Companion Notes — Chapter 1

Why Time Is a Problem

(Pedagogical companion to "Relational Time and Informational Arrows from Quantum Clocks")

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1. Why Do We Say There Is a "Problem of Time"?

Before asking how time *emerges*, we must understand why time is problematic in the first place.

In everyday life, time feels obvious:

- clocks tick,
- events happen in sequence,
- causes precede effects.

Yet, when we look carefully at our most successful physical theories, this intuitive picture starts to fracture.

The "problem of time" is not a single paradox. It is a **mismatch between intuitions and formalisms**.

1.1 Time in Classical Physics: A Silent Background

In classical mechanics, time is an external parameter:

- it flows uniformly,
- it is the same for all observers,
- it does not interact with the system.

Equations of motion tell us how physical quantities change *with respect to time*, but time itself is never questioned.

This works extremely well — until it doesn't.

1.2 Time in Relativity: No Universal “Now”

Einstein’s relativity removes the idea of a global present.

Key consequences:

- simultaneity becomes observer-dependent,
- different observers slice spacetime differently,
- there is no absolute temporal ordering of distant events.

Time is no longer a universal background; it is intertwined with space and motion.

Already here, the intuitive notion of a single, shared “now” disappears.

1.3 Time in Quantum Mechanics: An External Parameter Again

Quantum mechanics, surprisingly, goes back to treating time as external.

The Schrödinger equation reads:

$$i \frac{\partial}{\partial t} \left| \psi(t) \right\rangle = \hat{H} \left| \psi(t) \right\rangle .$$

Time appears as a parameter, not as an operator.

This creates a tension:

- quantum systems are described internally and relationally,
- but time is assumed to come from *outside* the theory.

Quantum mechanics works — but it does not explain where time comes from.

1.4 The Clash: Quantum Theory Meets Gravity

The problem becomes unavoidable in quantum gravity.

In canonical approaches, the fundamental equation takes the form of a *constraint*:

$$\hat{\mathscr{C}} \left| \Psi \right\rangle = 0 .$$

There is no time parameter at all.

The universe, at the most fundamental level, appears **static**.

This leads to a disturbing question:

> If the universe does not evolve, why do we experience change?

1.5 Why the Question Is Not “What Is Time?”

A common mistake is to ask:

> “What *is* time, fundamentally?”

The approach taken in this work is different.

We ask instead:

> “Under what physical conditions does *time appear* to an observer?”

This shift matters.

It moves the problem:

- from metaphysics to operations,
- from definitions to mechanisms,
- from universal statements to observer-relative descriptions.

1.6 Observers Are Part of the Problem

A crucial insight is often overlooked:

Observers are not external to the universe.

They are:

- physical systems,
- with limited access to information,
- embedded in the dynamics they try to describe.

Any account of time that ignores this is incomplete.

This does **not** mean that consciousness is fundamental.

It means that **information access matters**.

1.7 The Strategy of This Work

This paper — and these companion notes — adopt a clear strategy:

1. Assume no fundamental time.
2. Treat clocks as physical quantum systems.
3. Define time through conditioning on clock states.
4. Derive dynamics and arrows of time from restricted access.

Nothing exotic is added.

What changes is the *perspective*.

1.8 What This Chapter Established

By the end of this chapter, we have learned:

- time is treated inconsistently across our best theories,
- a global flowing time is not supported by relativity or quantum gravity,
- the real puzzle is not motion, but experienced change,
- observers and information access cannot be ignored.

With this motivation in place, we can now introduce the key idea that unlocks the framework:

> **conditioning replaces evolution.**

That step is the subject of Chapter 2.

Companion Notes — Chapter 2

Conditioning and Relational Dynamics

*(Pedagogical companion to **"The Observer as a Local Breakdown of Atemporality"**)*

What "Time Emergence" Really Means

This chapter explains the core conceptual move of the paper: **time does not emerge because systems evolve, but because observers condition on physical clocks**. Everything else follows from this shift.

2.1 Conditioning Is Not Evolution

In standard quantum mechanics, dynamics is usually written as

$$|\psi(t)\rangle = e^{-i\hat{H}t}|\psi(0)\rangle.$$

This equation suggests that a system *evolves in time*, where time is an external parameter shared by all systems.

In the Page–Wootters framework, this picture is reversed. The equation above is **not fundamental**. Instead, the universe is described by a *single, static* global state $|\Psi\rangle$. No parameter "t" appears at this level.

What replaces evolution is **conditioning**.

Rather than asking:

> "How does the system change as time passes?"

we ask:

> "What is the state of the system *given* that a physical clock shows a certain reading?"

This distinction is not semantic. It changes what counts as fundamental.

2.2 What Conditioning Means Physically

Conditioning is a standard operation in both probability theory and quantum mechanics. It always involves three ingredients:

1. A joint description of multiple subsystems.
2. A specific outcome (or subspace) associated with one subsystem.
3. A reduced description of the remaining subsystem *given* that outcome.

In relational time:

- the **joint state** is the global history state $|\Psi\rangle$,
- the **condition** is a clock readout $|t\rangle$,
- the **result** is the conditional system state $\rho_S(t)$.

Mathematically, this consists of projection, partial trace, and normalization. Nothing exotic is introduced.

Physically, however, this is the precise moment where *time appears*.

Globally, nothing moves. What changes is the *question being asked*.

2.3 Why Conditioning Produces an Ordered Sequence

A natural objection arises immediately:

> If the global state is static, why do conditional states form an *ordered sequence* rather than a random collection?

The answer lies in the **correlation structure** of the history state.

The global state is constructed so that:

- distinct clock states $|k\rangle$ are correlated with systematically related system states,
- neighboring clock labels correspond to nearby points along a unitary orbit of the system.

This structure induces continuity.

When the clock is sufficiently fine-grained and approximately covariant, conditioning on successive clock values produces a smooth trajectory in the system's state space. To an internal observer, this trajectory is operationally indistinguishable from time evolution.

2.4 Relational Time Is Observer-Relative by Construction

An important consequence follows directly from the formalism:

> Different clocks define different times.

The partition of the total Hilbert space into clock, system, and environment is an *operational choice*, not a fundamental one. Two observers may:

- condition on different physical clocks,
- obtain different effective dynamics,
- yet remain fully consistent with the same global state.

This is not a paradox. It is the precise meaning of *relational time*.

There is no universal temporal ordering imposed on the universe. There is a family of internally consistent temporal descriptions, each tied to a physical reference system.

2.5 What Emerges — and What Does Not

At this stage, it is crucial to be explicit.

What *emerges*

- an effective ordering of system states,
- continuity approximating Schrödinger dynamics,
- meaningful notions of “before” and “after” *within* an observer’s description.

What *does not* emerge

- a global flowing time,
- a universal present,
- any modification of fundamental unitary quantum mechanics.

Time is not added to the universe. It appears only inside conditioned descriptions.

2.6 Why This Is Not Merely Interpretational

One might worry that replacing evolution with conditioning is only a reinterpretation of standard quantum mechanics. The framework, however, has concrete operational consequences:

- finite clocks introduce measurable deviations from ideal Schrödinger dynamics,
- different clock choices lead to different effective evolutions,
- introducing inaccessible degrees of freedom generates an emergent arrow of time.

These effects are quantitative and testable. They are not matters of philosophical preference.

2.7 Chapter Summary

- Time does not flow; correlations are structured.
- Evolution is replaced by conditioning.
- Ordering arises from correlation structure, not from a background parameter.
- Time is observer-relative by construction, not by interpretation.

With this foundation in place, the next step is to ask a more concrete question:

> **What makes a physical system a good clock rather than a mere label?**

That question is addressed in Chapter 3.

Companion Notes — Chapter 3

What Makes a Good Quantum Clock

(Pedagogical companion to "The Observer as a Local Breakdown of Atemporality")

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3. What Makes a Good Quantum Clock

Why Not Everything Can Be a Clock

At this point in the companion, time has already emerged — but only in a very precise sense: as a **conditional description** of correlations between subsystems.

A natural next question is unavoidable:

> *If time emerges by conditioning on a physical system, what distinguishes a **good clock** from a bad one?*

The answer is subtle but crucial. **Not every subsystem can function as a clock**, even if it has many distinguishable states.

A clock is not just a label. It is a physical system whose correlations with another system generate:

- a stable ordering,
- approximate continuity,
- and minimal disturbance.

This chapter explains those requirements step by step.

3.1 Clock States vs Time States

A first conceptual pitfall is to confuse:

- a *basis of states* with
- a *basis of time readouts*.

In the Page–Wootters construction, the clock Hilbert space (H_C) may be spanned by states (lk). These states are **not yet time states**. They are simply orthogonal labels.

Time readouts arise only when:

- the clock Hamiltonian (H_C) has an approximately **equally spaced spectrum**, and
- the readout states are related by approximate **covariance** under time translation.

This distinction explains why a trivial label register does not automatically generate dynamics.

3.2 Resolution: How Fine Is the Clock?

A physical clock has finite resolution.

Operationally, resolution answers the question:

> *How well can two neighboring clock readings be distinguished?*

In finite-dimensional clocks, resolution is controlled by:

- the clock dimension (N),
- the spacing (dt) between effective time labels,
- the spread of the clock POVM associated with each readout.

A good clock satisfies:

$$\Delta t \ll \tau_S,$$

where τ_S is the characteristic timescale of the system Hamiltonian.

If this condition fails, the conditional dynamics becomes jagged or discontinuous.

3.3 Back-Action: The Cost of Asking the Time

Every clock measurement disturbs the clock.

In a relational framework, this disturbance feeds back into the system because clock and system are correlated.

We quantify this effect via the **clock back-action**:

$$\Delta E_C(t) := \left\langle \hat{H}_C \right\rangle_t - \left\langle \hat{H}_C \right\rangle_{\text{uncond}}.$$

Interpretation:

- small $|\Delta E_C(t)|$: clock is robust;
- large $|\Delta E_C(t)|$: clock degrades under conditioning.

Your QuTiP simulations show precisely this trade-off:

- better time resolution typically increases back-action,
- weaker coupling preserves the clock but reduces informational gain.

A good clock balances these competing effects.

3.4 Jitter and Deviation from Ideal Dynamics

Even when resolution is high and back-action is small, finite clocks introduce **jitter**.

Jitter refers to small deviations between:

- the ideal Schrödinger evolution (U_S(t)), and
- the actual conditional state (_S(t)).

Operationally, this deviation can be quantified using:

- fidelity,

- trace distance,
- or diamond-norm–based measures.

In your simulations, fidelity decreases smoothly as:

- clock dimension decreases,
- environment size increases,
- or coupling strength grows.

This behavior confirms that emergent dynamics is approximate — not exact.

3.5 Covariance and Continuity

A defining property of a good clock is **approximate covariance**:

> conditioning at $t + dt$ should produce a state close to the infinitesimal evolution of the state at t.

Mathematically, this means:

$$\rho_S(t + dt) \approx e^{-i\hat{H}_S dt} \rho_S(t) \, e^{i\hat{H}_S dt} .$$

This property fails for clocks with:

- irregular spectra,
- strong measurement back-action,
- insufficient dimensionality.

Covariance is what turns a discrete sequence of conditional states into something that *feels like continuous time*.

3.6 Why Clock Quality Matters for the Arrow of Time

Clock quality does more than recover Schrödinger dynamics.

It also determines how clearly an **arrow of time** can be identified.

Poor clocks:

- blur temporal ordering,
- amplify noise in entropy measurements,

- obscure monotonic trends.

Good clocks:

- cleanly separate system dynamics from environmental decoherence,
- allow the effective entropy (S_{}(t)) to be meaningfully tracked.

This is why clock quality must be established *before* discussing irreversibility.

3.7 Summary of the Chapter

- A clock is a physical system, not a label.
- Good clocks require resolution, covariance, and limited back-action.
- Finite clocks produce approximate — not exact — dynamics.
- Clock quality directly impacts the visibility of the arrow of time.

With this machinery in place, we are finally ready to address the central asymmetry:

> *Why does time appear to have a direction?*

That question is the subject of **Chapter 4: The Informational Arrow of Time**.

Companion Notes — Chapter 4

The Informational Arrow of Time

(Pedagogical companion to "The Observer as a Local Breakdown of Atemporality")

What This Chapter Is (and Is Not)

This chapter explains why an arrow of time appears in a framework where the global quantum description is stationary and unitary. It does not introduce new dynamics, collapse postulates, or phenomenological noise. Instead, it shows how irreversibility emerges from partial access to information.

The central claim is simple:

The arrow of time is not a property of the universe. It is a property of restricted descriptions.

4.1 Global Unitarity vs. Local Irreversibility

At the level of the full system (clock + system + environment), the state evolves unitarily or is globally stationary under a constraint. Nothing in this description prefers a direction.

Yet observers consistently report: • irreversibility, • loss of coherence, • growth of entropy, • a distinction between past and future.

This is not a contradiction. It reflects a mismatch of descriptive levels.

Unitarity holds for the whole. Irreversibility appears in the part.

4.2 Why Entropy Can Increase Without Violating Unitarity

Consider a pure global state l. If an observer has access only to a subsystem S, their description is

_S = _E(ll).

Even though lis pure, _S is generally mixed.

The von Neumann entropy

S_{} = -(_S _S)

quantifies how much information about the full state is inaccessible.

Crucially: • this entropy can increase over time, • without any fundamental entropy production, • and without breaking unitarity.

The arrow emerges from discarded correlations, not from fundamental dissipation.

4.3 Conditioning Creates a Temporal Ordering

In the Page–Wootters framework, states are indexed by conditioning on a clock:

_S(k) _E[klC lk_C].

As k increases, the system becomes increasingly entangled with the environment.

From the observer's point of view: • earlier clock readings correspond to low entanglement, • later clock readings correspond to higher entanglement, • entropy increases along the clock coordinate.

This induces a directionality.

The arrow is not tied to a fundamental time variable—it is tied to clock-indexed information loss.

4.4 Finite Environments and Recurrences

For finite environments, entropy growth is not perfectly monotonic.

One expects: • local oscillations, • partial revivals, • Poincaré recurrences.

This is not a flaw of the framework. It is a feature.

As the effective environment dimension d_E increases: • recurrence times scale roughly as d_E, • relaxation times scale roughly as $1/(g^2 d_E)$,

so for sufficiently large environments, entropy saturates long before recurrences become observable.

This is exactly what is seen in explicit simulations. Dedicated robustness tests (see `generate_structural_robustness.py`) confirm that breaking the symmetric coupling — which artificially forces exact recurrences — produces an exponentially growing number of distinct frequencies, suppressing recurrence depth to well above zero for $n_{env} \geq 3$. Additionally, 100 Haar-random initial states and all 10 single-qubit partitions were tested: the arrow appears generically, not as a fine-tuning artefact.

4.5 The Arrow Is Observer-Dependent

Different observers may: • trace out different degrees of freedom, • condition on different clocks, • define different effective subsystems.

As a result: • the experienced arrow can differ between observers, • even though the global description remains the same.

There is no contradiction.

The arrow is not universal—it is relational.

4.6 What the Arrow Is Not

It is essential to avoid common misunderstandings.

The informational arrow: • is not fundamental time asymmetry, • is not thermodynamic entropy of the universe, • is not collapse or decoherence postulated by hand, • does not require measurement or consciousness.

It is a bookkeeping consequence of partial access.

4.7 Reading the Simulations

In the QuTiP simulations: • global evolution is exactly unitary, • no Lindblad operators are introduced, • no noise is injected.

Yet one observes: • damping of coherent oscillations, • growth of effective entropy, • suppression of revivals with increasing environment size.

These are not added features.

They are the arrow of time, emerging exactly where the theory predicts.

4.8 Summary of the Chapter

- Unitarity and irreversibility coexist at different descriptive levels.
- Entropy growth reflects lost correlations, not fundamental dissipation.
- Conditioning on a clock induces directionality.
- The arrow of time is relational and observer-dependent.

With the arrow now understood, we can finally ask the deeper question:

What kind of physical system becomes an observer at all?

That question is addressed in the next chapter.

Companion Notes — Chapter 5

Reading the Simulations: When the Arrow Appears (and When It Does Not)

(Pedagogical companion to "The Observer as a Local Breakdown of Atemporality")

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5. Reading the Simulations

This chapter explains **how to read the numerical results** obtained with the QuTiP simulations, and—more importantly—**what they do and do not mean**. Nothing in this chapter introduces new assumptions or interpretations. We simply describe what follows from the unitary dynamics plus conditioning and partial access.

All simulations discussed here are performed using **QuTiP (Quantum Toolbox in Python)**, with exact unitary evolution. No Lindblad operators, stochastic noise, or phenomenological decoherence models are introduced at any point.

5.1 What Is Being Simulated (and What Is Not)

Before interpreting any plot, it is essential to be precise about the setup.

What is included:

- A finite-dimensional quantum clock C
- A qubit system S
- An environment E composed of a controllable number of qubits
- Exact unitary evolution of the full C–S–E system
- Conditioning on clock states $|k\rangle$ and partial trace over E

What is *not* included:

- No external time parameter beyond the clock labels
- No collapse postulate
- No Lindblad or master-equation dynamics
- No stochastic noise or coarse-grained averaging

Any irreversibility observed must therefore arise **solely** from conditioning and restricted access.

5.2 Version A: Coherent Dynamics Without Environment

We begin with the simplest case: **no environment**.

Observed result

- The expectation value ⟨σ_z⟩ oscillates sinusoidally as a function of the clock label k.
- The oscillation matches the Schrödinger prediction:

$$\left\langle \sigma_z \right\rangle(k) \approx \cos(\omega\, k\, dt)\,.$$

- Amplitude remains ≈ 1.
- No entropy growth is observed: the system remains in a pure state.

What this tells us

This result demonstrates that:

- Schrödinger-like dynamics can emerge **without external time**.
- The global state remains stationary.
- What looks like “evolution” is entirely due to **conditioning on the clock**.

Importantly, nothing in this behavior depends on an arrow of time. The dynamics is reversible and coherent.

5.3 Adding an Environment: What Changes

We now introduce an environment E and allow weak system–environment interaction.

The global evolution is still unitary.

Key observation

When we condition on the clock and **trace out the environment**:

- Oscillations of ⟨σ_z⟩ become damped.

- The reduced system state becomes mixed.
- The effective entropy

$$S_{\text{eff}}(k) = -\operatorname{Tr}[\rho_S(k) \ln \rho_S(k)]$$

increases on average.

This is the first appearance of an **arrow of time**.

5.4 Why This Is Not Decoherence by Assumption

It is crucial to understand what is *not* happening here.

- The global state never becomes mixed.
- No non-unitary dynamics is introduced.
- No thermodynamic limit is assumed.

The arrow emerges because:

1. The observer conditions on the clock subsystem.
2. The observer lacks access to E.
3. Correlations between S and E are dispersed into degrees of freedom that are not observed.

Irreversibility is therefore **informational**, not dynamical.

5.5 Effective Entropy and Environment Size

By increasing the number of environment qubits n_env, we observe:

- Faster saturation of S_eff(k)
- Reduced visibility of recurrences
- Increasing monotonicity of entropy growth

This behavior is consistent with standard results on thermalization in large Hilbert spaces:

- Poincaré recurrence times scale exponentially with environment dimension
- Local relaxation occurs on much shorter timescales

The arrow strengthens not because time flows faster, but because **information escapes more efficiently**.

5.5b Version C: Two-Clock Comparison — Pillar 3

The first two versions validate Pillars 1 and 2. But the formula contains a third structural feature: the clock C is a **local subsystem**, not a global parameter. A different observer using a different clock should obtain a different — but equally valid — temporal description.

Setup

We build two history states from the **same** global configuration (H_S, H_SE, n_env = 4, g = 0.1, $|\psi_0\rangle = |0\rangle$):

Parameter	Clock C_1	Clock C_2
dt	0.20	0.35
N	30	30
Total window	5.8	10.15

The only difference is the clock spacing. Both observers perform the **same three operations** on their respective history states:

1. ⟨k|_C — project onto their clock reading
2. Tr_E — trace out the inaccessible environment
3. Extract ⟨σ_z⟩ and S_eff

Results

The two observers produce quantitatively different temporal narratives from the same underlying physics:

Clock tick k	⟨σ_z⟩ (C_1)	S_eff (C_1)	⟨σ_z⟩ (C_2)	S_eff (C_2)
0	1.000	0.000	1.000	0.000
5	0.498	0.164	–0.139	0.348
10	–0.300	0.405	–0.320	0.633

Clock tick k	⟨σ_z⟩ (C_1)	S_eff (C_1)	⟨σ_z⟩ (C_2)	S_eff (C_2)
20	–0.154	0.665	0.001	0.693
29	0.023	0.693	–0.029	0.692

Key observations:

- **Dynamical profiles diverge immediately.** At k = 5, C_1 reports ⟨σ_z⟩ ≈ 0.50 while C_2 reports ⟨σ_z⟩ ≈ –0.14.
- **Entropy growth rates differ.** C_2 saturates faster because each tick covers a larger physical interval.
- **Both reach the same asymptotic entropy** (ln 2 ≈ 0.693), confirming that irreversibility is structural, not clock-dependent.

Interpretation

Neither observer is wrong. Neither description is more fundamental. The difference is not an error — it is the **content of Pillar 3**: the temporal narrative depends on the clock subsystem to which the observer has access.

This is operationally analogous to how different inertial frames in relativity assign different coordinates to the same event. The unified relational formula implements this principle informationally rather than geometrically.

(The full data is available in output/table_pillar3_two_clocks.csv; the plot is output/validation_pillar3_two_clocks.png. Script: generate_pillar3_plot.py)

5.5c Continuous Limit, Clock Covariance, and Emergent Group Structure

The two-clock comparison above (§5.5b) shows that different clocks produce different narratives. A natural follow-up is threefold: (i) does the discrete PaW construction converge to a continuum as the clock gets finer? (ii) is the thermodynamic content truly invariant across clocks? (iii) do the transformations between clocks form a mathematical group?

All three questions are answered affirmatively by explicit numerical simulation.

Part 1 — Continuous limit ($N \to \infty$)

We fix the total physical time $T = 2\pi/\omega$ and increase the number of clock ticks:

N	dt	Conv_σz	Conv_S_eff
32	0.1963	8.1×10^{-5}	1.1×10^{-5}
64	0.0982	5.0×10^{-6}	8.2×10^{-7}
128	0.0491	0	0
256	0.0245	— (reference)	— (reference)

Convergence metrics are computed by interpolating each curve onto the densest grid ($N = 256$) and measuring the maximum absolute difference. The monotonic decay to zero confirms that the discrete PaW construction admits a well-defined continuum — the physics does not depend on clock granularity.

Part 2 — Inter-clock covariance

Two clocks with different spacings ($dt_1 = 0.20 \to N_1 = 31$; $dt_2 = 0.35 \to N_2 = 17$; ratio $\alpha = dt_2/dt_1 = 1.75$) each produce a history state and extract ρ_S(t). Their temporal profiles differ visually (oscillation frequencies, damping rates), exactly as Pillar 3 predicts. However, the effective entropy at matched physical times agrees to within |ΔS_eff| ~ 10^{-6}. The temporal narrative is observer-dependent; the thermodynamic content is not.

This is the **informational analogue of general covariance**: a change of clock is a change of temporal coordinates, not a change of physics.

Part 3 — Emergent affine group Aff(ℝ)

Using three clocks (dt = 0.15, 0.20, 0.30) we construct pairwise transformation maps $t' = \alpha t + \beta$ and verify:

- **Closure:** composing $\alpha_{12} \circ \alpha_{23} = \alpha_{13}$ — composition error = 0
- **Identity:** $\alpha = 1$ (a clock compared with itself) — verified
- **Inverse:** $\alpha_{12} \cdot \alpha_{21} = 1$ — verified
- **Arrow-reversal involution:** $t \to T - t$ reverses the temporal ordering; applying it twice returns to the original — verified

These maps form the affine group $\mathrm{Aff}(\mathbb{R}) = \{ t \mapsto \alpha t + \beta : \alpha \neq 0 \}$. The group is not postulated; it **emerges** from the constraint algebra. This is the temporal analogue of the

Lorentz group in relativity: a symmetry structure generated by the theory rather than assumed.

(Data: output/table_continuum_limit.csv, output/table_clock_transformations.csv, output/table_group_structure.csv. Plots: output/continuum_limit_convergence.png, output/clock_transformation_fidelity.png, output/group_structure_composition.png. Script: generate_continuum_limit.py)

5.6 Back-Action: Why the Clock Still Works

One might worry that strong system–environment coupling would destroy the clock.

This is tested explicitly by measuring the conditional clock energy:

$$\Delta E_C(k) = \langle H_C \rangle_k - \langle H_C \rangle_{\text{uncond}} .$$

Results show:

- Back-action remains small in the weak-coupling regime
- Clock resolution remains sufficient to index conditional states

Thus, the arrow does **not** arise because the clock breaks.

5.7 What the Simulations Do *Not* Say

Equally important are the conclusions we *cannot* draw:

- The simulations do not show that time is fundamental.
- They do not introduce a preferred direction of global evolution.
- They do not imply any violation of unitarity.

They show something more subtle:

> **Irreversibility appears only inside restricted descriptions.**

5.8 Summary of the Chapter

From the simulations alone, we can state:

- Conditional dynamics reproduces Schrödinger evolution without external time.
- Adding inaccessible degrees of freedom generates an informational arrow.
- The arrow depends on access structure, not on fundamental asymmetry.
- The global universe remains stationary and unitary.

These results have been confirmed on IBM Quantum hardware (ibm_torino, 133 superconducting qubits): a 3-qubit Trotterised circuit reproduces the entropy growth predicted by the unified formula, with $S_{eff} = 0.583 \pm 0.005$ across three independent runs — 102.2% of the exact theoretical prediction. The slight over-estimation is physically expected: real QPU noise adds decoherence on top of entanglement-based entropy, *enhancing* rather than suppressing the arrow.

At this point, no ontological claims are required.

But they are now unavoidable.

The next chapter addresses the question that can no longer be postponed:

> *If the universe does not require time, why do observers experience it?*

That question leads directly to the observer as a **local breakdown of atemporality**.

Companion Notes — Chapter 6

The Observer as a Local Breakdown of Atemporality

(Pedagogical companion to "The Observer as a Local Breakdown of Atemporality")

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6. The Observer as a Local Breakdown of Atemporality

Up to this point, we have said nothing about observers beyond their operational role in conditioning and partial access. We have deliberately postponed any ontological interpretation.

After Chapter 5, this postponement is no longer possible.

The simulations demonstrate three facts simultaneously:

1. The global quantum description can remain stationary and unitary.
2. Effective time evolution emerges only inside conditioned descriptions.
3. An arrow of time emerges only when access to correlations is restricted.

The only remaining question is **where this restriction comes from**.

The answer is not abstract. It is physical.

6.1 The Universe Does Not Require Time

In the global description assumed throughout this work, the universe is represented by a state $|\Psi\rangle$ subject to a constraint

$$\hat{\mathscr{C}}\,|\Psi\rangle = 0\,.$$

Nothing in this equation refers to time, history, or experience.

This is not a deficiency of the description. It is a feature.

From this perspective:

- the universe does not evolve,
- nothing globally “happens”,
- no preferred ordering of events exists.

Yet structure, interaction, and correlation exist fully.

Time is not missing. It is simply not needed.

6.2 Why Observers Cannot Access the Global State

If the global state contains all correlations, why does any subsystem experience time at all?

Because no physical subsystem can:

- access all degrees of freedom,
- reverse all entanglement,
- store all correlations without loss.

An observer is therefore characterized not by consciousness, but by **physical limitation**.

This limitation is not accidental. It is unavoidable.

Any subsystem capable of recording information must:

- interact locally,
- decohere relative to other degrees of freedom,
- discard correlations into an environment.

This is where atemporality breaks.

6.3 Conditioning Creates the Temporal Bubble

The act of conditioning on a clock is not passive.

It does three things simultaneously:

1. It selects a subsystem as a reference.
2. It discards complementary correlations.
3. It induces an ordering of conditioned states.

This combination defines what we call a **temporal bubble**.

Inside this bubble:

- states are ordered,
- memory can persist,
- entropy can increase.

Outside it, none of these concepts apply.

Time does not flow into the observer.

The observer generates time by conditioning.

6.4 The Arrow Is Not a Property of the Universe

The simulations make this unambiguous:

- With full access, no arrow appears.
- With partial access, an arrow appears.

The arrow is therefore not a property of:

- the Hamiltonian,
- the global state,
- the laws of motion.

It is a property of **restricted descriptions**.

This explains why:

- microscopic laws are reversible,
- macroscopic experience is irreversible,
- no contradiction arises.

The contradiction was conceptual, not physical.

6.5 The Observer as an Anomaly, Not a Privilege

It is tempting to interpret the observer as central.

This framework suggests the opposite.

The observer is an **anomaly**:

- a local configuration where atemporality fails operationally,
- a subsystem forced to compress information,
- a structure that cannot avoid entropy production.

Stars do not experience time.

Galaxies do not remember their past.

Atoms do not anticipate their future.

Observers do—because they cannot do otherwise.

6.6 Why This Is Not Anthropocentrism

Nothing in this argument depends on humans.

Any physical system that:

- partitions degrees of freedom,

- uses a clock-like reference,
- stores records,
- discards correlations,

will generate:

- relational time,
- an informational arrow.

The observer is not special.

It is simply constrained.

6.7 What Changes — and What Does Not

What changes:

- time becomes relational,
- the arrow becomes informational,
- irreversibility becomes local.

What does not change:

- quantum mechanics remains unitary,
- no new dynamics are introduced,
- no preferred frame appears.

Nothing is added to physics.

Only a misinterpretation is removed.

6.8 Summary of the Chapter

The observer is not the center of the universe.

The observer is where the universe **cannot remain atemporal**.

Time and its arrow are not fundamental structures.

They are the inevitable consequence of being unable to access everything.

With this, the conceptual arc is complete.

The remaining task is not to add interpretation—but to explore consequences.

Companion Notes — Chapter 7

Consequences, Open Questions, and What Comes Next

(Pedagogical companion to "The Observer as a Local Breakdown of Atemporality")

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7. Consequences and Open Questions

With the conceptual arc now complete, this chapter addresses a different task. We no longer ask *how* time and its arrow emerge. That question has been answered operationally.

We now ask:

- What changes once we accept this framework?
- What familiar problems dissolve?
- Which questions remain genuinely open?

This chapter is not speculative. It maps consequences that follow directly from the previous chapters.

7.1 Consequence I — The Problem of Time Is Reframed, Not Solved

In this framework, the traditional "problem of time" does not receive a single unifying solution.

Instead, it is **reframed**.

- There is no conflict between a timeless global description and experienced dynamics.
- There is no need to quantize time or promote it to an operator.
- There is no need to introduce preferred foliations or external parameters.

The apparent contradiction arose from asking a global question ("Does time exist?") and expecting a local answer.

Once this mismatch is removed, the problem dissolves into a hierarchy of well-defined operational questions.

7.2 Consequence II — Relativity and Quantum Mechanics Are No Longer in Tension Over Time

Relativity already teaches us that:

- simultaneity is observer-dependent,
- no global present exists.

Quantum mechanics, in this framework, simply completes the picture:

- time itself becomes reference-frame dependent,
- different clocks generate different internal temporal descriptions.

This alignment removes a long-standing conceptual tension.

Relativity denies a universal "now".

Relational quantum time explains why no such "now" is ever observed.

7.3 Consequence III — The Arrow of Time Loses Its Mystery

The arrow of time has traditionally required special explanations:

- low-entropy initial conditions,
- cosmological boundary conditions,
- fundamental time asymmetry.

In this framework:

- no special initial state is required,
- no cosmological fine-tuning is invoked,
- no violation of unitarity occurs.

The arrow is a bookkeeping effect.

It tracks information that becomes inaccessible to an observer.

The mystery was never dynamical. It was epistemic.

7.4 Consequence IV — Measurement Loses Its Exceptional Status

Quantum measurement has often been treated as a special process.

Here, measurement becomes a particular case of:

- conditioning on a subsystem,
- discarding correlations into an environment.

The same mechanism that generates time and its arrow also explains why measurement appears irreversible.

No additional postulates are required.

Measurement is not an anomaly.

It is an instance of the same informational asymmetry.

7.5 Open Question I — Multi-Observer Consistency

If time is relational, a natural question arises:

> How do different observers agree on shared events?

This framework suggests:

- observers condition on different clocks,
- generate different internal times,
- yet interact and exchange records.

Consistency must therefore emerge from correlations between observers, not from a shared background time.

Explicit multi-clock simulations remain an important direction for future work.

7.6 Open Question II — Gravity and Spacetime Geometry

While the framework is compatible with relational ideas in quantum gravity, several issues remain open:

- How does spacetime geometry emerge from conditioned quantum states?
- Can gravitational redshift be recovered from clock–clock correlations?
- How does this framework interact with constraint quantization in general relativity?

Recent work suggests promising directions, but a complete synthesis remains unfinished.

Importantly, three gravitational robustness tests (see `generate_gravity_robustness.py`) demonstrate that the arrow of time is not an artefact of idealised assumptions:

- **Clock backreaction** (ε up to 1.0, i.e. 10× the S–E coupling): the arrow degrades but persists (strength 0.290).
- **Fuzzy subsystem boundaries** (partial SWAP between S and E_1): the arrow survives even a full swap (strength 0.882).
- **Clock uncertainty** (Gaussian smearing σ up to 4.0): the arrow is essentially immune (strength 0.997).

These results indicate that the mechanism does not require a fixed tensor-product structure or an ideal clock — precisely the conditions that would fail in a gravitational setting.

7.7 Open Question III — Complexity, Life, and Cognition

Nothing in this framework depends on biological observers.

However, it naturally raises questions about:

- why complex systems are particularly effective at generating temporal bubbles,
- how memory and prediction amplify the arrow,
- whether life exploits, rather than creates, temporal asymmetry.

These questions lie at the boundary between physics, information theory, and complex systems.

7.8 What This Framework Does *Not* Claim

Clarity requires stating limits explicitly.

This framework does *not* claim that:

- time is an illusion in everyday experience,
- consciousness collapses the wavefunction,
- physics requires new fundamental laws.

It claims something more restrained:

> Time and its arrow are emergent, local, and informational.

7.9 Summary of the Chapter

Accepting this framework leads to a coherent picture:

- the universe does not evolve globally,
- observers generate time locally,
- irreversibility reflects limited access,
- long-standing paradoxes lose their force.

What remains is not mystery, but work.

And now, the right questions can finally be asked.

Companion Notes — Chapter 8

Three Pillars, One Formula

8. The Unified Reading

Throughout these notes we have treated three aspects of the problem of time as separate threads:

- **Quantum dynamics** — how the Schrödinger equation emerges from conditioning on a clock (Chapters 2–3).

- **The thermodynamic arrow** — how irreversibility emerges from tracing out inaccessible degrees of freedom (Chapters 4–5).
- **Observer-dependent time** — how different clock choices yield different temporal descriptions, without requiring a global "now" (Chapter 6).

It would be natural to see these as three separate results that happen to coexist in the same framework. But the central finding of the technical paper is stronger: they are not three results. They are three readings of a single mathematical expression.

8.1 The Formula

The conditional reduced state of the system, given a clock reading t, is:

$$\rho_S(t) = \frac{\mathrm{Tr}_E\Big[\big\langle t\big|_C \ |\Psi\big\rangle\big\langle\Psi| \ |t\big\rangle_C\Big]}{p(t)}$$

This is the core equation of the entire framework. Every claim made in these notes reduces to an operation on this object.

8.2 Three Operations, Three Pillars

The formula contains exactly three operations, and each one generates one of the three pillars:

1. Projection: ⟨tl_C

Projecting the global state onto the clock reading t extracts a conditional state that depends on t. If we do this for successive values of t, we obtain a sequence of states that obeys an effective Schrödinger equation (in the good-clock limit). The projection is the sole source of dynamics. Without a clock to condition on, there is no temporal ordering to speak of.

This is Pillar 1: quantum mechanics emerges from conditioning.

2. Partial trace: Tr_E

Tracing out the environmental degrees of freedom that the observer cannot access produces a mixed state ρ_S(t) even though the global state IΨ⟩ is pure. The effective entropy S_eff(t) = −Tr[ρ_S(t) ln ρ_S(t)] grows with t because the observer progressively loses correlations to the environment. No non-unitary dynamics is required. No special initial conditions are invoked.

This is Pillar 2: the thermodynamic arrow emerges from the trace.

3. Locality of C

The clock C is a physical subsystem, not an absolute parameter. A different observer choosing a different clock C' would obtain a different $\rho_S(t')$, a different dynamical description, and potentially a different arrow. There is no contradiction because there is no global time to disagree about. Consistency between descriptions is expressed as transformations between relational clock choices — the temporal quantum reference frame program of Höhn, Smith, and Lock.

This is Pillar 3: observer-dependent time emerges from the locality of the clock.

8.3 Why This Unification Matters

Each of these three pillars has been studied individually in the literature:

- Page and Wootters (1983) for conditioning and emergent dynamics.
- Shaari (2026) for the informational arrow via partial trace.
- Höhn, Smith, and Lock (2021) for temporal quantum reference frames.

What had not been made explicit is that all three are already contained in a single expression applied to a single timeless object $|\Psi\rangle$. The three pillars of the problem of time — which have historically been treated as separate puzzles requiring separate solutions — turn out to be three facets of one conditional operation.

This is not a mathematical coincidence. It reflects a deep structural fact: if time is relational, then dynamics, irreversibility, and frame dependence must all arise from the same act of “looking at part of the universe from inside the universe.” The formula makes this precise.

8.4 A Summary for the Reader

Ingredient in the formula	What it produces	Which pillar
⟨t|_C (projection onto clock)	Temporal ordering, Schrödinger equation	Quantum mechanics
Tr_E (partial trace over environment)	Entropy growth, irreversibility	Thermodynamics
C is local (not global)	Observer-dependent time, no absolute "now"	Relativity (structural)
$\|\Psi\rangle$ with $\hat{C}\|\Psi\rangle = 0$	Atemporal base	Common ground

The unified formula does not derive all of physics. It does not prove Lorentz invariance, and it does not solve quantum gravity. But it establishes that the three most persistent tensions in the problem of time were never three separate problems. They were always three aspects of what it means to be a finite observer inside a timeless whole.

All three pillars have been validated numerically (`validate_formula.py`), their robustness tested against six physically motivated perturbations (`generate_gravity_robustness.py`, `generate_structural_robustness.py`), and the entropy growth mechanism confirmed experimentally on IBM Quantum hardware (`IBMquantum/run_ibm_enhanced.py`). The theory is not merely self-consistent — it is empirically testable and has passed its first hardware test.

Three pillars. One formula. One timeless state.

Closing Note

Where Time Ends — and Where It Begins

This companion was not written to solve the problem of time.

It was written to place the problem where it belongs.

Throughout these chapters, we have followed a strict discipline: nothing was introduced that could not be implemented, simulated, or operationally defined. Every conceptual step was anchored to a physical distinction—between conditioning and evolution, between access and inaccessibility, between global description and local experience.

The result is not a new theory of time, but a clarification of its status.

Time does not govern the universe.

The universe does not wait, advance, or remember. At the level of its most general description, it is consistent, correlated, and atemporal. Nothing is missing from that picture.

What is missing is the observer.

Not as a conscious subject, but as a physical configuration that cannot access everything it interacts with. Once such a configuration exists, three things become unavoidable — and, as Chapter 8 showed, they are three readings of a single formula:
1. Conditioning on internal reference systems (clocks) — projection $\langle t|_C$ yields dynamics 2. Loss of correlations into inaccessible degrees of freedom — partial trace Tr_E yields the arrow 3. The emergence of ordered records and irreversible descriptions — locality of C yields observer-dependent time

Time and its arrow appear precisely there—and nowhere else.

This reframes a long-standing confusion. The question was never whether time is fundamental or illusory. The question was why a fundamentally atemporal universe contains subsystems that experience temporal order.

The answer is now clear: because no subsystem capable of recording information can remain globally correlated.

The observer is not privileged.

It is constrained.

In this sense, the observer is not the center of the universe, nor its interpreter. The observer is a local breakdown of atemporality—a region where global consistency can no longer be maintained without compression, loss, and ordering.

Nothing in this framework diminishes lived experience. It explains why experience has the structure it does.

Time does not flow into us.

We generate it by the way we exist.

Reproducibility

All numerical simulations, output figures, and data tables referenced in these companion notes can be reproduced using the open-source code available at:

https://github.com/gabgiani/paw-toymodel

Scripts: * `validate_formula.py` — Three-pillar validation (Versions A and B) * `generate_pillar3_plot.py` — Version C: two-clock comparison (Pillar 3) * `generate_continuum_limit.py` — N→∞ convergence, inter-clock covariance, and emergent Aff(ℝ) group structure * `run_all.py` — Full pipeline * `run_essay_validation.py` — Essay-specific validations * `generate_gravity_robustness.py` — Three gravitational robustness tests * `generate_structural_robustness.py` — Poincaré recurrences, initial-state sensitivity, partition independence * `IBMquantum/run_ibm_validation.py` — IBM Quantum hardware validation (Pillar 2, single run) * `IBMquantum/run_ibm_enhanced.py` — Enhanced hardware validation with error bars, noise characterisation, and Pillar 1

Experimental Validation on IBM Quantum Hardware

Beyond numerical simulation, the entropy growth mechanism (Pillar 2) and pure dynamics (Pillar 1) have been experimentally confirmed on IBM's ibm_torino quantum processor (133 superconducting qubits). Using a 3-qubit Trotterised circuit with zero Trotter error (all Hamiltonian terms commute), three independent hardware runs yield S_eff = 0.583 ± 0.005 at step 20 — 102.2% of the exact theoretical value. The slight over-estimation is physically expected: real QPU noise (gate errors, readout errors, decoherence) adds decoherence on top of the model's entanglement-based entropy, enhancing rather than suppressing the arrow. Device noise was fully characterised: $T_1 \approx$ 148 μs, $T_2 \approx$ 162 μs, two-qubit gate error 0.25% (median), readout error 4.49%. This constitutes the first experimental confirmation on physical quantum hardware that the informational arrow of time — as derived from the unified relational formula — survives real-world noise.

Output tables (CSV) and publication-quality figures (PNG) are generated in the `output/` directory.