2022 Πathlon

Designed by Gabriel Weredyk

About the Πathlons

- The following page will contain 3 scenarios containing equations that feature π; your goal is to explain why π appears in the context of each problem. Explanations are to be written assuming that the reader knows all math required for the ACT or SAT, but no more than that.
- Each question will have some background information, all of which will be crucial for providing a direction to head in for answering these questions. In addition, you may use external resources such as a textbook or Wikipedia to gain a greater understanding of the topic at hand, but your submission must solely be in your own terms.
- Group work is allowed and even encouraged. If you choose to work in a group, your submission must have the names of all the members who participated listed alphabetically.
- Submissions may be handwritten or typed up. No matter the medium, make sure your submission is neat and uses proper notation (i.e. x^2 and not x^2)
- Submissions can be turned in at Dr. Caliendo's office (Room 321) or emailed to gabrielweredyk@duck.com. Submissions will be accepted no later than March 18th at 2:15 pm.
- Each explanation will be judged on how effectively it explains π's appearance. Each will receive a score from 1 to 10, with the final score being the sum of the individual question scores.
- The winning submission's author(s) will receive a $\$(10\pi)$ gift card[1] to Via Roma as well as a pat on the back.

An online version of this printout can be found at gabrielweredyk.com/2022piathlon.pdf

[1] Yet to be worked out with Via Roma. I, Gabriel Weredyk, am willing to provide the prize money in its entirety

Q1

The Normal Distribution is a commonly occurring data pattern within multiple data sets. Because of this natural phenomenon, it's crucial for statisticians to have a function resembling a normal distribution in order to effectively analyze situations. Functions derived from distributions are called probability density functions, and as the name implies can be used to determine the probability of a certain value appearing in our distribution. The probability density function for the Normal Distribution is[2]:

$$\mathcal{N}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$$

Where μ is the mean of the data, and σ is the standard deviation. Explain why $\sqrt{2\pi}$ appears in the probability density function for the Normal Distribution while incorporating the parent function of $f(x) = e^{-x^2}$.

Q2

Being an infinite sum both famous mathematicians frequently thought about, the Euler-Riemann zeta function is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} \; \{s \in \mathbb{C} : \Re(s) > 1\}$$

and shows up in more fields than you'd expect it to. The naive approach for calculating this series is simply impractical, so over the years, mathematicians have been trying to find other ways to evaluate the function. One prominent example of such a breakthrough was by Leonhard Euler himself, who defined the function for all even numbers as

$$\zeta(2n) = (-1)^{n+1} \frac{(2\pi)^{2n} B_{2n}}{2(2n)!}$$

Where B_n is the nth Bernoulli number, computed by

$$B_n = \sum_{k=0}^{n} \sum_{v=0}^{k} (-1)^v \binom{k}{v} \frac{(v+1)^n}{k+1}$$

Explain why 2π appears in the formula for even inputs of the Riemann Zeta function.

Q3

Two toy cars are placed tangentially on the edge of a frictionless, circular table with a radius of 1m. The two cars have a massless string attached between a post in the middle of the table and themselves to keep the cars in a circular motion. Car A is given an initial linear velocity of 1m/s while Car B remains at rest. All collisions between the cars are perfectly elastic and conserve momentum. However, on every other collision, twice the momentum of Car A is removed from the system. The formula for the number of collisions before Car A switches directions (or stops) is:

$$C(x) = \left\lceil \frac{\pi}{2\pi - 4\arctan\sqrt{x}} \right\rceil$$

Where x is the ratio between the mass of Car A and the mass of Car B. Explain why π appears in both the numerator and denominator of this equation.

[2] $\exp(x) = e^x$