Ampere & Biot-Savart's laws

Gabriel Weredyk

May 7, 2023

Notes are online at `https://gabrielweredyk.com/papers/PhysicsC/Ampere.pdf`

Ampere's Law

Ampere's Law states that the integrated magnetic field around a closed loop is directly proportional to the amount of electric current passing through the loop. This is mathematically represented as

$$\oint_C \vec{B} \cdot \mathrm{d}\vec{\ell} = \mu_0 I_{enc}$$

If this equation looks familiar, it's because it is essentially the Gauss's Law for magnetic fields. The most important difference between the two electromagnetic laws is that while Gauss's law uses Gaussian surfaces, Ampere's law uses Amperian loops. Unlike Gaussian surfaces, Amperian loops are two dimensional and have direction, as indicated by the $\mathrm{d}\vec{\ell}$. Like with Gaussian surfaces, Amperian loops should be chosen such that the magnetic field has a constant magnitude and is parallel to the $\mathrm{d}\vec{\ell}$ vectors. Problems involving Ampere's law aren't very complex, so we can cover all that you would find by doing a simple practice problem.

Let's say we had a wire with current I and radius R and we wanted to get an expression for the magnitude of magnetic field B a distance r away from the wire. For $r > R$, the derivation is easy enough using a circular path (set of points equidistant from one point on the wire). Note that instead of getting the surface area of a surface, we are getting the perimeter of a loop.

$$B \cdot 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

But for $r < R$, the problem gets a tad bit more complex, because not all of the current is enclosed in the Amperian loop. To deal with a radius smaller than the radius of the wire, we have to talk about current density. Much like two-dimensional charge density (σ), current density is the amount of current per unit of area. We represent current density as:

$$J = \frac{I}{A}$$

Using this notation, we can solve for the magnetic field in the wire by:

$$B \cdot 2\pi r = \mu_0 J(\pi r^2)$$
$$B \cdot 2\pi r = \mu_0 \frac{I}{\pi R^2}(\pi r^2)$$
$$B = \frac{\mu_0 I r}{2\pi R^2}$$

Biot–Savart Law

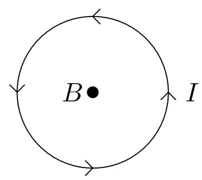
The Biot-Savart law is primarily used for determining the magnetic field created by asymmetrical wires. This law is primarily used for computations with vector-calculus, so its very rare to see it required on the exam. Mathematically, the law states

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{\ell} \times \hat{r}}{r^2}$$

Where r is the distance to the wire and $\hat{r}$ is the unit vector pointing in the direction of the point. Since $\hat{r}$ always has a magnitude of 1, it's not uncommon to see the Biot-Savart law written as

$$\mathrm{d}\vec{B} = \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\ell \sin\theta}{r^2}$$

Where θ is the angle between the $\mathrm{d}\vec{\ell}$ vector and the $\hat{r}$ vector. A simple application of the Biot–Savart Law is getting an expression for the magnetic field strength at the middle of a circular current carrying wire.



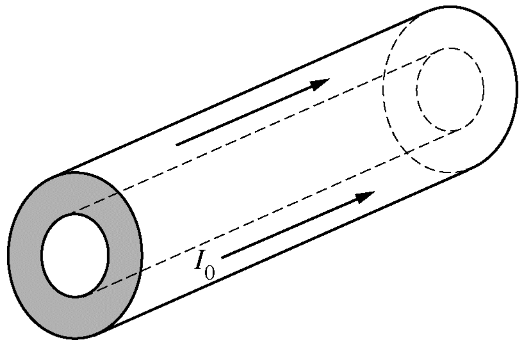
$$B = \oint_C \mathrm{d}\vec{B}$$
$$B = \oint_C \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\vec{\ell} \times \hat{r}}{r^2}$$

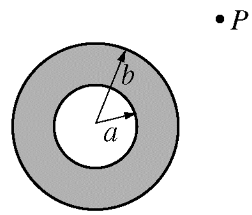
Since the vectors from the center of the loop to the loop its self are always perpendicular to the vectors in the loop, that means that we can simplify the integral to

$$B = \oint_C \frac{\mu_0}{4\pi}\frac{I\mathrm{d}\ell}{r^2}$$

Then we can simply take out the constants (Our radius included, because the loop is a circular path)

$$B = \frac{I\mu_0}{4\pi r^2}\oint_C \mathrm{d}\ell$$
$$B = \frac{I\mu_0}{4\pi r^2}(2\pi r)$$
$$B = \frac{I\mu_0}{2r}$$





Cross-sectional View
(current into page)

E&M. 3.

A section of a long conducting cylinder with inner radius a and outer radius b carries a current I_0 that has a uniform current density, as shown in the figure above.

(a) Using Ampère's law, derive an expression for the magnitude of the magnetic field in the following regions as a function of the distance r from the central axis.

i. $r < a$

---

ii. $a < r < b$

---

iii. $r = 2b$

(b) On the cross-sectional view in the diagram above, indicate the direction of the field at point P, which is at a distance $r = 2b$ from the axis of the cylinder.

(c) An electron is at rest at point P. Describe any electromagnetic forces acting on the electron. Justify your answer.

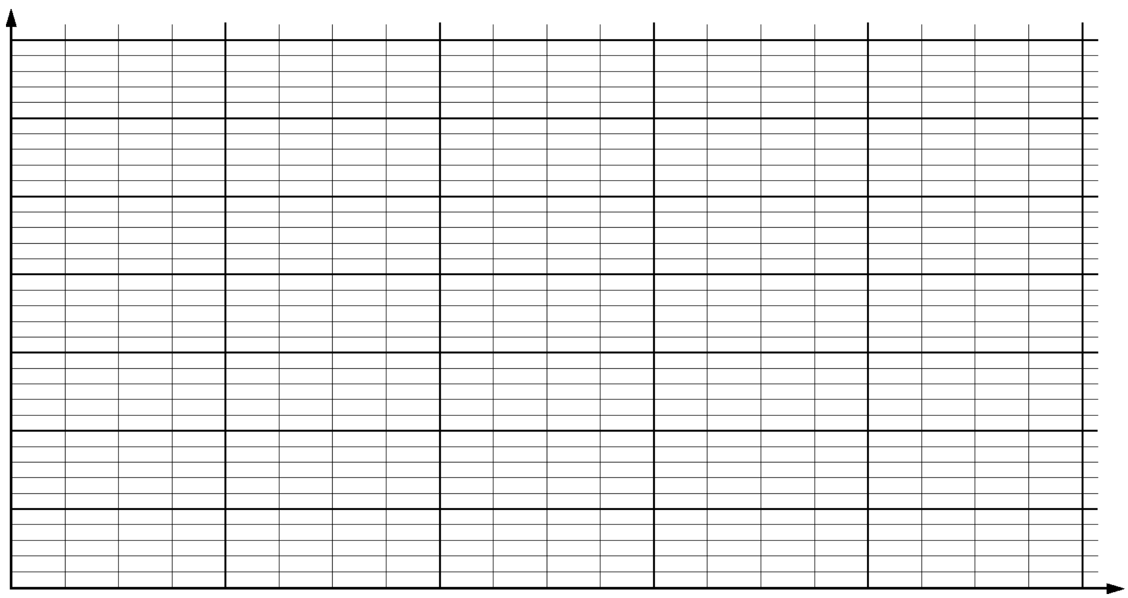
GO ON TO THE NEXT PAGE.

Now consider a long, <u>solid</u> conducting cylinder of radius b carrying a current I_0. The magnetic field inside this cylinder as a function of r is given by $B = \mu_0 I_0 r / 2\pi b^2$. An experiment is conducted using a particular solid cylinder of radius 0.010 m carrying a current of 25 A. The magnetic field inside the cylinder is measured as a function of r, and the data is tabulated below.

Distance r (m)	0.002	0.004	0.006	0.008	0.010
Magnetic Field B (T)	1.2×10^{-4}	2.7×10^{-4}	3.6×10^{-4}	4.7×10^{-4}	6.4×10^{-4}

(d)

i. On the graph below, plot the data points for the magnetic field B as a function of the distance r, and label the scale on both axes. Draw a straight line that best represents the data.



ii. Use the slope of your line to estimate a value of the permeability μ_0.

END OF EXAM