Homework 2

Important: To submit your homework, please create your GitHub repository on the AU-R-Programming organization by Monday (Sep. 21st 2020) at 2pm and use this repository to work on this assignment. Your final submission will be done by the group leader via Canvas in html format (in which you will specify the name of the corresponding GitHub repository) and is due by Friday, Oct. 2nd 2020 at 11.59pm (no late work is accepted). The version submitted on Canvas will have to correspond to the last version on the GitHub repository.

To start, create a **private** GitHub repository for your group, and start its name with **HW2**. This project **must** be done using GitHub and respect the following requirements:

(1) All members of the group must `commit` at least once.

(2) All commit messages must be reasonably `clear` and `meaningful`.

(3) Your GitHub repository must include at least one `issue` containing some form of TO DO list.

When in doubt, show your code in the output.

Assignment

Suppose that you are working in an investment bank as a quantitative analyst. Your boss asks you to create a portfolio for one of your clients. The client wants to find the portfolio with the smallest variance that satisfies the following constraints:

- Considers only 5 companies of the S&P500.
- The investment C is exactly \$1,000,000.
- You are allowed to invest in a maximum of 3 stocks.
- If needed, it is possible to consider weights with negative values. This means that you are allowed to short stocks.

Portfolio Weight:
The weight of an asset in a portfolio is simply the percentage of an investment portfolio that is held in a single asset. For example, a basic way (but not necessarily optimal) to determine the weight of an asset is by dividing the dollar value of a security by the total dollar value of the portfolio.

Your boss wants you to

1. compute all possible portfolios that satisfy the client's constraints,
2. for each portfolio size (i.e. 1, 2 and 3 stocks) create a matrix with understandable row and column names in which, for each portfolio, you will store
    - weights (when considering more than one stock in a portfolio),
    - expected returns and
    - risk,
3. represent them graphically as (for example) in the graph below, and
4. provide the weights of the best (i.e. minimum variance) portfolio with expected return and risk (i.e. variance).

For the last task, in order to find the optimal weights we denote the vector of weights as $\mathbf{w} \in \mathrm{R}^p$, where p is the number of stocks to invest, and denote the variance-covariance matrix of stocks as $\mathbf{\Sigma}$. Then given the constraint that $\mathbf{w}^T\mathbf{1} = 1$, where $\mathbf{1}$ is the vector of 1 of length p, we can construct the objective function with Lagrange multiplier as $f(\mathbf{w}) = \mathbf{w}^T\mathbf{\Sigma}\mathbf{w} + \lambda(\mathbf{w}^T\mathbf{1} - 1)$ with some λ. The optimal weights are therefore given by

$$\mathbf{w}^* = \frac{\mathbf{\Sigma}^{-1}\mathbf{1}}{\mathbf{1}^T\mathbf{\Sigma}^{-1}\mathbf{1}}.$$

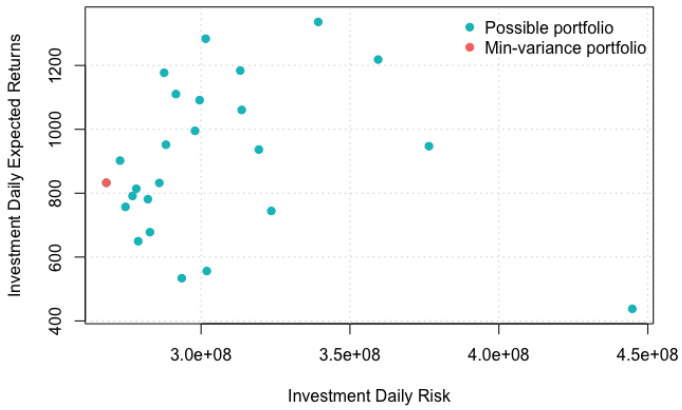
Based on these weights, the expected return and risk are respectively given by

$$\mu^* = (\mathbf{w}^{*T}\mu)\, C$$

and

$$\sigma^* = (\mathbf{w}^{*T}\mathbf{\Sigma}\mathbf{w}^*)\, C^2$$

where μ is the vector of expected (average) returns and C is the total investment.



To help complete this task your boss tells you to consider the data for the period from the 2017-04-01 to the 2020-04-01 and gives you this code to download the data you will need:

```
library(quantmod)
library(rvest)

# symbols of all S&P500 stocks
sp500 <- read_html("https://en.wikipedia.org/wiki/List_of_S%26P_500_companies")
sp500 %>%
html_nodes(".text") %>%
html_text() -> ticker_sp500

SP500_symbol <- ticker_sp500[(1:499)*2+1]
SP500_symbol[SP500_symbol == "BRK.B"] <- "BRK-B"
SP500_symbol[SP500_symbol == "BF.B"] <- "BF-B"

# example 5 companies of the S&P500
stocks_considered <- c("FB", "GOOGL", "AMZN", "AAPL", "MSFT")
```

He also mentioned that the function `getSymbols()` could be useful for this project and provides you with the example below:

```
# load data
library(quantmod)
three_year_ago <- seq(as.Date("2020-04-01"), length = 2, by = "-3 year")[2]
stocks_tickers <- c("AAPL", "MSFT")
getSymbols(stocks_tickers, from = three_year_ago, to = as.Date("2020-04-01"))
```

```
## [1] "AAPL" "MSFT"
```

```
# get covariance matrix of Apple and Microsoft returns
apple <- na.omit(ClCl(get(stocks_tickers[1])))
msft <- na.omit(ClCl(get(stocks_tickers[2])))
sigma_stocks <- var(cbind(apple, msft))
```

Required Assignment Outputs

1. Produce a short html file (not more than the equivalent of two pages) representing a brief financial report for the client (no code details) describing what was done and what your conclusions are.
2. Provide an html file for your boss which would allow them to replicate your analysis based on your explanations, codes and