double_pendulum

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[1]:
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import sympy as sp
from IPython.display import display, Latex

F12x, F12y, F32x, F32y, m2, m3, I2, I3, w2, w3 = sp.symbols(r'F_{12x} F_{12y}␣
↪F_{32x} F_{32y} m_2 m_3 I_2 I_3 w_2 w_3', real=True)
T12, l2, l3, t2_ddot, t2_dot, t3_ddot, t3_dot = sp.symbols(r'T_{12} l_2 l_3␣
↪\ddot{\theta_2} \dot{\theta_2} \ddot{\theta_3} \dot{\theta_3}', real=True)
t2, t3 = sp.symbols(r'\theta_2 \theta_3', real=True)

# Using the complex form of vectors to get the accelerations
a2 = l2/2*t2_ddot*sp.I*sp.exp(sp.I*t2) - l2/2*t2_dot**2*sp.exp(sp.I*t2)
a2x = sp.re(a2)
a2y = sp.im(a2)

ap = l2*t2_ddot*sp.I*sp.exp(sp.I*t2) - l2*t2_dot**2*sp.exp(sp.I*t2)
a3 = l3/2*t3_ddot*sp.I*sp.exp(sp.I*t3) - l3/2*t3_dot**2*sp.exp(sp.I*t3) + ap
a3x = sp.re(a3)
a3y = sp.im(a3)

f1 = sp.Eq(F12x - F32x, m2*a2x)
f2 = sp.Eq(F12y - F32y - w2, m2*a2y)
f3 = sp.Eq(T12 - F12y*(l2/2*sp.cos(t2)) + F12x*(l2/2*sp.sin(t2)) + F32x*(l2/
↪2*sp.sin(t2)) - F32y*(l2/2*sp.cos(t2)), I2*t2_ddot)
f4 = sp.Eq(F32x, m3*a3x)
f5 = sp.Eq(F32y - w3, m3*a3y)
f6 = sp.Eq(F32x*l3/2*sp.sin(t3) - F32y*l3/2*sp.cos(t3), I3*t3_ddot)

for i, eq in enumerate([f1, f2, f3, f4, f5, f6]):
    display(eq)
```

$$F_{12x} - F_{32x} = m_2\left(-\frac{\ddot{\theta}_2 l_2 \sin(\theta_2)}{2} - \frac{\dot{\theta}_2^{\,2} l_2 \cos(\theta_2)}{2}\right)$$

$$F_{12y} - F_{32y} - w_2 = m_2\left(\frac{\ddot{\theta}_2 l_2 \cos(\theta_2)}{2} - \frac{\dot{\theta}_2^{\,2} l_2 \sin(\theta_2)}{2}\right)$$

$$\frac{F_{12x} l_2 \sin(\theta_2)}{2} - \frac{F_{12y} l_2 \cos(\theta_2)}{2} + \frac{F_{32x} l_2 \sin(\theta_2)}{2} - \frac{F_{32y} l_2 \cos(\theta_2)}{2} + T_{12} = I_2\ddot{\theta}_2$$

$$F_{32x} = m_3 \left(-\ddot{\theta}_2 l_2 \sin(\theta_2) - \frac{\ddot{\theta}_3 l_3 \sin(\theta_3)}{2} - \dot{\theta}_2^{\,2} l_2 \cos(\theta_2) - \frac{\dot{\theta}_3^{\,2} l_3 \cos(\theta_3)}{2} \right)$$

$$F_{32y} - w_3 = m_3 \left(\ddot{\theta}_2 l_2 \cos(\theta_2) + \frac{\ddot{\theta}_3 l_3 \cos(\theta_3)}{2} - \dot{\theta}_2^{\,2} l_2 \sin(\theta_2) - \frac{\dot{\theta}_3^{\,2} l_3 \sin(\theta_3)}{2} \right)$$

$$\frac{F_{32x} l_3 \sin(\theta_3)}{2} - \frac{F_{32y} l_3 \cos(\theta_3)}{2} = I_3 \ddot{\theta}_3$$

[2]:
```
f32x = sp.solve(f4, F32x)[0]
f32y = sp.solve(f5, F32y)[0]
f12x = sp.solve(f1, F12x)[0].subs(F32x, f32x)
f12y = sp.solve(f2, F12y)[0].subs(F32y, f32y)
t3_ddot_ = sp.solve(f6.subs(F32x, f32x).subs(F32y, f32y), t3_ddot)[0]
t2_ddot_solved = sp.solve(f3.subs(F12x, f12x).subs(F12y, f12y).subs(F32x, f32x).
 ↪subs(F32y, f32y).subs(t3_ddot, t3_ddot_), t2_ddot)[0]
t2_ddot_solved.simplify()
```

[2]:
$$\frac{16 I_3 T_{12} - 8 I_3 \dot{\theta}_3^{\,2} l_2 l_3 m_3 \sin(\theta_2 - \theta_3) - 8 I_3 l_2 w_2 \cos(\theta_2) - 16 I_3 l_2 w_3 \cos(\theta_2) + 4 T_{12} l_3^2 m_3 - 2 \dot{\theta}_2^{\,2} l_2^2 l_3^2 m_3^2 \sin(2\theta_2 - 2\theta_3)}{16 I_2 I_3 + 4 I_2 l_3^2 m_3 + 4 I_3 l_2^2 m_2 + 16 I_3 l_2^2 m_3 + l_2^2 l_3^2 m_2 m_3 -}$$

[3]:
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t2_ddot_ = sp.solve(f3.subs(F12x, f12x).subs(F12y, f12y).subs(F32x, f32x).
 ↪subs(F32y, f32y), t2_ddot)[0]
t3_ddot_solved = sp.solve(f6.subs(F32x, f32x).subs(F32y, f32y).subs(t2_ddot,␣
 ↪t2_ddot_), t3_ddot)[0]
t3_ddot_solved.simplify()
```

[3]:
$$\frac{2 l_3 \left(4 I_2 \dot{\theta}_2^{\,2} l_2 m_3 \sin(\theta_2 - \theta_3) - 4 I_2 w_3 \cos(\theta_3) - 4 T_{12} l_2 m_3 \cos(\theta_2 - \theta_3) + \dot{\theta}_2^{\,2} l_2^3 m_2 m_3 \sin(\theta_2 - \theta_3) + 4 \dot{\theta}_2^{\,2} l_2^3 m_3^2 \sin(\theta_2\right.}{16 I_2 I_3 + 4 I_2 l_3^2 m_3 + 4 I_3 l_2^2 m_2 + 1}$$