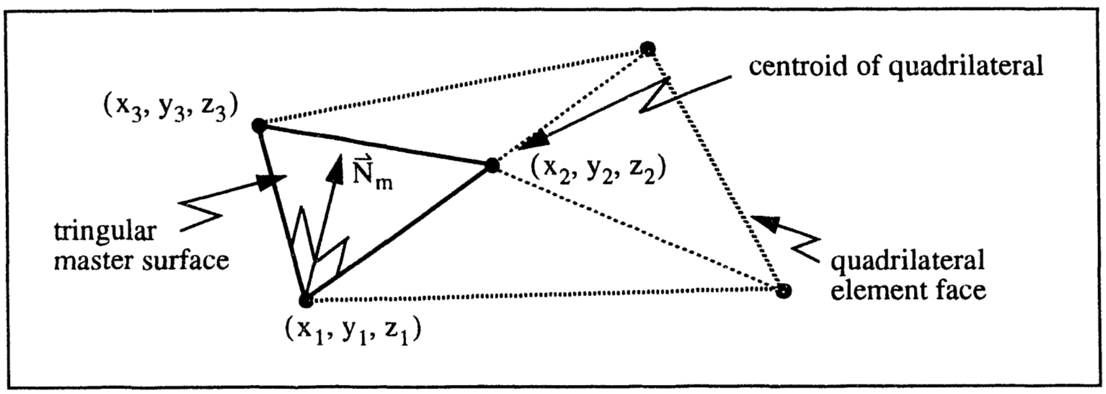
Velocity Based Contact Check

February 15, 2024

[1]:
```
import sympy as sp
from sympy.abc import x, y, z
```



Above is the definition for a master surface. See Appendix 1 for the more rigorous details about the procedure. The purpose of this walkthrough is to verify the math in the appendix.

[2]:
```
# Making symbols for the nodes
x1, x2, x3 = sp.symbols('x1:4')
y1, y2, y3 = sp.symbols('y1:4')
z1, z2, z3 = sp.symbols('z1:4')

# Node coordinate points
n1 = sp.Matrix([x1, y1, z1])
n2 = sp.Matrix([x2, y2, z2])
n3 = sp.Matrix([x3, y3, z3])

x_vec = n1 - n3
y_vec = n2 - n1

N_m = x_vec.cross(y_vec)
a, b, c = N_m
display(x_vec, y_vec, N_m)
```

$$\begin{bmatrix} x_1 - x_3 \\ y_1 - y_3 \\ z_1 - z_3 \end{bmatrix}$$

$$\begin{bmatrix}-x_1 + x_2\\ -y_1 + y_2\\ -z_1 + z_2\end{bmatrix}$$

$$\begin{bmatrix}-\left(-y_1 + y_2\right)\left(z_1 - z_3\right) + \left(y_1 - y_3\right)\left(-z_1 + z_2\right)\\ \left(-x_1 + x_2\right)\left(z_1 - z_3\right) - \left(x_1 - x_3\right)\left(-z_1 + z_2\right)\\ -\left(-x_1 + x_2\right)\left(y_1 - y_3\right) + \left(x_1 - x_3\right)\left(-y_1 + y_2\right)\end{bmatrix}$$

The definition of the plane then becomes the following,

[3]:
```
master_pln = (a*(x - x1) + b*(y - y1) + c*(z - z1)).expand().collect((x, y, z))
master_pln = sp.Eq(master_pln, 0)
master_pln
```

[3]:

$$x\left(y_1 z_2 - y_1 z_3 - y_2 z_1 + y_2 z_3 + y_3 z_1 - y_3 z_2\right) - x_1 y_2 z_3 + x_1 y_3 z_2 + x_2 y_1 z_3 - x_2 y_3 z_1 - x_3 y_1 z_2 + x_3 y_2 z_1 + y\left(-x_1 z_2 + x_1 z_3 + x_2 z_1 - x_2 z_3 - x_3 z_1 + x_3 z_2\right) + z\left(x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 y_1 - x_3 y_2\right) = 0$$

The above is the equation of the triangular master surface plane in terms of the two master nodes and the centroid node. If we consider the fact that the position of the plane changes with velocity, then we can know the plane equation after some Δt by using the following:

$$\langle x_i(t + \Delta t), y_i(t + \Delta t), z_i(t + \Delta t)\rangle = \langle x_i(t) + \dot{x}_i \Delta t, y_i(t) + \dot{y}_i \Delta t, z_i(t) + \dot{z}_i \Delta t\rangle$$

[4]:
```
# Making substitution
x1_dot, y1_dot, z1_dot = sp.symbols(r'\dot{x}_1 \dot{y}_1 \dot{z}_1')
x2_dot, y2_dot, z2_dot = sp.symbols(r'\dot{x}_2 \dot{y}_2 \dot{z}_2')
x3_dot, y3_dot, z3_dot = sp.symbols(r'\dot{x}_3 \dot{y}_3 \dot{z}_3')
del_t = sp.Symbol(r'\Delta t')

del_t_sub = master_pln.lhs.subs([
    (x1, x1 + x1_dot*del_t),
    (y1, y1 + y1_dot*del_t),
    (z1, z1 + z1_dot*del_t),
    (x2, x2 + x2_dot*del_t),
    (y2, y2 + y2_dot*del_t),
    (z2, z2 + z2_dot*del_t),
    (x3, x3 + x3_dot*del_t),
    (y3, y3 + y3_dot*del_t),
    (z3, z3 + z3_dot*del_t)
])

del_t_sub = sp.Eq(del_t_sub.expand().collect((x, y, z)), 0)
a0 = del_t_sub.lhs.subs([(x, 0), (y, 0), (z, 0)])
a1 = del_t_sub.lhs.coeff(x)
a2 = del_t_sub.lhs.coeff(y)
a3 = del_t_sub.lhs.coeff(z)
```

This plane equation in terms of Δt is expressed as the following,

$$a_0 + a_1 x + a_2 y + a_3 z = 0$$

where

$a_0 =$

$-\Delta t^3 \dot{x}_1 \dot{y}_2 \dot{z}_3 + \Delta t^3 \dot{x}_1 \dot{y}_3 \dot{z}_2 + \Delta t^3 \dot{x}_2 \dot{y}_1 \dot{z}_3 - \Delta t^3 \dot{x}_2 \dot{y}_3 \dot{z}_1 - \Delta t^3 \dot{x}_3 \dot{y}_1 \dot{z}_2 + \Delta t^3 \dot{x}_3 \dot{y}_2 \dot{z}_1 - \Delta t^2 \dot{x}_1 \dot{y}_2 z_3 + \Delta t^2 \dot{x}_1 \dot{y}_3 z_2 + \Delta t^2 \dot{x}_1 \dot{z}_2 y_3 - \Delta t^2 \dot{x}_1 \dot{z}_3 y_2 + \Delta t^2 \dot{x}_2 \dot{y}_1 z_3 - \Delta t^2 \dot{x}_2 \dot{y}_3 z_1 - \Delta t^2 \dot{x}_2 \dot{z}_1 y_3 + \Delta t^2 \dot{x}_2 \dot{z}_3 y_1 - \Delta t^2 \dot{x}_3 \dot{y}_1 z_2 + \Delta t^2 \dot{x}_3 \dot{y}_2 z_1 + \Delta t^2 \dot{x}_3 \dot{z}_1 y_2 - \Delta t^2 \dot{x}_3 \dot{z}_2 y_1 - \Delta t^2 \dot{y}_1 \dot{z}_2 x_3 + \Delta t^2 \dot{y}_1 \dot{z}_3 x_2 + \Delta t^2 \dot{y}_2 \dot{z}_1 x_3 - \Delta t^2 \dot{y}_2 \dot{z}_3 x_1 - \Delta t^2 \dot{y}_3 \dot{z}_1 x_2 + \Delta t^2 \dot{y}_3 \dot{z}_2 x_1 - \Delta t \dot{x}_1 y_2 z_3 + \Delta t \dot{x}_1 y_3 z_2 + \Delta t \dot{x}_2 y_1 z_3 - \Delta t \dot{x}_2 y_3 z_1 - \Delta t \dot{x}_3 y_1 z_2 + \Delta t \dot{x}_3 y_2 z_1 + \Delta t \dot{y}_1 x_2 z_3 - \Delta t \dot{y}_1 x_3 z_2 - \Delta t \dot{y}_2 x_1 z_3 + \Delta t \dot{y}_2 x_3 z_1 + \Delta t \dot{y}_3 x_1 z_2 - \Delta t \dot{y}_3 x_2 z_1 - \Delta t \dot{z}_1 x_2 y_3 + \Delta t \dot{z}_1 x_3 y_2 + \Delta t \dot{z}_2 x_1 y_3 - \Delta t \dot{z}_2 x_3 y_1 - \Delta t \dot{z}_3 x_1 y_2 + \Delta t \dot{z}_3 x_2 y_1 - x_1 y_2 z_3 + x_1 y_3 z_2 + x_2 y_1 z_3 - x_2 y_3 z_1 - x_3 y_1 z_2 + x_3 y_2 z_1$

$a_1 =$

$\Delta t^2 \dot{y}_1 \dot{z}_2 - \Delta t^2 \dot{y}_1 \dot{z}_3 - \Delta t^2 \dot{y}_2 \dot{z}_1 + \Delta t^2 \dot{y}_2 \dot{z}_3 + \Delta t^2 \dot{y}_3 \dot{z}_1 - \Delta t^2 \dot{y}_3 \dot{z}_2 + \Delta t \dot{y}_1 z_2 - \Delta t \dot{y}_1 z_3 - \Delta t \dot{y}_2 z_1 + \Delta t \dot{y}_2 z_3 + \Delta t \dot{y}_3 z_1 - \Delta t \dot{y}_3 z_2 - \Delta t \dot{z}_1 y_2 + \Delta t \dot{z}_1 y_3 + \Delta t \dot{z}_2 y_1 - \Delta t \dot{z}_2 y_3 - \Delta t \dot{z}_3 y_1 + \Delta t \dot{z}_3 y_2 + y_1 z_2 - y_1 z_3 - y_2 z_1 + y_2 z_3 + y_3 z_1 - y_3 z_2$

$a_2 =$

$-\Delta t^2 \dot{x}_1 \dot{z}_2 + \Delta t^2 \dot{x}_1 \dot{z}_3 + \Delta t^2 \dot{x}_2 \dot{z}_1 - \Delta t^2 \dot{x}_2 \dot{z}_3 - \Delta t^2 \dot{x}_3 \dot{z}_1 + \Delta t^2 \dot{x}_3 \dot{z}_2 - \Delta t \dot{x}_1 z_2 + \Delta t \dot{x}_1 z_3 + \Delta t \dot{x}_2 z_1 - \Delta t \dot{x}_2 z_3 - \Delta t \dot{x}_3 z_1 + \Delta t \dot{x}_3 z_2 + \Delta t \dot{z}_1 x_2 - \Delta t \dot{z}_1 x_3 - \Delta t \dot{z}_2 x_1 + \Delta t \dot{z}_2 x_3 + \Delta t \dot{z}_3 x_1 - \Delta t \dot{z}_3 x_2 - x_1 z_2 + x_1 z_3 + x_2 z_1 - x_2 z_3 - x_3 z_1 + x_3 z_2$

$a_3 =$

$\Delta t^2 \dot{x}_1 \dot{y}_2 - \Delta t^2 \dot{x}_1 \dot{y}_3 - \Delta t^2 \dot{x}_2 \dot{y}_1 + \Delta t^2 \dot{x}_2 \dot{y}_3 + \Delta t^2 \dot{x}_3 \dot{y}_1 - \Delta t^2 \dot{x}_3 \dot{y}_2 + \Delta t \dot{x}_1 y_2 - \Delta t \dot{x}_1 y_3 - \Delta t \dot{x}_2 y_1 + \Delta t \dot{x}_2 y_3 + \Delta t \dot{x}_3 y_1 - \Delta t \dot{x}_3 y_2 - \Delta t \dot{y}_1 x_2 + \Delta t \dot{y}_1 x_3 + \Delta t \dot{y}_2 x_1 - \Delta t \dot{y}_2 x_3 - \Delta t \dot{y}_3 x_1 + \Delta t \dot{y}_3 x_2 + x_1 y_2 - x_1 y_3 - x_2 y_1 + x_2 y_3 + x_3 y_1 - x_3 y_2$

Note that the math in the appendix is missing the negative signs in this step.

Furthermore, the slave node also moves at some velocity,

$$\langle x_s(t+\Delta t), y_s(t+\Delta t), z_s(t+\Delta t) \rangle = \langle x_s(t) + \dot{x}_s \Delta t, y_s(t) + \dot{y}_s \Delta t, z_s(t) + \dot{z}_s \Delta t \rangle$$

A polynomial of Δt can be found by substituting the slave node motion into the master slave plan equation. The solution of Δt is the time is takes for intersection.

[9]:

```
xs, ys, zs = sp.symbols('x_s y_s z_s')
xs_dot, ys_dot, zs_dot = sp.symbols(r'\dot{x}_s \dot{y}_s \dot{z}_s')

slave_sub = del_t_sub.subs([
```

```
    (x, xs + xs_dot*del_t),
    (y, ys + ys_dot*del_t),
    (z, zs + zs_dot*del_t)
])
slave_sub = sp.Eq(slave_sub.lhs.expand().collect((del_t, del_t**2, del_t**3)),
  0)
b0 = slave_sub.lhs.subs(del_t, 0)
b1 = slave_sub.lhs.coeff(del_t)
b2 = slave_sub.lhs.coeff(del_t**2)
b3 = slave_sub.lhs.coeff(del_t**3)
```

The resulting polynomial of Δt is,

$$b_0 + b_1\Delta t + b_2\Delta t^2 + b_3\Delta t^3 = 0$$

where

$b_0 =$

$$-x_1\left(y_2z_3 - y_2z_s - y_3z_2 + y_3z_s + y_sz_2 - y_sz_3\right) + x_2\left(y_1z_3 - y_1z_s - y_3z_1 + y_3z_s + y_sz_1 - y_sz_3\right) - x_3\left(y_1z_2 - y_1z_s - y_2z_1 + y_2z_s + y_sz_1 - y_sz_2\right) + x_s\left(y_1z_2 - y_1z_3 - y_2z_1 + y_2z_3 + y_3z_1 - y_3z_2\right)$$

$b_1 =$

$$-\dot{x}_1\left(y_2z_3 - y_2z_s - y_3z_2 + y_3z_s + y_sz_2 - y_sz_3\right) + \dot{x}_2\left(y_1z_3 - y_1z_s - y_3z_1 + y_3z_s + y_sz_1 - y_sz_3\right) - \dot{x}_3\left(y_1z_2 - y_1z_s - y_2z_1 + y_2z_s + y_sz_1 - y_sz_2\right) + \dot{x}_s\left(y_1z_2 - y_1z_3 - y_2z_1 + y_2z_3 + y_3z_1 - y_3z_2\right) - x_1\left(\dot{y}_2z_3 - \dot{y}_2z_s - \dot{y}_3z_2 + \dot{y}_3z_s + \dot{y}_sz_2 - \dot{y}_sz_3 - \dot{z}_2y_3 + \dot{z}_2y_s + \dot{z}_3y_2 - \dot{z}_3y_s - \dot{z}_sy_2 + \dot{z}_sy_3\right) + x_2\left(\dot{y}_1z_3 - \dot{y}_1z_s - \dot{y}_3z_1 + \dot{y}_3z_s + \dot{y}_sz_1 - \dot{y}_sz_3 - \dot{z}_1y_3 + \dot{z}_1y_s + \dot{z}_3y_1 - \dot{z}_3y_s - \dot{z}_sy_1 + \dot{z}_sy_3\right) - x_3\left(\dot{y}_1z_2 - \dot{y}_1z_s - \dot{y}_2z_1 + \dot{y}_2z_s + \dot{y}_sz_1 - \dot{y}_sz_2 - \dot{z}_1y_2 + \dot{z}_1y_s + \dot{z}_2y_1 - \dot{z}_2y_s - \dot{z}_sy_1 + \dot{z}_sy_2\right) + x_s\left(\dot{y}_1z_2 - \dot{y}_1z_3 - \dot{y}_2z_1 + \dot{y}_2z_3 + \dot{y}_3z_1 - \dot{y}_3z_2 - \dot{z}_1y_2 + \dot{z}_1y_3 + \dot{z}_2y_1 - \dot{z}_2y_3 - \dot{z}_3y_1 + \dot{z}_3y_2\right)$$

$b_2 =$

$$-\dot{x}_1\left(\dot{y}_2z_3 - \dot{y}_2z_s - \dot{y}_3z_2 + \dot{y}_3z_s + \dot{y}_sz_2 - \dot{y}_sz_3 - \dot{z}_2y_3 + \dot{z}_2y_s + \dot{z}_3y_2 - \dot{z}_3y_s - \dot{z}_sy_2 + \dot{z}_sy_3\right) + \dot{x}_2\left(\dot{y}_1z_3 - \dot{y}_1z_s - \dot{y}_3z_1 + \dot{y}_3z_s + \dot{y}_sz_1 - \dot{y}_sz_3 - \dot{z}_1y_3 + \dot{z}_1y_s + \dot{z}_3y_1 - \dot{z}_3y_s - \dot{z}_sy_1 + \dot{z}_sy_3\right) - \dot{x}_3\left(\dot{y}_1z_2 - \dot{y}_1z_s - \dot{y}_2z_1 + \dot{y}_2z_s + \dot{y}_sz_1 - \dot{y}_sz_2 - \dot{z}_1y_2 + \dot{z}_1y_s + \dot{z}_2y_1 - \dot{z}_2y_s - \dot{z}_sy_1 + \dot{z}_sy_2\right) + \dot{x}_s\left(\dot{y}_1z_2 - \dot{y}_1z_3 - \dot{y}_2z_1 + \dot{y}_2z_3 + \dot{y}_3z_1 - \dot{y}_3z_2 - \dot{z}_1y_2 + \dot{z}_1y_3 + \dot{z}_2y_1 - \dot{z}_2y_3 - \dot{z}_3y_1 + \dot{z}_3y_2\right) - x_1\left(\dot{y}_2\dot{z}_3 - \dot{y}_2\dot{z}_s - \dot{y}_3\dot{z}_2 + \dot{y}_3\dot{z}_s + \dot{y}_s\dot{z}_2 - \dot{y}_s\dot{z}_3\right) + x_2\left(\dot{y}_1\dot{z}_3 - \dot{y}_1\dot{z}_s - \dot{y}_3\dot{z}_1 + \dot{y}_3\dot{z}_s + \dot{y}_s\dot{z}_1 - \dot{y}_s\dot{z}_3\right) - x_3\left(\dot{y}_1\dot{z}_2 - \dot{y}_1\dot{z}_s - \dot{y}_2\dot{z}_1 + \dot{y}_2\dot{z}_s + \dot{y}_s\dot{z}_1 - \dot{y}_s\dot{z}_2\right) + x_s\left(\dot{y}_1\dot{z}_2 - \dot{y}_1\dot{z}_3 - \dot{y}_2\dot{z}_1 + \dot{y}_2\dot{z}_3 + \dot{y}_3\dot{z}_1 - \dot{y}_3\dot{z}_2\right)$$

$b_3 =$

$$-\dot{x}_1\left(\dot{y}_2\dot{z}_3 - \dot{y}_2\dot{z}_s - \dot{y}_3\dot{z}_2 + \dot{y}_3\dot{z}_s + \dot{y}_s\dot{z}_2 - \dot{y}_s\dot{z}_3\right) + \dot{x}_2\left(\dot{y}_1\dot{z}_3 - \dot{y}_1\dot{z}_s - \dot{y}_3\dot{z}_1 + \dot{y}_3\dot{z}_s + \dot{y}_s\dot{z}_1 - \dot{y}_s\dot{z}_3\right) - \dot{x}_3\left(\dot{y}_1\dot{z}_2 - \dot{y}_1\dot{z}_s - \dot{y}_2\dot{z}_1 + \dot{y}_2\dot{z}_s + \dot{y}_s\dot{z}_1 - \dot{y}_s\dot{z}_2\right) + \dot{x}_s\left(\dot{y}_1\dot{z}_2 - \dot{y}_1\dot{z}_3 - \dot{y}_2\dot{z}_1 + \dot{y}_2\dot{z}_3 + \dot{y}_3\dot{z}_1 - \dot{y}_3\dot{z}_2\right)$$
