System Dynamics Homework 4

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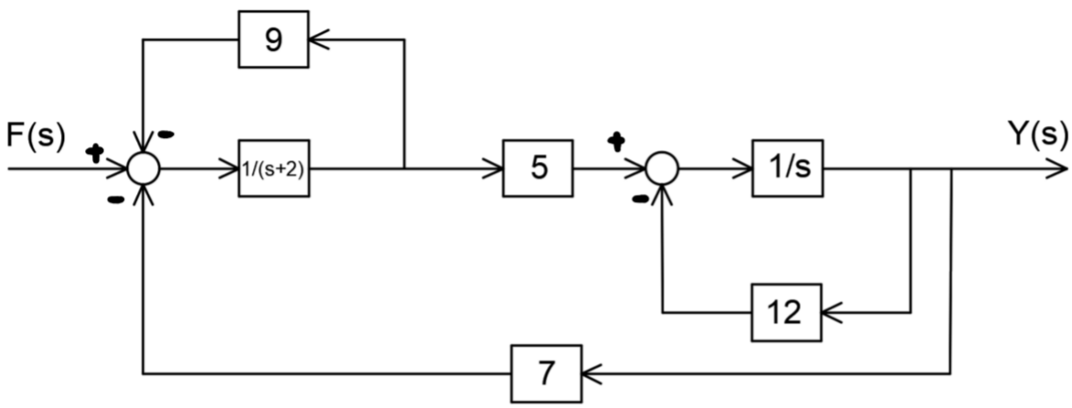
[1]:
```python
import sympy as sp
import control as ct
import matplotlib.pyplot as plt
import numpy as np
from scipy.integrate import odeint

plt.style.use('../maroon_ipynb.mplstyle')
```

Contents

1 Problem 1

1.1 Given



1.2 Find

Find the transfer function $\frac{Y(s)}{F(s)}$ for the block diagram.

1.3 Solution

The solution can be determined using two different methods. The first is an algebraic solution where B is the expression after the first block seen above. The second can be determined using the feedback and series functions.

[2]:
```
# Using algebra
from sympy.abc import F, Y, B, s

eq1 = sp.Eq((F - 9*B - 7*Y)*1/(s + 2), B)
eq2 = sp.Eq((B*5 - 12*Y)*1/s, Y)
display(eq1, eq2)
```

$$\frac{-9B + F - 7Y}{s + 2} = B$$

$$\frac{5B - 12Y}{s} = Y$$

[3]:
```
sol = sp.solve([eq1, eq2], [Y, B], dict=True)[0]
sol[Y]/F
```

[3]:

$$\frac{5}{s^2 + 23s + 167}$$

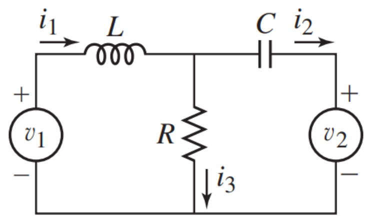
[4]:
```
sys1 = ct.feedback(ct.tf(1, [1, 0]), 12)
sys2 = ct.series(5, sys1)
sys3 = ct.feedback(ct.tf(1, [1, 2]), 9)
sys4 = ct.series(sys3, sys2)
sys5 = ct.feedback(sys4, 7)
sys5
```

[4]:

$$\frac{5}{s^2 + 23s + 167}$$

2 Problem 2

2.1 Given



$$L = 500\,mH,\ C = 100\,\mu F,\ R = 300\,\Omega$$

$$v_1 = 5e^{-t}\sin(6t)\,V,\ v_2 = 10\sin(t)\,V$$

All initial conditions are zero.

2.2 Find

a. The system of ODE's (should be two equations if using mesh currents).
b. Solve the system for i_1, i_2, and i_3 as seen the figure above. Use any method to find the result and plot up to 6 seconds.

2.3 Solution

2.3.1 Part A

[5]:
```
L_, C_, R_ = 500e-3, 100e-6, 300

L, C, R = sp.symbols('L C R')
t = sp.Symbol('t')
V1 = sp.Function('v_1')(t)
V2 = sp.Function('v_2')(t)
iA = sp.Function('i_A')(t)  # Mesh currents
iB = sp.Function('i_B')(t)  # Mesh currents

eq1 = sp.Eq(L*iA.diff() + R*(iA - iB), V1)  # KVL around iA
eq2 = sp.Eq(1/C*iB + R*(iB.diff() - iA.diff()) + V2.diff(), 0)  # KVL around iB
display(eq1, eq2)
```

$$L\frac{d}{dt}i_A(t) + R\left(i_A(t) - i_B(t)\right) = v_1(t)$$

$$R\left(-\frac{d}{dt}i_A(t) + \frac{d}{dt}i_B(t)\right) + \frac{d}{dt}v_2(t) + \frac{i_B(t)}{C} = 0$$

2.3.2 Part B

The state variable solution is the easiest since we already have a system of first order ODE's.

[6]:
```
# Solving using state variables
state_sol = sp.solve([eq1, eq2], [iA.diff(), iB.diff()], dict=True)[0]
for key, value in state_sol.items(): display(sp.Eq(key, value))
```

$$\frac{d}{dt}i_A(t) = -\frac{Ri_A(t)}{L} + \frac{Ri_B(t)}{L} + \frac{v_1(t)}{L}$$

$$\frac{d}{dt}i_B(t) = -\frac{\frac{d}{dt}v_2(t)}{R} - \frac{Ri_A(t)}{L} + \frac{Ri_B(t)}{L} + \frac{v_1(t)}{L} - \frac{i_B(t)}{CR}$$

[7]:
```
# Getting the analytical solution
subs = [
    (L, sp.S(str(L_))),
    (R, sp.S(str(R_))),
    (C, sp.S(str(C_))),
    (V1, 5*sp.exp(-t)*sp.sin(6*t)),
    (V2.diff(), 10*sp.cos(t))
]

eq1_subs = eq1.subs(subs)
eq2_subs = eq2.subs(subs)
display(eq1_subs, eq2_subs)
```

$$300i_A(t) - 300i_B(t) + 0.5\frac{d}{dt}i_A(t) = 5e^{-t}\sin(6t)$$

$$10000.0i_B(t) + 10\cos(t) - 300\frac{d}{dt}i_A(t) + 300\frac{d}{dt}i_B(t) = 0$$

[8]:
```
d_sol = sp.dsolve([eq1_subs, eq2_subs], ics={
    iA.subs(t, 0): 0,
    iB.subs(t, 0): 0
})
display(*d_sol)
```

$$i_A(t) = -1.66682871529138 \cdot 10^{-6}\sin(t)\sin^2(140.435829552939t) + 8.470329472543 \cdot 10^{-22}\sin(t)\sin(140.435829552939t)\cos(140.435829552939t) - 1.66682871529138 \cdot 10^{-6}\sin(t)\cos^2(140.435829552939t) - 0.00100004722431337\sin^2(140.435829552939t)\cos(t) - 0.00100004722431337\cos(t)\cos^2(140.435829552939t) - 0.000894252621170149e^{-16.6666666666665t}\sin(140.435829552$$
$$0.0018569732844178e^{-16.6666666666665t}\cos(140.435829552939t) + 0.0162490402135116e^{-1.0t}\sin(6.0t)\sin^2(140.43582$$
$$0.0162490402135116e^{-1.0t}\sin(6.0t)\cos^2(140.435829552939t) + 0.00285702050873119e^{-1.0t}\sin^2(140.435829552939t)$$
$$2.71050543121376 \cdot 10^{-19}e^{-1.0t}\sin(140.435829552939t)\cos(6.0t)\cos(140.435829552939t) + 0.00285702050873119e^{-1.0t}\cos(6.0t)\cos^2(140.435829552939t)$$

$$i_B(t) = -8.33414357648159 \cdot 10^{-11}\sin(t)\sin^2(140.435829552939t) + 8.470329472543 \cdot 10^{-22}\sin(t)\sin(140.435829552939t)\cos(140.435829552939t) - 8.33414357646041 \cdot 10^{-11}\sin(t)\cos^2(140.435829552939t) - 0.00100005000236123\sin^2(140.435829552939t)\cos(t) - 0.00100005000236123\cos(t)\cos^2(140.435829552939t) - 0.000434769631157325e^{-16.6666666666665t}\sin(140.435829552$$

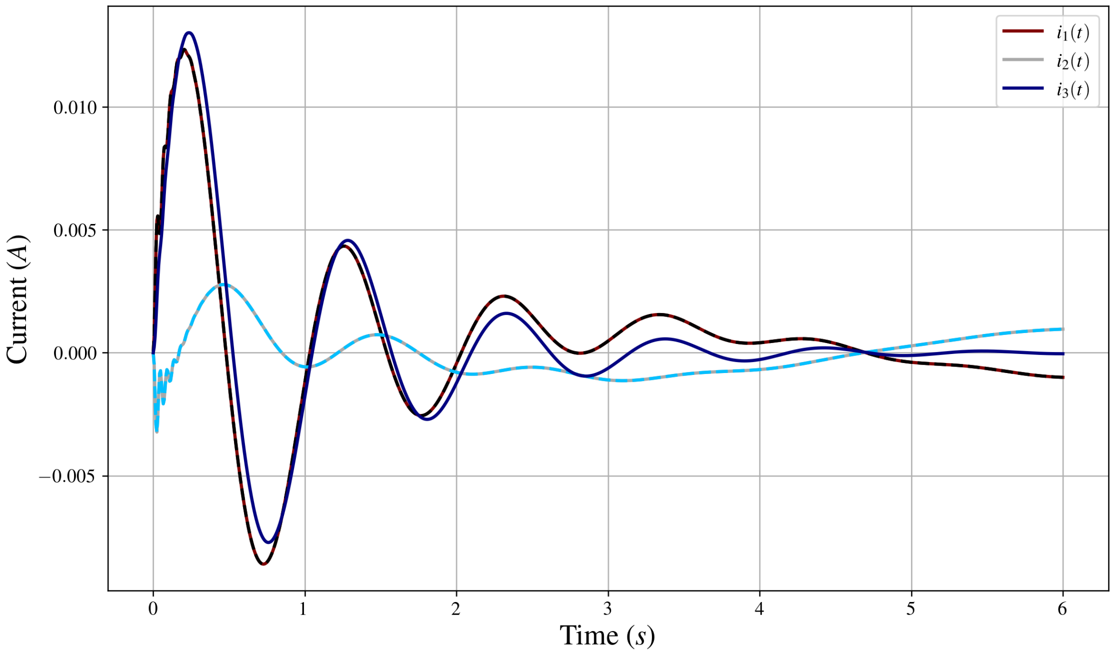
$0.00201469920765718e^{-16.6666666666665t} \cos(140.435829552939t) - 0.000473278391931575e^{-1.0t} \sin(6.0t) \sin^2(140.43$
$0.000473278391931576e^{-1.0t} \sin(6.0t) \cos^2(140.435829552939t) + 0.00301474921001842e^{-1.0t} \sin^2(140.435829552939$
$2.71050543121376 \cdot 10^{-19} e^{-1.0t} \sin(140.435829552939t) \cos(6.0t) \cos(140.435829552939t) +$
$0.00301474921001842e^{-1.0t} \cos(6.0t) \cos^2(140.435829552939t)$

[9]:
```
v1 = lambda t_: 5*np.exp(-t_)*np.sin(6*t_)
v2_diff = lambda t_: 10*np.cos(t_)
iA_lamb = sp.lambdify(t, d_sol[0].rhs, modules='numpy')
iB_lamb = sp.lambdify(t, d_sol[1].rhs, modules='numpy')

def state_vars(i, t_):
    return [
        (v1(t_) + R_*i[1] - R_*i[0])/L_,
        -v2_diff(t_)/R_ - R_/L_*i[0] + R_/L_*i[1] + v1(t_)/L_ - i[1]/(C_*R_)
    ]


t_array = np.linspace(0, 6, 1000)
sol = odeint(state_vars, (0, 0), t_array)
iA, iB = sol[:, 0], sol[:, 1]

plt.plot(t_array, iA, label='$i_1(t)$')
plt.plot(t_array, iA_lamb(t_array), ls='--')
plt.plot(t_array, -iB, label='$i_2(t)$')
plt.plot(t_array, -iB_lamb(t_array), ls='--')
plt.plot(t_array, iA - iB, label='$i_3(t)$')
plt.legend()
plt.xlabel('Time ($s$)')
plt.ylabel('Current ($A$)')
plt.show()
```



$i_1(t)$
$i_2(t)$
$i_3(t)$
0.010
0.005
0.000
−0.005
Current (A)
0
1
2
3
4
5
6
Time (s)