ME 8213 – Fall 2023
Exam #1

- Exam will be available for 36 hours before it is due.
- You may consult online resources but **NOT** other students
- Recommended format is 1 or more files of raw code for each problem + 1 word document containing a brief report of your results and comments. Code may be in any language/software.

Problem 1:

A wire carrying an electric current is surrounded by rubber insulation of outer radius r. The resistance of the wire generates heat, which is conducted through the insulation and convected into the surrounding air. The temperature of the wire in the steady state can be shown to be:

$$T = \frac{q}{2\pi}\left(\frac{\ln(r/a)}{k} + \frac{1}{hr}\right) + T_\infty$$

Where q=50W/m is the rate of heat generation, a=5mm is the radius of the wire, k=0.16W/(m K) is the thermal conductivity of rubber, h=20W/(m^2K) is the convective heat transfer coefficient, and T_∞ = 280K is the ambient temperature. Find r that minimizes T. Recall that the minimum of a function must occur at a zero of its derivative.

Problem 2:

The gamma function is an extension of the factorial function to the real numbers such that for all positive integers:

$$\Gamma(n) = (n-1)!$$

Defined by:

$$\Gamma(n) = \int_0^\infty t^{n-1}e^{-t}dt$$

So the following are true:

$\Gamma(1) = 0! = 1$

$\Gamma(2) = 1! = 1$

$\Gamma(3) = 2! = 2$

$\Gamma(4) = 3! = 6$

$\Gamma(5) = 4! = 24$

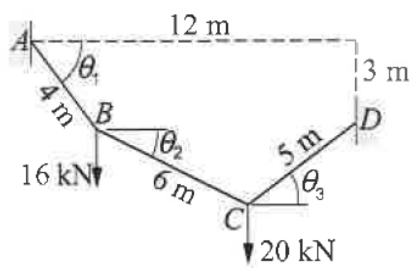
Using a cubic spline and the above values to approximate the gamma function, estimate (a) $\Gamma(1.5)$ and (b) the minimum of the gamma function between 1 and 2. How do these predictions compare to the actual values of (a) $\frac{\sqrt{\pi}}{2}$ and (b) 0.885603?

Problem 3: The specific heat of Cu, C_p, depends on temperature as follows:

T(K)	5	10	16	20	30	40
c_p (J/mol·K)	0.00943	0.0555	0.225	0.462	1.693	3.74

According to Solid State Theory at low temperatures, the specific heat capacity should vary as $c_p(T)=\gamma T+\beta T^3$ if $\frac{\partial cp}{\partial T}>0$, and is constant at higher temperatures. Fit γ and β and then find the value of c_p at 50K.

Problem 4:



A 15-m cable is suspended from A and D and carries concentrated loads at B and C. The vertical equilibrium equations of joints B and C are:

$$T(-\tan\theta_2 + \tan\theta_1) = 16kN$$

$$T(-\tan\theta_3 + \tan\theta_2) = 20kN$$

where T is the horizontal component of the cable tension. In addition, there are two geometric constraints due to the position of the supports:

$$-4\sin\theta_1 - 6\sin\theta_2 + 5\sin\theta_3 = -3m$$

$$4\cos\theta_1 + 6\cos\theta_2 + 5\cos\theta_3 = 12m$$

Determine θ_1, θ_2, and θ_3.