Disk and Bulge Intensity Profile Evolution During the MW-M31 Major Merger

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ABSTRACT

The predicted future merger of the Milky Way (MW) and Andromeda (M31) galaxies would transform each dramatically. In this work, we examine the intensity profile evolution of the disk and bulge components of the MW and M31 galaxies through a simulated major merger. We find clues to the redistribution of stars that leads to these spiral disk galaxies' melding into one elliptical remnant. The bulges of our galaxies are well fit by Sérsic profiles before, during, and after the major merger. While the disks of the MW and M31 are well described by Sérsic profiles before and after the merger, throughout it their intensity profiles are changed dramatically. Importantly, we find that post-merger all four components can be described as having a Sérsic index $n \approx 3.6$. This quantifies the product of the merger as being a single elliptical galaxy. This result has implications for other galactic major mergers, in particular, it supports the hypothesis that some or many elliptical galaxies could have formed from major mergers. We also find that the disks of the MW and M31 in our simulations are better fit by Sérsic profiles than exponential disks, even at the present epoch. This could indicate an issue with the creation of our simulation or how we sample stellar particles from it. When fitting exponential disk profiles, we see an increase in scale length from the merger. This indicates that the remnant is more extensive than either of the disks of the MW or M31.

Keywords: Local Group(929) — Galaxy mergers(608) — Galaxy stellar disks(1594) — Galaxy bulges(578) — Milky Way evolution(1052)

1. INTRODUCTION

1.1. Topic

A galaxy merger occurs when two galaxies collide. Major mergers, those between large galaxies of comparable luminosities, are the most transformative events that galaxies may undergo after their formation (Lambas, D. G. et al. 2012). It is understood that the predominant members of the Local Group, the MW and M31 galaxies, will likely merge in several billion years. This is hypothesized to cause the two now-spiral galaxies to ultimately coalesce into a single elliptical galaxy.

The Local Group is a galaxy group containing over 100 gravitationally bound galaxies, most of which are satellites of the MW and M31 galaxies. Using measurements of the velocities of the centers of mass (COMs) of the MW and M31 galaxies (van der Marel et al. 2012b), van der Marel et al. (2012a) found an expected merger of the MW and M31 in ~6 Gyr after a first pericenter in ~4 Gyr. Collisionless interactions between the stars of the MW and M31 galaxies are believed to lead to largely

random orbits, in line with observed elliptical galaxies and juxtaposed with the ordered rotation of the present MW and M31 spirals.

The stellar bodies of spiral galaxies can be decomposed into bulges and disks. Stellar bulges are relatively dense groups of stars found within the central regions of their galactic hosts. Bulges generally come in one of two forms: classical bulges or pseudobulges. M31 has a large classical bulge (Kormendy et al. 2010), which is similar in form to an elliptical galaxy. The MW has a smaller pseudobulge. Pseudobulges are not spherically symmetric, are rotation-supported, and have Sérsic indices $n \leq 2$.

We can examine the evolutions of the MW and M31 disks and bulges throughout a major merger by analyzing their radial intensity profiles. Radial intensity profiles describe how the luminosity per unit area in a two-dimensional projection of a galaxy varies as a function of radius from its center. These can be fit in terms of Sérsic or exponential disk profiles (see Equations 1, 2) to characterize the profile's shape with a Sérsic index n (Sérsic 1963; Sérsic 1968) or disk scale length h_r . The

"law" of exponential spiral disks is first described in de Vaucouleurs (1959).

1.2. Motivation

The evolutions of disks and bulges during major mergers are important to understanding the galaxies we see today, including those in the Local Group. From Willman & Strader (2012): "A galaxy is a gravitationally bound set of stars whose properties cannot be explained by a combination of baryons and Newton's laws of gravity." Galaxy evolution involves how galaxies change with time, which we can see by observing them at varied look-back times (redshifts z) to get statistical or population-level results for how galaxies have come to their current forms. The field of galaxy evolution includes studying how galaxies may lose gas ("quenching"), form stars, change in color (a proxy for the age of stellar populations), grow their central supermassive black holes, and, importantly, merge.

The MW-M31 system is unique in that we can directly measure the proper motions of stars in each galaxy (Sohn et al. 2012), which has informed our simulated merger model. Presently, the MW and M31 are large spiral galaxies with masses $M \sim 10^{12}$. Understanding how spiral galaxies like these may evolve to form ellipticals through major mergers informs our understanding of elliptical galaxy formation. By analyzing the evolutions of the disks and bulges of the MW and M31 through their future simulated merger, we can track the bulk migrations of these galaxies' stars over billions of years to see how they redistribute themselves in such transformative events.

1.3. Contemporary Understanding

It is thought that many elliptical galaxies could have formed through major mergers. Additionally, major mergers have been considered as a pathway to transform spiral galaxies into S0 (lenticular) galaxies. This is an alternative to viewing S0s as "faded spirals" which cannot explain all their observed properties (Querejeta et al. 2015). See Fig 1 for a showcasing of the striking similarities between other simulated merger remnants and observed S0 galaxies.

Further, it is believed that mergers (including minor mergers) could be the source of classical bulges (Brooks & Christensen 2016). Most relevant to this work is that galaxy mergers have been examined as possible creators of elliptical galaxies for several decades now (Toomre 1977). However, there are concerns that mergers alone may be unable to reproduce all relations observed for elliptical galaxies (Brooks & Christensen 2016). In sum, examining major mergers of spiral galaxies is important

to understanding the formation of classical bulges along with elliptical and S0 galaxies. All three of these can often be characterized and distinguished by their intensity profiles and fitted Sérsic or exponential disk profiles (Equations 1, 2).

1.4. Open Questions

There are still many open questions concerning galaxy evolution and major mergers: did some elliptical galaxies form from major mergers like that of the future MW-M31 merger? If so, could all of them have formed this way? Are some or all classical bulges formed from galaxy mergers? Are some or all S0 galaxies formed by major mergers? How do bulges and disks evolve during mergers? How are stars redistributed in mergers? Further examination of simulated major mergers will help bring us closer to answering some or all of these questions. Querejeta et al. (2015) explore the possibility of mergers as progenitors of S0 galaxies. They accomplish this with a comparison of the properties of simulated merger remnants with those of observed S0 galaxies (see Figure 1). Additionally, Brooks & Christensen (2016) examine how mergers may produce classical bulges by modeling the effects on galactic gas that can, in turn, influence star formation in such a way that changes bulge morphology.

2. THIS PROJECT

2.1. Project Introduction

In this paper, we examine the evolutions of the radial intensity profiles of the MW and M31 disks and bulges throughout their simulated future merger. We analyze the stellar components of these two galaxies only, ignoring M33 and dark matter particles (also simulated). The evolutions of the bulge and disk components of the MW and M31 throughout the merger were not explored in detail by van der Marel et al. (2012a), where the final state of the merged remnant was a larger focus. We fit intensity profiles to Sérsic and exponential disk profiles, where applicable, at all time snapshots in the simulation. More specifically, we fit to the Sérsic index n and/or the disk scale length h_r , as any other functional parameters can be calculated explicitly. Assuming a mass-to-light ratio of 1 allows us to trivially convert areal mass densities to intensities $(M_{\odot} \to L_{\odot})$.

After fitting, we examine how well our idealized functions fit our simulated profiles, and how this evolves through the merger. We also analyze how the nominal values of our fitted parameters, Sérsic index n and/or disk scale length h_r , change in time. Results are compared and contrasted for the four galaxy components.

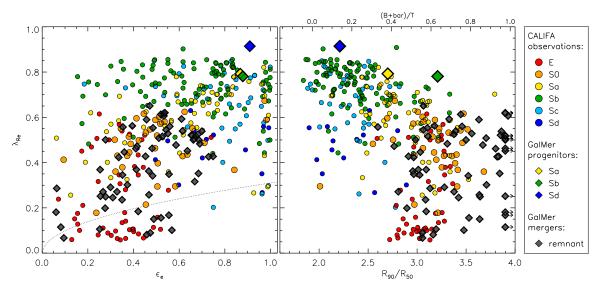


Figure 1. Figure 1 from Querejeta et al. (2015). Here we see the stellar angular momentum λ_{R_e} plotted against both the ellipticity ϵ_e and concentration R_{90}/R_{50} for simulated merger progenitors and remnants from the GalMer (Chilingarian et al. 2010) simulations (galmer.obspm.fr) as well as observed Calar Alto Legacy Integral Field Area Survey (CALIFA) galaxies (Sánchez et al. 2012). We see that the observed CALIFA S0 galaxies as orange circles generally coincide well with the GalMer merger remnants as grey diamonds in these regions of galactic parameter space. This hints at the possibility of galactic mergers having formed the S0 galaxies that we see today.

2.2. Questions to Address

We will address the evolution of the luminosity profiles of the bulge and disk components of the MW and M31 throughout the merger. This describes one aspect of the evolution of disks and bulges during mergers. Of particular interest is the question of how Sérsic indices change throughout the simulated merger, and when they change the most. A larger Sérsic index generally indicates that more light is concentrated in the center of a galaxy and that at large radii it will fall off more gradually than for a low Sérsic index. Sérsic indices n > 2 are indicative of elliptical galaxies and classical bulges, where lower Sérsic indices $(n \sim 1)$ generally better describe spirals.

2.3. Importance, Project Relevance

Many of the galaxies that we see may have already undergone, will undergo, or are currently undergoing a major merger. To understand what properties we should expect from merger remnants, we require simulations. Intensity profile evolution is a proxy for the evolution of stellar density as a function of radius and will tell us how stars redistribute themselves in major mergers. Therefore, changes in profile parameters (n, h_r) will help us to describe how two spiral galaxies can come to form an elliptical galaxy.

3. METHODOLOGY

3.1. Simulation

We use an N-body simulation from van der Marel et al. (2012a) for this analysis. It is initialized so as to

represent the components of the two galaxies in their current states. This simulation follows collisionless, massive particles which interact only through gravity. These particles are meant to represent stars, though due to computational limitations, we restrict our resolution to particles each of masses on the order of $10^7 \, \mathrm{M}_{\odot}$. The MW, M31, and M33 are each modeled with disk, bulge, and dark matter particles. While we are not analyzing dark matter or M33 in this work, their gravitational effects have still implicitly impacted our results. In our simulation, the MW is modeled with a classical bulge more similar to that of M31, as opposed to its truer pseudobulge.

3.2. Approach

Sérsic and exponential disk profiles are fit using $scipy.optimize.curve_fit$ for radii between 1 and 40 kpc. Sérsic profiles as a function of radius r are given by the equation,

$$I(r) = I_e \exp(-7.67[(r/R_e)^{1/n} - 1])$$
 (1)

where $I_e = L/7.2\pi R_e^2$. I_e and R_e are the equivalent (half-light) intensity and radius, respectively. I_e and R_e are calculated explicitly while n is varied to best fit the simulated data. In addition to fitting both the bulges and disks of M31 and the MW to Sérsic profiles, we also fit the disks to an exponential disk profile as given by,

$$I(r) = I_0 \exp(-r/h_r) \tag{2}$$

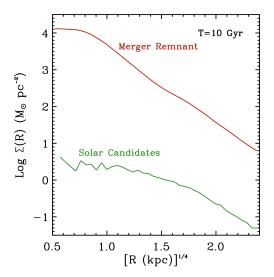


Figure 2. Figure 7 from van der Marel et al. (2012a). Here we see surface mass density Σ in M_{\odot}/pc^2 plotted against radius to the power of 1/4 in $kpc^{1/4}$ for the remnant and its solar candidates. Surface mass density Σ can be converted to intensity I by assuming a mass-to-light ratio. The straight lines past \sim 1 kpc indicate accordance with a de Vaucouleurs, n=4 Sérsic profile typical of elliptical galaxies.

where I and r are as before for the Sérsic profile, I_0 is another (i.e. not half-light) effective surface brightness and h_r is a scale length. I_0 is given by $I_0 = L/2\pi h_r^2$ for a total luminosity L. In this case, we fit for the scale length h_r though note that this profile is very similar to a Sérsic profile with n=1. See Figures 2 and 3 for examples of fitted profiles. In Figure 3, Sérsic profiles were fit explicitly, and in Figure 2 the surface mass density Σ was plotted against $R^{1/4}$ such that a straight line is indicative of a de Vaucouleurs profile (de Vaucouleurs 1948). A de Vaucouleurs profile can be described with a Sérsic index n=4 and is characteristic of elliptical galaxies and classical bulges.

3.3. Calculations

To compute intensity profiles, we read in .txt simulation data for the galaxies' components at each snapshot. The data files contain the particle types, masses, positions, and velocities. The positions and velocities are given with respect to the initial position of the MW disk and, therefore, we move to center-of-mass coordinates for each galaxy at each snapshot. After rotating to a face-on plane, intensities are computed in annuli. This is done by calculating the mass enclosed in each annulus and subsequently using our assumed mass-to-light ratio of 1 to get an annulus luminosity. By dividing this luminosity by the area of the annulus, we find an intensity.

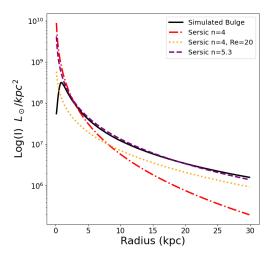


Figure 3. Plot from Lab 6 showing Sérsic profiles fit to the simulated bulge of M31 at t=0 (the present epoch). Intensity in L_{\odot}/kpc^2 is plotted against radius in kpc. Here we see how the effective radius R_e and Sérsic index n affect the profiles' morphologies.

3.4. Plots

All programming and plotting were completed in Python with the Matplotlib, Numpy, Astropy, and SciPy modules (Hunter 2007; Harris et al. 2020; Astropy Collaboration et al. 2013, 2018, 2022; Virtanen et al. 2020). We also used the functionality of IPython (Perez & Granger 2007). The aforementioned intensity profiles were plotted versus the annuli midpoint radii before fitting to Sérsic and/or exponential disk profiles (see Equations 1, 2) with scipy.optimize.curve_fit. See Figure 4 for intensity profiles of all four components at selected snapshots. After fitting profiles, we plot the fitted Sérsic indices of each component as a function of simulation time in Figure 5. Similarly, in Figure 7 we see the fitted disk scale lengths plotted as a function of time. Movies of the fitted profiles through the simulation were also created¹. All together, these plots show us how the stars are being redistributed in the merger and how that leads to a single, homogeneous elliptical remnant.

3.5. Hypothesized Results

We expect the Sérsic indices for both the bulge and disk particles of M31 and the MW to generally increase with time as the merger comes to form a single elliptical galaxy. Relaxation over time through collisionless two-

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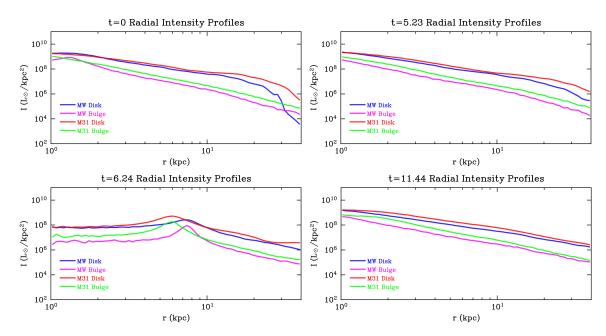


Figure 4. Selected intensity profiles for times of 0, 5.23, 6.24, and 11.44 Gyr. Intensity in L_{\odot}/kpc^2 is plotted against radius in kpc. t=0 Gyr corresponds to the beginning of the simulation at the present epoch, and t=11.44 Gyr to the end of the simulation. t=5.23 is about 1 Gyr after the first pericenter, and we see minimal changes in the profiles from the initial conditions. t=6.24 Gyr is right in the heart of the merger, where we see the largest change in intensity profiles occur, with notable "waves" visible for both stellar components of each galaxy. At t=11.44 Gyr, all profiles have similar slopes, indicative of the particle mixing and ultimate homogeneity of the elliptical remnant.

body interactions should drive this process of increasing n. Disk scale lengths may either increase or decrease depending on how the extent of the galaxy changes with the merger. However, we would not expect an elliptical remnant to be fit well by an exponential disk profile (Equation 2). We would also expect the change in the luminosity profiles/Sérsic indices to be greatest at or just after pericenters in the merger, when stars may shift the most. This is motivated by the low Sérsic indices of observed spiral galaxies, and the higher indices found for observed elliptical galaxies.

4. RESULTS

4.1. Figure Descriptions

At the end of the simulation, more than 11 Gyr in the future and about 5 Gyr post-merger, all four galaxy components have effectively been combined into a single elliptical that is well-fit with a Sérsic index of $n \approx 3.6$. This is slightly lower than the Sérsic index expected for an elliptical galaxy of n=4, possibly due to some excess rotational support present in our merger remnant that may not be typical of ellipticals formed through monolithic collapse.

In Figure 4, we see the calculated intensity profiles for our four galaxy components at selected times. These times are not arbitrary, but instead, we see the beginning and end of the simulation at t=0 Gyr and

t=11.44 Gyr, respectively, along with mid-merger snapshots at t=5.23 Gyr and t=6.24 Gyr.

See Figure 5 for a plot of all the components' fitted Sérsic indices throughout the simulation. The varying indices for the bulges can be more easily seen in Figure 6. We have smoothed all of our fitted parameter plots with a Savitzky-Golay filter (Savitzky & Golay 1964), manifested as fitting a moving box of 19 points with a quartic polynomial. The extremely high values of n for the two disks during the merger are really indicative of the profiles at those times not being well described by Sérsic profiles.

When fitting exponential disk profiles given by Equation 2, we do not achieve as good of a fit to our disks, even before the merger (see Figure 8). We do see a similar trend in the evolution of h_r for the disks as with the Sérsic index n, however (Figure 7). The scale lengths are relatively constant before and after the merger and increase significantly during the major merger. There are, however, some significant characteristics that we see by fitting to h_r in our disk profile than n in the Sérsic profile. For one, the scale length of the disk of M31 begins significantly larger than that of the MW, demonstrating the greater extent of M31's disk. After the merger, both scale lengths have increased but converged to the same (greater) value of \sim 3 kpc.

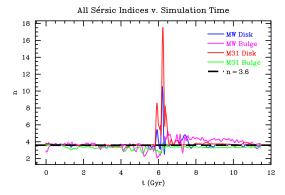


Figure 5. Sérsic indices n plotted versus time in Gyr for the bulges and disks of both the MW and M31 galaxies throughout the course of the merger simulation. We see that the disks are more strongly changed during the merger than the bulges, though the bulges do see their Sérsic indices temporarily lower before the merger and raise again after it. All components start and end at $n \sim 3.6$.

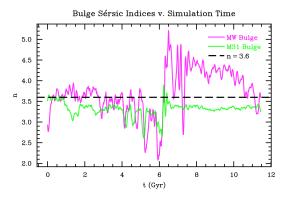


Figure 6. Sérsic indices n plotted versus time in Gyr for the bulges of the MW and M31 galaxies throughout the course of the merger simulation. We see that the indices are not strongly changed by the merger, though they do temporarily lower before the merger and raise again after it. The two bulges start and end at $n \sim 3.6$.

That Sérsic profiles fit the disks better than exponential disks even at t=0 Gyr (the present epoch) is a surprising result (see Figure 8). It is possible that this is a problem with the simulation itself or with the sampling of particles from it. The great Sérsic fit at the end of the simulation (t=11.44 Gyr) is expected, as we should be left with a mixed-up, homogeneous elliptical remnant. The Sérsic fits for the other three galaxy components are similar at t=11.44 Gyr, which can be inferred from the similarity of the calculated profile shapes shown in the bottom right of Figure 4.

5. DISCUSSION

 Paragraph 1: Summarize one result from the previous section. ● Does this result

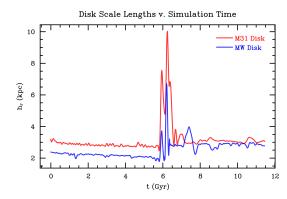


Figure 7. Fitted disk scale lengths h_r in kpc plotted versus time in Gyr for the MW and M31 disk particles through the simulated merger. The exponential disk profile used for fitting with scipy.optimize.curve_fit is found in Equation 2. Note that the baseline scale lengths for both galaxies' disks have increased from the merger, indicating more extended stellar distributions. The convergence in scale length at the end of the simulation is also indicative of the homogeneity of the elliptical remnant.

agree or disagree with your hypothesis? •
How does this result relate to existing
work in the literature? • What is the
importance/meaning of this result for our
understanding of galaxy evolution? 2.
Paragraph 2: What are the uncertainties in
your analysis? 3. Subsequent Paragraphs:
Repeat the above if you have a 2nd result
(etc.)

5.1. Summary

We see that the profiles of the two bulges are effectively unchanged by the first MW-M31 pericenter at $t \sim 4$ Gyr, while the disks are slightly modified to as to have some more stars at large radii, presumably kicked outward by the encounter (Figure 5). At t=6.24 Gyr after the full merger, we see the largest change in intensity profiles. There are remarkable "waves" which appear to travel outwards in all components, though there is a slight phase shift between the two galaxies (but not between their respective components).

When analyzing the disk scale lengths for exponential disk profiles (Figure 7), we find that the scale lengths increase for both the MW and M31 from the merger. They temporarily increase dramatically during the merger before settling down to a single baseline that is higher than either of them started at. The disk of M31 does begin the simulation with a greater disk scale length, meaning that its disk is more extended than the MW disk. Both then converge to a single scale length of $\sim 3~\rm kpc$, evi-

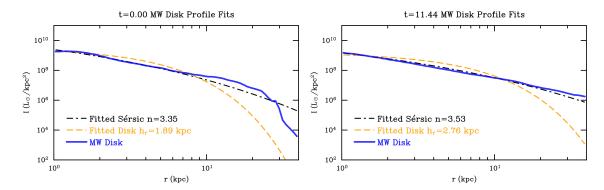


Figure 8. Comparison of Sérsic and exponential disk fits for the MW disk at t = 0 Gyr (the present epoch) and t = 11.44 Gyr (the end of the simulation). Intensity in L_{\odot}/kpc^2 is plotted against radius in kpc. The two fitted profiles are given by Equations 1 and 2. We can see that while neither profile is an excellent fit for the simulated galaxy's initial disk, the Sérsic profile does provide a better match. This is an unexpected result, and the outcome for M31's disk is similar.

dence of the remnant's homogeneity and the completion of the major merger.

6. CONCLUSIONS

6.1. Introduction summary

A simulated major merger of the MW and M31 does show marked changes occurring to the stars of the two galaxies. We have examined here how these powerful major mergers can transform the properties of two galaxies, and combine them into one remnant. In this paper, we have specifically analyzed the changes in radial intensity profiles for the bulges and disks of the MW and M31 throughout their simulated future merger. This has given us clues as to how these galaxies' remnant will form, and what is happening to the distribution of stars with time that tracks this evolution and elliptical galaxy formation.

6.2. A Key Finding

We found that the intensity profiles for both galaxies' disks cannot be well fit by the standard exponential disk model of Equation 2. This does not fit our hypothesized result, where we would expect the disk particles to be well fit by the model before the merger, and poorly fit by it after the merger. Similarly, we would expect the disks to be well fit by a low-n Sérsic profile (or not well fit by a Sérsic profile at all) at the beginning of the simulation, and then well fit by an $n \sim 4$ Sérsic profile

6.3. Future Directions

In the future, further investigation into why the disk components of the MW and M31 in our simulation are

not well fit by an $n \approx 1$ –2 Sérsic profile and/or an exponential disk profile given by Equation 2. It is possible that this is an issue with either the simulation or the sampling of stellar particles from it. If the stars in the disk components of these simulated galaxies were not generated to follow an exponential disk profile in the first place, then of course we would not expect to be able to retroactively fit an exponential disk profile to the particles sampled from the simulation.

Similarly, if we are not properly sampling from the simulation (e.g. if particles at greater radii are more likely to be selected for some reason), then we may not expect to be able to fit the proper profile to the sampled data. Our analysis may also be improved slightly by using a higher-resolution sampling from the simulation, though at a much greater computational cost which may require high-performance computing (HPC). An analysis of the stellar particle kinematics in the simulation would be very complimentary to our work here, and may help to explain better the specific motions of stars in these galaxies which manifest here as changes in radial intensity profiles.

Software: Astropy (Astropy Collaboration et al. 2013, 2018, 2022), NumPy (Harris et al. 2020), Matplotlib (Hunter 2007), SciPy (Virtanen et al. 2020), IPython (Perez & Granger 2007)

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