ME 8213 – Fall 2023
Exam #3

- You may consult online resources but **NOT** other students
- Recommended format is 1 or more files of raw code for each problem + 1 word document containing a brief report of your results and comments. Code may be in any language/software although Python is preferred.
- You may use code available at the course website on Canvas but no additional solvers or libraries beyond the basic math tools

Problem 1:

Find the value of *z* such that the smallest eigenvalue of the following matrix is 1.0

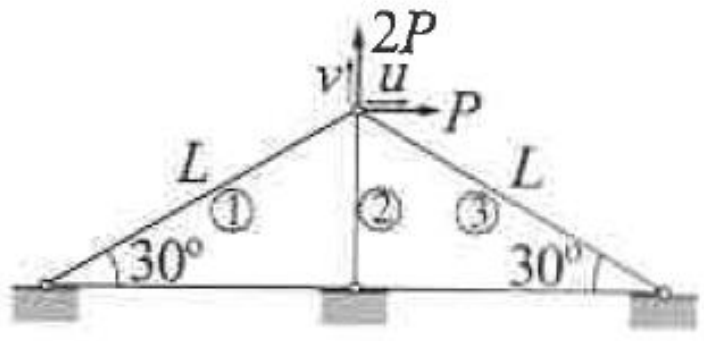
$$\begin{bmatrix} z & 4 & 3 & 5 & 2 & 1 \\ 4 & z & 2 & 4 & 3 & 4 \\ 3 & 2 & z & 4 & 1 & 8 \\ 5 & 4 & 4 & z & 2 & 5 \\ 2 & 3 & 1 & 2 & z & 3 \\ 1 & 4 & 8 & 5 & 3 & z \end{bmatrix}$$

Problem 2:

The below truss system results in the following equations for the displacements *u* and *v*

$$\frac{E}{4L}\begin{bmatrix} 3A_1 + 3A_3 & \sqrt{3}A_1 + \sqrt{3}A_3 \\ \sqrt{3}A_1 + \sqrt{3}A_3 & A_1 + 8A_2 + A_3 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} P \\ 2P \end{bmatrix}$$

where *E* is the elastic modulus A_i is the cross-sectional area of member *i* and *P* is an applied load. If A_1=A_3, **find the cross-sectional areas** that minimize the structural volume assuming *u* and *v* cannot exceed some critical displacement *δ*. **What is the minimum volume?** Your answer will be in terms of *E*, *L*, *P*, and *δ*.



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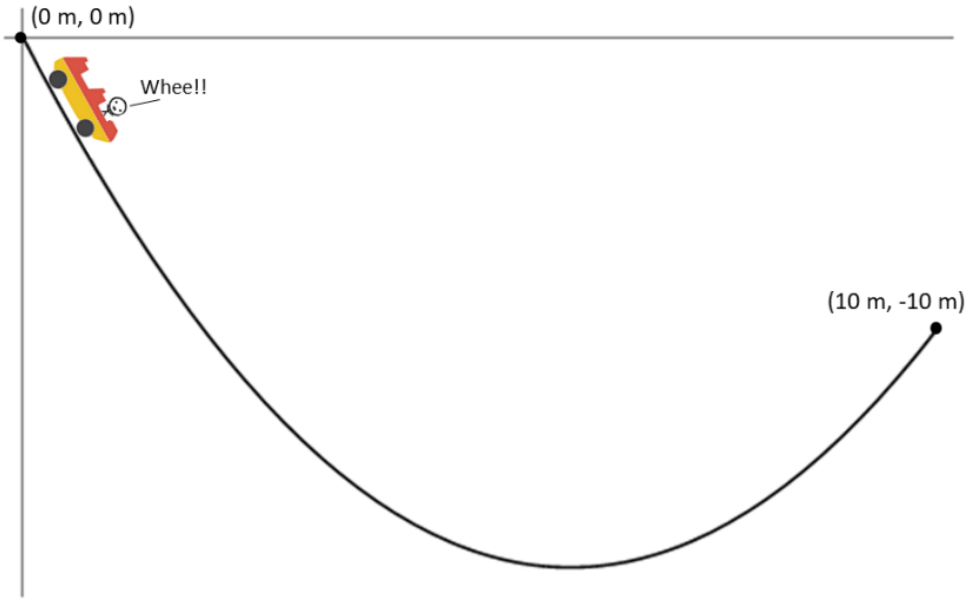
Problem 3:

Consider a rollercoaster car starting from rest sliding down a parabolic track. The height of the track, *y*, as a function of distance from the starting location, *x*, is given by $y = ax^2 + bx$. If the destination for the car is 10.0 m below the starting point and 10.0 m away, **find the value of *a***, the curvature of the parabola, that minimizes the travel time, and **give that total time**. Assume the force of gravity, *g* = 10 m/s^2. **Be sure to consider the accuracy of your answer** and provide some discussion of that accuracy. Hint: The travel time is given by the integral $\int \frac{ds}{v}$ where s is the arc along which the car travels and *v* is the instantaneous velocity which depends on the current height of the car relative to its origin: $v = \sqrt{-2gy}$



Bonus:

Consider problem 3 above but the track no longer needs to be parabolic, and can instead take on any shape. Find the shape of the track which minimizes the travel time.