Harder Circuits Problem

November 1, 2023

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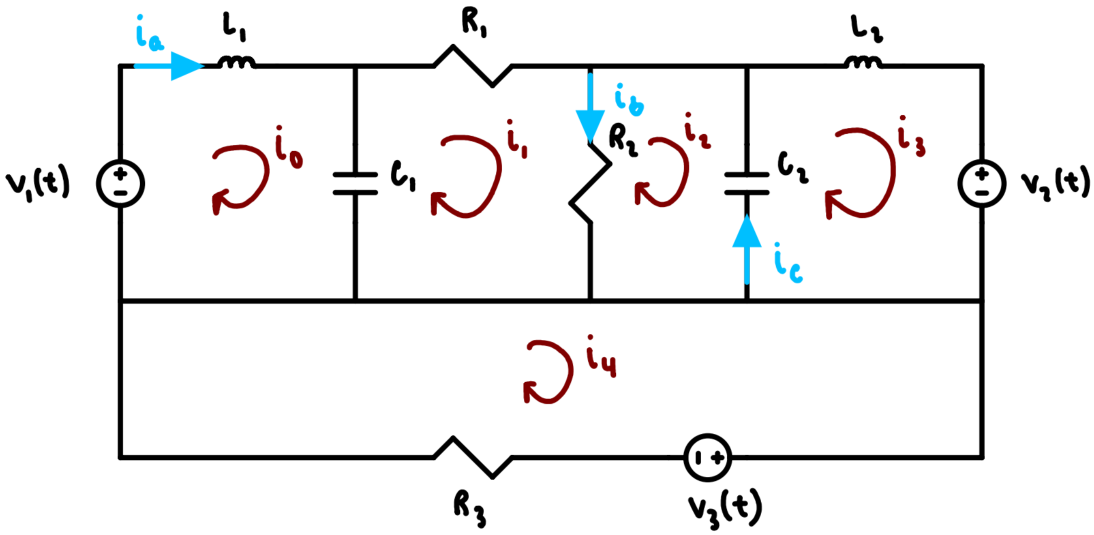
[1]:
```
import matplotlib.pyplot as plt
import numpy as np
import sympy as sp
from scipy.integrate import odeint

plt.style.use('../maroon_ipynb.mplstyle')
```

Contents

1 Given



$$L_1 = 5\,mH,\ L_2 = 2\,mH$$
$$C_1 = 80\,mF,\ C_2 = 50\,mF$$
$$R_1 = 1\,k\Omega,\ R_2 = 600\,\Omega,\ R_3 = 300\,\Omega$$
$$v_1(t) = 120\sin(5t),\ v_2(t) = 120\sin(5t)e^{-10t},\ v_3(t) = 120\sin(5t)e^{-20t}$$

2 Find

Solve for the actual currents i_a, i_b, and i_c by

a. Constructing the ODE's of the system.
b. Put the system in the state-variable form.
c. Solve using `odeint` and plot $i_a(t)$, $i_b(t)$, and $i_c(t)$ up to 2 seconds.

3 Solution

3.1 Part A

[2]:
```
L1, L2 = sp.symbols('L1 L2')
C1, C2 = sp.symbols('C1 C2')
R1, R2, R3 = sp.symbols('R1 R2 R3')
t = sp.Symbol('t')

L1_, L2_ = 5e-3, 2e-3
C1_, C2_ = 80e-3, 50e-3
R1_, R2_, R3_ = 1_000, 600, 300
```

```python
sub_values = [
    (L1, L1_),
    (L2, L2_),
    (C1, C1_),
    (C2, C2_),
    (R1, R1_),
    (R2, R2_),
    (R3, R3_)
]

v1_ = 120*sp.sin(5*t)
v2_ = 120*sp.sin(5*t)*sp.exp(-10*t)
v3_ = 120*sp.sin(5*t)*sp.exp(-20*t)

v1_diff = sp.lambdify(t, v1_.diff(), modules='numpy')
v2_diff = sp.lambdify(t, v2_.diff(), modules='numpy')
v3_diff = sp.lambdify(t, v3_.diff(), modules='numpy')

v1, v2, v3 = sp.Function('v1')(t), sp.Function('v2')(t), sp.Function('v3')(t)
i0, i1, i2, i3, i4 = [sp.Function(f'i{i}')(t) for i in range(5)]

eq1 = sp.Eq(L1*i0.diff(t, 2) + 1/C1*(i0 - i1), v1.diff())
eq2 = sp.Eq(R1*i1.diff() + R2*(i1.diff() - i2.diff()) + 1/C1*(i1 - i0), 0)
eq3 = sp.Eq(1/C2*(i2 - i3) + R2*(i2.diff() - i1.diff()), 0)
eq4 = sp.Eq(L2*i3.diff(t, 2) + v2.diff() + 1/C2*(i3 - i2), 0)
eq5 = sp.Eq(v3.diff() + R3*i4.diff(), 0)

eqs = [eq1, eq2, eq3, eq4, eq5]
display(*eqs)
```

$$L_1 \frac{d^2}{dt^2} i_0(t) + \frac{i_0(t) - i_1(t)}{C_1} = \frac{d}{dt} v_1(t)$$

$$R_1 \frac{d}{dt} i_1(t) + R_2 \left(\frac{d}{dt} i_1(t) - \frac{d}{dt} i_2(t)\right) + \frac{-i_0(t) + i_1(t)}{C_1} = 0$$

$$R_2 \left(-\frac{d}{dt} i_1(t) + \frac{d}{dt} i_2(t)\right) + \frac{i_2(t) - i_3(t)}{C_2} = 0$$

$$L_2 \frac{d^2}{dt^2} i_3(t) + \frac{d}{dt} v_2(t) + \frac{-i_2(t) + i_3(t)}{C_2} = 0$$

$$R_3 \frac{d}{dt} i_4(t) + \frac{d}{dt} v_3(t) = 0$$

3.2 Part B

[3]:
```
i5, i6 = sp.Function('i5')(t), sp.Function('i6')(t)

eq6 = sp.Eq(i0.diff(), i5)
eq7 = sp.Eq(i3.diff(), i6)

sub_states = [
    (i0.diff(), i5),
    (i3.diff(), i6)
]

eqs_subs = [eq.subs(sub_states) for eq in eqs]
state_sol = sp.solve(eqs_subs + [eq6, eq7],
                     [i0.diff(), i1.diff(), i2.diff(), i3.diff(), i4.diff(), i5.
 ↪diff(), i6.diff()])

funcs = []
for key, value in state_sol.items():
    display(sp.Eq(key, value))
    funcs.append(sp.lambdify(
        (i0, i1, i2, i3, i4, i5, i6, v1.diff(), v2.diff(), v3.diff()),
        value.subs(sub_values),
        modules='numpy'
    ))
```

$$\frac{d}{dt}i_0(t) = i_5(t)$$

$$\frac{d}{dt}i_1(t) = -\frac{i_2(t)}{C_2R_1} + \frac{i_3(t)}{C_2R_1} + \frac{i_0(t)}{C_1R_1} - \frac{i_1(t)}{C_1R_1}$$

$$\frac{d}{dt}i_2(t) = -\frac{i_2(t)}{C_2R_2} + \frac{i_3(t)}{C_2R_2} - \frac{i_2(t)}{C_2R_1} + \frac{i_3(t)}{C_2R_1} + \frac{i_0(t)}{C_1R_1} - \frac{i_1(t)}{C_1R_1}$$

$$\frac{d}{dt}i_3(t) = i_6(t)$$

$$\frac{d}{dt}i_4(t) = -\frac{\frac{d}{dt}v_3(t)}{R_3}$$

$$\frac{d}{dt}i_5(t) = \frac{\frac{d}{dt}v_1(t)}{L_1} - \frac{i_0(t)}{C_1L_1} + \frac{i_1(t)}{C_1L_1}$$

$$\frac{d}{dt}i_6(t) = -\frac{\frac{d}{dt}v_2(t)}{L_2} + \frac{i_2(t)}{C_2L_2} - \frac{i_3(t)}{C_2L_2}$$

3.3 Part C

```
[4]: def state_vars(i, t_):
         return [func(i[0], i[1], i[2], i[3], i[4], i[5], i[6], v1_diff(t_),
     ↪v2_diff(t_), v3_diff(t_)) for func in funcs]


     t_array = np.linspace(0, 2, 1000)
     sol = odeint(state_vars, (0, 0, 0, 0, 0, 0, 0), t_array)
     ia = sol[:, 0]
     ib = sol[:, 1] - sol[:, 2]
     ic = sol[:, 3] - sol[:, 2]

     plt.plot(t_array, ia, label='$i_a(t)$')
     plt.plot(t_array, ib, label='$i_b(t)$')
     plt.plot(t_array, ic, label='$i_c(t)$')
     plt.xlabel('Time ($s$)')
     plt.ylabel('Current ($A$)')
     plt.legend()
     plt.show()
```