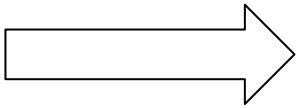
exploring partisan symmetry

gabe schoenbach
mggg redistricting lab | tufts university | nov. 2020

Background



This talk

What is gerrymandering?

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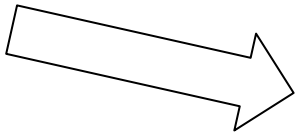

Partisan symmetry

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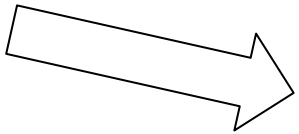
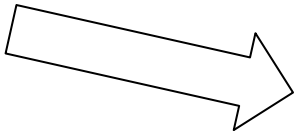


Partisan symmetry



Problems and paradoxes!

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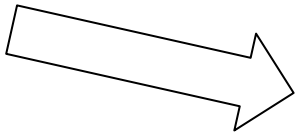

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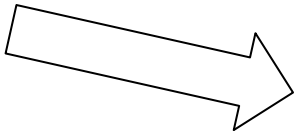
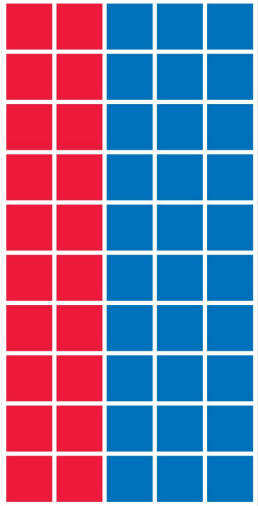


Problems and paradoxes!

How we talk about gerrymandering

Redistricting (and Gerrymandering)



50 PRECINCTS
60% BLUE
40% RED

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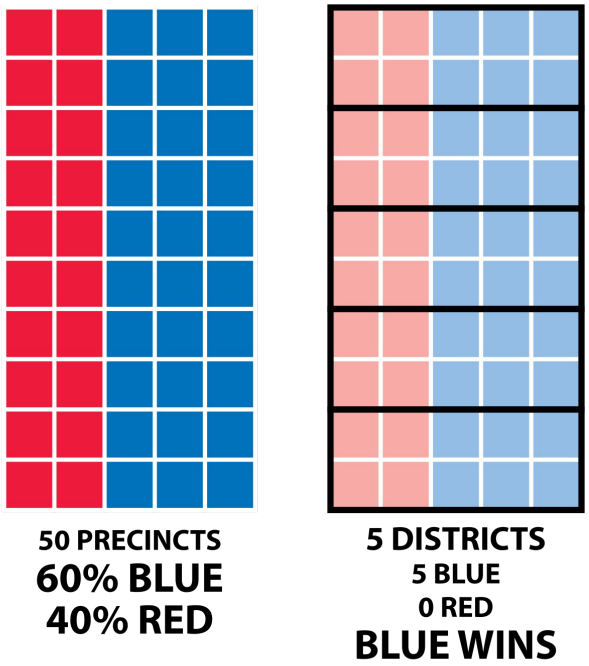

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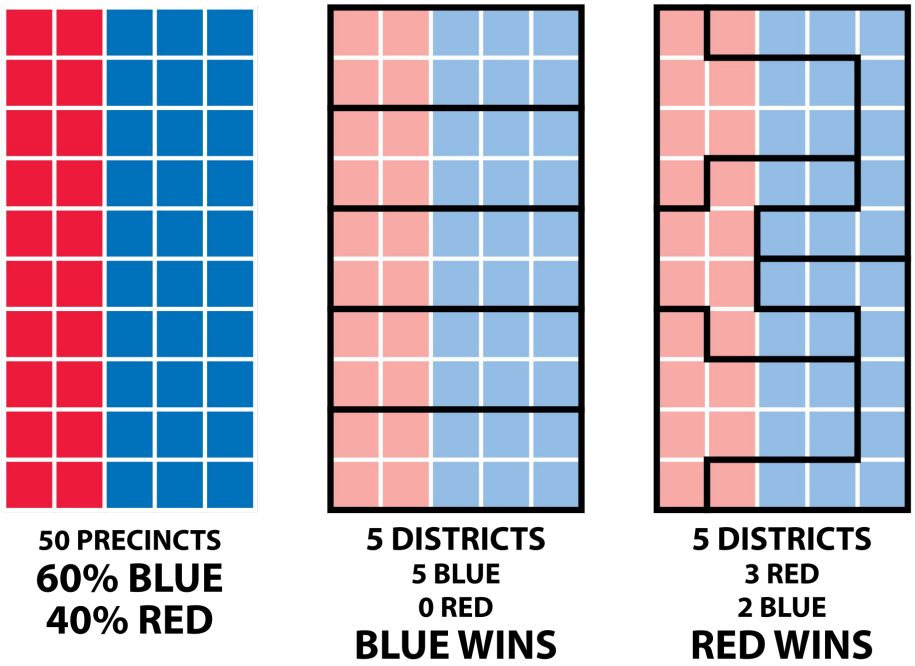

Redistricting (and Gerrymandering)

Def. Partisan gerrymandering is the manipulation of district lines to secure as many seats as possible for one's political party.

What is partisan symmetry?

Election		
Party	Vote Share	Seat Share
Republicans	60%	80%
Democrats	40%	20%

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Election			Hypothetical Election		
Party	Vote Share	Seat Share	Party	Vote Share	Seat share
Republicans	60%	80%	Republicans	40%	20%
Democrats	40%	20%	Democrats	60%	80%

Definition of fairness: the share of representation awarded to one party with its share of the vote *should* also have been secured by the other party, had the vote shares been exchanged.

Why partisan symmetry?

- Metrics based on partisan symmetry are being used to fight gerrymandering
- “Fighting” means IDing gerrymandered plans and throwing them out

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- They have claimed the metrics are “reliable and difficult to manipulate” and U.S. states are signing on — Utah has explicitly included partisan symmetry as a criterion to be considered in evaluating new district plans:

> “The Legislature and the Commission shall use judicial standards and the best available data and scientific and statistical methods, including measures of partisan symmetry, to assess whether a proposed redistricting plan abides by and conforms to the redistricting standards" that bar party favoritism.

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- Stakeholders may use these scores to constrain extreme seat outcomes. **Will that work?**

How do we measure partisan symmetry?

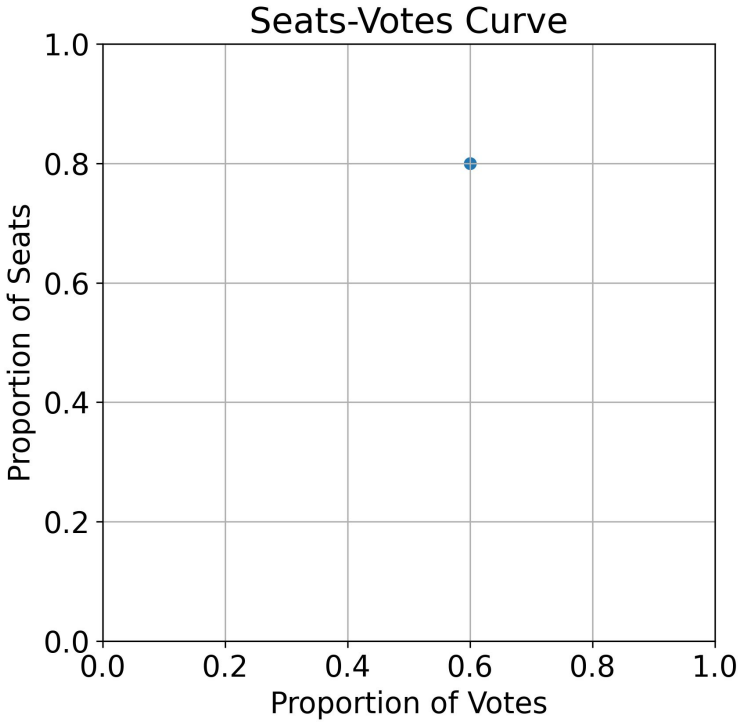
There are three metrics (Mean Median, Partisan Bias, Partisan Gini) that we will focus on, each premised upon the ideal of partisan symmetry

Each metric can be read off of the *Seats-Votes Curve*

How do we measure partisan symmetry?

Seats-Votes Curve γ: shows a relationship between vote share and seat share, starting from one observed election and extrapolating outward under the Uniform Partisan Swing assumption

Here, we use a made-up vote vector *v*:
v = (.40, .**55**, **.62**, **.68**, **.75**)



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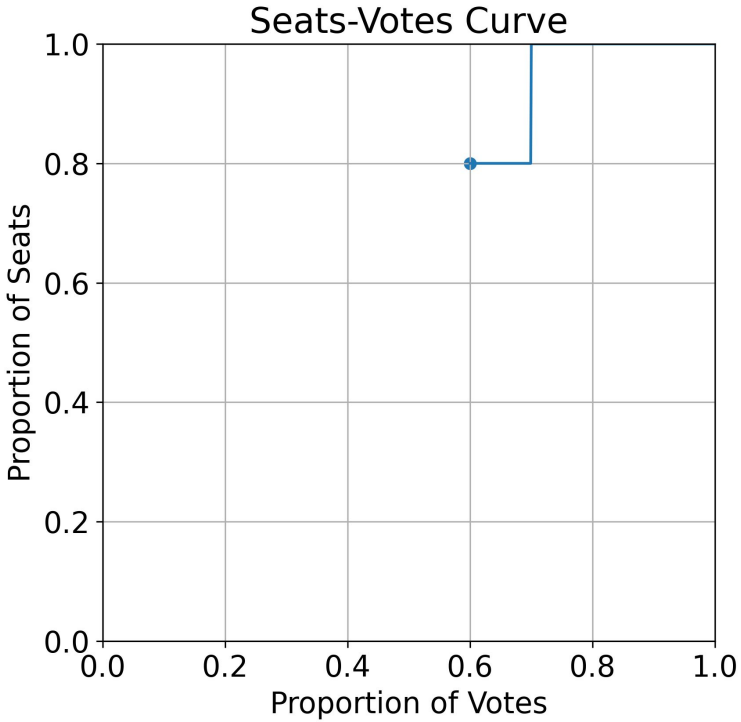

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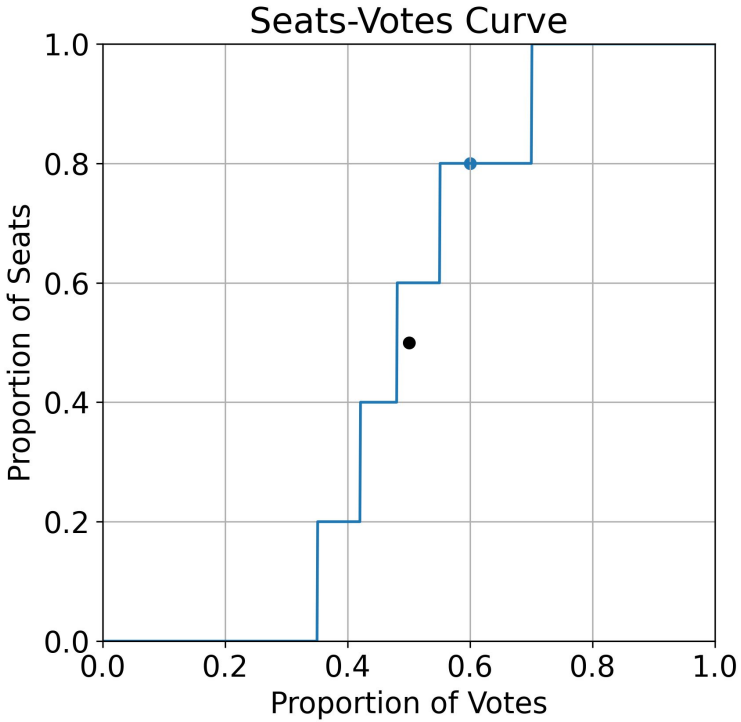
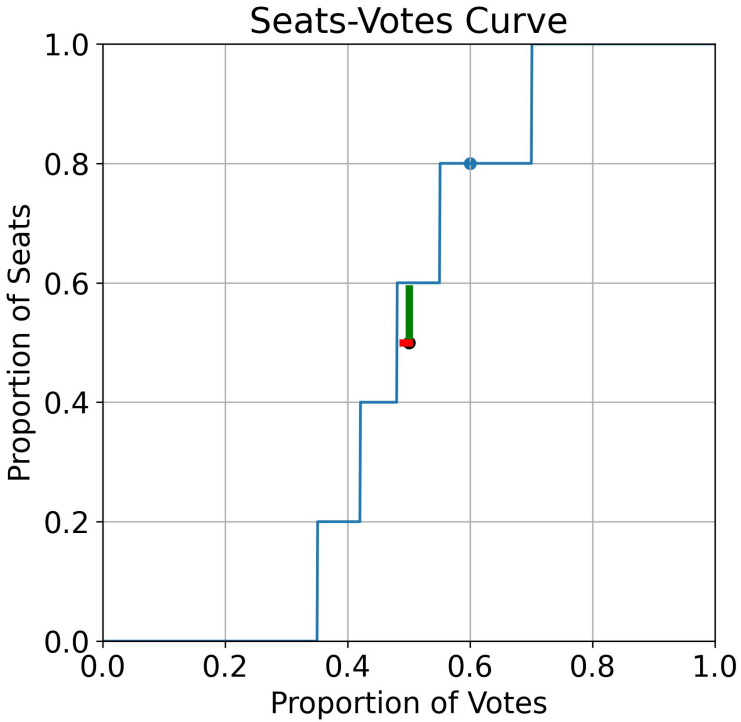


How do we measure partisan symmetry?

Mean-Median (MM): How many percentage points under 50% of the vote do you need to win >50% of the seats?

- *v_med* - *v_mean* = 0.02 (> 0 is good!)



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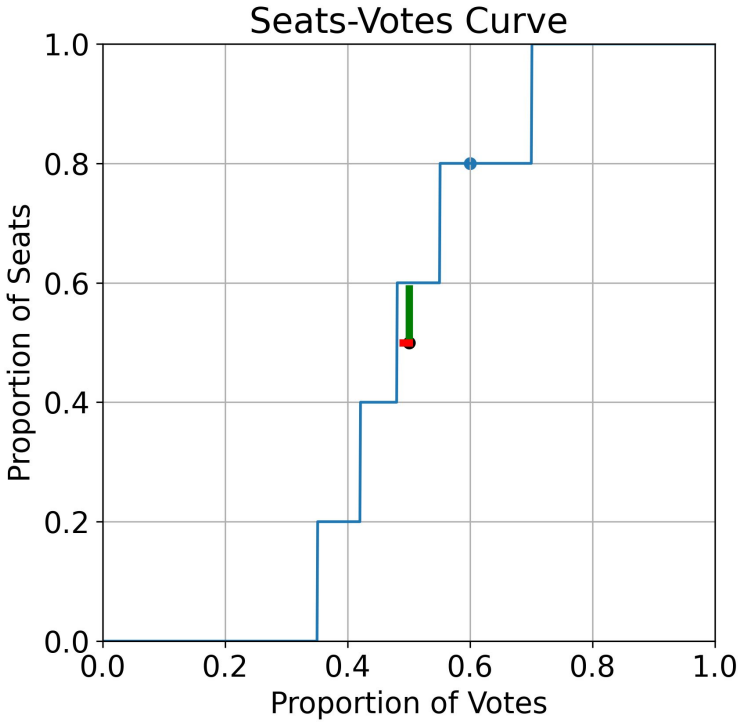
Partisan Bias (PB): At 50% of the votes, how much higher than 50% of the seats do you get?

- γ(0.50) - 0.50 = 0.60 - 0.50 = 0.10 (> 0 is good!)



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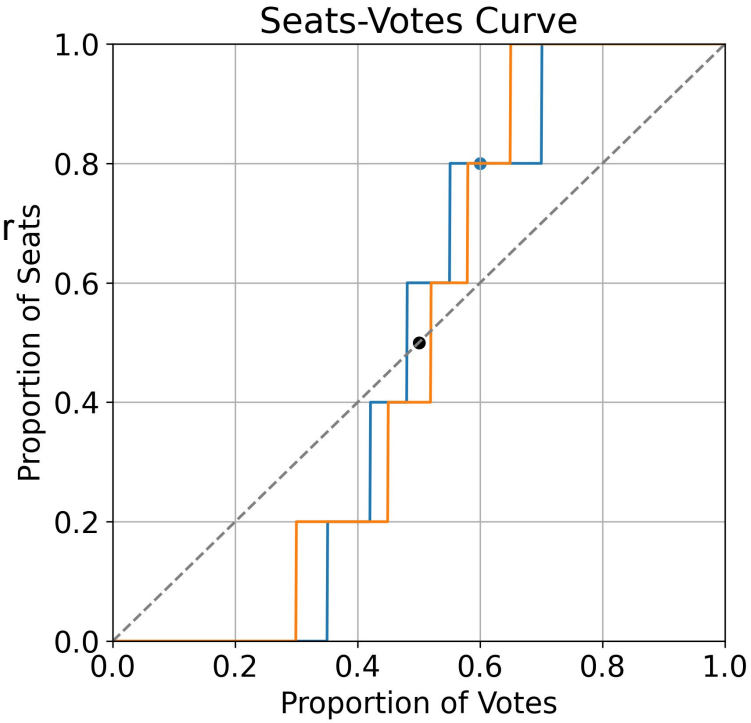
Partisan Gini (PG): rotate γ 180 degrees around center point, take the sum of the area between the curves

Note:

- MM, PB signed
- PG = 0 => MM, PB = 0 — ideal value
- Proportional representation (dotted line) is symmetric, but proportionality is not necessary for PG = 0.



When does PG = 0?

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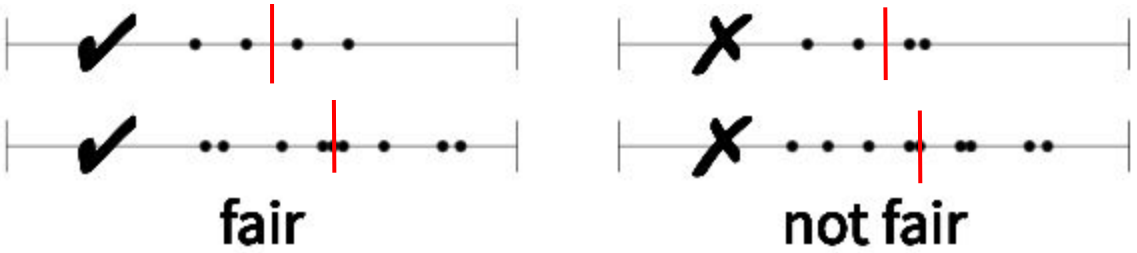


Figure 3. Four election outcomes, shown as vote shares by district. On the left-hand side, the v_i are symmetric about their center, so all partisan symmetry scores are perfect. On the right-hand side, non-symmetric outcomes. The partisan symmetry standard can be eyeballed by a glance at the vote shares in the districts.

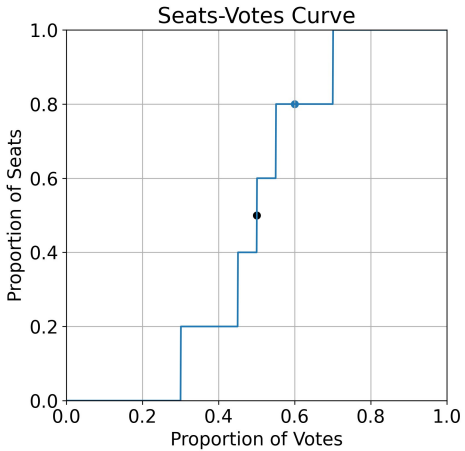
Partisan symmetry characterization

PG = 0

v = (.40, .55, .60, .65, .80)

d = (.15, .05, .05, .15)

j = (.30, .45, .50, .55, .70)

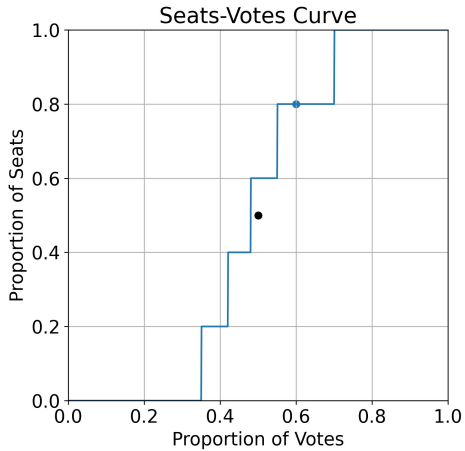


PG > 0

v = (.40, .55, .62, .68, .75)

d = (.15, .07, .06, .07)

j = (.35, .42, .48, .55, .70)



Theorem 3 (Partisan Symmetry Characterization) *Given k districts with vote shares* v, *jump vector* j, *and gap vector* δ, *the following are equivalent:*

v = (.40, .55, .60, .65, .80)

$$PG(v) = 0 \qquad \text{(Partisan Symmetry Standard)}$$

j = (.30, .45, .50, .55, .70)

$$j_i + j_{k+1-i} - 1 = 0 \quad \forall i \qquad \text{(jumps)}$$

$$\frac{1}{2}(v_i + v_{k+1-i}) = \overline{v} \quad \forall i \qquad \text{(mean vote)}$$

$$\frac{1}{2}(v_i + v_{k+1-i}) = v_{\text{med}} \quad \forall i \qquad \text{(median vote)}$$

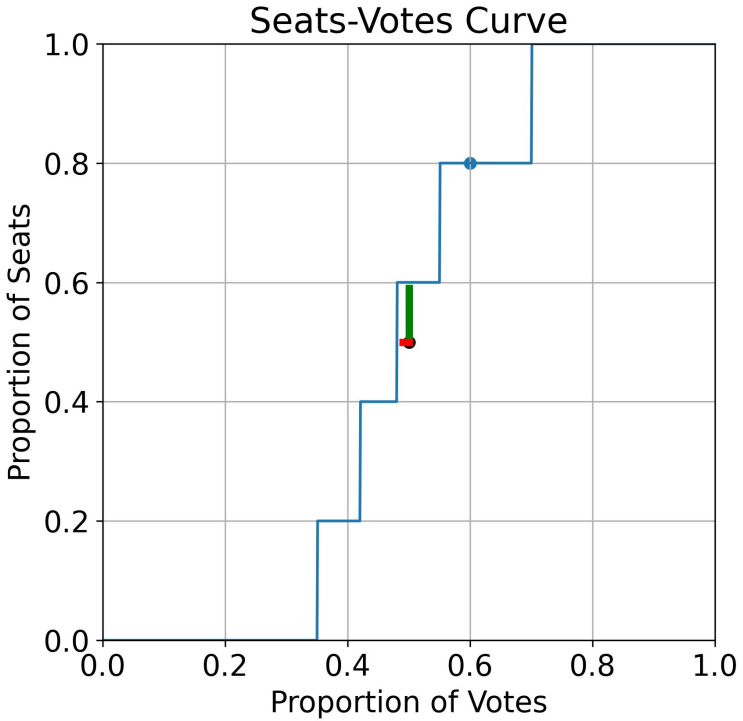
d = (.15, .05, .05, .15)

$$\delta_i = \delta_{k-i} \quad \forall i \qquad \text{(gaps)}$$

Paradoxes with mean-median and partisan bias

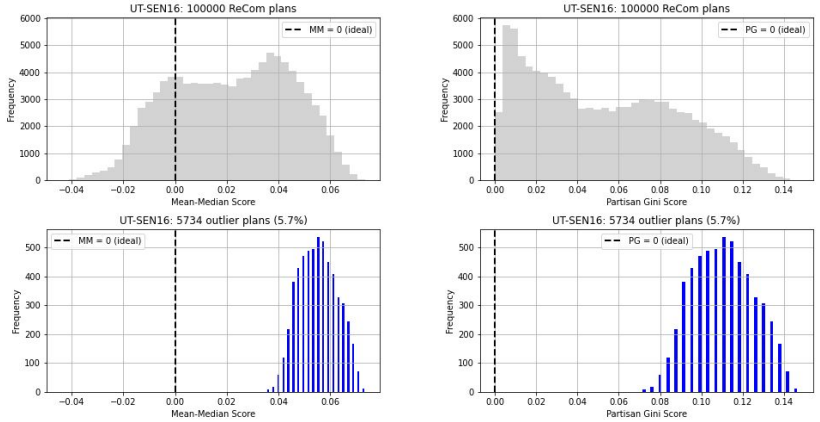
For both MM and PB, positive scores indicates an advantage for the POV party (the one whose vote shares are being reported)

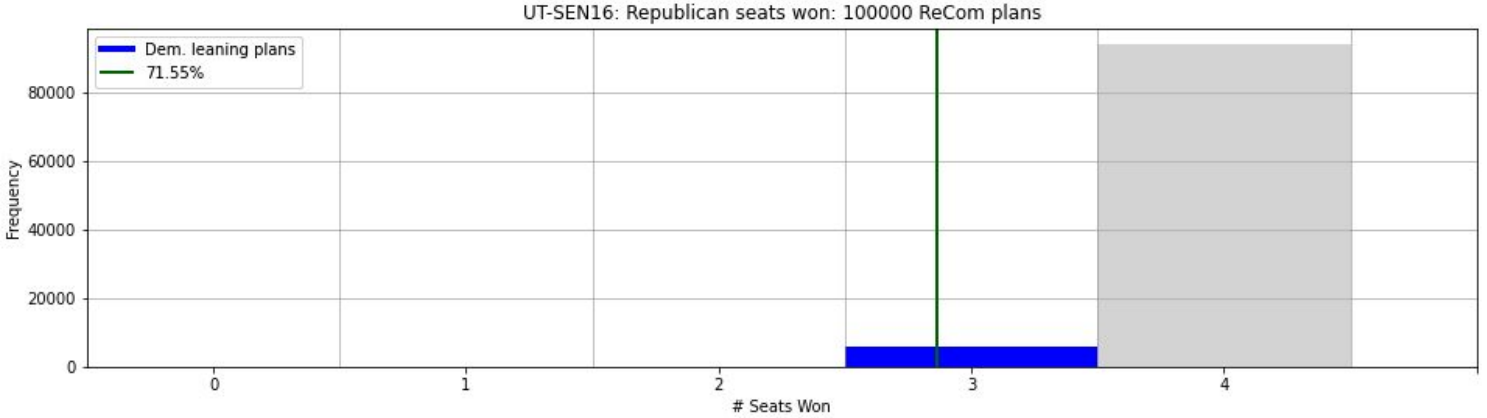
Def. A **paradox** occurs when a signed partisan symmetry score indicates an advantage for one party even though that party has a very low number of seats.



The Utah Paradox

- Signed symmetry scores are supposed to indicate which party is advantaged
- MM and PB report each plan with any D representation as significant R gerrymanders!





Paradoxes with mean-median and partisan bias

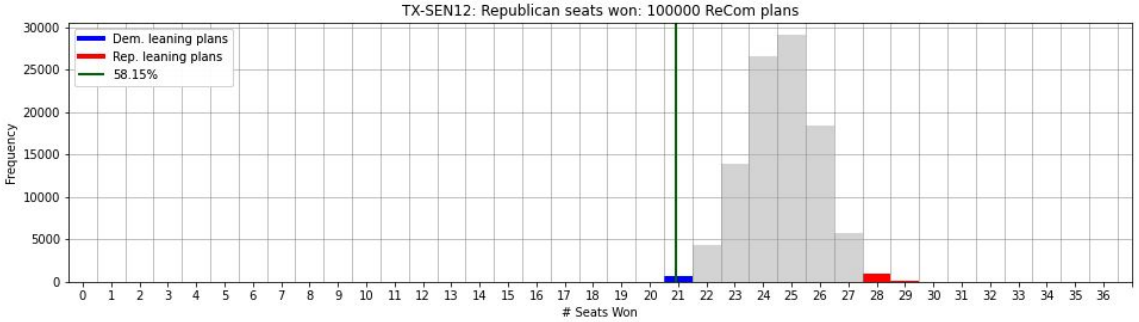
Example 4 (Paradoxes forced by arithmetic) *Suppose we have $k = 4$ districts and an extremely skewed election in favor of Party A, achieving $75\% < \overline{v} < 87.5\%$. With equal turnout, Party B can get at most one seat. However, every vote vector* v *achieving this outcome (one B seat) yields* MM $\geq \overline{v} - \frac{3}{4} > 0$. *In particular, such districting plans all have positive* MM *and* PB, *paradoxically indicating an advantage for Party A in every case where Party B gets representation.*

The demonstration is simple arithmetic. Since $\frac{1}{2}(v_2 + v_3) = v_{\text{med}}$, we have

$$\overline{v} = \frac{1}{4}(v_1 + v_2 + v_3 + v_4) = \frac{v_1 + v_4}{4} + \frac{v_2 + v_3}{4} \implies v_{\text{med}} - \overline{v} = \overline{v} - \frac{v_1 + v_4}{2}.$$

Since $v_1 \leq \frac{1}{2}$ (for B to win a seat) and $v_4 \leq 1$, we get $\text{MM} = v_{\text{med}} - \overline{v} \geq \overline{v} - \frac{3}{4}$, as needed.

Texas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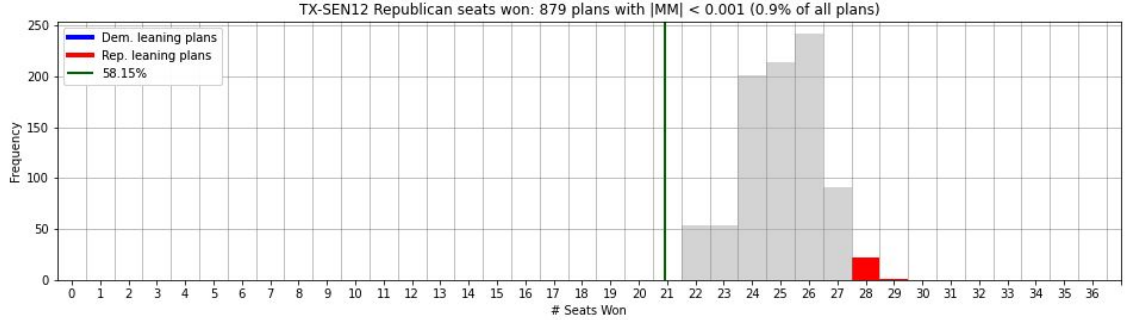
Dem. leaning plans
Rep. leaning plans
58.15%
Frequency
Seats Won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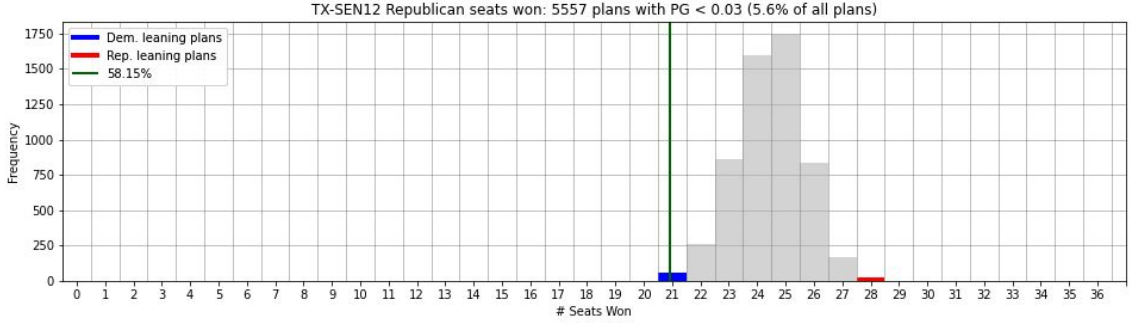
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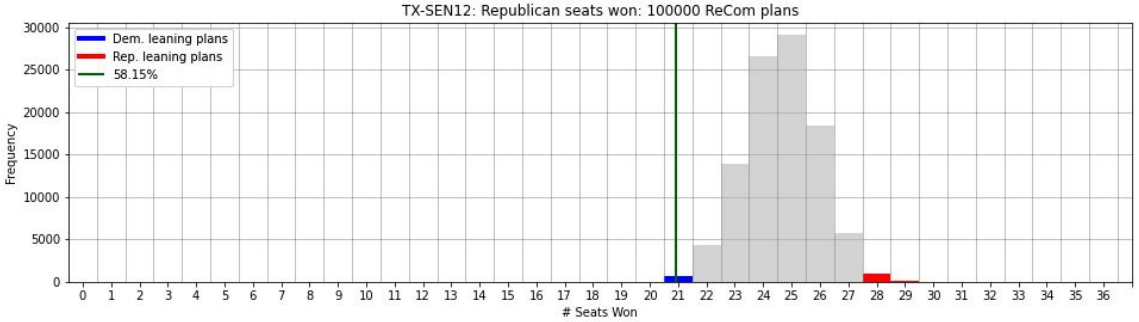
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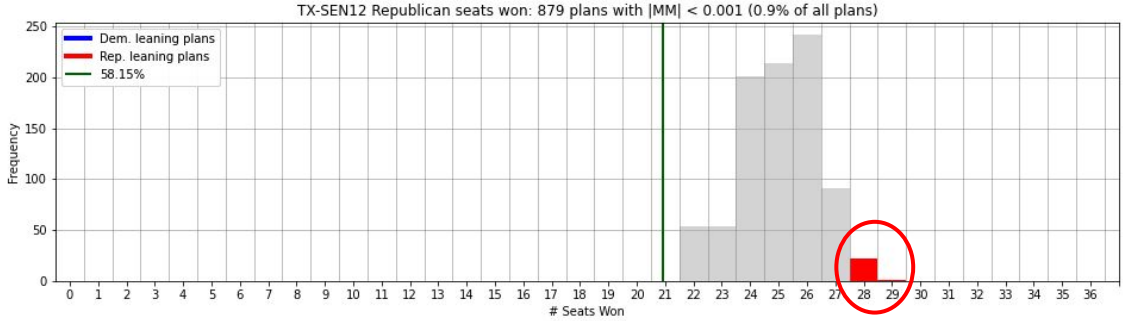
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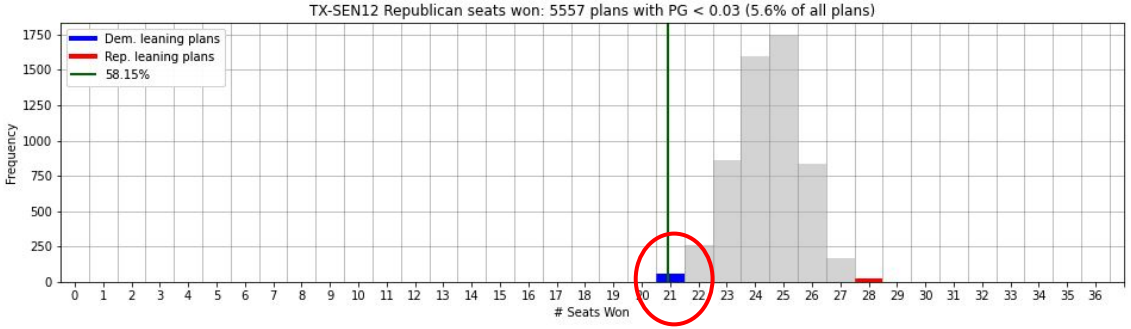
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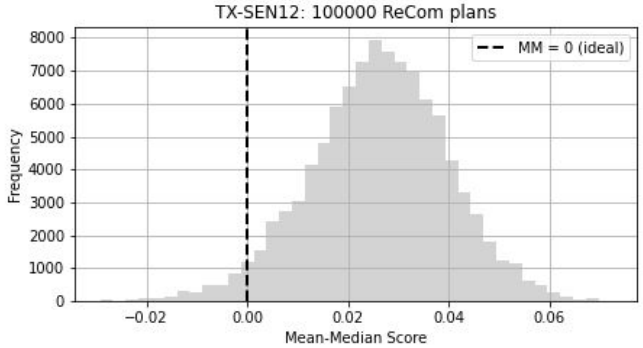
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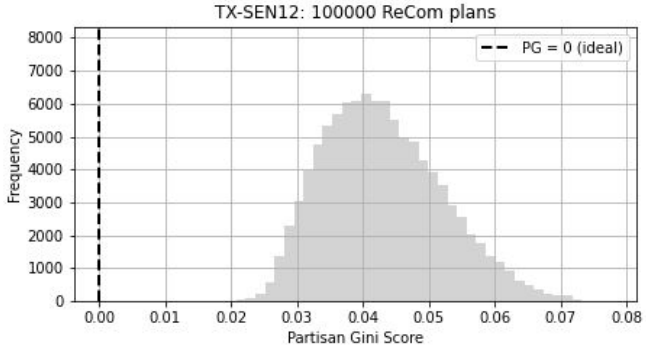
MM = 0 (ideal)
Frequency
Mean-Median Score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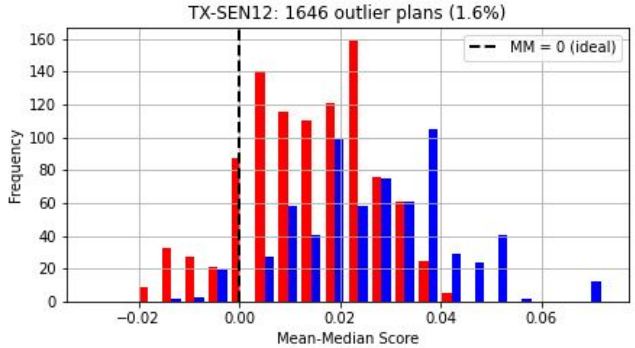
Partisan Gini Score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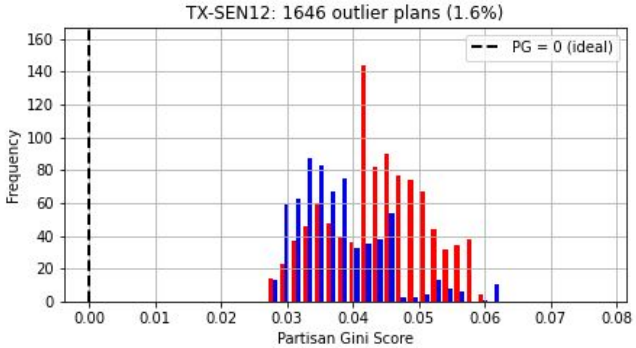
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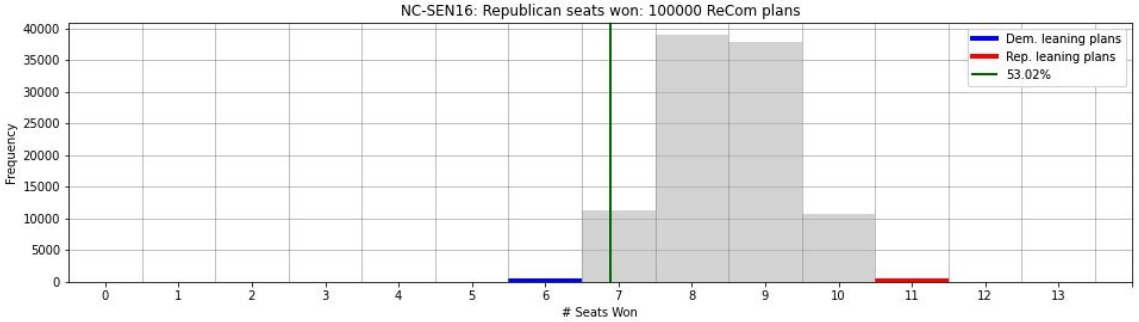

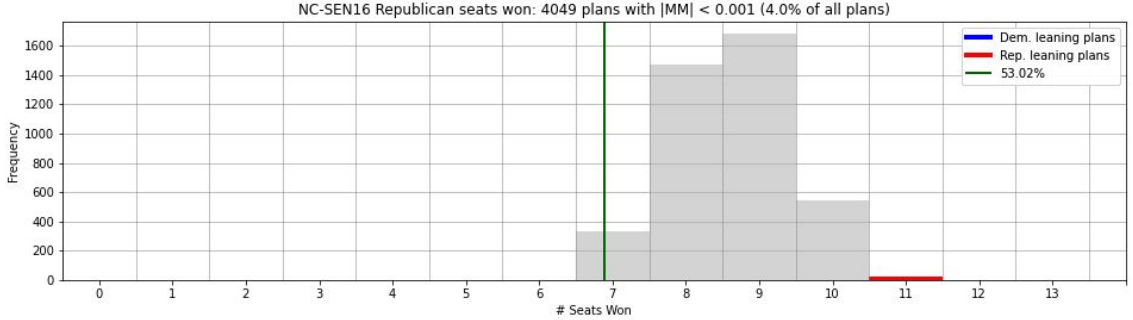

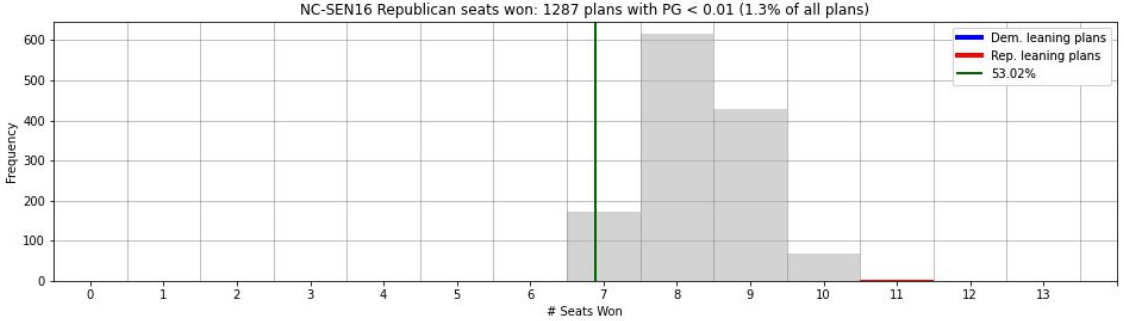

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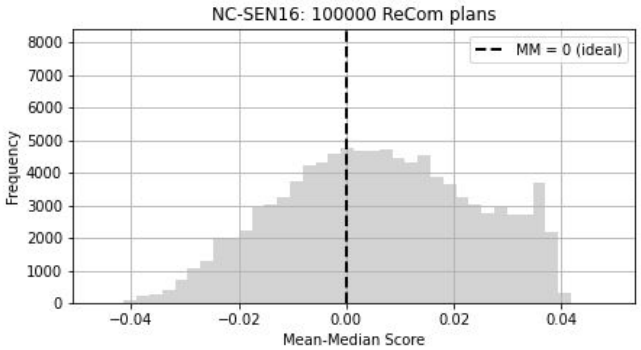
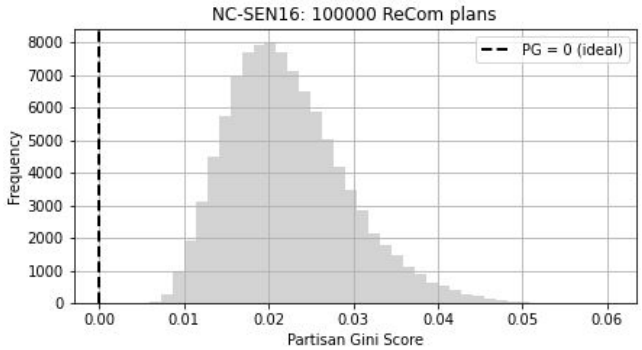
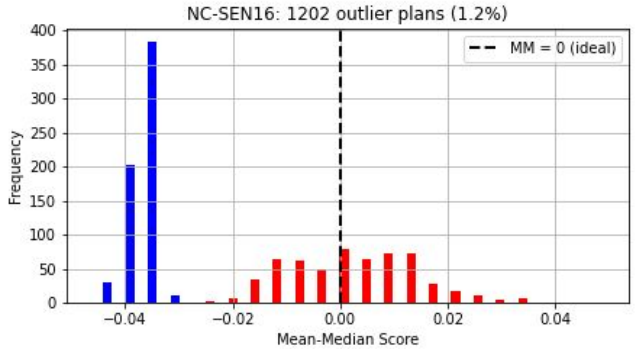
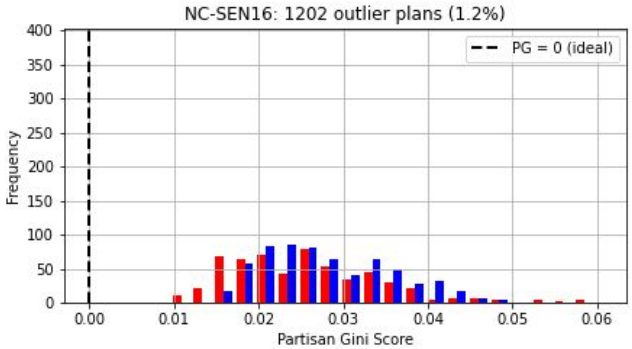




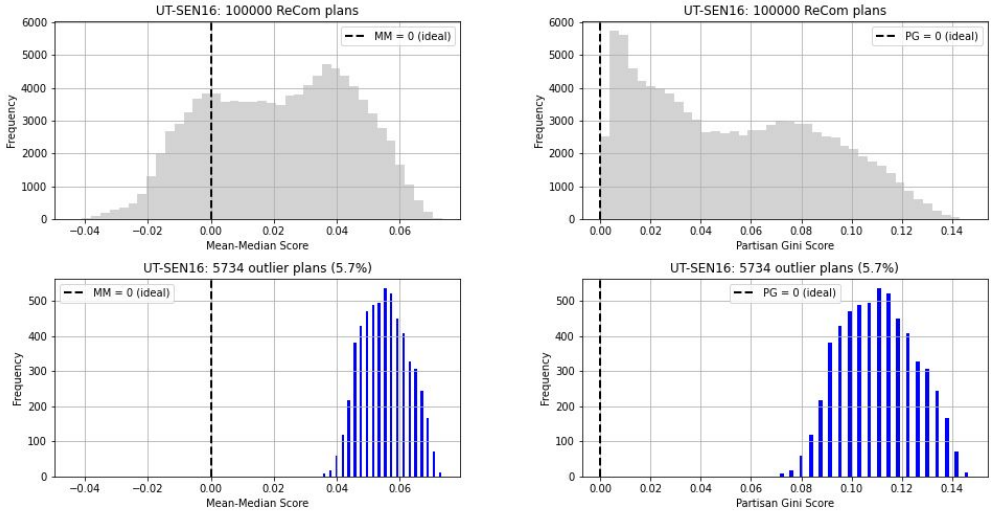




Conclusion

Adoption of symmetry standards:

- Fails to prevent extreme partisan outcomes
- Can plainly mis-identify which party is advantaged by a plan



...what do we tell Democrats in Utah?

Questions?