

ME 4990/6990 02 – Spring 2026  
Homework #6 – Physics Informed Neural Networks

- Please submit any Python code used as a separate file in addition to a text document (Word) which provides answers to the below problem. Writing should be in complete sentences with enough detail to demonstrate a thoughtful consideration of what is asked

A function  $u(x, y)$  is defined on a unit square  $x \in [0,1], y \in [0,1]$ , and obeys the following partial differential equation:

$$\nabla^2 u(x, y) = e^{-x}(x - a + y^3 + by)$$

Where  $a$  and  $b$  are unknown parameters.  $u(x, y)$  is also subject to the following Dirichlet boundary conditions:

$$u(0, y) = y^3, u(1, y) = (1 + y^3)/e, u(x, 0) = xe^{-x}, u(x, 1) = e^{-x}(1 + x)$$

For particular values of  $a$  and  $b$  the analytic solution is:

$$u(x, y) = e^{-x}(x + y^3)$$

Construct a physics informed neural network (PINN) to determine the unknown constants  $a$  and  $b$  which give this solution. Try to minimize the number of points away from the boundary which explicitly use the analytic solution. Plot how the prediction of  $a$  and  $b$  evolves with the number of training epochs.