Computationally Solving a Partial Differential Equation using a Single Logical Operator

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Abstract

The purpose of this paper is A) to serve as a concrete demonstration that you can solve a high-order mathematical proof using only binary logic; and B) to serve as a tutorial on how to both represent and compute high-order expressions in low-order languages.

Keywords: Second-Order Predicate Logic, Partial Differential Equations, Computational Mathematics

1 Introduction

To solve our PDE, we will use only the NAND operator—the Sheffer stroke (↑):

A	B	$A \uparrow B$
T	T	F
T	F	T
F	T	T
F	F	T

Using the Sheffer stroke, we will build to the rules required to perform this proof.

The PDE we will solve is the Navier-Stokes Equation, used to describe the motion of viscous fluid substances,

$$0 = -\frac{dP}{dx} + \mu \frac{d^2u}{dx^2} \tag{1}$$

where P is the pressure, μ is the dynamic viscosity, and $u(x)$ is the velocity of the fluid as a function of position x. For simplicity, assume P is a constant gradient, so $\frac{dP}{dx} = C$, where C is a constant.

The equation simplifies to:

$$\mu \frac{d^2u}{dx^2} = C \tag{2}$$

Integrating both sides with respect to x gives:

$$\mu \frac{du}{dx} = Cx + K_1 \tag{3}$$

where K_1 is an integration constant. Integrating again with respect to x:

$$\mu u = \frac{C}{2}x^2 + K_1 x + K_2 \tag{4}$$

where K_2 is another integration constant. Finally, solving for $u(x)$:

$$u(x) = \frac{C}{2\mu}x^2 + \frac{K_1}{\mu}x + \frac{K_2}{\mu} \tag{5}$$

This expression for $u(x)$ represents the velocity profile of the fluid under the given assumptions and conditions. The constants K_1 and K_2 can be determined based on boundary conditions specific to a problem.

2 Justification

The language from which we will begin is second-order predicate logic. This is because the ninth of the Peano axioms (as they are typically expressed) is a second-order axiom, since it involves induction over sets.

From first-order logic, we may assign terms, which are either quantifier-bound variables or constants, to predicates. We are permitted to use the existential and universal quantifiers.

From second-order logic,

3 Results

Present the findings of your study. Use figures, tables, and equations to support your results.

4 Conclusion

Summarize the main findings and their implications. Discuss the limitations of your study and suggest future research directions.

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