ME 8213 – Fall 2023
Homework #5

- Recommended format is 1 or more files of raw code for each problem + 1 word document containing a brief report of your results and comments. Code may be in any language/software.

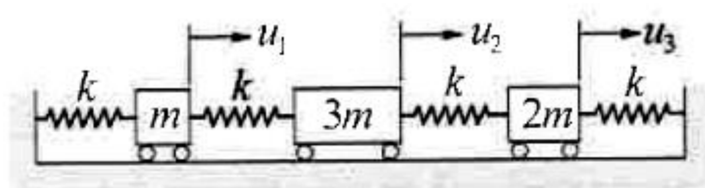
Problem 1:

A projectile of mass *m* experiences a drag force $F_d = cv^2$ where *v* is the magnitude of the velocity. The equations of motion in the x- and y-directions are:

$$\ddot{x} = \frac{-c}{m} v\dot{x} \qquad\qquad \ddot{y} = \frac{-c}{m} v\dot{y} - g$$

If the projectile is to hit a target 8 km away after flying for 10 seconds, determine the launch velocity and angle of inclination, θ. Use *m* = 20 kg, *c* = 3.2 x 10^{-4} kg/m and *g* = 9.80665 m/s^2

Problem 2:

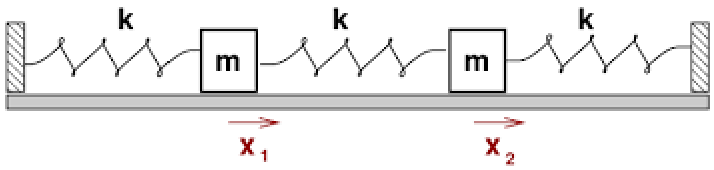


The differential equations for the motion for the mass-spring system above are:

$$k(-2u_1 + u_2) = m\ddot{u}_1 \qquad k(u_1 - 2u_2 + u_3) = 3m\ddot{u}_2 \qquad k(u_2 - 2u_3) = 2m\ddot{u}_3$$

Where u_i is the displacement of mass *i* from its equilibrium position and k is the spring stiffness. Determine the frequencies of vibration and the corresponding mode shapes for this system. Use k = 30 N/m and m = 5 kg.

Problem 3:



Assume a long system of *n* masses constrained to move in the x-direction with mass *m* with adjacent neighbors connected by springs of spring constant *k*. The normal modes and frequencies of this system can be found by finding the eigenvectors and eigenvalues of the mass-weighted Hessian matrix, whose elements are defined by:

$$H_{i,j}^{m} = \frac{1}{\sqrt{m_i m_j}} \frac{\partial^2 E}{\partial x_i \partial x_j}$$

Where *E* is the potential energy of the system and x_i is the position of mass *i*. The eigenvalues of H are proportional to the square of the mode frequency. Show how the frequency of the lowest and highest frequency modes change as n increases and describe the shape of these modes. The potential energy of a spring is ½ k x^2. Use k = 1 N/m and m = 1 kg.