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Modeling Evapotranspiration using Bootstrap based Quadratic Synaptic Neural Networks

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Abstract: This study evaluated the combined bootstrap resampling and quadratic synaptic neural (QSN) network models for estimating daily grass reference crop evapotranspiration (ET_o). Two different artificial neural network (ANN) models, QSN and bootstrap based QSN (BQSN), were developed for estimating ET_o. Two-input combinations were tried for carrying out this study. The ensemble ET_o was estimated by averaging the output of 200 individual BQSN models. A Mann-Whitney U test was performed to compare the observed and ensemble of bootstrap resamplings for the training data of the ET_o. The uncertainty associated with ET_o estimation using BQSN model was evaluated. Results indicated that BQSN models could improve confidence in ET_o estimation and that ensemble ET_o using BQSN models is more stable and reliable than that using QSN models. The conventional multiple regression (MLR) models was applied for the test period and compared with the optimal QSN and BQSN models. It was found that BQSN models performed better than optimal QSN and MLR models.

Keywords: *Neural networks, Evapotranspiration, Bootstrap, Quadratic, Uncertainty*

1. INTRODUCTION

Accurate and reliable estimation of daily grass reference crop evapotranspiration (ET_o) in irrigated lands is vital to improve the planning and efficient use of water resources. For example, one mm loss of water through ET_o across one ha land/area is equivalent to 10 m^3 (10,000 liters) of water (Allen *et al.* 1998). Thus, if one may overestimate the ET_o value by one mm, the farmer will have to pay for 10,000 liters of water unnecessarily to one ha field which ultimately lead to increase in cost of cultivation and depletion of water resources. Further, most of hydrological simulation models need ET_o as one of input data to predict or simulate other hydrological variables. The amount of ET_o can be determined through direct or indirect methods. Direct methods include measuring ET_o using eddy covariance, lysimeters, and scintillometers are limited to some primary experiments due to cumbersome, high resolution, and expensive construction, and these require well trained personal to install, check, and monitor the equipment. Indirect methods employ relationships between ET_o amount and weather variables are having drawbacks of local calibration, complexity and non-linearity.

Due to their ease of application and simple architecture, artificial neural network (ANN) models have gained popularity in the last few decades for ET_o modeling (Adamala *et al.* 2014a & b). Quadratic synaptic neural (QSN) network is a type of ANN, which extracts second-order correlations between the inputs and the synaptic weight vectors by employing a quadratic synaptic operation (Giles and Maxwell, 1987; Gupta *et al.* 2003).In a developing country like India with higher spatial variation in climate, the required climatic data for ET_o estimation may be extremely difficult to obtain at all locations due to the difficulty in observation. The most readily available data for India may be the maximum air temperature (T_{max}) and minimum air temperature (T_{min}). This shows the need of developing QSN models with limited input data i.e. temperature based Hargreaves method (Hargreaves and Samani, 1985).

One of the limitation with the ET_o based QSN models is the uncertainity in measured lysimeter and climatic data. To quantify the uncertainty in the computation of QSN models, the bootstrap resampling method is employed (Zio, 2006). The combining strength of QSN models with the bootstrap technique, constitutes the bootstrap quadratic synaptic neural (BQSN) network models. The bootstrap resampling method is a computational procedure that uses intensive resampling with replacement to reduce uncertainty (Efron and Tibshirani, 1993). In addition, it is the simplest approach since it does not require complex computations (Dybowski and Roberts, 2001).The bootstrap resampling method has been successfully used in hydrological modeling (Abrahart, 2003; Han *et al.* 2007; Jeong and Kim, 2005; Jia and Culver, 2006; Maier and Dandy, 2000; Seo *et al.* 2013; Sharma and Tiwari, 2009; Srivastav *et al.* 2007; Tiwari and Chatterjee, 2010a&b) and is a topic of current research. However, attempts using BQSN models for estimating ET_o have been not yet done. Therefore, the objective of this study is to develop BQSN models and compare the performance of BQSN models with the QSN and multiple linear regression (MLR) models.

2. MATERIALS AND METHODS

2.1 Bootstrap Resampling Method

The bootstrap resampling method is based on resampling with replacement of the available data and training an individual model using each resampling instead of the original data. The bootstrap resampling method can control the training data to generate different outputs and obtain an aggregated predictor using different outputs (Efron and Tibshirani, 1993). This has been found to increase the accuracy of neural network models. The bootstrap resampling method assumes that the training data are a representation of the population, and multiple resamplings of the population can be simulated from a single data source. This is done by repeated resampling of the original data with n size to obtain B bootstrap resamplings. Each bootstrap resampling contains a different data, which results in B neural network models. It is assumed that the data consist of a random resampling $T_n = \{(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)\}$ of n size, where $t_i = (x_i, y_i)$ consists of a predictor variable x_i and the corresponding output variable y_i. The set of B bootstrap resamplings can be represented as $T^1, T^2, \ldots, T^b, \ldots, T^B$, where B is the total number of bootstrap resamplings and usually ranges from 50 to 200 (Efron and Tibshirani, 1993).

The output of BQSN models can be represented as $f_{QSNs}(x, w_b/T^b)$, where x is the specific input, T^b the bootstrap resampling, and w_b the connection weight. For new input x, the estimates, $\hat{f}_{\text{bootstarp}}(x)$, using BQSN models can be calculated by the average of the individual estimates using B bootstrap resamplings as:

$$\hat{f}_{\text{bootstarp}}(x) = \frac{1}{B}\sum_{b=1}^{B} f_{QSNs}\left(x, w_b \middle| T^b\right) \tag{1}$$

In repeated application of the bootstrap resampling method, the confidence interval (CI) at the $\alpha\%$ significance level covering the range of BQSN models estimation can be written as[6]:

$$CI = \hat{f}_{\text{bootstarp}}(x) \pm t_{n-p}^{\alpha/2}\sigma(x) \tag{2}$$

Where $\sigma(x)$ = standard deviation for estimates of B bootstrap resampling; $t_{n-p}^{\alpha/2} = \alpha/2$ percentile for Student's t distribution with *n-p* degree of freedom; n = total number of ET_o values considered; and p = total number of parameters. A typical value of α is 0.05, which corresponds to $(1-0.05)\times 100\% = 95\%$ confidence limits. The ensemble ET_o means the average ET_o values aggregated from individual BQSN outputs.

The methodology adopted to develop bootstrap based models is shown in Fig. 1.

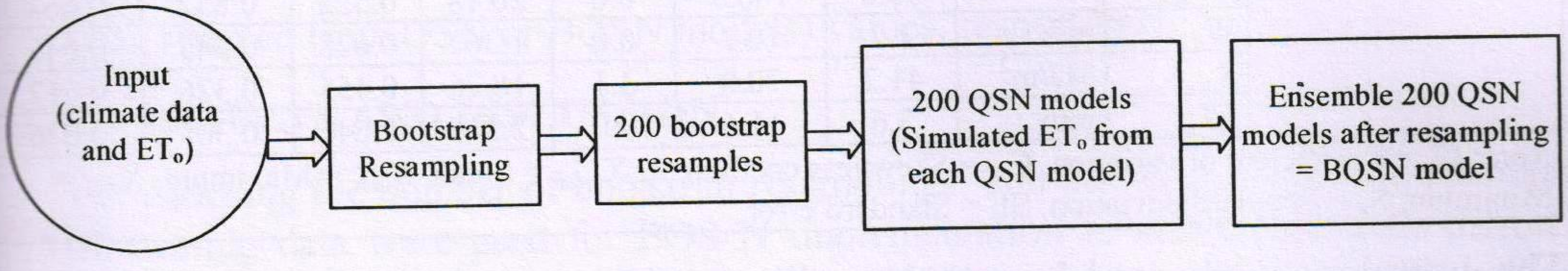

Fig. 1: Methodology for Developing BQSN Models

2.2 Data Used

For the purpose of this study, daily climatic data of minimum and maximum temperature (T_{min} and T_{max}), minimum and maximum relative humidity (RH_{min} and RH_{max}), wind speed (W_s), and solar radiation (S_{ra}) for Davis California Irrigation Management Information System station were collected for the period of January 1, 2000 to December 31, 2014(Set 1). Daily ET_o values were estimated using the FAO-56 PM method because the lysimeter measured ET_o values were not available for the period. The FAO-56 PM estimated ET_o values were considered as standard and used for training and testing of different architectures of QSN and BQSN models. The FAO-56 PM method is considered as standard because it ranked first for both humid and arid regions (Jensen *et al.* 1990).To compare the QSN predicted ET_o with the FAO-56 PM estimated ET_o, daily lysimeter measured grass evapotranspiration along with climatic data (T_{min}, T_{max}, RH_{min}, RH_{max}, W_s and S_{ra}) from January 1, 1960 to December 31, 1963 (Set 2) were collected for Davis (Kumar *et al.* 2009). Table 1 shows a summary of inputs for QSN and BQSN models along with the detailed models. Table 2 shows a summary of statistical indices of daily data used.

Table 1: Inputs for QSN and BQSN Models

Input	Target	Corresponding QSN Models	Corresponding BQSN Models	Set
T_{max}, T_{min}, RH_{max}, RH_{min}, W_S, S_{ra}	Lysimeter ET_o	$QSN1_L$	$BQSN1_L$	Set 2
T_{max}, T_{min}, RH_{max}, RH_{min}, W_S, S_{ra}	FAO-56 PM ET_o	$QSN1_F$	$BQSN1_F$	Set 1
T_{max}, T_{min}, R_a	Lysimeter ET_o	$QSN2_L$	$BQSN2_L$	Set 2
T_{max}, T_{min}, R_a	FAO-56 PM ET_o	$QSN2_F$	$BQSN2_F$	Set 1
Note: $_L$ Indicates Lysimeter ET_o as Target Data; $_F$ Indicates FAO-56 PM ET_o as Target Data				

Table 2: Daily Statistical Parameters of Davis Site Data

Davis Station	Data	Unit	X_{mean}	X_{max}	X_{min}	S_x	C_v	C_{sx}	SE
Set I (2000-14)	T_{max}	°C	23.25	43.6	2.60	8.34	0.359	0.0183	0.113
	T_{min}	°C	8.27	21.8	-6.9	4.87	0.589	-0.328	0.065
	RH_{max}	%	84.35	100	19	12.05	0.142	-0.659	0.162
	RH_{min}	%	39.91	100	7	20.49	0.513	0.719	0.277
	W_s	m/s	2.534	10.2	0.4	1.188	0.469	0.592	0.016
	S_{ra}	MJ/m^2	17.70	39.225	0	8.804	0.497	-0.149	0.119
	ET_o	mm	3.91	12.62	0.34	2.296	0.587	0.168	0.031
Set II (1960-63)	T_{max}	°C	22.6	43.3	-0.6	8.612	0.381	-0.071	0.283
	T_{min}	°C	7.1	23.3	-17.8	5.196	0.733	-0.399	0.171
	RH_{max}	%	92.9	100.0	18.0	13.62	0.147	-1.566	0.448
	RH_{min}	%	38.6	100.0	0.0	20.18	0.522	0.832	0.664
	W_s	m/s	2.2	10.2	0.0	1.361	0.628	0.545	0.045
	S_{ra}	MJ/m^2	41.2	70.0	3.1	18.76	0.455	0.326	0.617
	ET_o	mm	3.6	11.6	0.0	2.316	0.646	0.301	0.076
Note: C_v = Coefficient of variation, C_{sx} = Skewness coefficient, X_{mean} = Mean, X_{max} = Maximum, X_{min} = Minimum, S_x = Standard deviation, SE = Standard error									

The neural network models use the split sample approach to examine the model generalization. To apply the split sample approach, the data used are split into training and testing sets, respectively. The training set is used to train the model to estimate the model parameters; the testing data are used to test the generalization capability of the model. In all of these applications, the first70% of data was applied for training and the last 30%of data for testing.

2.3 Performance Measures

The performance evaluation of all the developed models is carried out for both the training and testing periods in order to examine their effectiveness in estimating ET_o. The performance indices used for evaluating the models are; root mean squared error (RMSE), mean absolute error (MAE), coefficient of determination (R^2), ratio of average output to the average target ET_o values (R_{ratio}), and Nash-Sutcliffe efficiency (NSE, %). A description of the aforementioned indices is provided below.

$$MSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(T_i - O_i)^2} \tag{3}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|T_i - O_i\right| \tag{4}$$

$$R^2 = \frac{\left[\sum_{i=1}^{n}\left(O_i - \bar{O}\right)\left(T_i - \bar{T}\right)\right]^2}{\sum_{i=1}^{n}\left(O_i - \bar{O}\right)^2 \sum_{i=1}^{n}\left(T_i - \bar{T}\right)^2} \tag{5}$$

$$R_{ratio} = \frac{\bar{O}}{\bar{T}} \tag{6}$$

$$NSE = \left[1 - \frac{\sum_{i=1}^{n}(T_i - O_i)^2}{\sum_{i=1}^{n}\left(T_i - \bar{T}\right)^2}\right] * 100 \tag{7}$$

Where T_i and O_i = target (FAO-56 PM ET_o or lysimeter ET_o) and output (ET_o resulted from QSN or BQSN models) values at the i^{th} step, respectively; n = number of data points, $\bar{T}$ and $\bar{O}$ = average of target (FAO-56 PM ET_o or lysimeter ET_o) and output (ET_o resulted from QSN or BQSN models) values, respectively.

3. RESULTS AND DISCUSSION

For choosing the number of bootstrap resamplings, 200 individual bootstrap resamplings of training data were used for BQS N implementation in accordance with (Efron and Tibshirani, 1993). Tiwari and Chatterjee (2010b) recommended that 25 individual bootstrap resamplings be added to the ensemble formation, at least to reduce the generalization errors. He suggested that the final output was generated by the average values from the aggregation of all the bootstrap resamplings. Tiwari and Chatterjee (2010a) developed the bootstrap based ANNs for hourly flood forecasting using50 individual bootstrap resamplings. Seo *et al.* (2013) applied bootstrap based ANNs for the uncertainty assessment and ensemble forecasting of water level using 50 individual bootstrap resamplings.

BQSN models were developed to evaluate the uncertainty associated with the ensemble ET_o. Individual bootstrap resampling was used to develop a BQSN, and many BQSN models were developed and then combined to approximate the relationship between input and output nodes. The confidence bands represent the uncertainty interval of ET_o estimation using BQSN that use a wide range of parameters instead of a single parameter. The confidence bands show the limits to which the ET_o estimation could have varied based on the parameters used. The final ET_o values are the average of these models. A Mann-Whitney U test, one of the homogeneity analyses, was performed to compare the observed and the ensemble of some bootstrap resamplings for the ET_o training data to evaluate the confidence level of the bootstrap resampling method. It is a nonparametric alternative to the two-sample t test for two independent samples and can be used to test whether two independent samples were taken from the same population (Tiwari and Chatterjee, 2010b).The critical value of the z statistic was computed for the level of significance (5%). If the computed value of the z statistic is greater than the critical value of the z statistic (1.96), the null hypothesis, that the two independent samples are from the same population, should be rejected and the alternative hypothesis accepted.

Table 3 shows the results of the Mann-Whitney U test between the observed and ensemble of some bootstrap resamplings (corresponding to 50, 100, 150, and 200 resamplings) for the ET_o training data. The critical value of the z statistic was computed as ($z_{0.05}$) 1.960 for the 5% level of significance. Since, the computed values of the z statistic for both sets were not significant, the null hypothesis, that the two independent samples are from the same population, was accepted for both stations. The primary sources of uncertainty are input data, model parameters, and the structure of neural network models. A few applications to quantify the uncertainty associated with the neural network models have focused on the Bayesian approach (Han *et al.* 2007). However, this approach is computationally expensive since it requires Monte Carlo solutions of the integrals. The bootstrap resampling method, however, is the simplest approach since it does not require complex computations of derivatives and Hessian-matrix or Monte Carlo solutions.

Table 3: Results of Mann-Whitney U Test

Data	No. of Resamplings	Mann-Whitney U test	
		Z Computes Statistic	Null Hypothesis
Set 1	50	-0.765	Accept
	100	-0.123	
	150	-0.981	
	200	-0.123	
Set 2	50	-0.065	
	100	-0.843	
	150	-0.145	
	200	-0.653	

Tables 4 and 5 summarizes the statistical indices (RMSE, MAE, R^2, R_{ratio}, and NSE) of QSN and BQSN models during the training and testing periods for the set 2 data. It is clear from Table 4 and 5that the statistical results of the BQSN models are generally better than those of the QSN models during the train and test periods for both the input combinations. It can also be seen from Tables 4 and 5 that the statistical results of QSN1 and BQSN1 models (with six climate variables) are better than those of QSN2 and BQSN2 (3 input variables) models for the set 2.

Table 4: Performance of QSN and BQSN Models during Training (70% of 1960–63)

Performance Indicator	Inputs (T_{max}, T_{min}, RH_{max}, RH_{min}, W_S, S_{ra})				Inputs (T_{max}, T_{min}, R_a)			
	$QSN1_L$	$BQSN1_L$	$QSN1_F$	$BQSN1_F$	$QSN2_L$	$BQSN2_L$	$QSN2_F$	$BQSN2_F$
RMSE	0.399	0.361	0.584	0.365	0.837	0.676	1.003	0.843
MAE	0.257	0.243	0.383	0.255	0.586	0.489	0.712	0.607
R^2	0.970	0.992	0.934	0.992	0.865	0.914	0.939	0.961
R_{ratio}	0.993	0.997	0.997	0.999	0.997	0.998	1.001	1.009
NSE	97.06	99.21	93.43	99.21	86.48	91.45	93.97	96.11

Table 5: Performance of QSN and BQSN Models during Testing (30% of 1960–63)

Performance Indicator	Inputs (T_{max}, T_{min}, RH_{max}, RH_{min}, W_S, S_{ra})				Inputs (T_{max}, T_{min}, R_a)			
	$QSN1_L$	$BQSN1_L$	$QSN1_F$	$BQSN1_F$	$QSN2_L$	$BQSN2_L$	$QSN2_F$	$BQSN2_F$
RMSE	0.510	0.428	0.545	0.445	0.834	0.654	0.911	0.857
MAE	0.381	0.295	0.422	0.300	0.565	0.484	0.651	0.611
R^2	0.962	0.986	0.955	0.990	0.884	0.927	0.952	0.960
R_{ratio}	0.979	0.997	0.995	1.006	1.001	0.992	1.003	0.997
NSE	96.14	98.39	95.43	98.97	0.834	0.654	0.911	0.857

Tables 6 and 7 summarizes the statistical indices (RMSE, MAE, R^2, R_{ratio}, and NSE) of optimal QSN and BQSN models during the training and testing periods for set 1. It is clear from Tables 6 and 7 that the statistical results of the BQSN models are generally better than those of the QSN models during the train and test periods for both the input combinations (set 1). It can also be seen from Tables 6 and 7 that the statistical results of QSN1 and BQSN1 models (with six climate variables) are better than those of QSN2 and BQSN2 (3 input variables) models for the set 2. It can be concluded that greater uncertainties, such as considerable changes in weather variables at the Davis weather station, result in a relatively small improvement in model performance using the bootstrap resampling method.

Table 6: Performance of QSN and BQSN Models during Training (70% of 2000–14)

Performance Indicator	Inputs (T_{max}, T_{min}, RH_{max}, RH_{min}, W_S, S_{ra})		Inputs (T_{max}, T_{min}, R_a)	
	$QSN1_F$	$BQSN1_F$	$QSN2_F$	$BQSN2_F$
RMSE	0.153	0.112	0.729	0.701
MAE	0.110	0.083	0.479	0.460
R^2	0.995	0.997	0.900	0.906
R_{ratio}	0.996	1.001	1.007	1.000
NSE	99.55	99.76	90.03	90.61

Table 7: Performance of QSN and BQSN Models during Testing (30% of 2000–14)

Performance Indicator	Inputs (T_{max}, T_{min}, RH_{max}, RH_{min}, W_S, S_{ra})		Inputs (T_{max}, T_{min}, R_a)	
	$QSN1_F$	$BQSN1_F$	$QSN2_F$	$BQSN2_F$
RMSE	0.214	0.135	0.843	0.729
MAE	0.152	0.101	0.554	0.461
R^2	0.992	0.996	0.860	0.900
R_{ratio}	0.993	1.004	1.000	0.998
NSE	99.10	99.64	86.09	90.08

It is clear from above Tables that the statistical results of the set 2 data (lysimeter measurements) are generally better than those of the set 1 data (FAO-56 PM estimated ET_o) during the train and test periods for QSN and BQSN models. In addition, conventional multiple regression (MLR) models were developed and compared with the QSN and BQSN models. Due to the worst performance of MLR models as compared to QSN and BQSN models, the results pertaining to them are not shown here.

4. CONCLUSIONS

This study evaluates ensemble ET_o using the bootstrap resampling method in neural network modeling, and the uncertainty associated with ET_o estimation is evaluated using BQSN implementation. Two different QSN and BQSN models, are developed for two sets of data set 1 (FAO-56 PM estimated ET_o) and set 2 (Lysimeter ET_o), respectively. Two-input combination models (QSN1or BQSN1 and QSN2 or BQSN2) are tried in this study to develop QSN and BQSN models for 2 sets of data. A Mann-Whitney U test is performed to compare the observed and the ensemble of some bootstrap resamplings (corresponding to 50, 100, 150, and 200 resamplings) for ET_o training data. The computed values of the z statistic for both sets are not significant for the training data. The null hypothesis that the two independent samples are from the same population is accepted for both sets. The statistical results of BQSN implementation are better than those of QSN models. The statistical results of the set 2 data are more improved than those of set 1 for BQSN and QSN models. In all of the cases studied, BQSN1 and QSN1 models (with 6 inputs) produced promising results for estimating ET_o values as compared to BQSN2 and QSN2 models (with 3 inputs).

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