Determining the 3D Spin Axis from Statcast Data
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A technique is developed to determine the direction of the spin axis for pitched baseballs. The method utilizes the measurement of the magnitude of the spin from Statcast and the magnitude and direction of the spin-induced deflection from the ball trajectory.

I. A QUICK REVIEW OF BASEBALL TRAJECTORIES

When a baseball travels through the air, it experiences various forces that determine the trajectory. The most familiar of these forces is the downward pull of gravity F_G. Less familiar are the aerodynamic forces, namely the drag force F_D and the Magnus force F_M. The drag force, or in everyday language "air resistance," is due to the fact that the ball has to push the air out of the way. Rather than talk about the forces, it is more convenient to talk about the accelerations, since those are actually measured by the tracking systems; they are related to the forces by Newton's famous Third Law $F = ma$. The conventional way to express the drag acceleration a_D is through the expression

$$\vec{a}_D = -KC_D v^2 \hat{v} \,, \tag{1}$$

where the factor K is given by

$$K = \frac{1}{2}\frac{\rho A}{m} \,, \tag{2}$$

where m and A are the mass and cross sectional area of the ball, respectively, and ρ is the density of the air. The direction of the drag is $-\hat{v}$, which is a unit vector pointing opposite to the direction of the velocity. Thus the drag reduces the speed of the ball but does not change its direction. The factor C_D is called the drag coefficient. If the baseball is spinning,

it also experiences the Magnus acceleration a_M, which is conventionally written as

$$\vec{a}_M \;=\; KC_L v^2 \frac{\hat{\omega} \times \hat{v}}{|\hat{\omega} \times \hat{v}|}\,, \tag{3}$$

where C_L is called the lift coefficient. The direction of the Magnus force is given by the vector cross product $(\hat{\omega} \times \hat{v})$, which is perpendicular both to the velocity vector and the spin axis, the latter indicated by the unit vector $\hat{\omega}$. A handy mnemonic is that the Magnus force is in the direction that the leading edge of the ball is turning. Note that the cross product vanishes when the spin is either parallel or antiparallel to the direction of the velocity, so that component of spin does not contribute to the Magnus force. The component of spin perpendicular to the velocity contributes to the Magnus force, which deflects the trajectory from a straight line path but does not change the speed. Of course, the gravitational acceleration $\vec{g}$ changes both the speed and the direction of the ball.

In the following sections, it will be assumed that the pitched ball trajectory is parameterized by the so-called "nine-parameter" fit to a constant-acceleration model. Although an approximation, such a description has been shown to be an excellent description of the full trajectory.[1] It does not matter which of the two tracking systems are used to obtain the trajectory (PITCHf/x or Trackman). Note that since the start of the 2017 season, the Statcast data exclusively uses Trackman for pitch tracking. The PITCHf/x coordinate system will be used in which the origin is located at the corner of home plate, the y axis runs toward the pitcher, the z axis points vertically upward, and the x axis points to the catcher's right ($\hat{x} = \hat{y} \times \hat{z}$). Therefore a pitched baseball is primarily moving in the $-\hat{y}$ direction.

II.   THE TECHNIQUE

A.   The transverse and gyro components of spin

According to Eq. 3, the Magnus acceleration is only sensitive to the component of spin that is perpendicular to (or transverse to) the velocity. We call this part the "transverse spin" and denote it by $\vec{\omega}_\perp$. As mentioned above, the Magnus acceleration is not sensitive to any component of spin that is along the velocity direction. We call this part the "gyrospin"[2] and denote it by $\vec{\omega}_\parallel$. The total spin $\vec{\omega}$ is the vector sum of the transverse and gyrospin parts:

$\vec{\omega} = \vec{\omega}_\perp + \vec{\omega}_\parallel$. Since the transverse and gyrospin parts are perpendicular to each other, then

$$\omega_\parallel = \sqrt{\omega^2 - \omega_\perp^2}\,, \tag{4}$$

where I have purposely rearranged the order since the right-hand side of the equation now contain measurable quantities. Namely, ω is determined by the Trackman measurement system while $\omega_\perp$ is determined by the trajectory. Eq. 4 then tells us how to compute the magnitude of $\vec{\omega}_\parallel$. By definition, the direction of $\vec{\omega}_\parallel$ is either parallel or antiparallel to $\langle \hat{v} \rangle$, and this method does not distinguish those two possibilities. One might guess, however, that $\omega_\parallel$ is parallel or antiparallel to $\langle \hat{v} \rangle$ for right-handed and left-handed pitchers, respectively. We next outline how to determine $\vec{\omega}_\perp$ from the tracking data.

B.   Determining the Magnus acceleration

The first step in determining $\vec{\omega}_\perp$ from the tracking data is to isolate the part of the acceleration that is due to the Magnus force. The 9-parameter constant acceleration fit to the data is utilized, where the acceleration vector $\vec{a} = (a_x, a_y, a_z)$ is given by

$$\vec{a} = \vec{a}_D + \vec{a_M} + \vec{g} \tag{5}$$

and the downward gravitational acceleration $\vec{g} = -g\hat{z}$. We first remove the gravitational acceleration to obtain $\vec{a^*} \equiv \vec{a} - \vec{g} = \vec{a}_D + \vec{a}_M$ which has the remaining drag and Magnus parts. We next remove the drag. That part is a bit tricky since the drag depends on the square of the velocity, which is not constant. The approximation adopted here is that the appropriate velocity to use in Eq. 1 is the mean velocity $\langle \vec{v} \rangle$ over the trajectory. The drag acceleration is simply the projection of $\vec{a^*}$ along the $-\langle \hat{v} \rangle$ direction. This follows from the fact that the Magnus acceleration has *no* component along that direction whereas the drag is *entirely* along that direction. Therefore the drag is given by $\vec{a}_D = -\left[\vec{a^*} \cdot \langle \hat{v} \rangle\right] \langle \hat{v} \rangle$. As a byproduct, one can easily obtain the drag coefficient from Eq. 1, $C_D = a_D/(K\langle v \rangle^2)$, although that is not essential to the problem at hand. Once the drag has been removed, the remaining acceleration is due entirely to the Magnus force. Putting all of this together, the Magnus acceleration is related to the total acceleration by

$$\vec{a}_M = \vec{a} - \vec{g} + \left[(\vec{a} - \vec{g}) \cdot \langle \hat{v} \rangle\right] \langle \hat{v} \rangle\,. \tag{6}$$

C. Determining the transverse spin from the Magnus acceleration

Once the Magnus acceleration is determined from Eq. 6, it's magnitude can be inserted into Eq. 3, which can then be rearranged to find the lift coefficient C_L:

$$C_L = \frac{a_M}{K \langle v \rangle^2} . \tag{7}$$

There is no first-principles theory that relates C_L to the transverse spin, so it must be done experimentally. The experiments that have been done fire a spinning baseball and track it with high-speed video, which is used to measure both the trajectory (which determines C_L) and the spin. A number of such experiments have been done in the past decade[3] and a summary of the results is presented in Fig. 1. The C_L values determined from these experiments are fitted to a smooth curve given by

$$\omega_\perp = \frac{\langle v \rangle}{R} \frac{0.40 C_L}{1 - 2.32 C_L} , \tag{8}$$

where R is the radius of the ball. Eq. 8 gives us the magnitude of $\omega_\perp$; the direction is determined by a rearrangement of Eq. 3 to obtain

$$\hat{\omega}_\perp = \langle \hat{v} \rangle \times \hat{a}_M , \tag{9}$$

where $\hat{a}_M$ is the direction of the Magnus acceleration. It is important to keep in mind that the actual experimental data scatter about the smooth curve (see Fig. 1), so that the values of $\omega_\perp$ are probably not determined to better than $\pm 20\%$.

D. Putting it all together

We now have everything we need to find the direction of the total spin. The TrackMan system determines ω. The trajectory determines both the magnitude and direction of $\vec{\omega}_\perp$ via Eqs. 8 and 9, respectively. Then Eq. 4 is used to calculate the magnitude of $\vec{\omega}_\parallel$, the direction of which is $\pm \langle \hat{v} \rangle$. In component form, we write

$$\begin{aligned}
\omega_x &= \omega_{\perp,x} + \omega_{\parallel,x} \\
\omega_y &= \omega_{\perp,y} + \omega_{\parallel,y} \\
\omega_z &= \omega_{\perp,z} + \omega_{\parallel,z} ,
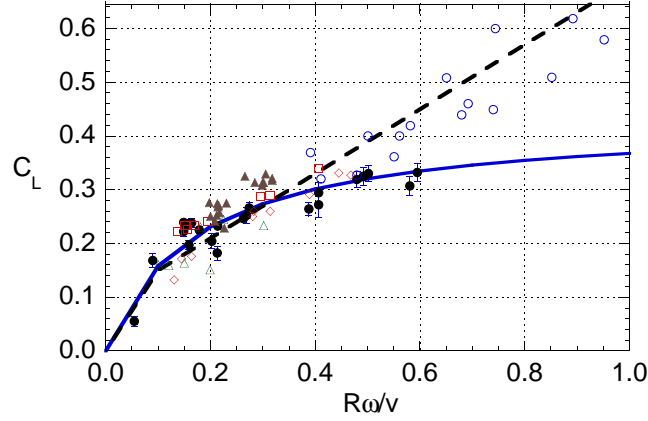\end{aligned} \tag{10}$$

FIG. 1: Experimental values of C_L as a function of $R\omega_\perp/v$. Different plotting symbols correspond to different experiments, with full details given by Nathan.[3] The dashed and solid curves are different parametrizations of the data and give similar results for $R\omega_\perp/v \leq 0.4$, which is the regime most relevant for pitched baseballs. For values of $R\omega_\perp/v > 0.4$, the data and curves show two different trajectories. The open blue circles are from wind tunnel data taken at speeds less than 40 mph and are best fit with the dashed curve. The remaining points are from trajectory-tracking data taken at speeds in the range 80-100 mph and are best fit with the solid curve, Eq. 8.

where everything on the right-hand side of Eq. 10 is known. Thus the components of $\vec{\omega}$ are all determined and the direction can be calculated. In doing so, it is helpful to define a pair of angles. First, θ is the angle of the spin with respect to the $x - z$ plane, so that $\theta \approx 0°$ for the case where the spin is all transverse, whereas $\theta \approx 90°$ for a "gyroball". In fact, $\cos(\theta) \approx \omega_\perp/\omega$, which is just the ratio of "useful" to total spin. Second, ϕ is the angle with respect to the x axis of the projection of the spin in the $x - z$ plane. In terms of the components, the angles are given as follows:

$$
\begin{aligned}
\theta &= \arctan\left(\frac{\omega_y}{\sqrt{\omega_x^2 + \omega_z^2}}\right) \\
\phi &= \arctan\left(\frac{\omega_z}{\omega_x}\right).
\end{aligned}
\tag{11}
$$

This completes the determination of the direction of the total spin vector.

E. Some caveats

NOTE: This is the section that was updated on March 31, 2015

When comparing $\omega_\perp$, which is extracted from the trajectory data, to the total spin ω, it is important to keep in mind the role played by random measurement errors. For example, suppose one were to compare $\omega_\perp$ with ω for a large collection of four-seam fastballs, for which one expects the two quantities will be equal. One would likely find that, on average, $\omega_\perp = \omega$, as expected. However one would also likely find a considerable spread in the distribution about the mean value, with as many pitches with $\omega_\perp - \omega > 0$ (a physical impossibility) as with $\omega_\perp - \omega < 0$. How does one account for the spread in these values? It is almost surely due to random measurement error in the pitch trajectory. The latter results in random measurement error in the accelerations, which in turn leads to random measurement error in $\omega_\perp$. It could also be the case that there is a significant random error in the total spin ω. However a careful study comparing the spin measured by Trackman to spin directly measured using high-speed video shows that possibility to be very unlikely.[4] The role of random measurement error in this type of analysis was discussed in a recent article.[5]

III. THE EXCEL SPREADSHEET

NOTE: This is the section that was updated on August 31, 2018

In this section I will relate the specific calculations in the accompanying Excel spreadsheet[6] to the formulas in this document. It is important to read the "Readme" tab on the spreadsheet before using it, as it contains important information on the input parameters. I will discuss this briefly here, referring to specific columns of the spreadsheet.

All of the columns of the spreadsheet that I have added are highlighted in yellow; the original columns from Statcast are not highlighted. The important columns of original data are as follows:

- The x and z locations at release, columns H and J

- The extenstion, column X

- The total spin rate, column W

- The velocity components from the 9P fit at $y = 50$ ft, columns Q-S

- The mean acceleration components from the 9P fit, columns T-V

None of the remaining columns of original data are needed (and, in fact, could be deleted).

The extension is used to calculate the release point y_R, column I. The 9P velocity components are at $y = 50$ and need to be calculated at y_R, and this is done in columns D-F. The components of the average velocity vector $\langle \vec{v} \rangle$ are in Z-AB. The three components and magnitude of the Magnus acceleration are in AF-AH and AI, respectively; the x and z movements are in AJ-AK; and C_L is in AL. Although not essential in the present context, the drag coefficient C_D is calculated in AE. The components of transverse spin $\vec{\omega}_\perp$ are in AO-AQ, while the magnitude $\omega_\perp$ is in AN. The ratio of transverse to total spin ("spin efficiency") is in column AT, and the angles θ and ϕ are in AU and AS, respectively. Whenever the calculated transverse spin exceeds the total spin, a physical impossibility, the angle θ is left blank.

Note that to calculate the transverse spin requires knowledge of the numerical factor K (see Eq. 7), which depends on the air density. In turn the air density depends on the temperature and elevation above sea level. The calculation is done in tab K, where the temperature in Fahrenheit should be entered into cell B3 and the elevation in feet into cell B4. The factor K is automatically computed in D3.

IV. A VISUAL TOOL

If the baseball were oriented so that the spin axis passes through a seam, the batter would observe a red dot. That turns out to be an excellent way to visualize the orientation of the spin axis. In Fig. 2 the ball is represented by a circle of unit radius, as seen by the catcher looking along the y axis, with the $\hat{x}$ axis pointing to the right and the $\hat{z}$ axis pointing up. The distance of the spin axis from the center of the circle is given by $1 - \sin(\theta)$, whereas ϕ gives the orientation of the dot about a circumference at that radius. Seven pitches are labeled on the plot. Pitch 1 is a gyroball, with the spin perfectly aligned with the y axis. Pitches 2 and 3 have their axes completely in the $x - z$ plane (i.e., no y or gyrospin component), and correspond to the transverse spin being purely backspin and topspin, respectively. Pitches 4 and 5 have a small gyrospin component and correspond to a

tailing fastball (armside break) thrown by left-handed and right-handed pitcher, respectively. Pitch 6 has a small gyrospin component and corresponds to a curveball thrown by a right-handed pitcher. Finally pitch 7 has a large gyrospin component, with the transverse spin being purely sidespin and corresponds to a slider thrown by a right-handed pitcher.

The direction of the break can be determined from the tool by making a fist with your right hand placing it near the center of the circle. Orient your hand so that your thumb points to the red dot. Then your fingers will point in the direction of the break, as observed by the catcher. Alternately, you can just apply the following simple rules: pitches lying in the upper half of the circle break to the right, those in the lower half to the left, those in the left half break up, those in the right half break down. And of course, those in the center of the circle do not break at all. As an example, if we apply these rules to pitch 6, we see that it breaks down and to the right (catcher's view), exactly as expected for a curveball thrown by a right-handed pitcher. For a given total spin and velocity, the magnitude of the break will be greatest the farther the dot is from the center. For example, the maximum break will be achieved by pitches 2 and 3 and the minimum by pitch 1. Generally speaking, a slider (7) has less break than pitches of comparable total spin, since it usually has a large gyrospin component.
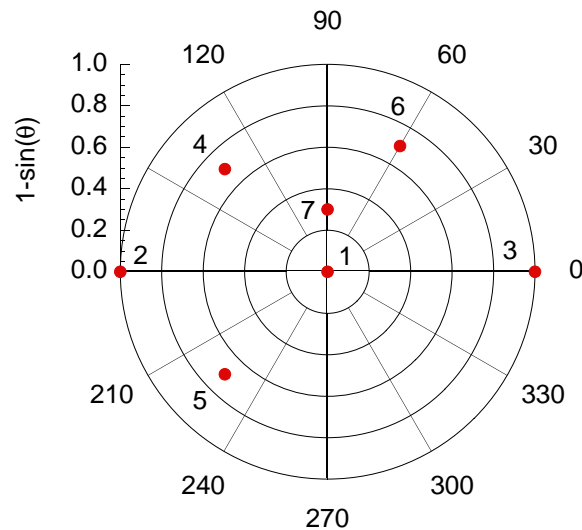
FIG. 2: Visual representation of the spin axis, as seen by the catcher. The radial coordinate is $1 - \sin(\theta)$ and the azimuthal coordinate is the angle ϕ, as defined in Eq. 11. The numbers refer to specific pitches, as described in the text.

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[1] http://baseball.physics.illinois.edu/MCAnalysis.pdf.

[2] The origin of this notation is that a pitched baseball with its spin axis perfectly aligned with its velocity is conventionally called a "gyroball."

[3] See a summary in A. M. Nathan, Am. J. Phys. **76**, 119-124 (2008). A copy of the paper can be downloaded at http://baseball.physics.illinois.edu/AJPFeb08.pdf. Fig. 2 of that paper is shown here as Fig. 1.

[4] http://www.baseballprospectus.com/article.php?articleid=23202

[5] http://www.baseballprospectus.com/article.php?articleid=25915

[6] http://baseball.physics.illinois.edu/trackman/MovementSpinEfficiencyTemplate.xlsx.