Implementation of a Spring Reverberation Model in MATLAB

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ABSTRACT

This paper provides an overview and application of a spring reverberation modelling technique proposed by Välimäki, Parker et al [1]. The process of the model was conducted through observations and analysis of recorded impulse responses from two spring reverberation tanks – a Leem Pro KA-1210 and a Sansui RA-700. Based on characteristics presented in these responses, their spectrograms both displayed a similar basis, as follows: a sequence of decaying chirp-like pulses progressively blurred over time into a reverberant tail. In the digital signal processing, a modelling of the latter can thereby be implemented using techniques familiar from other forms of digital reverberation effects: a spectral delay filter, comprised of a cascade of identical all-pass filters, with a temporal smearing and attenuation of the signal, using a modulated multi-tap delay line in a feedback loop.

The paper will be organized as follows. First, the basic model for producing the initial low and high frequency chirps will be covered. Thereafter, the synthesis behind the temporal sequence of the chirp pulses, by the introduction of feedback delay structures. Finally, the interpolation of the various blocks that became the basis of a signal processing algorithm for a spring reverberation model in MATLAB.

The parameters, block diagrams, and impulses responses used in this project are all derived from the paper by Välimäki, Parker et al. [1]

Author Keywords

Spring Reverberation, Digital Signal Processing, Impulse Response, MATLAB

1. INTRODUCTION

First, proposed by Hammond in the 1930s, the spring reverberation was used as a method for adding an acoustic quality to the sound of his electromechanical organs [2]. Unlike from those produced by either real acoustic spaces or artificial reverberation algorithms, the synthetic sound produced by a spring reverberation presents a uniquely distinctive and rudimentary quality[1]. Due to its compact nature and low cost, this technique led to its wide adoption both within studio environments and integrated into amplification systems [1].

Although previous attempts at producing a digital model of spring reverberation have been made, the first of these attempts was based on digital-waveguide techniques in combination with the optimization of a high- order all-pass filter to fit measured data from a spring reverberation unit [3], [4]. Later work has concentrated on the discretization of a continuous model of spring vibration via finite difference techniques [5], [6].

2. MODELING OF CHIRPS

A. Low frequency chirps - Interpolated stretched all-pass filters, dc blocking high-pass filters, tenth-order elliptic IIR low-pass filter, & an equalization second order IIR filter

A spectral delay filter can produce a chirp similar in shape to a single spring response, and the stretching can be realized by replacing the unit delay in a first-order all-pass filter with a delay line of K samples. The following structure is thereby used for producing low frequency chirps, of stretching factor K, related to the transition frequency f_C of chirps by

$$K = \frac{f_s}{2 f_C}$$

where f_s is the sampling frequency ($44.1\,kHz$), and f_C the maximum frequency of the first sequence of pulses ($\approx 4,3kHz$). The result of the fraction can be a rational number; therefore, an interpolated delay line is applied in the stretched all-pass filters. The transfer function of the interpolated stretched all-pass filters then becomes

$$A_{low}(z) = \frac{a1 + \frac{a2 + z^{-1}}{1 + a2\, z^{-1}} z^{-K1}}{1 + a1 \frac{a2 + z^{-1}}{1 + a2\, z^{-1}} z^{-K1}}$$

where $K1 = round(K) - 1$ is one less than the integral of K, coefficient $a1$ affects the shape of the chirp and coefficient $a2$ determines the fractional delay

$$a2 = \frac{1 - d}{1 + d}$$

where $d = K - K1$ is one larger than the fractional (decimal) part of K. The range of delay parameter values is thus $0.5 \leq d \leq 1.5$.

The difference equation of the interpolated stretched all-pass filter is thereby calculated from the transfer function using an inverse Z-transform as follows:

$$A_{low}(z) = \frac{\frac{a1 + a1\, a2\, z^{-1} + a2\, z^{-K1} + z^{-1} z^{-K1}}{1 + a2\, z^{-1}}}{\frac{1 + a2\, z^{-1} + a1\, a2\, z^{-K1} + a1\, z^{-1} z^{-K1}}{1 + a2\, z^{-1}}}$$

$$\frac{Y(z)}{X(z)} = \frac{a1 + a1\,a2\,z^{-1} + a2\,z^{-K1} + z^{-1}z^{-K1}}{1 + a2\,z^{-1} + a1\,a2\,z^{-K1} + a1\,z^{-1}z^{-K1}}$$

$$\begin{aligned} y[n] = {} & a1\,x[n] + a1\,a2\,x[n-1] + \\ & a2\,x[n-K1] + x[n-1-K1] - \\ & a2\,y[n-1] - a1\,a2\,y[n-K1] - \\ & a1\,y[n-1-K1]; \end{aligned}$$

The cascade of M_{low}(with $M = 100$) identical interpolated stretched all-pass filters is realized by feeding the output of the first filter to the input of the second one, and so forth.

Moreover, the low frequency chirps can be suppressed by applying a low-pass filter at the output of the filter chain. Indeed, a tenth-order elliptic IIR low-pass filter with a cutoff frequency of 4750Hz, 1-dB passband ripple, and 60-dB stopband suppression would produce a good imitation of the analyzed chirps [1]. However, since the transfer function of this filter was not provided in the paper, the filter coefficients were thereby obtained in MATLAB [8] using the *ellip* () function (Cf. Elliptic_IIR_Lowpass_Filter.m):

$$\begin{aligned} y[n] = {} & b1\,x[n] + b2\,x[n-1] + b3\,x[n-2] + \\ & b4\,x[n-3] + b5\,x[n-4] + b6\,x[n-5] + \\ & b5\,x[n-6] + b4\,x[n-7] + b3\,x[n-8] + \\ & b2\,x[n-9] + b1x[n-10] - a1\,y[n-1] - \\ & a2\,y[n-2] - a3\,y[n-3] - a4\,y[n-4] - \\ & a5\,y[n-5] - a6\,y[n-6] - a7\,y[n-7] - \\ & a8\,y[n-8] - a9\,y[n-9] - a10\,y[n-10] \end{aligned}$$

The measured spring responses were observed to be lacking low frequencies near dc [1]. The chirp sequences also show an increased delay at very low frequencies, displayed as a smearing effect at the foot of each chirp [1]. To model this effect within the feedback delay structure of the low frequency chirps, a dc blocker is inserted in the feedback loop with the following transfer function:

$$Hdc(z) = \frac{1 + adc}{2}\,\frac{1 - z^{-1}}{1 - adc\,z^{-1}}$$

where $(1 + adc)/2$ is a scaling factor to ensure a maximum gain of 0dB, and the coefficient adc depends on the -3 dB frequency f_{cutoff}

$$adc = tan\left(\frac{\pi}{4} - \frac{\pi f_{cutoff}}{fs}\right)$$

The difference equation of the dc blocking high-pass filter is thereafter calculated from the $Hdc(z)$ transfer function using inverse Z-transform as follows:

$$\frac{Y(z)}{X(z)} = \frac{1 - z^{-1} + adc - adcz^{-1}}{2 - 2\,adcz^{-1}}$$

$$y[n] = \frac{1}{2}(x[n] - x[n-1] + adcx[n] - adcx[n-1]) + adcy[n-1]$$

Finally, for an equalization of the low frequency chirps that approximates the spectral shape of the first chirp, a second order IIR filter was designed by Välimäki, Parker et al. [1] with zeros at 0Hz and at the Nyquist limit. Just as the abovementioned spectral delay filter, it is stretched by replacing each unit delay by a sequence of Keq unit delays to obtain enough suppression around the transition frequency f_C .

$$Heq(z) = \frac{A0}{2}\,\frac{1 - z^{-2K_{eq}}}{1 + aeq1\,z^{-K_{eq}} + aeq2\,z^{-2K_{eq}}}$$

where $aeq1 = -2R\cos\theta$, $aeq2 = R^2$, the cos θ term sets the pole angle, R is the pole radius, and the scaling factor A0 that sets the peak gain to 0 dB is defined as $A0 = 1 - R^2$. R is calculated from:

$$R = 1 - \frac{\pi BKeq}{f_s}$$

where $B = 130\ Hz$. The cos θ term can be computed from the desired peak frequency f_{peak} in the following way:

$$\cos\theta = \frac{1 + R^2}{2R}\cos\left(\frac{2\pi f_{peak} K_{eq}}{f_s}\right)$$

where $f_{peak} = 95Hz$. The difference equation of the stretched second order IIR filter for equalization is calculated from the prementioned transfer function $Heq(z)$ using the inverse Z-transform as follows:

$$\frac{Y(z)}{X(z)} = \frac{A0 - A0\,z^{-2K_{eq}}}{2 + 2aeq1\,z^{-K_{eq}} + 2aeq2\,z^{-2K_{eq}}}$$

$$y[n] = \left(\frac{A0}{2}\right)\left(x[n] - x[n - 2K_{eq}]\right) - aeq1\,y[n - K_{eq}] - aeq2\,y[n - 2K_{eq}]$$

B. *High frequency chirps – Cascading first-order all-pass filters*

To model the high-frequency chirps above $4kHz$, we can also apply a spectral delay filter. The shape of this high frequency chirp sequence is simpler than that observed for the low-frequency chirps. The first frequencies to arrive are at the Nyquist limit and the last frequencies to arrive are toward dc. A cascade of first order all-pass filters can thereby be applied to approximate its shape, given the following transfer function:

$$A_{high}(z) = \left(\frac{ahigh + z^{-1}}{1 + ahigh\,z^{-1}}\right)$$

where $ahigh = -0.6$. A negative coefficient value is chosen due to the need for the filter to be dispersive over a relatively wide range of frequencies, with the maximum group delay occurring near dc. The difference equation calculated from the transfer is thereby:

$$y[n] = ahigh\,x[n] + x[n-1] - ahigh\,y[n-1];$$

The cascade of M_{high} of these filters are realized in similar fashion as in the low-frequency identical interpolated stretched all-pass filters, by feeding the output of the first filter to the input of the second one and so on, until M_{high} number of all-pass filters are interpolated.

3. SYNTHESIS OF CHIRP SEQUENCES

In this section two feedback structures were proposed by Välimäki, Parker et al [1] to produce the sought out decaying chirp sequences of a spring reverberation– a low frequency feedback delay structure *Clf* , and high frequency feedback delay structure *Chf* .

A. Low frequency feedback delay structure

A proposed feedback structure for modeling the chirp sequence below the transition frequency f_C by *Clf* , where *lf* refers to low frequency. As aforementioned, the low frequency chirps are comprised of a cascade of all-pass filters, a long delay, a loop gain coefficient, and a dc blocking filter $Hdc(z)$ in a feedback loop.

The delay line length can be calculated based on the delay time T_D of the spring unit and the group delay of the all-pass filter chain. The delay time T_D can be estimated from the spectrogram of the recorded impulse response as the period of pulses at low frequencies. The delay line *L* is then obtained by subtracting the contribution of the all-pass filter chain's group delay at dc from the delay time in samples:

$$L = T_D f_s - \tau(0)$$

Where the group delay of the interpolated stretched all-pass filter chain at dc is:

$$\tau(0) = KM\frac{1 - a1}{1 + a1}$$

where T_D is estimated to be 56ms or 2470 samples.

The delay line in the feedback path of the structure *Clf* can be supplied with additional output taps at a number of places along the delay line [1]. This can be thought of as splitting the delay line into several separate sections, with feedforward paths bypassing each. This is technique is only applied to the low-frequency chirp sequence model.

Varying the length of the delay line skipped by these feedforward paths allows to produce specific effects, such as the equivalent of a comb filter within the feedback path of the structure [1] – a process also known as the ripples filter. If we connect the length L_{ripple} to the value of *K* used for the stretched all-pass filter,

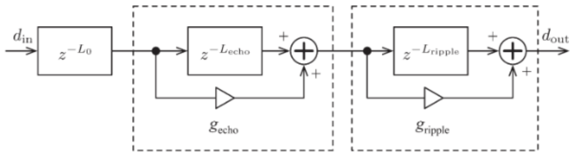
$$L_{ripple} = 2KN_{ripple}$$

The parameter N_{ripple} is gained, which allows to specify the number of ripples in the loop gain below the transition frequency f_C , and the ripple filter coefficient g_{ripple} produces a variety of different frequency-dependent reverberation time characteristics.

As observed in the spectrograms, faint "preechoes" can be seen preceding each of the main echoes below the transition frequency [1]. This feature can be replicated by introducing a feedforward path, which skips a relatively large part of the delay line. Measured spring responses show that this preecho arrives around 10ms before the main echo, when then main echoes are spaced by around 50ms [1]. A feedforward path that bypasses one-fifth of the delay line is thereby constructed where g_{echo} corresponds to the feedforward coefficient:

$$L_{echo} = L/5,\ L_{ripple} = 2KN_{ripple},\ N_{ripple} = 0.5,$$
$$g_{ripple} = 0.1,\ g_{echo} = 0.1,\ L0 = L - L_{echo} - L_{ripple}$$

with a difference equation of the delay line is calculated from the transfer function as followed:



$$H_{md}(z) = z^{-L0}\left(g_{echo} + z^{-L_{echo}}\right)\left(g_{ripple} + z^{-L_{ripple}}\right)$$

$$y[n] = g_{echo}g_{ripple}x[n - L0] + g_{echo}x[n - L_{ripple} - L0] + g_{ripple}x[n - L_{echo} - L0] + x[n - L_{echo} - L_{ripple} - L0]$$

As for the effect of diffusing and blurring each successive echo produced by the spring, the approach of modulating the length of the main delay line with a filtered noise signal should be considered. Indeed, the linear interpolation is used to produce fractional delay line lengths [7], which thereby produces gradual blurring and spreading of the individual chirps, in both time and frequency domain – thereby sounding reverberant.

The chosen white noise modulation signal was generated using the *rand* () function, which is normalized with a leaky integrator of the for

$$H_{int}(z) = \frac{1 - aint}{1 - aint\, z^{-1}}$$

where *aint* = 0.93. The difference equation of the normalized leaky integrator is calculated from transfer function $H_{int}(z)$ using an inverse Z-transform as follows:

$$\frac{Y(z)}{X(z)} = \frac{1 - aint}{1 - aint\, z^{-1}}$$

$$y_{noise}[n] = x_{noise}[n] - aint\, x_{noise}[n] + aint\, y_{noise}[n - 1]$$

which output signal is multiplied by the modulation gain g_{mod} and varying this modulation depth thereby varies the rate at which the system descends into its reverberant tail.

B. High frequency feedback delay structure

The high frequency chirps modeled by the *Chf* structure is present at a significantly lower level in the impulse response of a real spring and therefore are less perceptually important. Indeed, observations reveal that the high frequency chirps repeat at around three times the rate of low frequency chirps [1]. The delay line in the *Chf* structure is defined by the difference equation

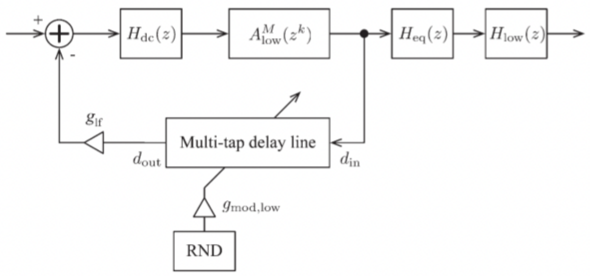
$$y[n]=x\left[n-L_{high}\right]$$

where $L_{high}=round(L/2.3)$ and the length of the delay line is modulated by the abovementioned filtered white noise – normalized leaky integrator.

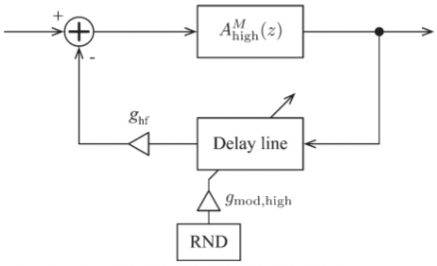
4. CONNECTING THE BLOCKS

The main features visible in the impulse response of a spring are modeled into two structures *Clf* and *Chf* :

Block diagram of Clf structure, where $g_{lf}=-0.8$

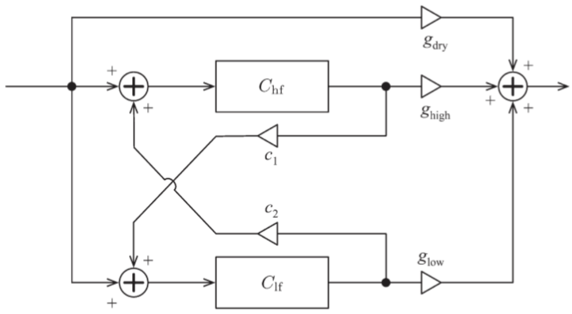


Block diagram of Chf structure, where $g_{hf}=-0.77$



These models must be combined to produce a complete spring model, by placing the two in parallel, and applying cross feedback paths between them, controlled by coupling coefficients c_1 = 0.1 (feedback loop gain from *Chf* to *Clf*) and c_2 = 0 (feedback loop gain from *Clf* to *Chf*)[1].

Block diagram of the spring reverberation model, where the gain of Clf structure $g_{low}=0.95$*, and the gain of Chf structure* $g_{high}=g_{low}/1000$*, and the gain of the dry input signal* $g_{dry}=1-g_{low}-g_{high}$



5. MATLAB IMPLEMENTATION

The model implemented through a signal processing algorithm in MATLAB [8] calculates the signal sample-by-sample (f_s =44.1kHz). The code can be seen in the attached ‘Spring_Reverberation_Effect_Model.m’ file in the submitted folder.

6. REFERENCES

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