System Dynamics Homework 5

November 28, 2023

First Last

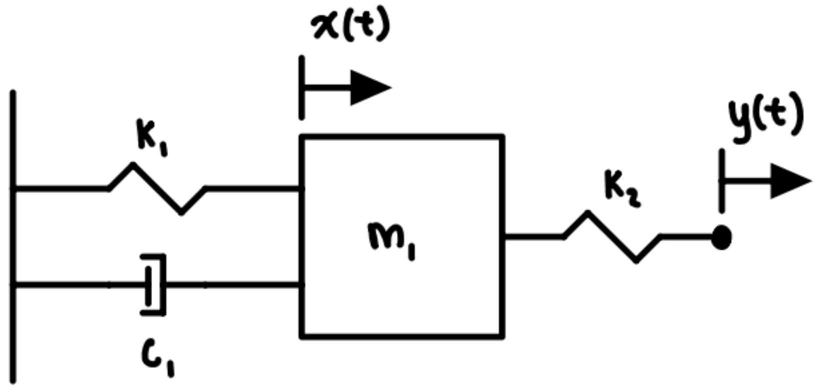
[1]:
```python
import control as ct
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import fsolve

# Use whichever pertains to your set-up
# plt.style.use('maroon_ipynb.mplstyle')
# plt.style.use('../maroon_ipynb.mplstyle')
```

Contents

1 Problem 1

1.1 Given



The mass above is being controlled by the input position $y(t)$. Take $k_1 = 10\,\frac{lbf}{in}$, $k_2 = 100\,\frac{lbf}{in}$, $c_1 = 1\,\frac{lbf\,s}{in}$, and $m_1 = 0.0518\,goobs$ where $1\,goob = \frac{lbs\,s^2}{in}$. The number comes from the mass weighing $20\,lbf$.

$$W = mg$$
$$20\,lbf = m \cdot 32.2\,\frac{ft}{s^2}$$
$$m = \frac{20}{32.2}\,slugs = 0.621\,\frac{lbf\,s^2}{ft} \cdot \frac{ft}{12\,in} = 0.0518\frac{lbf\,s^2}{in} = 0.0518\,goobs$$

1.2 Find

For $y(t) = 1.5\sin(\omega_r t)$,

a. Find the equation of motion.
b. Find the transfer function $\frac{X(s)}{Y(s)}$.
c. Plot the Magnitude ($M(\omega)$ - not in decibels) and Phase Response for $1 \leq \omega \leq 1000\,rad/s$. Use the `bode()` function for checking. Note, the `bode()` function will produce a log-log plot on the y and x-axis, so the usual plot of $M(\omega)$ will look different, since we use a linear scale for the y-axis.
d. Find the resonant frequency ω_r.
e. Find the steady state function $x_{ss}(t)$ at the resonant frequency and plot the forced response on top of the steady state response up to 1 second.

1.3 Solution

1.3.1 Part A - Finding the Equation of Motion

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1.3.2 Part B - Finding the Transfer Function

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1.3.3 Part C - Plotting the Frequency Response

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1.3.4 Part D - Finding the Resonant Frequency

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1.3.5 Part E - Finding the Response at the Resonant Frequency

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2 Problem 2

2.1 Given

A certain series RLC circuit has the following transfer function.

$$T(s) = \frac{I(s)}{V(s)} = \frac{Cs}{LCs^2 + RCs + 1}$$

Suppose that $L = 300\,H$, $R = 10^4\,\Omega$, and $C = 10^{-6}\,F$.

2.2 Find

Determine the filtering properties by

a. Deriving the equations for the magnitude and the phase.
b. Plot the magnitude ($m(\omega)$) and phase and use the analytical solution above for checking.
c. Find the bandwidth from ω_1 to ω_2. What kind of filter is this (i.e. low-pass, band-pass, or high-pass)? Note: The relationship for finding ω_r is not valid for this example because of the presence of the s in the numerator of the transfer function. So, just find the peak M_{peak} by using the `max()` function.

2.3 Solution

2.3.1 Part A - Finding Magnitude and Phase Equations

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2.3.2 Part B - Plotting the Frequency Response

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2.3.3 Part C - Finding the Bandwidth

[ ]:

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