

HW 1 - ME 8613
Numerical solution of State variable models and source load synthesis

Read sections 2.1-2.3 in the textbook (pages 15-48).

Problem 1)

a) Express the differential equation below in state variable form

$$\frac{d^2x}{dt^2} - \mu(1 - x^2)\frac{dx}{dt} + x - A \sin(\omega t) = 0,$$

$$\mu = 1, \quad \omega = \frac{2\pi}{10}, \quad \text{and} \quad A = 10$$

b) Use Euler's explicit method to obtain a numerical solution of the state variable equations from part (a). Assume zero initial conditions and that $t \leq 20$. Find the smallest value of Δt that gives a converged solution (solution doesn't change as time step decreases).

c) Repeat part (b) using a 4th order runge kutta solver with a fixed time step.

d) The results in parts b) and c) should be the same, but the value of Δt chosen for part c) should be smaller than that for part b). If the method used in part c) requires 4 times the computational resources per time step, compare the computational resources for each case.

Problem 2) Solve textbook problem 2.1 (page 24)

Problem 3) Solve textbook problem 2.2 (page 24).

Problem 4) Solve textbook problem 2.3 (page 24).

Problem 5) Consider an impeller pump with the following source curve

$$P_s(Q) = 83 - 0.004901 \cdot Q - 0.000324 \cdot Q^2$$

This source drives a load with a load curve of $P_L(Q) = 20 + 0.1 \cdot Q$. In both the source and load curves pressure has units of psi and Q has units of $\frac{\text{in}^3}{\text{s}}$. Determine the following: a) Estimate the equilibrium pressure and flow; b) Assuming the load can be changed arbitrarily, find the maximum power point on the source curve.