# FEA Homework 2

February 14, 2022

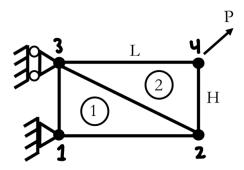
#### Gabe Morris

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## 1 Problem 1

## 1.1 Given



 $P=150\,lb,\,L=5\,in,\,H=2\,in,\,t=0.5\,in,\,E=30\cdot 10^6\,psi,\,{\rm and}\,\,\nu=0.30.$  The angle of P is assumed to be  $45^\circ.$ 

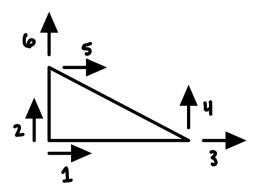
Notice that the global nodes have been rearranged. This was done to make the mapping easier.

#### 1.2 Find

- a. The global stiffness matrix
- b. The displacements at each node
- c. The stresses within each element
- d. Plot the undeformed and deformed shape

#### 1.3 Solution

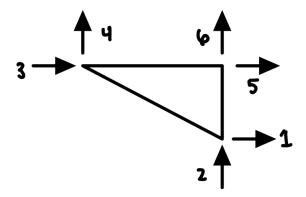
For the first element,



$$\delta_1 = \delta_2 = \delta_5 = 0$$
$$\delta_3 = u_2$$

$$\delta_4 = v_2$$
$$\delta_6 = v_3$$

For the second element,



$$\delta_3 = 0$$

$$\delta_3 = 0$$

$$\delta_1 = u_2$$

$$\delta_2 = v_2$$

$$\delta_4 = v_3$$

$$\delta_5 = u_4$$

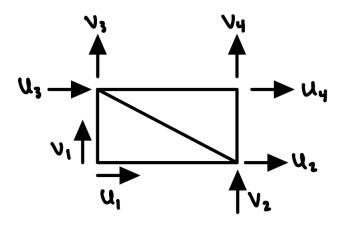
$$\delta_2 = v_2$$

$$\delta_4 = v_3$$

$$\delta_5 = u_4$$

$$\delta_6 = v_4$$

The global displacements are,



## 1.3.1 Part A

[2]: # Define numerical inputs here

$$P_{-} = 150$$

$$t_{-} = 0.5$$

$$E_{nu} = 30e6, 0.3$$

```
# Define local coordinates for each element
     x1 = [0, L_{-}, 0]
     y1 = [0, 0, H_]
     x2 = [L_, 0, L_]
     y2 = [0, H_, H_]
     # The area
     A_1 = ((x1[1] - x1[0])*(y1[2] - y1[0]) - (x1[2] - x1[0])*(y1[1] - y1[0]))/2
     A_2 = -1*((x2[1] - x2[0])*(y2[2] - y2[0]) - (x2[2] - x2[0])*(y2[1] - y2[0]))/2
     A_1, A_2
[2]: (5.0, 5.0)
[3]: # Define a symbolic B
     A, y_23, y_31, y_12, x_32, x_13, x_21 = sp.symbols('A <math>y_{23} y_{31} y_{12}
      \rightarrow x_{32} x_{13} x_{21}')
     B = 1/(2*A)*sp.Matrix([
         [y_23, 0, y_31, 0, y_12, 0],
          [0, x_32, 0, x_13, 0, x_21],
         [x_32, y_23, x_13, y_31, x_21, y_12]
     ])
     В
[3]: \lceil \frac{y_{23}}{2A} \rceil
           0
                              0
                0
                         0
[4]:  # Numeric B
     # Remember that indices start at 0
     B1 = B.subs([
         (A, A_1),
         (y_23, y1[1] - y1[2]),
         (y_31, y1[2] - y1[0]),
         (y_12, y1[0] - y1[1]),
         (x_32, x1[2] - x1[1]),
         (x_13, x1[0] - x1[2]),
         (x_21, x1[1] - x1[0])
     ])
     В1
[4]: \Gamma-0.2
             0
                  0.2
                                0 -
            -0.5 0
                       0
                           0
                               0.5
     -0.5 -0.2 0 0.2 0.5 0
[5]: B2 = B.subs([
         (A, A 2),
         (y_23, y2[1] - y2[2]),
```

```
(y_31, y2[2] - y2[0]),
          (y_12, y2[0] - y2[1]),
          (x_32, x2[2] - x2[1]),
          (x_13, x2[0] - x2[2]),
          (x_21, x2[1] - x2[0])
     ])
     В2
[5]: <sub>[5]</sub>
               0.2
                     0
                        -0.2
                                0
                     0
                          0
                               -0.5
       0
          0.5
               0
                    0.2 -0.5 -0.2
     0.5
[6]: E = E_{/(1 - nu_{**2})*sp.Matrix([
          [1, nu_, 0],
          [nu_, 1, 0],
          [0, 0, (1 - nu_{})/2]
     round_expr(E, 3)
[6]: <sub>[32967032.967</sub>
                     9890109.89
                                        0
       9890109.89
                    32967032.967
                                        0
           0
                          0
                                  11538461.538
[7]: k1_local = t_*A_1*sp.transpose(B1)*E*B1
     round_expr(k1_local, 3)
[7]: \(\Gamma_{10508241.758}\)
                      5357142.857
                                     -3296703.297
                                                   -2884615.385
                                                                  -7211538.462
                                                                                  -2472527.473
       5357142.857
                      21758241.758
                                     -2472527.473
                                                   -1153846.154
                                                                  -2884615.385
                                                                                 -20604395.604
                                                                                  2472527.473
      -3296703.297
                     -2472527.473
                                     3296703.297
                                                         0
                                                                        0
      -2884615.385
                     -1153846.154
                                          0
                                                    1153846.154
                                                                   2884615.385
                                                                                       0
      -7211538.462
                     -2884615.385
                                          0
                                                    2884615.385
                                                                   7211538.462
      -2472527.473 \quad -20604395.604
                                     2472527.473
                                                         0
                                                                        0
                                                                                  20604395.604
[8]: k2\_local = t_*A\_2*sp.transpose(B2)*E*B2
     round_expr(k2_local, 3)
[8]: <sub>7211538.462</sub>
                           0
                                          0
                                                    2884615.385
                                                                  -7211538.462
                                                                                  -2884615.385
            0
                                     2472527.473
                      20604395.604
                                                         0
                                                                  -2472527.473
                                                                                 -20604395.604
            0
                      2472527.473
                                                         0
                                     3296703.297
                                                                  -3296703.297
                                                                                  -2472527.473
       2884615.385
                                                                  -2884615.385
                                                                                  -1153846.154
                           0
                                          0
                                                    1153846.154
      -7211538.462
                     -2472527.473
                                     -3296703.297
                                                   -2884615.385
                                                                  10508241.758
                                                                                  5357142.857
      -2884615.385
                    -20604395.604
                                    -2472527.473
                                                   -1153846.154
                                                                   5357142.857
                                                                                  21758241.758
[9]: k1_global = k1_local.col_insert(6, sp.zeros(rows=6, cols=2)).row_insert(6, sp.

→zeros(rows=2, cols=8))
     round_expr(k1_global, 3)
[9]:
```

```
10508241.758
                       5357142.857
                                     -3296703.297 -2884615.385 -7211538.462
                                                                                  -2472527.473
                                                                   -2884615.385
                                                                                                 0
                                                                                                    0
        5357142.857
                      21758241.758
                                     -2472527.473
                                                    -1153846.154
                                                                                 -20604395.604
       -3296703.297
                      -2472527.473
                                      3296703.297
                                                          0
                                                                        0
                                                                                   2472527.473
                                                                                                 0
                                                                                                    0
                                                     1153846.154
                                                                    2884615.385
                                                                                                 0
                                                                                                    0
       -2884615.385
                      -1153846.154
                                           0
                                                                                        0
       -7211538.462
                                           0
                                                     2884615.385
                                                                    7211538.462
                                                                                        0
                                                                                                 0
                                                                                                    0
                      -2884615.385
                                                                                                    0
       -2472527.473 \quad -20604395.604
                                      2472527.473
                                                          0
                                                                        0
                                                                                  20604395.604
                                                                                                 0
                                                          0
                                                                        0
                                                                                                 0
                                                                                                    0
             0
                            0
                                           0
                                                                                        0
             0
                            0
                                           0
                                                          0
                                                                        0
                                                                                        0
                                                                                                 0
                                                                                                    0
[10]: k2 global = k2 local.col insert(0, sp.zeros(rows=6, cols=2)).row insert(0, sp.
       ⇒zeros(rows=2, cols=8))
      round_expr(k2_global, 3)
[10]:
         0
                   0
                                  0
                                                 0
                                                                0
                                                                              0
      Γ0
       0 \quad 0
                   0
                                  0
                                                 0
                                                                0
                                                                              0
       0 0
                                  0
              7211538.462
                                                 0
                                                           2884615.385
                                                                         -7211538.462
                                                                                        -2884615.385
       0 \quad 0
                  0
                            20604395.604
                                            2472527.473
                                                                0
                                                                                        -20604395.604
                                                                         -2472527.473
       0 0
                   0
                             2472527.473
                                                                0
                                            3296703.297
                                                                         -3296703.297
                                                                                        -2472527.473
       0 0
              2884615.385
                                  0
                                                 0
                                                           1153846.154
                                                                         -2884615.385
                                                                                        -1153846.154
       0 \quad 0 \quad -7211538.462
                            -2472527.473
                                           -3296703.297
                                                                                         5357142.857
                                                          -2884615.385
                                                                         10508241.758
            -2884615.385
                           -20604395.604
                                           -2472527.473
                                                          -1153846.154
                                                                         5357142.857
                                                                                        21758241.758
[11]: k_global = k1_global + k2_global
      round_expr(k_global, 1)
[11]:
      Γ10508241.8
                                                                                          0
                     5357142.9
                                 -3296703.3
                                              -2884615.4
                                                            -7211538.5
                                                                        -2472527.5
                    21758241.8
                                 -2472527.5
                                              -1153846.2
                                                            -2884615.4
                                                                        -20604395.6
                                                                                          0
        5357142.9
                                                                                                   -2884615.4
       -3296703.3
                    -2472527.5
                                 10508241.8
                                                   0
                                                                0
                                                                         5357142.9
                                                                                      -7211538.5
       -2884615.4
                    -1153846.2
                                      0
                                               21758241.8
                                                            5357142.9
                                                                             0
                                                                                      -2472527.5
                                                                                                  -20604395.6
                    -2884615.4
                                               5357142.9
       -7211538.5
                                      0
                                                            10508241.8
                                                                             0
                                                                                      -3296703.3
                                                                                                   -2472527.5
       -2472527.5
                    -20604395.6
                                  5357142.9
                                                   0
                                                                         21758241.8
                                                                                      -2884615.4
                                                                                                   -1153846.2
                                                                0
            0
                         0
                                 -7211538.5
                                              -2472527.5
                                                            -3296703.3
                                                                         -2884615.4
                                                                                      10508241.8
                                                                                                    5357142.9
            0
                         0
                                 -2884615.4
                                              -20604395.6
                                                           -2472527.5
                                                                        -1153846.2
                                                                                      5357142.9
                                                                                                   21758241.8
     1.3.2 Part B
[12]: F 1, F 2, F 5 = sp.symbols('F 1 F 2 F 5')
      F = sp.Matrix([
```

0

0

```
[F<sub>1</sub>],
      [F<sub>2</sub>],
      [0],
      [0],
      [F_5],
      [0],
      [P_*sp.sqrt(2)/2],
      [P_*sp.sqrt(2)/2]
]).n()
F
```

[12]:

```
F_1
               0
               F_5
       106.066017177982\\
       106.066017177982
[13]: u2, v2, v3, u4, v4 = sp.symbols(u_2 v_2 v_3 u_4 v_4)
       d = sp.Matrix([
           [0],
           [0],
           [u2],
           [v2],
           [0],
           [v3],
           [u4],
           [v4]
       ])
       d
[13]: [0
        0
       u_2
       v_2
        0
       v_3
       u_4
       v_4
[14]: system = sp.Eq(F, k_global*d)
       round_expr(system, 3)
[14]:
                                      -3296703.297u_2 - 2884615.385v_2 - 2472527.473v_3
                                     -2472527.473u_2 - 1153846.154v_2 - 20604395.604v_3
                              10508241.758u_2 - 7211538.462u_4 + 5357142.857v_3 - 2884615.385v_4 \\
          0
                                     -2472527.473u_4 + 21758241.758v_2 - 20604395.604v_4
                                      -3296703.297u_4 + 5357142.857v_2 - 2472527.473v_4
                              5357142.857u_2 - 2884615.385u_4 + 21758241.758v_3 - 1153846.154v_4
       106.066
                     -7211538.462u_2 + 10508241.758u_4 - 2472527.473v_2 - 2884615.385v_3 + 5357142.857v_4
       106.066
                    -2884615.385u_2 + 5357142.857u_4 - 20604395.604v_2 - 1153846.154v_3 + 21758241.758v_4
[15]: solved = sp.solve(system)
       for key, value in solved.items():
           display_latex(f'${sp.latex(key)}={sp.latex(value)}$')
      F_1 = -265.165042944938
      F_2 = -106.066017177975
```

```
F_5 = 159.099025766958
      u_2 = 2.42131523003677 \cdot 10^{-5}
      u_4 = 6.89954607183987 \cdot 10^{-6}
      v_2 = 6.54725387051039 \cdot 10^{-5}
      v_3 = -1.4243030764922 \cdot 10^{-6}
      v_4 = 6.83110553439691 \cdot 10^{-5}
      1.3.3 Part C
[16]: strain1 = B1*sp.Matrix([
            [0],
            [0],
            [solved[u2]],
            [solved[v2]],
            [0],
            [solved[v3]]
       ])
       strain1
[16]: \boxed{4.84263046007354 \cdot 10^{-6}}
        -7.12151538246102 \cdot 10^{-7}
       1.30945077410208 \cdot 10^{-5}
[17]: stress1 = E*strain1
       stress1
24.4166241684383\\
       151.090473934855
[18]: strain2 = B2*sp.Matrix([
            [solved[u2]],
            [solved[v2]],
            [0],
            [solved[v3]],
            [solved[u4]],
            [solved[v4]]
       ])
       strain2
[18]: \lceil -1.37990921436797 \cdot 10^{-6} \rceil
       -1.41925831943259 \cdot 10^{-6}
       -5.29026856982834 \cdot 10^{-6}
[19]: stress2 = E*strain2
       stress2
[19]:
```

```
\begin{bmatrix} -59.5281333032225 \\ -60.4361895739445 \\ -61.0415604210962 \end{bmatrix}
```

#### 1.3.4 Part D

Here is a plot of the deformed shape.

