# Untouched NMT Topics

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# Grade 12 Topics[1]

- Geometry
  - Graphs of Polar Coordinates and Equations
  - Conics & Asymptotes
- Algebra
  - Function Composition and Inverse
  - Solving Higher-Degree Polynomial Equations
  - Infinite Geometric Series
  - Parametric Equations
- Trigonometry
  - Polar and Rectangular Coordinate Systems
  - Polar Forms of Complex Numbers & DeMoivre's Theorem
- Calculus
  - Limits & Derivatives
  - Equation of Tangent Line
  - Function Optimization

[1] Source: `http://nciml.com/nmt/NMTtopics.pdf`

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# Conics

All conics can be represented by the following equation:

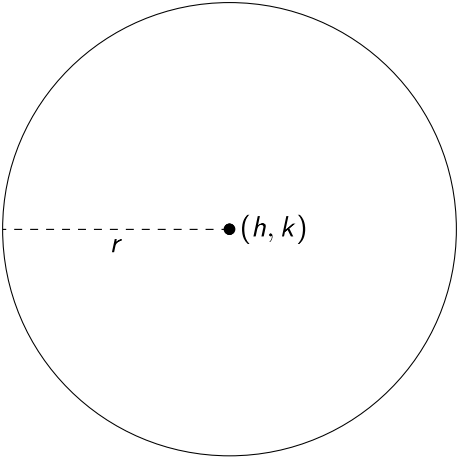
$$Ax^2 + By^2 + Cx + Dy + F = 0$$

# Circles

- Technically a conic with one focal point
- If $A = B$, then the conic is a circle
- Equation based on center:

$$(x-h)^2 + (y-k)^2 = r^2$$

# Diagram of a Circle

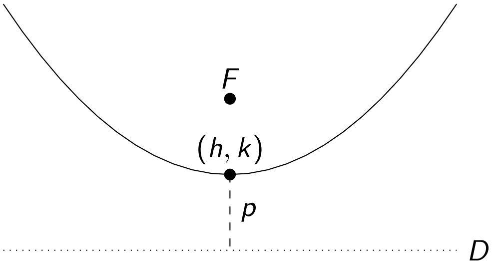


# Parabolas

- Has one focal point and an directrix
- If $A = 0$ or $B = 0$, then the conic is a parabola
- The vertex is at $(h, k)$, and a distance $p$ away from the focal point and directrix
- Focus based formula:

$$y = \frac{1}{4p}(x - h)^2 + k$$

# Diagram of a Parabola



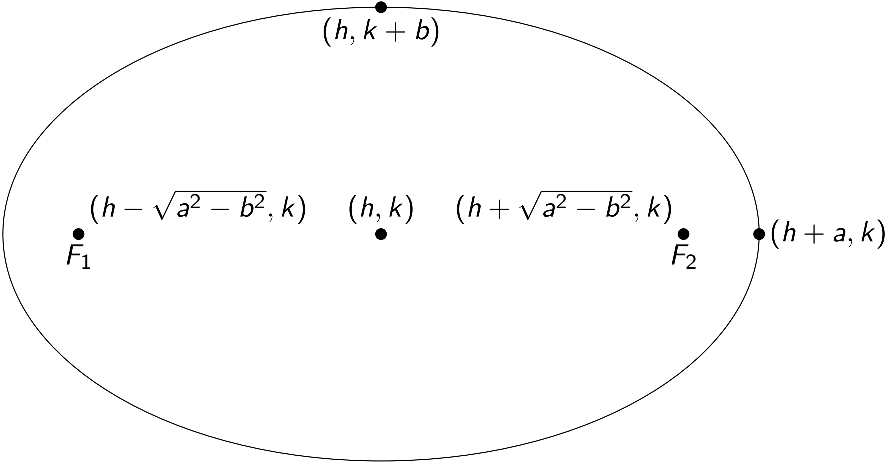
# Ellipses

- More popular conic with two foci
- If $A$ and $B$ have the <u>same</u> sign, then the conic is an ellipse
- Center $(h, k)$ based equation:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

- $a > b$ and are the extrema of the ellipse.
- $a$ and $b$ are not the same as $A$ and $B$!
- Foci are located at: $h \pm \sqrt{a^2 - b^2}$

# Diagram of an Ellipse



# Hyperbolas

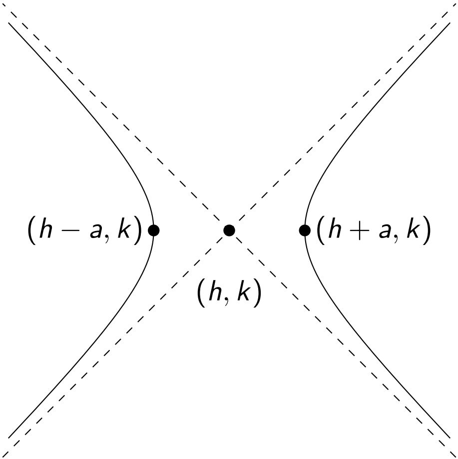
- Basically the inverse of an ellipse
- Also has two foci
- If $A$ and $B$ have different signs, then the conic is a hyperbola
- Center $(h, k)$ based equation:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

- Vertices are located at $h \pm a$
- Foci are located at $h \pm \sqrt{a^2 + b^2}$
- Equations for the asymptotes:

$$y - k = \pm\frac{b}{a}(x-h)$$

# Diagram of a Hyperbola



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# Function Composition & Inverses

- $f \circ g = f(g(x))$
- If $f \circ g = x$, then $f$ and $g$ are inverses.

# Properties of the roots of polynomials

$$p(x) = ax^n + bx^{n-1} + cx^{n-2} + \cdots + z$$

- Sum of the roots: $-\frac{b}{a}$
- Product of the roots: $(-1)^n \cdot \frac{z}{a}$

# Geometric Series

Recall that the sum of a geometric series is given by:

$$S_n = \frac{a_0(1 - r^n)}{1 - r}$$

So assuming that $r < 1$, we get the following formula for an infinite sum:

$$S = \lim_{n \to \infty} \frac{a_0(1 - r^n)}{1 - r}$$
$$S = \frac{a_0}{1 - r}$$

Where $a_0$ represents the first term in the series

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# Polar Curves as Parametric Curves

If you had a polar function of $\theta$ called $r$, then you could represent the curve parametrically as:

$$\vec{s}(t) = \begin{bmatrix} r(t)\cos t \\ r(t)\sin t \end{bmatrix}$$

# Polar Circles

Circles take one of three forms on the *xy*-plane:

- $r = a$: A circle centered at the origin of radius $a$
- $r = a\sin\theta$: A circle centered at $\left(0, \frac{a}{2}\right)$ with radius $\left|\frac{a}{2}\right|$
- $r = a\cos\theta$: A circle centered at $\left(\frac{a}{2}, 0\right)$ with radius $\left|\frac{a}{2}\right|$

# Polar Roses

Roses are equations that fit the form:

$$r = a\sin(k\theta) \vee r = a\cos(k\theta)$$

- $a$ determines the maximum radius of the rose
- If $k$ is even, there will be $2k$ petals
- If not, there will only be $k$ petals

# Limaçons

Limaçons are equation that fit the form:

$$r = a + b \sin\theta \vee r = a + b\cos\theta$$

- If $a = \pm b$, then the limaçon will pinch at the origin
- If $|b| > |a|$, then the limaçon will have a loop
- If $|b| < |a|$, then the limaçon will not touch the origin and bow out

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# Conjugate

Complex numbers typically use the variable $z$ where

$$z = a + bi$$

The conjugate of a complex number, denoted by $\overline{z}$ or $z^*$ is simply:

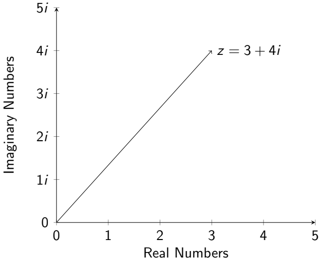
$$z^* = a - bi$$

Some properties of conjugates are:

$$\begin{aligned}(z+w)^* &= z^* + w^* \\ (zw)^* &= z^* w^* \\ zz^* &= |z|^2 = a^2 + b^2\end{aligned}$$

# The Complex Plane

Geometrically, complex numbers can be thought of as vectors on a plane consisting of the real number line and the imaginary number line.




This means that the $a$ is the $x$-value of the vector, and $b$ is the y-value of the vector.

# Polar Forms of Complex Numbers and Euler's Formula

Since polar numbers are really just vectors, we could represent them by their magnitude and direction

$$z = r(\cos\theta + i\sin\theta)$$

Euler's formula expands the domain and range of the exponential function into the complex domain, and states that

$$e^{ix} = \cos x + i\sin x$$

Which means we can finally represent our complex number as:

$$z = re^{i\theta}$$

# DeMoivre's Theorem

DeMoivre's Theorem states that for any $n$,

$$(cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$$

We can see that this holds true by substituting the two trig equations into Euler's Formula:

$$(e^{i\theta})^n = e^{i(n\theta)}$$

# Links

- Slideshow:
  `gabrielweredyk.com/papers/NMTtopicsSeniors.pdf`
- Practice Questions:
  `gabrielweredyk.com/papers/NMTpractice.pdf`