules='numpy')
w = np.linspace(6.5, 8, 1000)

plt.plot(w, mode_lamb(w), zorder=2)
plt.scatter(nat1, np.float64(mode1), marker='o', label='Mode 1', zorder=3,␣
↪color='black')
```

```
plt.scatter(nat2, np.float64(mode2), marker='s', label='Mode 2', zorder=3,␣
↪color='black')
plt.legend()
plt.xlabel(r'$\omega$')
plt.ylabel(r'$\frac{x}{\theta}$')
plt.show()
```



Comment on the values. A positive value means that the node is behind the vehicle and vice versa. A larger value means that the mode is predominantly a bounce motion since $x > \theta$ and vice versa. We want the bounce to be behind the rear axle and the pitch value to be near the front axle. *See more on page 847.*