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# Some applications of mathematics in golf

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Review

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At its core, like many other sports, golf is a game of integers. The minimization of the number of strokes played is generally what determines the winner, whether each of these are associated with the shortest of putts or the longest of drives. The outcomes of these shots are influenced by very slight changes, but hopefully in a deterministic sense. Understanding the mechanics of golf necessitates the development of models and this is coupled more often than not to the use of statistics. In essence, the individual aspects of the sport can be modelled adequately via fairly simplistic models, but the presence of a human at one end of the kinematic chain has a significant impact on the variability of the entire process. In this paper, we will review some of the ways that mathematics has been used to develop the understanding of the physical processes involved in the sport, including some of the analysis which is exploited within the Equipment Rules. We will also discuss some of the future challenges.

## 1. Introduction

The exact origins of golf are unclear, but it is evident that St Andrews in Fife has played a significant role in the development of the game's history [1]. Furthermore, the golf club (The Royal and Ancient Golf Club of St Andrews) and since 2004 the company The R&A (via R&A Rules Limited) have played a large role in the determination of the Rules of Golf, in close collaboration with the United States Golf Association (USGA). In May 2002, the organizations published the Joint Statement of Principles, which conveys the ethos under which equipment rulings are made. Of particular relevance is the concept that while embracing technology it is believed that it should be skill which determines success rather than technology.

The Rules of Golf are under continual review and this encompasses the Equipment Rules. At the time of writing, these form Rules 4 and 5 of the playing Rules,

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which together with Appendices II, III and IV give guidance on the conformance status of equipment. A further publication [2] gives details on the interpretations used to make rulings. At the time of writing, there are 8000 drivers on the conforming list and 12 000 irons on the informational list for grooves. In addition, there are 1200 models of ball on the conforming list and these balls are resubmitted for evaluation against the Rules each year. The evaluation against the Rules can only be made by R&A Rules Limited or the United States Golf Association. As stated in the preamble to Appendix II [3], a manufacturer runs the risk of an item of equipment being found non-conforming if they fail to submit or fail to wait for a ruling.

The evaluation of equipment against the Rules is a straightforward matter for the majority of items, but as with many things it is the exceptions which prove to be the most challenging.

## 2. Prediction of ball speed and the Equipment Rules

The basic physics of normal impact can be captured using the definition of the coefficient of restitution ($e$) and the conservation of momentum

$$e(u_c - u_b) = v_b - v_c \tag{2.1}$$

and

$$m_b u_b + m_c u_c = m_b v_b + m_c v_c. \tag{2.2}$$

The velocities pre-impact are denoted by $u$ and those post impact by $v$, with the subscripts of b and c indicating quantities pertaining to ball and club, respectively. The mass ($m_b$) of the ball is generally close to the limit value of 45.93 g and the mass of the clubhead ($m_c$) ranges from around 180/200 g for a driver up to close to 300 g for a wedge. Obviously for a stationary ball being hit by a moving club $u_b = 0$, however, within the evaluation of the Rules there are instances where a ball is propelled from a cannon against a stationary club. Trivial manipulation of equations (2.1) and (2.2) yields the quantity often referred to as the smash factor (the ratio between incoming clubhead speed and outgoing ball speed), namely

$$\frac{v_b}{u_c} = \frac{m_c(1+e)}{m_b + m_c}.$$

In 1998, a Rule was introduced by the USGA which limited the coefficient of restitution to a value of 0.822 for drivers, which means that this smash factor is typically just under 1.5. It is possible to increase this ratio by increasing clubhead mass, but experimentation over the last 50 years has shown that the optimum clubhead mass for a driver has remained around 200 g [4]. Increasing clubhead mass (and the first moment—dubbed the swingweight in golf) causes a disproportionate decrease in clubhead velocity. Since its initial inception in this form, this Rule has been through various incarnations and has been adopted worldwide since 2003 and now even pertains to higher lofted clubs (up to 35°). Simple models using combinations of springs and dashpots [5] have been used to model the dynamics of impact. These provide very useful insights into the underlying physics. This simplistic model tends to work well in understanding the basic physics, however, even quantities like clubhead speed require further scrutiny, since the clubhead is obviously not a point mass.

The coefficient of restitution is obviously a measure of the efficiency of the impact between two bodies and as such it is dependent on both clubs and balls. In the evaluation of a piece equipment for this Rule, it is necessary to adopt a standard. This is accomplished by using a titanium plate weighing close to 200 g designed to have the requisite flexibility to represent 0.822 at a particular speed and for a given golf ball. Golf balls are highly nonlinear in their response at different strains and strain rates, as such it is possible to use the plate, which essentially behaves in a linear elastic manner to enable the test to be conducted at different speeds and for different balls. This test is destructive and for the majority of clubs it was replaced in 2004 by the pendulum test [6].

The pendulum test can be conducted on full clubs both within the laboratory and if necessary at tournaments. The central aspect of the test is the impact against the face of the club by the front of a steel object whose radius is designed to match a golf ball. The impact is modelled as a

Hertzian impact between two spheres [7] and the impact is repeated at different speeds to capture the effect of strain rate [8]. The outcome from the test is a set of characteristic times, which are generated from the accelerations during impact gathered from an accelerometer attached to the rear of the impacting object. These signals are filtered using a truncated Fourier series and then the characteristic time is calculated using a combination of analytical methods and a simple multi-dimensional Secant method. Even after filtering there is an aspect of uncertainty and statistical methods are used to provide a prediction interval, together with analysis on leverages and Cook's distances [9].

There are other aspects of the Rules which use fairly simple concepts including: Archimedes Principle used in the measurement of the volume of clubheads; and the parallel axis theorem, used in the calculation of the moment of inertia about a vertical axis through the centre of gravity of a driver. Any degree of obliquity or lack of alignment with the centre of gravity is going to generate spin and this can also be modelled by fairly simple methods, see for instance [10].

## 3. Ball flight

The Rules on Golf balls involve limitations on size, weight, efficiency (quantified by a test dubbed the initial velocity) and the Overall Distance Standard, together with a restriction on symmetry and the requirement that the ball is in some sense traditional and customary [3]. The subject of modelling golf ball flight has been tackled by several authors, see [11] for example.

The tests within the evaluation against the Rules involve some degree of statistics and also aspects of modelling. The Overall Distance Standard uses a robot to launch a test ball with a swing which is preset to hit a calibration ball at a launch angle of 10° and a back spin rate of 42 revolutions per second, with a clubhead speed of 120 miles per hour. The launch of the ball to be tested is measured using image analysis to yield the ball's actual launch conditions. The actual launch conditions are fed into a model which contains the aerodynamic characterization of the ball's performance, which have been obtained from an optimization problem. This simulation software uses models to predict the total distance the ball will travel and this is limited within the Rules.

## 4. Variability of humans

The previous two sections have focused on the distal end of the kinematic chain and the Equipment Rules. The human at the proximal end of the kinematic chain leads to inconsistency and the analysis of this effect has evolved significantly over recent years. It is important to understand the role of variability and to understand the difference between variance and determinism/predictability. The swing of a human can be modelled using a simple pendulum to a double pendulum and onwards [12]. A certain amount of understanding can be gleaned from these models, however, it is necessary to measure real swings and this can produce interesting challenges.

At its simplest, the variability could be quantified by statistical analysis of outcomes, however, this somewhat trivializes the complexity of the subject and also neglects the possibility that variability might be a seen as a positive attribute [13]. The use of mean-based statistics is not always appropriate in quantification of variability. Instead median-based measures such as the median absolute deviation (MAD) can be preferable to the more common standard deviation. For a dataset $X_1, \ldots, X_n$, the MAD defined as

$$\mathrm{MAD} = \mathrm{median}|X_i - \mathrm{median}(X)|.$$

It is noted that the MAD and the standard deviation are linked by a simple multiplicative factor of 1.4826 for normally distributed data. The earliest mention of this statistic seems to be in a paper by Gauss [14]. The MAD statistic also has the advantage that it can be used within the identification of outliers [15]. This is perhaps illustrated by a very simple example, consider a player who hits their sand wedge nine times 100 yards and then on their 10th shot they hit the ball 110 yards.

The value of their standard deviation is 3.1 yards, whereas the MAD for these data are 0 yards. The criteria that any point which is beyond five times the MAD value from the median clearly identifies the 110 yard shot as an outlier. The choice of which measure to use is down to the individual experiment and where the focus of the investigator lies.

Studies within the field of biomechanics frequently exploit event-based analysis in the quantification of variability, for instance, looking at the X factor (in essence the angle between thorax and hips, although this requires further definition) at a point 40 ms before impact. This can provide very useful information; however, it is often preferable to perform a spectral analysis. For instance, consideration of the variability of one of the wrist angles during the entire downswing as opposed to the variance of this quantity during the swing. This analysis can involve aspects like the calculation of entropy [16] or even calculations of quantities like Lyapunov exponents (although these are more useful over repetitive actions as might occur in gait analysis).

## 5. Concluding remarks

The use of models within our work is crucial and the closer these stay to core physics the better; this often involves the minimization of the number of parameters. The move to more complex models is more likely to lead to a data-fitting exercise rather than permitting the models to be used as predictive tools.

We will always try to ensure that the definition of the question receives almost as much work as the determination of the answer. This theme runs into our use of statistics. As an example, merely demonstrating something is statistically significant is insufficient, we also need to consider whether the effect is practically significant and as such the quantification of effect size is often critical, see for example, Cohen's *d* [17].

The fitting of equipment to a player's swing is a very interesting topic, but the key is to define the desired outcome. The preferred outcome could legitimately be one or more of the following: to reduce outcome variability; to improve the best outcome; to remove bad outliers or to improve the central tendency. The schema used to achieve this outcome is likely to be different for all of these. Developing the understanding of a player's inherent variability is crucial is designing the method for achieving the outcome, whether this is inter- or intra-session.

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