Multiple Inputs

October 23, 2023

[1]:
```python
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
import control as ct

plt.style.use('../maroon_ipynb.mplstyle')
```

This is an extension to Example 2.5.1 in the book. This is how you get the response to a system that has multiple inputs. You add the forced response from each input function. For this example, the first input is $f(t) = t^2$, and the second input is $g(t) = e^{-5t}$.

[2]:
```python
t = sp.Symbol('t')
x = sp.Function('x')(t)
f = t**2
g = sp.exp(-5*t)

eq = sp.Eq(5*x.diff(t, 2) + 30*x.diff() + 40*x, 6*f - 20*g)
eq
```

[2]:

$$40x(t) + 30\frac{d}{dt}x(t) + 5\frac{d^2}{dt^2}x(t) = 6t^2 - 20e^{-5t}$$

[3]:
```python
sol = sp.dsolve(eq, ics={
    x.subs(t, 0): 0,
    x.diff().subs(t, 0): 0
})
sol
```

[3]:

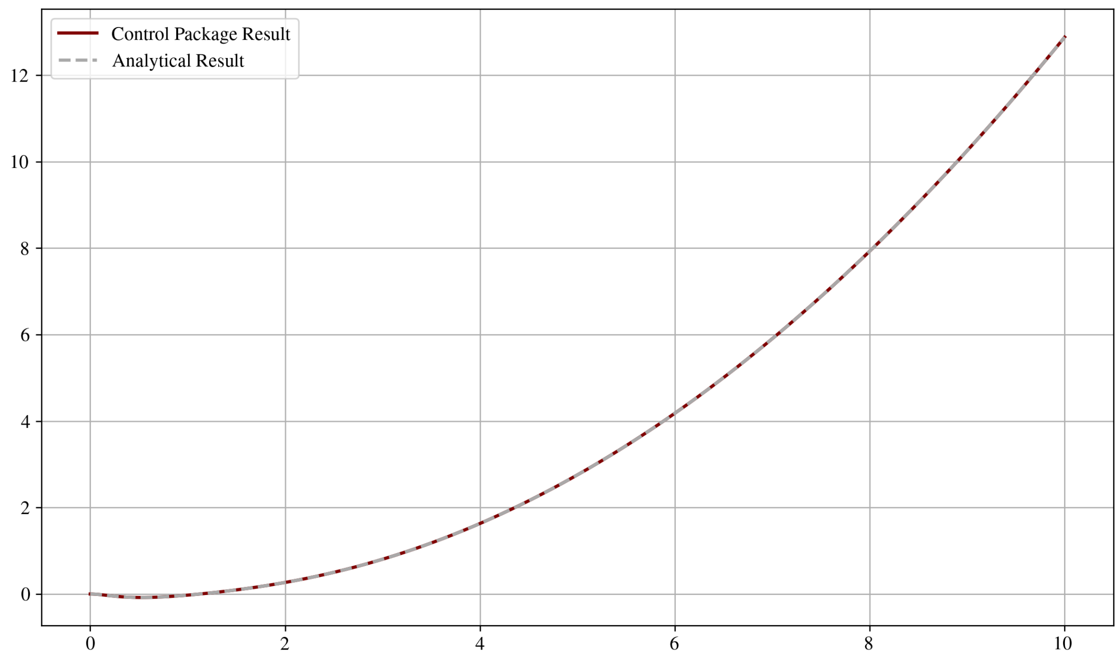
$$x(t) = \frac{3t^2}{20} - \frac{9t}{40} + \frac{21}{160} - \frac{49e^{-2t}}{60} + \frac{323e^{-4t}}{160} - \frac{4e^{-5t}}{3}$$

[4]:
```python
x_lamb = sp.lambdify(t, sol.rhs, modules='numpy')
t_array = np.linspace(0, 10, 500)

den = [5, 30, 40]  # all transfer functions have the same denominator␣
 ↪(characteristic equation)
X_F = ct.tf(6, den)
X_G = ct.tf(-20, den)
display(X_F, X_G)
```

$$\frac{6}{5s^2 + 30s + 40}$$

$$\frac{-20}{5s^2 + 30s + 40}$$

[5]:
```
_, f_response = ct.forced_response(X_F, T=t_array, U=t_array**2)
_, g_response = ct.forced_response(X_G, T=t_array, U=np.exp(-5*t_array))
response = f_response + g_response

plt.plot(t_array, response, label='Control Package Result')
plt.plot(t_array, x_lamb(t_array), label='Analytical Result', ls='--',␣
 ↪color='darkgrey')
plt.legend()
plt.show()
```



For a more complicated ODE or systems of ODE's, then it would be wise to use the state-variable form or state-space form. For example:

$$\dddot{x}_0 + 5\ddot{x}_0 + 7\dot{x}_0 + 3x_0 = f(t) + g(t)$$

[6]:
```
x0 = sp.Function('x0')(t)
eq1 = sp.Eq(x0.diff(t, 3) + 5*x0.diff(t, 2) + 7*x0.diff() + 3*x0, f + g)
eq1
```

[6]:

$$3x_0(t) + 7\frac{d}{dt}x_0(t) + 5\frac{d^2}{dt^2}x_0(t) + \frac{d^3}{dt^3}x_0(t) = t^2 + e^{-5t}$$

[7]:
```
d_sol = sp.dsolve(eq1, ics={x0.diff(t, 2).subs(t, 0): 0, x0.diff().subs(t, 0):
 ↪0, x0.subs(t, 0): 0})
x0_lamb = sp.lambdify(t, d_sol.rhs, modules='numpy')
d_sol
```

[7]:

$$x_0(t) = \frac{t^2}{3} - \frac{14t}{9} + \left(-\frac{7t}{8} - \frac{83}{32}\right)e^{-t} + \frac{68}{27} + \frac{23e^{-3t}}{216} - \frac{e^{-5t}}{32}$$

[8]:
```
# Putting in state variable form
x1, x2 = sp.Function('x1')(t), sp.Function('x2')(t)

eq2 = sp.Eq(x0.diff(), x1)
eq3 = sp.Eq(x1.diff(), x2)

sub_states = [
    (x0.diff(t, 3), x2.diff()),
    (x0.diff(t, 2), x1.diff()),
    (x1.diff(), x2)
]

eq1 = eq1.subs(sub_states)
state_sol = sp.solve([eq1, eq2, eq3], [x0.diff(), x1.diff(), x2.diff()],
 ↪dict=True)[0]
for key, value in state_sol.items(): display(sp.Eq(key, value))
```

$$\frac{d}{dt}x_0(t) = x_1(t)$$

$$\frac{d}{dt}x_1(t) = x_2(t)$$

$$\frac{d}{dt}x_2(t) = t^2 - 3x_0(t) - 7x_1(t) - 5x_2(t) + e^{-5t}$$

[9]:
```
A = [
    [0, 1, 0],
    [0, 0, 1],
    [-3, -7, -5]
]

B = [
    [0, 0],
    [0, 0],
    [1, 1]
]

C = [
```

```
    [1, 0, 0]
]

D = [[0, 0]]

ss1 = ct.ss(A, B, C, D)
ss1
```

[9]:

$$\left(\begin{array}{ccc|cc} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -3 & -7 & -5 & 1 & 1 \\ \hline 1 & 0 & 0 & 0 & 0 \end{array}\right)$$

[10]:

```
f_lamb = lambda t_: t_**2
g_lamb = lambda t_: np.exp(-5*t_)

t_array = np.linspace(0, 5, 1000)
inp = np.stack([f_lamb(t_array), g_lamb(t_array)])  # Input has to be stacked␣
 ↪and be 2x1000

_, sol_con = ct.forced_response(ss1, T=t_array, U=inp)

plt.plot(t_array, sol_con[0], label='Control Package Result')
plt.plot(t_array, x0_lamb(t_array), ls='--', label='Analytical Result',␣
 ↪color='darkgrey')
plt.legend()
plt.show()
```