Read section 4.1, 4.5-4.6 from the textbook (pages 155-169, 197-233)

Problem 1. Consider the system shown below and the corresponding free body diagram.

a. Develop a set of 5 state variable equations using the following variables: $\Delta x, \Delta\theta, v, \omega_1, \omega_2$. Define the state variable using the general matrix form: $\dot{S} = A \cdot S + B \cdot U$, where A and B are matrices and S is a vector of state variables and U is a vector of inputs. Be careful with labeling since one of the damping coefficients is also labeled B in the figure.
b. Assuming that r=0.05, k_1=1000, k_2=500, b=100, B=2.5, J_1=J_2=1, and m=1, where all units are SI, solve the state variable equations using a 4^{th} order Runge-Kutta numerical method for a total simulation time of 20 seconds and assuming that all initial velocities and spring forces are zero and that f(t)=10. Make sure to check that the number of points used in the simulation is adequate.
c. Verify your results using energy conservation. Show that at a given time, t, the total energy that has crossed the boundary through inputs and dampers is equal to the total energy stored in the system through mass/inertias and springs.
d. Use the output matrix form $Y = C \cdot S + D \cdot U$, where Y is the chosen output, C and D are matrices. Use this form to define the contact force between the rack and the pinion using the state variable simulation results. Then plot the contact force.
e. Find input-output equation between the input force and the contact force. One approach to doing this is to use the ss2tf function in the scipy.signal library in python.
f. Use the Laplace transform approach to solve the input-output equation and compare with results from part d. Comment on how the roots of the CP affect the response.