

1. Let \mathcal{R} be the region in the complex plane consisting of all complex numbers z that can be written as the sum of complex numbers z_1 and z_2 , where z_1 lies on the segment with endpoints 3 and $4i$, and z_2 has magnitude at most 1. What is the nearest integer to the area of \mathcal{R} ?

2. Find the least positive value of t such that

$$\arcsin(\sin(t)), \arccos(\cos(t)), \arctan(\tan(t))$$

form (in some order) a three-term arithmetic progression with a nonzero common difference.

3. A single die is tossed only as many times as is necessary until a five occurs. Find the probability that an odd number of tosses is required to roll a five.

4. If $x^3 + x^2 + x = -1$, compute x^{2016}

5. Let $f(x) = \log_b x$ and let $g(x) = x^2 - 4x + 4$. Given that $f(g(x)) = g(f(x)) = 0$ has exactly one solution and $b > 1$, compute b

6. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y -axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \dots, T_n$ returns the point $(1, 0)$ back to itself?

7. Find a value of n such that complex number z with imaginary part 164 satisfies

$$\frac{z}{z+n} = 4i$$

8. Compute the sum of the solutions of the equation $z^3 - 8i = 0$

9. In the right triangle $\triangle ACE$, we have $AC = 12$, $CE = 16$, and $EA = 20$. Points B , D , and F are located on AC , CE , and EA , respectively so that $AB = 3$, $CD = 4$, and $EF = 5$. What is the ratio of the area of $\triangle DBF$ to that of $\triangle ACE$?

10. Compute

$$\lim_{n \rightarrow \infty} \frac{1}{2} n \sin\left(\frac{360^\circ}{n}\right)$$

11. On the interval $[0, 1]$, what point along the curve $y = x^2$ minimizes the combined area of the rectangle formed between the point and the origin and the rectangle formed between the point and $(1, 1)$