



Iterated local search for single machine total weighted tardiness batch scheduling

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Abstract

This paper presents an iterated local search (ILS) algorithm for the single machine total weighted tardiness batch scheduling problem. To our knowledge, this is one of the first attempts to apply ILS to solve a batching scheduling problem. The proposed algorithm contains a local search procedure that explores five neighborhood structures, and we show how to efficiently implement them. Moreover, we compare the performance of our algorithm with dynamic programming-based implementations for the problem, including one from the literature and two other ones inspired in biased random-key genetic algorithms and ILS. We also demonstrate that finding the optimal batching for the problem given a fixed sequence of jobs is \mathcal{NP} -hard, and provide an exact pseudo-polynomial time dynamic programming algorithm for solving such problem. Extensive computational experiments were conducted on newly proposed benchmark instances, and the results indicate that our algorithm yields highly competitive results when compared to other strategies. Finally, it was also observed that the methods that rely on dynamic programming tend to be time-consuming, even for small size instances.

Keywords Batch scheduling · Total weighted tardiness · Iterated local search · Metaheuristics

1 Introduction

Let $J = \{1, 2, \dots, n\}$ be a set of jobs to be processed by a single machine.

Each job $j \in J$ has the following attributes: processing time p_j , size s_j , release date r_j (earliest time that job j can begin to be processed), due date d_j and a positive weight w_j . The single machine total weighted tardiness batch scheduling problem (TWTBSP) involves two significant decisions: partitioning the jobs into batches (each job must be in exactly one of the batches) and sequencing the batches. A batch-

Table 1 A small TWTBSP instance

Job	p_j	d_j	s_j	w_j	r_j
1	2	3	1	1	4
2	2	2	1	1	0
3	1	3	2	2	2
4	2	4	2	2	1
5	3	2	3	1	0
6	1	1	1	3	1
$Q = 4$					

processing machine (or batching machine) can process several jobs simultaneously as a batch, and the processing time of an each of them is equal to the longest processing time among all jobs in the corresponding batch. For a given sequence of m batches $\mathcal{B} = (B_1, B_2, \dots, B_m)$, the jobs in B_i are processed together and therefore such batch can only start to be processed at time $r(B_i) = \max_{j \in B_i} \{r_j\}$, and has a processing time $p(B_i) = \max_{j \in B_i} \{p_j\}$. In addition, $C(B_1) = r(B_1) + p(B_1)$ denotes the completion time of the first batch and $C(B_i) = \max\{r(B_i), C(B_{i-1})\} + p(B_i)$ the completion time of B_i , for $i = 2, 3, \dots, m$.

The machine has a capacity Q which must be satisfied, i.e., $s(B_i) = \sum_{j \in B_i} s_j \leq Q$ for each batch B_i . The tardiness of a job j is given by $T_j = \max\{0, C_j - d_j\}$, where C_j is completion time of job j (which is equal to the completion time of the batch that j belongs). The objective is to find a collection of batches over J and a sequence of these batches, which minimizes $\sum_{j \in J} w_j T_j$. We will hereafter denote the total weighted tardiness (TWT) of a solution \mathcal{B} as $TWT(\mathcal{B})$.

According to the 3-field notation widely used in the scheduling literature (Graham et al. 1979), TWTBSP can be denoted as $1|batch, r_j, s_j, compt|\sum w_j T_j$. The term *compt* indicates that any pair of jobs can be in the same batch, meaning that they are all compatible with each other, as opposed to similar versions of the problem that assume some incompatible job families (Perez et al. 2005; Tangudu and Kurz 2006; Erramilli and Mason 2008; Kurz and Mason 2008). The particular case of TWTBSP in which $r_j = 0$ and $s_j = 1, \forall j \in J$, was proven \mathcal{NP} -hard in the strong sense (Brucker et al. 1998), thus motivating the use of heuristic approaches for solving medium and large instances of the problem.

Table 1 presents an example containing a 6-job instance, whose optimal solution is illustrated in Fig. 1.

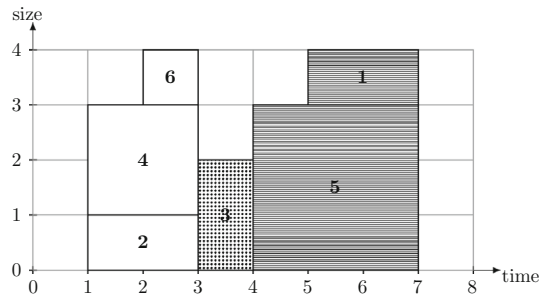
According to Potts and Kovalyov (2000), there are two main classes of batch scheduling: (i) the family scheduling models (Potts and Wassenhove 1992; Kress et al. 2018); and (ii) the batching machine models (Brucker et al. 1998; Pinedo 2016). In the first (a.k.a. serial-batching), jobs are grouped into families, and there are no setup costs when processing two consecutive jobs that belong to the same family. In this type of model, jobs from the same family are scheduled contiguously to avoid setup

Fig. 1 Graphical representation of a solution for the instance presented in Table 1. The sequence

$\mathcal{B} = (\{2, 4, 6\}, \{3\}, \{1, 5\})$

corresponds to the optimal solution with

$$TWT(\mathcal{B}) = \sum_{j \in J} w_j T_j = (w_2 T_2 + w_4 T_4 + w_6 T_6) + (w_3 T_3) + (w_1 T_1 + w_5 T_5) = 18$$



operations. In the second model, jobs are allowed to be scheduled simultaneously, which is the case of the problem considered in this paper.

Motivated by many applications in the manufacturing industry (Mathirajan and Sivakumar 2006; Mönch et al. 2011; Blazewicz et al. 2019), batch scheduling problems have been intensively studied over the past decades. In Lee et al. (1992), a batch scheduling application for semiconductor burn-in operations in device production (e.g., microprocessors and DRAM) was introduced. The purpose of burn-in operations is to subject the chips to thermal stress for an extended period of time in order to bring out latent defects known as infant mortality. A set of chips (jobs) of the same product is placed on one or more boards (the job size is given by the number of boards), and the boards are loaded into the oven. Hence, boards related to different products are grouped into a batch to be processed together in the same oven considering the batch capacity. The processing time of each product is the minimum testing time that it should be exposed to a high temperature. The processing time of a batch is equal to the longest time processing time among all products in the batch because the burn-in phase cannot be interrupted until the longest testing time is performed (other products may remain in the oven longer than necessary). Minimizing the total weighted tardiness

can significantly reduce the costs involved in the production process of circuits, due to the fact that it is a long-term test compared to other testing operations. The main benefit of using the TWT as the objective function is that, in many scenarios, the industry requires metrics focusing on due dates.

TWTBSP was approached by relatively few works in the literature. Chou and Wang (2008) presented a mathematical formulation and two hybrid heuristics using a dynamic programming (DP) procedure, one of them based on a genetic algorithm (GA), which in turn was the one that found the best results. Mathirajan et al. (2010) designed greedy constructive heuristics and a simulated annealing (SA) metaheuristic, which were tested on instances with up to 100 jobs. Wang (2011) proposed a two-phase heuristic called iterated heuristic (IH), where the first phase consists of a population-based reasoning (PBR) procedure which generates a set of job sequences, and the second one consists of forming and sequencing the batches for each sequence by means of the DP algorithm proposed by Chou and Wang (2008). In addition, the same author also employed an iterative approach to improve the solution obtained by the two-phase procedure. In this case, better solutions were obtained than those achieved by the two algorithms proposed in Chou and Wang (2008). Vélez-Gallego et al. (2011) put forward a variable neighborhood search (VNS) metaheuristic, which

found optimal solutions for instances with up to 15 jobs, whereas Zinouri et al. (2012) developed a greedy randomized adaptive search procedure (GRASP) for the problem. More recently, Kohn (2015) devised a VNS-based framework for the general batch scheduling problem which makes use of neighborhood structures involving operations over jobs (insertion and swap) and batches (insertion, swap, split and merge).

When considering TWT batch scheduling problems on a parallel machine environment, very few works addressed the problem taking into account compatible jobs (Hulett et al. 2017). In contrast, there are several references that addressed the problem with families of incompatible jobs, e.g., Balasubramanian et al. (2004), Chiang et al. (2010), Almeder and Mönch (2011), Mönch et al. (2005), Mönch and Roob (2018).

Furthermore, other single machine batch scheduling works considered different objectives, namely: completion time (Chang and Wang 2004), flow time (Zee 2007) and makespan (Li et al. 2005; Chou et al. 2006; Lu et al. 2010; Velez-Gallego et al. 2011; Xu et al. 2012; Zhou et al. 2014).

The main contributions of this paper are as follows:

- We propose a metaheuristic algorithm based on iterated local search (ILS) for solving TWTBSP. To the best of our knowledge, this is one of the first attempts to apply ILS (Lourenço et al. 2019) to solve a batch scheduling problem. In contrast, ILS-based procedures have been successfully implemented to solve other classes of scheduling problems (Grosso et al. 2004; Subramanian et al. 2014; Subramanian and Farias 2017; Lourenço et al. 2019). We thus attempt to fill this gap in the present work, as ILS appears to be a highly promising strategy to efficiently solve different families of scheduling problems.
- We show how to efficiently implement the local search procedures used in our algorithm by avoiding unnecessary operations.
- We demonstrate that finding the optimal batching for TWTBSP given a fixed sequence of jobs is \mathcal{NP} -hard, and provide an exact pseudo-polynomial time DP algorithm for solving such problem. Moreover, we also compare the performance of our algorithm with DP-based implementations for the problem, including the heuristic by Wang and Chou (2013) and two other ones inspired in biased random-key genetic algorithms (BRKGA) and ILS. The results show that the former achieves a far better performance when compared to the DP approaches. In addition, we tried to replace the non-exact DP procedure (Chou and Wang 2008) used in the existing heuristics (as a decoder for a job sequence) with the proposed exact DP approach, but the CPU time turned out to be prohibitively expensive, and the solutions obtained using the latter were similar to those found when utilizing the former.
- We introduce a new set of benchmark instances for TWTBSP, as those used in previous works are no longer available.

The remainder of this paper is organized as follows. In Sect. 2, we provide a comprehensive description of our ILS algorithm. Section 3 discusses the DP-based implementations. Section 4 contains the results obtained, whereas Section 5 presents the concluding remarks of this work.

2 Proposed metaheuristic algorithm

ILS is a metaheuristic that explores the space of local optima in order to find a global optimum (Lourenço et al. 2019). It works by applying perturbations to the current local optimum solution, followed by a local search over the perturbed solution. If the solution obtained after the local search satisfies some conditions, it is accepted as the new current solution. This process is repeated until a given stopping criterion is met. In this work, we use a randomized variable neighborhood descent (RVND) procedure in the local search phase of the ILS algorithm. The basic idea of the traditional variable neighborhood descent (VND) approach is to successively explore a set of neighborhoods to improve the current solution (Hansen et al. 2010) using a predefined neighborhood ordering. In the case of RVND, the order in which the neighborhoods are explored is random. The latter is shown to yield promising results when compared to the former strategy (Penna et al. 2013). The combination of ILS and RVND has been successfully employed in solving several combinatorial optimization problems, such as vehicle routing (Penna et al. 2013), scheduling (Subramanian et al. 2014), and manufacturing cell formation (Martins et al. 2015).

Algorithm 1 describes the proposed multi-start metaheuristic, called ILS_{batch} . The algorithm receives four input parameters: I_R is the total number of restarts, I_{ILS} is the number of consecutive ILS iterations without improvement, I_P is the maximum number of perturbation moves, and α is a parameter used in the constructive heuristic $CHEDD$ (line 4). ILS_{batch} follows the standard ILS framework (Lourenço et al. 2019), by iteratively applying local search and perturbation moves (lines 8 and 12).

```

1 Procedure  $ILS_{batch}(I_R, I_{ILS}, I_P, \alpha)$ 
2    $TWT^* \leftarrow \infty$ 
3   for  $iter = 1 \dots I_R$  do
4      $B \leftarrow CHEDD(\alpha)$ 
5      $B' \leftarrow B$ 
6      $iter_{ILS} \leftarrow 0$ 
7     while  $iter_{ILS} < I_{ILS}$  do
8        $B \leftarrow LocalSearch(B);$ 
9       if  $TWT(B) < TWT(B')$  then
10         $B' \leftarrow B$ 
11         $iter_{ILS} \leftarrow 0$ 
12        $B \leftarrow Perturbation(B', I_P)$ 
13        $iter_{ILS} \leftarrow iter_{ILS} + 1$ 
14     if  $TWT(B') < TWT^*$  then
15        $B^* \leftarrow B'$ 
16        $TWT^* \leftarrow TWT(B')$ 
17   return  $B^*$ 

```

Algorithm 1: ILS_{batch}

In what follows, we describe the main components of ILS_{batch} .

2.1 Constructive procedure

The constructive heuristic CH_{EDD} , described in Algorithm 2, is based on the *earliest due date* (EDD) rule, which is a well-known in the scheduling literature (Vepsäläinen 1987). Basically, the algorithm combines greediness and randomness by selecting one of the first α jobs in L (denoted by $F(\alpha, L)$). If the chosen job does not fit in the batch, the algorithm tries to place the job with smaller s_j among the first α jobs (lines 10–16).

```

1 Procedure  $CH_{EDD}(J, Q, \alpha)$ 
2   Let  $L$  be the list of jobs in  $J$  ordered in increasing order of their EDD rule
3   Let  $\mathcal{B} \leftarrow \emptyset$  be a batch sequence solution
4   Let  $B' \leftarrow \emptyset$  be a new batch to be created
5   while  $L \neq \emptyset$  do
6     Select a random job  $j$  from  $F(\alpha, L)$ 
7     if  $s_j + s(B') \leq Q$  then
8        $B' \leftarrow B' \cup \{j\}$ ;  $L \leftarrow L \setminus \{j\}$ 
9     else
10       $j' \leftarrow \arg \min_{i \in F(\alpha, L) \setminus \{j\}} s_i$ 
11      if  $s_{j'} + s(B') \leq Q$  then
12         $B' \leftarrow B' \cup \{j'\}$ ;  $L \leftarrow L \setminus \{j'\}$ 
13      else
14        add the batch  $B'$  at the end of list  $\mathcal{B}$ 
15         $B' \leftarrow \emptyset$  // creating an empty batch
16         $B' \leftarrow B' \cup \{j\}$ ;  $L \leftarrow L \setminus \{j\}$ 
17   return  $\mathcal{B}$ 

```

Algorithm 2: Constructive heuristic using EDD rule.

2.2 Local search

Algorithm 3 shows the pseudocode of the VND-based procedure, which employs the *best improvement* strategy, where each neighborhood of a solution \mathcal{B} is completely examined to determine the move (neighbor) that yields the best TWT improvement.

A neighborhood from set \mathcal{N} is systematically selected (line 11) to continue the search whenever the current neighborhood fails to improve the current solution. In case of improvement, the search continues from the first neighborhood, randomly redefined, over the new solution found (lines 8–9). Here, at least one neighborhood structure is necessary, but it is interesting if more neighborhoods are available. Generally, the larger the number of distinct neighborhoods, the better the results.

```

1 LocalSearch( $\mathcal{B}$ )
2   initialize the set of neighborhoods  $\mathcal{N}$  in a random order
3    $k \leftarrow 1$  // neighborhood structure selector
4   while  $k \leq |\mathcal{N}|$  do
5     // find the best improvement neighbor  $\mathcal{B}'$  of  $\mathcal{B}$  in  $\mathcal{N}_k(\mathcal{B})$ 
6      $\mathcal{B}' \leftarrow \text{BestImprovement}(\mathcal{N}_k, \mathcal{B})$ 
7     if  $TWT(\mathcal{B}') < TWT(\mathcal{B})$  then
8       reinitialize  $\mathcal{N}$  in a random order
9        $k \leftarrow 1$ 
10       $\mathcal{B} \leftarrow \mathcal{B}'$ 
11    else
12       $k \leftarrow k + 1$ 
13  return  $\mathcal{B}$ 

```

Algorithm 3: VND-based local search.

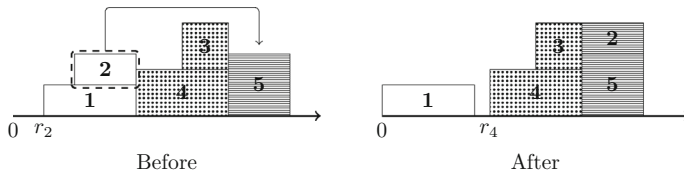


Fig. 2 Let $\mathcal{B} = (B_1, B_2, B_3)$ be a solution. Job 2 was moved from B_1 to B_3

The $\text{ILS}_{\text{batch}}$ operates over five neighborhood structures, which will be thoroughly described in the next subsections.

2.2.1 Job insertion

In this neighborhood, hereafter referred to as $\text{insert}_{\text{job}}$, a job is removed from its current batch and inserted in another batch (see Fig. 2).

A move is valid only if the capacity Q is not exceeded, and the number of batches is maintained (if a job is the only one in the batch, it cannot be moved). There are at most $n \times (m - 1)$ neighbors for a solution \mathcal{B} , where $n = |J|$ and $m = |\mathcal{B}|$. This neighborhood has complexity $\mathcal{O}(n^2)$, since in the worst case we have $m = n$ (all batches having only one job). Algorithm 4 describes the $\text{insert}_{\text{job}}$ move evaluation in $\mathcal{O}(n)$ time.

Therefore, the best $\text{insert}_{\text{job}}$ move can be determined in $\mathcal{O}(n^3)$ time.

2.2.2 Job swap

In this neighborhood, hereafter referred to as swap_{job} , two jobs from different batches are swapped (see Fig. 3).

```

1 Procedure EvalJobInsertion( $\mathcal{B}, k, j, l$ )
2   Let  $m = |\mathcal{B}|$  and  $\mathcal{L} = \min\{k, l\}$ 
3    $B_k \leftarrow B_k \setminus \{j\}$  and  $B_l \leftarrow B_l \cup \{j\}$  // performs the move
4   Update processing time and release date for  $B_k$  and  $B_l$ 
5   Let  $\delta \leftarrow 0$  be the improvement induced by the move
6   Let  $c'(B_{\mathcal{L}-1}) = c(B_{\mathcal{L}-1})$  if  $\mathcal{L} > 1$ .
7   for  $i = \mathcal{L}, \mathcal{L} + 1, \dots, m$  do
8     if  $i = 1$  then
9        $c'(B_i) \leftarrow r(B_i) + p(B_i)$  // new completion time
10    else
11       $c'(B_i) \leftarrow \max\{c'(B_{i-1}), r(B_i)\} + p(B_i)$  // new completion time
12      // improvement induced by  $j$ 
13    if  $i = l$  then
14       $\delta \leftarrow \delta + (w_j \times \max\{0, c(B_k) - d_j\}) - (w_j \times \max\{0, c'(B_l) - d_j\})$ 
15      // improvement induced by other jobs
16    forall the  $q \in B_i$  do
17      if  $q \neq j$  then
18         $\delta \leftarrow \delta + (w_q \times \max\{0, c(B_i) - d_q\}) - (w_q \times \max\{0, c'(B_i) - d_q\})$ 
19   $B_l \leftarrow B_l \setminus \{j\}$  and  $B_k \leftarrow B_k \cup \{j\}$  // undo the move
20  Update processing time and release date for  $B_k$  and  $B_l$ 
21  return ( $TWT(\mathcal{B}) - \delta$ )

```

Algorithm 4: Evaluation of a job insertion. The solution evaluated is obtained by moving the job j from B_k to B_l .

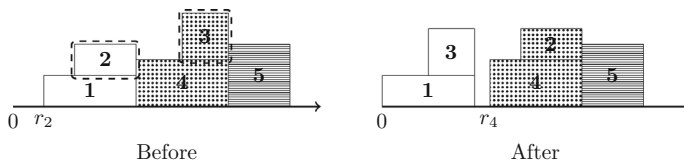


Fig. 3 Let $\mathcal{B} = (B_1, B_2, B_3)$ be a solution. Jobs $2 \in B_1$ and $3 \in B_2$ were swapped

A move is valid only if the capacity Q is not exceeded in both batches. When $n = m$, there are at most $n(n-1)/2$ neighbors, thus this neighborhood has complexity $\mathcal{O}(n^2)$. Algorithm 5 describes the swap_{job} move evaluation in $\mathcal{O}(n)$ time. Therefore, the best swap_{job} move can be found in $\mathcal{O}(n^3)$ time.


```

1 Procedure EvalJobSwap ( $\mathcal{B}, k, i, l, j$ )
2   Let  $m = |\mathcal{B}|$ 
3    $B_k = (B_k \setminus \{i\}) \cup \{j\}$  and  $B_l = (B_l \setminus \{j\}) \cup \{i\}$  // performs the move
4   Update processing time and release date for  $B_k$  and  $B_l$ 
5   Let  $\delta = 0$  be the improvement induced by the move
6   Let  $c'(B_{k-1}) = c(B_{k-1})$  if  $k > 1$ .
7   for  $o = k, k + 1, \dots, m$  do
8     if  $o = 1$  then
9        $c'(B_o) \leftarrow r(B_o) + p(B_o)$  // new completion time
10    else
11       $c'(B_o) \leftarrow \max\{c'(B_{o-1}), r(B_o)\} + p(B_o)$  // new completion time
12    if  $o = k$  then
13      // improvement induced by  $j$ 
14       $\delta \leftarrow \delta + (w_j \times \max\{0, c(B_l) - d_j\}) - (w_j \times \max\{0, c'(B_k) - d_j\})$ 
15    if  $o = l$  then
16      // improvement induced by  $i$ 
17       $\delta \leftarrow \delta + (w_i \times \max\{0, c(B_k) - d_i\}) - (w_i \times \max\{0, c'(B_l) - d_i\})$ 
18      // improvement induced by other jobs
19      forall the  $q \in B_o$  do
20        if  $q \neq i$  and  $q \neq j$  then
21           $\delta \leftarrow \delta + (w_q \times \max\{0, c(B_o) - d_q\}) - (w_q \times \max\{0, c'(B_o) - d_q\})$ 
22     $B_k = (B_k \setminus \{j\}) \cup \{i\}$  and  $B_l = (B_l \setminus \{i\}) \cup \{j\}$  // undo the move
23    Update processing time and release date for  $B_k$  and  $B_l$ 
24  return ( $TWT(\mathcal{B}) - \delta$ )

```

Algorithm 5: Evaluation of a job swap. The solution evaluated is obtained by swapping the jobs $i \in B_k$ and $j \in B_l$, where $k < l$.

2.2.3 Batch insertion

In this neighborhood, hereafter referred to as $\text{insert}_{\text{batch}}$, a batch is removed from its current position and inserted in another one (see Fig. 4).

Removing and inserting a batch from the k -th position to l -th position is only valid if $|k - l| > 1$, avoiding intersection with the $\text{swap}_{\text{batch}}$ described in Sect. 2.2.4.

This neighborhood has complexity $\mathcal{O}(n^2)$, since in the worst case we have $m = n$ and for each batch there are $\mathcal{O}(n)$ moves. Algorithm 6 describes the $\text{insert}_{\text{batch}}$ move evaluation in $\mathcal{O}(n)$ time. Therefore, the best $\text{insert}_{\text{batch}}$ move can be found in $\mathcal{O}(n^3)$ time.

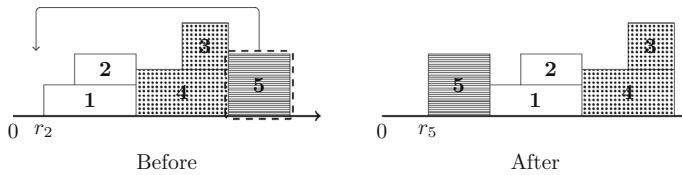


Fig. 4 Let $B = (B_1, B_2, B_3)$ be a solution. Batch B_3 was moved to the first position

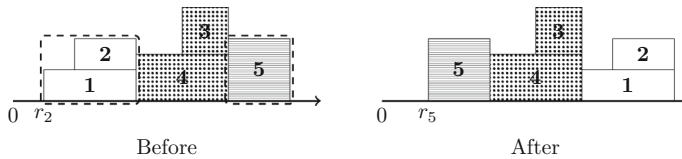


Fig. 5 Swap between batches B_1 and B_3

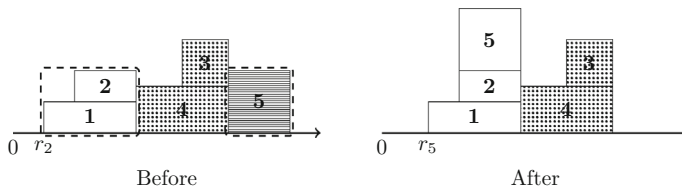


Fig. 6 Batches B_1 and B_3 are merged into one

2.2.4 Batch swap

In this neighborhood, hereafter referred to as $\text{swap}_{\text{batch}}$, two different batches are swapped (see Fig. 5).

This neighborhood has complexity $\mathcal{O}(n^2)$, since there are $n(n-1)/2$ possible swaps when $m = n$. Algorithm 7 describes the $\text{swap}_{\text{batch}}$ move evaluation in $\mathcal{O}(n)$ time. Therefore, the best $\text{swap}_{\text{batch}}$ move can be found in $\mathcal{O}(n^3)$ time.

2.2.5 Batch merge

In this neighborhood, hereafter referred to as $\text{merge}_{\text{batch}}$, two different batches are merged (see Fig. 6). The new batch is positioned in place of the leftmost batch between the two involved in the move.

A merge is only valid if the capacity Q is not violated. This neighborhood has complexity $\mathcal{O}(n^2)$, since there are at most $n \times (n-1)/2$ moves when $n = m$. Algorithm 8 describes the move evaluation in $\mathcal{O}(n)$ time. Therefore, the best move can be found in $\mathcal{O}(n^3)$ time.

```

1 Procedure EvalBatchInsertion( $\mathcal{B}, k, l$ )
2   Let  $m = |\mathcal{B}|$ 
3   Let  $\mathcal{B}(i)$  be the  $i$ -th batch of  $\mathcal{B}$ , where initially  $\mathcal{B} = (B_1, B_2, \dots, B_m)$ .
4   Let  $\delta \leftarrow 0$  be the improvement induced by the move.
5   if  $k < l$  then
6      $\mathcal{B} = (\dots, B_{k-1}, B_{k+1}, \dots, B_l, B_k, \dots)$  // performs the move
7     Let  $c'(B_{k-1}) = c(B_{k-1})$  if  $k > 1$ .
8     for  $i = k, k+1, \dots, m$  do
9       if  $i = 1$  then
10         $c'(\mathcal{B}(i)) \leftarrow r(\mathcal{B}(i)) + p(\mathcal{B}(i))$ 
11       else
12         $c'(\mathcal{B}(i)) \leftarrow \max\{c'(\mathcal{B}(i-1)), r(\mathcal{B}(i))\} + p(\mathcal{B}(i))$ 
13       forall the  $q \in \mathcal{B}(i)$  do
14         $\delta \leftarrow \delta + (w_q \times \max\{0, c(\mathcal{B}(i)) - d_q\}) - (w_q \times \max\{0, c'(\mathcal{B}(i)) - d_q\})$ 
15      $\mathcal{B} = (\dots, B_{k-1}, B_k, \dots, B_l, B_{l+1}, \dots)$  // undo the move
16   else
17      $\mathcal{B} = (\dots, B_{l-1}, B_k, B_l, \dots, B_{k-1}, B_{k+1}, \dots)$  // performs the move
18     Let  $c'(B_{l-1}) = c(B_{l-1})$  if  $l > 1$ .
19     for  $i = l, l+1, \dots, m$  do
20       if  $i = 1$  then
21         $c'(\mathcal{B}(i)) \leftarrow r(\mathcal{B}(i)) + p(\mathcal{B}(i))$ 
22       else
23         $c'(\mathcal{B}(i)) \leftarrow \max\{c'(\mathcal{B}(i-1)), r(\mathcal{B}(i))\} + p(\mathcal{B}(i))$ 
24       forall the  $q \in \mathcal{B}(i)$  do
25         $\delta \leftarrow \delta + (w_q \times \max\{0, c(\mathcal{B}(i)) - d_q\}) - (w_q \times \max\{0, c'(\mathcal{B}(i)) - d_q\})$ 
26      $\mathcal{B} = (\dots, B_{l-1}, B_l, \dots, B_{k-1}, B_k, B_{k+1}, \dots)$  // performs the move
27   return  $(TWT(\mathcal{B}) - \delta)$ 

```

Algorithm 6: Evaluation of a batch insertion. The solution evaluated is obtained by moving the batch B_k to position l .

2.3 Perturbation

Perturbation is a key ILS feature which aims at slightly modifying a local optimal solution, thus allowing the local search to converge to a different, and possibly better, local optimum. In this work, we consider the following perturbation mechanisms:

```

1 Procedure EvalBatchSwap( $\mathcal{B}, k, l$ )
2   Let  $m = |\mathcal{B}|$ 
3   Let  $\mathcal{B}(i)$  the  $i$ -th batch of  $\mathcal{B}$ , where initially  $\mathcal{B} = (B_1, B_2, \dots, B_m)$ .
4   Let  $\delta \leftarrow 0$  be the improvement induced by the move.
5    $\mathcal{B} = (\dots, B_{k-1}, B_l, B_{k+1}, \dots, B_{l-1}, B_k, B_{l+1}, \dots)$  // performs the move
6   Let  $c'(B_{k-1}) = c(B_{k-1})$  if  $k > 1$ .
7   for  $i = k, k+1, \dots, m$  do
8     if  $i = 1$  then
9        $c'(\mathcal{B}(i)) \leftarrow r(\mathcal{B}(i)) + p(\mathcal{B}(i))$ 
10    else
11       $c'(\mathcal{B}(i)) \leftarrow \max\{c'(\mathcal{B}(i-1)), r(\mathcal{B}(i))\} + p(\mathcal{B}(i))$ 
12    forall the  $q \in \mathcal{B}(i)$  do
13       $\delta \leftarrow \delta + (w_q \times \max\{0, c(\mathcal{B}(i)) - d_q\}) - (w_q \times \max\{0, c'(\mathcal{B}(i)) - d_q\})$ 
14   $\mathcal{B} = (\dots, B_{k-1}, B_k, B_{k+1}, \dots, B_{l-1}, B_l, B_{l+1}, \dots)$  // undo the move
15  return  $(TWT(\mathcal{B}) - \delta)$ 

```

Algorithm 7: Evaluation of a batch swap. The solution evaluated is obtained by exchanging the batches B_k and B_l , such that $k < l$.

1. *Swap of jobs*—A random move from the neighborhood swap_{job} is applied.
2. *Batch split*—At first, a batch $B_i \in \mathcal{B}$ and a cut point $t \in [2, |B_i| - 1]$ are selected. Next, assuming that B_i is in non-ascending order w.r.t. the processing time, B_i is divided into two new batches B'_i and B''_i , where B'_i receives the first t jobs of B_i and B''_i receives the remaining ones. B'_i and B''_i are then included, in this order, in place of B_i in the sequence \mathcal{B} (B_i is replaced by the new batches).

At each call of the perturbation procedure, one of the two first mechanisms is randomly chosen, and at most I_P moves associated to the selected mechanism are applied.

If the swap of jobs perturbation is selected and at least one feasible move could not be performed due to the capacity constraints, then the batch split mechanism is applied.

3 Dynamic programming-based approaches

In the heuristic by Wang (2011), a candidate solution is represented by a sequence of jobs, and the DP algorithm proposed by Chou and Wang (2008) is used as a *solution decoder*, i.e., to define a batching from the sequence and to compute the TWT. The problem of finding an optimal batching over a given fixed sequence of jobs (hereafter referred to as $\text{TWTBSP}_{\text{seq}}$) is formally defined and proved to be \mathcal{NP} -hard in

```

1 Procedure EvalBatchMerge( $\mathcal{B}, k, l$ )
2   Let  $m = |\mathcal{B}|$ 
3   Let  $\mathcal{B}(i)$  the  $i$ -th batch of  $\mathcal{B}$ , where initially  $\mathcal{B} = (B_1, B_2, \dots, B_m)$ .
4   Let  $\delta \leftarrow 0$  be the improvement induced by the move.
5   Let  $B_{kl}$  be the batch obtained by merging  $B_k$  and  $B_l$ .
6    $\mathcal{B} = (\dots, B_{k-1}, B_{kl}, B_{k+1}, \dots, B_{l-1}, B_{l+1}, \dots)$  // performs the move
7   Let  $c'(B_{k-1}) = c(B_{k-1})$  if  $k > 1$ .
8   for  $i = k, k+1, \dots, (m-1)$  do
9     if  $i = 1$  then
10       $c'(\mathcal{B}(i)) \leftarrow r(\mathcal{B}(i)) + p(\mathcal{B}(i))$ 
11     else
12       $c'(\mathcal{B}(i)) \leftarrow \max\{c'(\mathcal{B}(i-1)), r(\mathcal{B}(i))\} + p(\mathcal{B}(i))$ 
13     forall the  $q \in \mathcal{B}(i)$  do
14       $\delta \leftarrow \delta + (w_q \times \max\{0, c(\mathcal{B}(i)) - d_q\}) - (w_q \times \max\{0, c'(\mathcal{B}(i)) - d_q\})$ 
15    $\mathcal{B} = (\dots, B_{k-1}, B_k, B_{k+1}, \dots, B_{l-1}, B_l, B_{l+1}, \dots)$  // undo the move
16   return  $(TWT(\mathcal{B}) - \delta)$ 

```

Algorithm 8: Evaluation of a batch merge. The solution evaluated is obtained by merging the batches B_k and B_l , such that $k < l$.

Appendix A. We describe in **Appendix B** the DP algorithm by Chou and Wang (2008), whose complexity is $\mathcal{O}(n^3)$, and a counterexample that proves that the algorithm does not necessarily yield an optimal solution. An exact DP algorithm for TWTBSP_{seq} is provided in **Appendix C**.

In order to compare the performance of our method described in Sect. 2 with those based on DP for TWTBSP, we implemented the heuristic developed in Wang (2011) and also another two DP-based procedures that are inspired on ILS and BRKGA, respectively. Although the DP algorithm by Chou and Wang (2008) is not exact for TWTBSP_{seq}, preliminary experiments revealed that its performance in terms of solution quality is similar to the exact DP described in **Appendix C**. We thus decided to use the DP procedure by Chou and Wang (2008) because our exact DP led to prohibitively large CPU times in practice.

3.1 ILS_{DP}

ILS_{DP} basically consists of embedding the DP algorithm in the proposed ILS_{batch} framework by using neighborhoods and perturbations over a sequence of jobs. Given a solution $\mathcal{B} = (B_1, B_2, \dots, B_m)$, we will assume that the jobs of each batch are in descending order w.r.t. the processing time. Given that B_i^j is the j -th job of the batch B_i ,

the job sequence is defined as $\pi = (B_1^1, B_1^2, \dots, B_1^{|B_1|}, B_2^1, B_2^2, \dots, B_2^{|B_2|}, \dots, B_m^1, B_m^2, \dots, B_m^{|B_m|})$. ILS_{DP} has the following neighborhood structures over π :

- Insert_{seq}: move a job $\pi_{[j]}$ (j -th element of π) to the i -th position of π , such that $j \neq i$.
- Swap_{seq}: exchange two jobs $\pi_{[i]}$ and $\pi_{[j]}$ in π , such that $i \neq j$.

The two perturbation mechanisms of ILS_{DP} apply random moves (according to the Sect. 2.3) of insert_{seq} and swap_{seq}. After each local search or perturbation move, the DP algorithm is called to generate the resulting batching.

3.2 BRKGA_{DP}

BRKGA_{DP} employs the DP procedure as a solution decoder within the well-known BRKGA approach (Gonçalves and Resende 2011). We made use of the BRKGA C++ framework developed by Toso and Resende (2015), where we specified the chromosome decoder procedure. To decode the random keys as feasible solutions, we sort them in ascending order, where the sorted random keys correspond to the sequence of the jobs. We then apply the DP procedure to obtain a new batch scheduling in the genetic algorithm.

4 Computational experiments

The experiments were conducted on an Intel Xeon CPU E5-2650 v4 with 2.20 GHz, 128 GB of RAM, running Ubuntu 16.04 LTS. The algorithms were coded in C++ and compiled with g++ 5.4 and ‘-O3’ flag. ILS_{batch} and ILS_{DP} were run with the following parameter values: $I_R = 20$, $I_{ILS} = 4n$, $I_p = 2$, $\alpha = 4$. The first value was taken from Subramanian and Farias (2017), the second from Subramanian et al. (2014); Subramanian and Farias (2017), while the third and fourth parameters were calibrated after experimenting with different values ranging from 1 to 5.

IH was run with the same parameters used by Wang (2011), whereas the default parameters suggested by Toso and Resende (2015) were adopted for BRKGA_{DP}. Finally, we performed long runs of ILS_{batch} to obtain upper bounds to be used as reference values while comparing the methods. In this case, a time limit of 1 h was imposed as a stopping criterion as opposed to the number of maximum restarts. Detailed results of ILS_{batch} are reported in the Appendix.

4.1 Instances

Because the TWTBSP instances used in previous works are no longer available, we decided to generate a new benchmark dataset to test the performance of the methods. In view of this, we used the procedure described in Chou and Wang (2008) and Mathirajan et al. (2010) to create the instances. It is worth mentioning that Wang (2011) used the approach presented in Mathirajan et al. (2010) to generate the instances for the problem.

Table 2 Distribution of the parameters used by the literature for instance generation

Parameters	Levels (Chou and Wang 2008)	Levels (Mathirajan et al. 2010)
n	5, 7, 9, 11, 13, 15, 50, 100, 150, 200	10, 12, 20, 50, 100
r_j	U(0, 48)	U(1, 20), U(1, 30)
p_j	U(8, 48)	U(1, 10), U(1, 15)
d_j	$r_j + p_j + I(R_1, T)$	$r_j + p_j + U(1, 30), U(1, 45)$
s_j	U(1, 30) U(15, 35)	U(4, 10), U(4, 14)
w_j	U(1, 11)	s_j
Q	40	20

Both Chou and Wang (2008) and Mathirajan et al. (2010) used a uniform distribution over a predefined interval to define the values of the attributes of the problem.

However, no information was provided by Mathirajan et al. (2010) concerning the weight parameter distribution. We then assumed that the authors determined the weights of each job proportionally to their sizes.

Table 2 presents the distribution of the parameters adopted by Chou and Wang (2008) and Mathirajan et al. (2010).

Note that for Chou and Wang (2008), the distribution of the parameters is mostly similar.

However, there is a considerable difference in the due dates distribution, where in this case the interval is not known *a priori*. Instead, it is based on an expected makespan C_{\max}^* obtained by applying the full batch longest processing time (FBLPT) algorithm by Lee (1999). Once the C_{\max}^* is computed, due dates d_j are distributed along the interval $I(R_1, T) = [\mu(1 - R_1/2), \mu(1 + R_1/2)]$, such that $\mu = (1 - T)(\min\{r_i \mid i \in J\} + C_{\max}^*)$. The possible values for T and R_1 are $\{0.3, 0.6\}$ and $\{0.5, 2.5\}$, respectively.

We performed preliminary experiments on several instances generated using the two approaches described above. This allowed us to derive some insights on their difficulty. In fact, we were able to identify the following shortcomings:

- Instances turned out to be sometimes very easy, generally when using the configuration by Chou and Wang (2008). For the instances with up to 15 jobs, it is not uncommon to have optimal solutions with associated TWT of 0, and easy to find even with rather basic local search methods. This happens because the distribution of the due dates often leads to instances with very large values of d_j , which are seldom violated in practice.
- The definition of the release dates also tends to decrease the difficulty of the instances. Although the release dates are uniformly distributed within a predefined interval, and they seem somewhat homogeneously distributed over the scheduling horizon for very small instances, one can verify that they become irrelevant for large size problems.

Table 3 Distribution of the parameters for the new benchmark instances

Parameters	Proposed values
n	10, 15, 20, 25, 30, 35, 50, 75, 100
r_j	$U(0, R_2 \cdot C_{\max}^*)$
p_j	$U(1, 30)$
d_j	$r_j + p_j + U(1, 20)$
s_j	$U(1, 10)$, $U(5, 15)$ and $U(1, 20)$ if $Q = 20$ $U(1, 20)$, $U(10, 30)$ and $U(1, 40)$ if $Q = 40$
w_j	$U(1, 9)$
Q	$[20, 40]$

The possible values for R_2 are $\{0.1, 0.4, 0.8\}$

Table 4 Instance groups and their parameter settings

Group	$Q = 20$	$Q = 40$
G1	$s_j \in [1, 10]; R_2 = 0.1$	$s_j \in [1, 20]; R_2 = 0.1$
G2	$s_j \in [1, 10]; R_2 = 0.4$	$s_j \in [1, 20]; R_2 = 0.4$
G3	$s_j \in [1, 10]; R_2 = 0.8$	$s_j \in [1, 20]; R_2 = 0.8$
G4	$s_j \in [5, 15]; R_2 = 0.1$	$s_j \in [10, 30]; R_2 = 0.1$
G5	$s_j \in [5, 15]; R_2 = 0.4$	$s_j \in [10, 30]; R_2 = 0.4$
G6	$s_j \in [5, 15]; R_2 = 0.8$	$s_j \in [10, 30]; R_2 = 0.8$
G7	$s_j \in [1, 20]; R_2 = 0.1$	$s_j \in [1, 40]; R_2 = 0.1$
G8	$s_j \in [1, 20]; R_2 = 0.4$	$s_j \in [1, 40]; R_2 = 0.4$
G9	$s_j \in [1, 20]; R_2 = 0.8$	$s_j \in [1, 40]; R_2 = 0.8$

Therefore, it was thought advisable to adopt different values for some parameters in order to make the instances more challenging, as presented in Table 3. For example, we adopted three different values for parameter $R_2 \in [0, 1]$ to represent scenarios in which the values of the release date correspond to a small (10%), medium (40%) and large (80%) percentage of the expected makespan C_{\max}^* . The idea is to generate distinct scenarios with different levels of difficulty so one can evaluate the impact of the release dates on the algorithm's performance.

We provided 5 instances for each configuration of parameters, leading to a total of 810 instances. For each problem size, i.e., $\forall n \in [10, \dots, 100]$, and for each capacity value $Q = [20, 40]$, there are 9 groups of 5 instances, with varying settings, as detailed in Table 4. The instances are available at <http://professor.ic.ufal.br/rian/Instances/TWTBSP.rar>.

Table 5 Impact of adding each neighborhood structure at a time

Setting	Neighborhoods	Avg. improvement	#Improved solutions
(0)	None	—	—
(1)	insert _{job}	54.38%	162
(2)	(1) + swap _{job}	13.13%	139
(3)	(2) + insert _{batch}	2.67%	105
(4)	(3) + swap _{batch}	0.20%	35
(5)	(4) + merge _{batch}	0.81%	58

4.2 Impact of the neighborhood structures

Table 5 shows the effect of adding each neighborhood structure at a time using the order that they were presented in Sect. 2.2. We consider the first of the 5 instances of each group of the newly proposed benchmark dataset, thus resulting in a total of 162 instances. Five executions were performed for every instance when testing each setting. The first row represents the basic configuration, where no local search is performed, i.e., the algorithm only executes I_R iterations of CH_{EDD} .

The other rows represent the addition of a neighborhood to the subset composed of those considered in the previous rows. The average improvement and the number of improvements are computed w.r.t. the previous configuration, thus measuring the benefits of adding the corresponding neighborhood.

The results suggest that all neighborhoods play a relevant role in helping the algorithm to obtain better solutions. For example, after including neighborhood insert_{batch}, there is an average improvement of 2.67% on the solution quality, and the best result of 105 instances was improved, when compared to the setting that considers only neighborhoods insert_{job} + swap_{job}.

4.3 Impact of the perturbation mechanisms

Table 6 shows the impact of the two perturbation mechanisms presented in Sect. 2.3. Each setting was tested on the same subset of instances considered in Sect. 4.2. For each setting we computed: (i) the average gap w.r.t. the best solution obtained during the experiment (including all settings); (ii) the number of best solutions found; and (iii) the average CPU time in seconds. In this work, the gap between the objective value of two solutions \mathcal{B}_1 and \mathcal{B}_2 , where $TWT(\mathcal{B}_1) \geq TWT(\mathcal{B}_2)$, is computed as $100 \times (TWT(\mathcal{B}_1) - TWT(\mathcal{B}_2)) / TWT(\mathcal{B}_1)$. The results clearly suggest that using both perturbation mechanisms yields a superior performance than the other two settings.

Table 6 Impact of the perturbation mechanisms

Setting	Avg. gap (%)	#Best solutions	Avg. time (s)
Swap of jobs	3.78	81	15.11
Batch split	0.16	146	15.40
Swap of jobs + Batch split	0.12	154	15.08

4.4 Performance comparison

All methods were run 10 times on each instance. However, because of the large CPU times, only a single run was performed for ILS_{DP} on the instances with $|J| \geq 75$, and a time limit of 2 h was imposed.

Tables 7 and 8 present aggregate results obtained by the four algorithms on each group, according to the number of jobs (n), for $Q = 20$ and $Q = 40$, respectively. We report the average (Avg) and minimum (Min) gaps w.r.t. the best known solution (BKS) found after performing long runs of ILS_{batch} , as well as the CPU time in seconds. In addition, we also provide two additional columns for ILS_{batch} , reporting the average and minimum gaps w.r.t. the lower bound (LB) provided by an exact algorithm (Pessoa et al. 2020), except for 5 groups involving 100 jobs, in which the LB values were not available.

On average, the results clearly indicate that ILS_{batch} was capable of outperforming the other heuristics both in solution quality as well as CPU time. ILS_{DP} obtained the second best average gap, but at the expense of high CPU times, especially for larger instances. $BRKGA_{DP}$, on the other hand, scaled much better than ILS_{DP} , but at the expense of solution quality, even failing to achieve the best solutions for 15-job instances. Finally, IH presented a very poor performance, even for 10-job instances.

Furthermore, in 46 groups (including both tables), the average gap obtained by ILS_{batch} equaled the LB, meaning that the proposed method was capable of systematically finding the optimal solutions for the instances of such groups. This happened, in particular, to all 10- and 15-job instances and to those containing 25 jobs associated with group G9 and $Q = 40$.

Tables 9 and 10 present the number of cases where the BKS (found by performing long runs of ILS_{batch}) was achieved by each method, for $Q = 20$ and $Q = 40$, respectively. We can observe that ILS_{batch} (considering regular runs) found most of the BKSs both for $Q = 20$ and $Q = 40$. Overall, the proposed algorithm managed to find 91.60% of the BKSs, against 68.40%, 59.01%, 8.15% of ILS_{DP} , $BRKGA_{DP}$ and IH, respectively.

The results obtained by ILS_{batch} can also be analyzed through Fig. 7, which shows, for every group, the average (a) percentage gap; (b) CPU time in seconds; (c) the number of batches; and (d) number of singletons, which consists of batches composed of a single job. Recall from Table 3 that the job size distribution is equal for the first three groups ([1, 10]). The same happens for groups G4, G5, and G6, whose job distribution is [5, 15], as well as for the last three groups ([1, 20]). From Fig. 7a, we can verify that

Table 7 Aggregate results obtained by ILS_{batch}, ILS_{Dp}, BRKG_{App} and IH for $Q = 20$

n	Group	ILS _{Shach}					ILS _{DP}					BRKG _{DP}					IH																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																									
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																								
10	G1	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 7 continued

n	Group	IL _{Search}				IL _{Spp}				BRKG _{ADP}				IH			
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)		
20	G1	0.00	0.00	0.2	0.00	0.00	0.00	0.00	6.6	0.00	0.00	24.1	38.61	33.06	26.0		
	G2	0.00	0.00	0.2	0.49	0.00	0.03	0.00	6.8	0.94	0.00	23.8	47.51	38.46	26.0		
	G3	0.00	0.00	0.3	0.00	0.00	0.09	0.00	6.6	0.00	0.00	23.2	64.40	48.95	26.1		
	G4	0.00	0.00	0.2	0.71	0.00	0.00	0.00	6.4	0.12	0.00	25.1	28.35	23.60	25.9		
	G5	0.00	0.00	0.3	0.71	0.00	0.12	0.00	6.4	1.33	0.00	24.8	50.96	40.38	25.9		
	G6	0.00	0.00	0.3	0.00	0.00	0.00	0.00	5.6	0.42	0.00	24.8	75.13	64.59	25.8		
	G7	0.00	0.00	0.2	0.80	0.00	0.02	0.00	6.0	0.47	0.00	25.4	37.80	28.73	25.8		
	G8	0.00	0.00	0.3	0.57	0.00	0.00	0.00	6.2	0.52	0.00	24.1	48.27	42.52	25.7		
	G9	0.00	0.00	0.3	0.00	0.00	0.00	0.00	5.4	0.00	0.00	23.7	74.73	61.39	25.8		
25	G1	0.03	0.00	0.3	1.52	0.00	0.48	0.00	16.3	0.71	0.00	26.0	44.49	40.97	39.8		
	G2	0.00	0.00	0.4	0.60	0.00	0.15	0.00	15.7	0.21	0.00	26.3	60.59	48.83	39.9		
	G3	0.00	0.00	0.5	0.09	0.00	0.13	0.00	14.6	0.18	0.00	26.2	77.08	70.07	39.6		
	G4	0.00	0.00	0.5	0.70	0.00	0.00	0.00	17.0	0.96	0.00	27.6	41.84	38.29	39.5		
	G5	0.00	0.00	0.6	2.17	0.00	0.12	0.00	12.6	0.33	0.00	28.2	66.16	56.84	39.6		
	G6	0.00	0.00	0.6	1.10	0.00	0.00	0.00	13.4	0.74	0.00	28.6	77.20	65.09	39.4		
	G7	0.02	0.00	0.5	0.26	0.00	0.03	0.00	14.1	0.53	0.12	29.2	41.19	33.33	39.1		
	G8	0.00	0.00	0.5	2.55	0.00	0.45	0.00	11.8	0.74	0.00	29.1	64.89	54.68	39.6		
	G9	0.00	0.00	0.6	0.70	0.00	0.12	0.00	12.4	1.69	0.00	30.7	68.16	55.88	39.5		

Table 7 continued

<i>n</i>	Group	ILS _{batch}				ILS _{DP}				BRKG _{DP}				IH			
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)		
30	G1	0.00	0.00	0.6	1.80	0.90	0.40	0.08	32.4	0.38	0.19	31.0	52.11	46.06	56.7		
	G2	0.02	0.00	0.6	2.15	1.24	0.95	0.05	30.7	0.83	0.17	31.7	61.32	53.09	56.7		
	G3	0.08	0.00	0.7	1.34	0.00	0.19	0.00	28.7	0.52	0.00	31.4	76.53	71.82	56.7		
	G4	0.00	0.00	0.9	1.04	0.09	0.04	0.00	30.9	0.65	0.17	34.0	47.48	38.62	56.3		
	G5	0.00	0.00	0.9	1.71	0.00	0.14	0.00	27.1	2.08	0.02	34.1	61.47	47.29	55.9		
	G6	0.00	0.00	1.2	2.66	0.00	0.20	0.00	24.5	2.44	0.00	35.2	82.97	78.01	56.4		
	G7	0.00	0.00	0.7	1.49	0.00	0.18	0.00	28.0	1.21	0.20	34.1	49.13	41.76	56.3		
	G8	0.00	0.00	1.1	1.90	0.00	0.00	0.00	24.1	1.61	0.23	36.0	63.05	57.95	56.6		
	G9	0.00	0.00	1.0	0.79	0.00	0.02	0.00	23.9	0.65	0.00	34.6	78.16	65.59	56.5		
35	G1	0.08	0.00	0.7	2.20	0.72	1.33	0.51	66.6	2.13	0.61	37.6	54.65	52.61	76.7		
	G2	0.00	0.00	1.0	2.43	0.00	1.90	0.37	62.7	2.38	0.33	37.7	72.08	61.94	76.7		
	G3	0.06	0.00	1.3	3.15	0.00	1.41	0.00	58.9	2.16	0.20	36.9	80.96	78.52	76.6		
	G4	0.00	0.00	1.4	1.06	0.41	0.53	0.04	68.4	1.46	0.43	38.4	48.31	43.40	75.9		
	G5	0.00	0.00	1.9	2.21	0.63	0.38	0.00	58.1	2.17	0.78	39.1	65.82	55.98	76.0		
	G6	0.00	0.00	2.1	1.63	0.00	0.08	0.00	48.6	3.67	1.18	37.7	86.44	82.08	76.2		
	G7	0.06	0.00	1.3	0.72	0.00	0.21	0.00	55.8	0.79	0.15	37.7	50.82	46.89	76.0		
	G8	0.00	0.00	1.7	1.59	0.28	0.59	0.20	51.2	2.27	0.67	40.9	65.70	59.69	76.3		
	G9	0.00	0.00	2.2	4.45	0.00	0.15	0.00	45.8	1.63	0.00	39.5	83.54	65.79	76.0		

Table 7 continued

n	Group	ILS _{batch}				ILS _{Dp}				BRKG _{Adp}				IH			
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)		
50	G1	0.28	0.03	3.1	2.14	0.87	2.92	2.37	304.6	4.75	3.58	47.2	60.87	57.84	153.9		
	G2	0.32	0.00	3.6	3.43	2.30	3.96	2.11	284.0	6.16	3.88	43.1	72.32	71.00	154.5		
	G3	0.05	0.00	4.7	5.86	2.50	1.43	0.15	251.3	12.23	9.20	42.6	88.61	84.96	154.9		
	G4	0.00	0.00	5.8	0.81	0.00	0.48	0.02	333.4	3.14	2.13	46.2	55.45	49.29	153.7		
	G5	0.04	0.00	6.6	1.52	0.65	1.74	0.18	274.3	6.70	4.34	50.4	72.37	66.94	153.8		
	G6	0.00	0.00	7.6	4.82	3.73	1.35	0.19	212.9	9.23	3.17	47.1	89.50	85.22	153.6		
	G7	0.04	0.00	5.8	0.97	0.13	0.94	0.08	255.9	2.77	2.07	47.2	56.91	52.26	153.8		
	G8	0.06	0.00	6.4	2.10	1.09	1.82	0.75	228.4	5.68	4.78	46.1	74.85	72.13	152.6		
	G9	0.25	0.00	8.3	2.67	0.84	2.53	0.71	198.3	10.19	6.39	47.5	90.46	87.67	153.3		
75	G1	0.55	0.24	14.6	2.16	1.46	4.65	3.60	1863.5	8.30	6.58	63.8	65.07	62.02	347.5		
	G2	0.95	0.41	16.3	5.39	4.73	8.00	5.91	1901.6	15.73	13.96	64.7	80.44	77.85	347.8		
	G3	0.50	0.00	22.5	6.36	3.78	7.81	2.58	1479.7	21.95	18.27	63.7	90.96	88.14	346.9		
	G4	0.07	0.00	29.2	0.55	0.15	1.65	0.96	2451.1	6.39	5.41	65.8	57.56	55.14	344.5		
	G5	0.23	0.00	35.7	1.78	0.91	4.82	2.52	1813.0	13.08	9.86	65.2	77.59	73.59	345.5		
	G6	0.32	0.00	42.1	4.51	1.16	6.59	5.83	1249.6	28.70	22.81	66.9	92.03	87.73	344.9		
	G7	0.31	0.13	26.3	1.45	0.84	2.15	1.19	1684.7	6.51	5.56	75.2	62.50	60.71	337.6		
	G8	0.20	0.05	34.3	2.41	1.54	4.07	2.03	1452.2	10.60	6.58	79.8	76.93	74.51	321.1		
	G9	0.18	0.00	36.3	4.35	1.70	3.81	0.86	1139.2	21.57	16.81	79.3	93.24	91.83	325.0		

Table 7 continued

<i>n</i>	Group	ILShatch				ILSpp				BRKGApp				IH			
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)
100	G1	0.99	0.67	41.8	3.04	2.35	4.86	3.06	7078.0	11.30	10.41	93.8	67.20	65.87	620.6		
	G2	1.56	0.44	51.2	5.54	4.07	8.83	7.36	6649.1	21.13	19.18	93.6	79.84	78.43	617.9		
	G3	1.59	0.73	66.8	12.35	11.58	11.75	6.33	5383.2	40.70	33.62	93.0	93.61	91.69	623.5		
	G4	0.16	0.01	89.3	0.84	0.48	1.98	1.57	7200.1	8.78	7.55	101.5	61.43	57.68	621.7		
	G5	0.57	0.26	105.1	2.70	1.37	6.92	5.25	7200.1	19.61	15.69	95.0	78.64	71.06	620.2		
	G6	0.73	0.44	136.1	6.55	5.26	7.92	6.68	4683.3	43.31	35.63	101.2	93.99	92.72	619.2		
	G7	0.36	0.21	86.4	1.21	0.81	1.97	1.20	7090.3	8.19	6.75	104.0	62.24	57.91	621.6		
	G8	0.37	0.14	113.7	2.58	2.02	6.64	4.02	5121.8	18.29	13.53	104.0	81.13	76.70	621.9		
	G9	0.34	0.00	126.9	7.72	5.29	10.12	5.62	4173.8	41.91	32.88	103.0	94.49	91.76	621.3		
Mean		0.14	0.05	14.27			1.63	0.92	900.35	5.51	4.03	41.13	57.59	51.15	148.36		

Table 8 Aggregate results obtained by ILS_{batch}, ILS_{Dp}, BRKG_{Adp} and IH for $Q = 40$

n	Group	ILS _{batch}				ILS _{Dp}				BRKG _{Adp}				IH			
		Avg (%)	Min (%)	Time (s)	Time (%)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Time (s)
10	G1	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	14.4	0.30	0.00	6.6	6.6
	G2	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	14.2	2.78	1.23	6.6	6.6
	G3	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	14.4	8.36	3.77	6.6	6.6
	G4	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	14.2	4.74	1.44	6.6	6.6
	G5	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	13.8	6.61	2.55	6.6	6.6
	G6	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	14.3	11.01	5.91	6.6	6.6
	G7	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.3	0.00	0.00	15.4	2.42	0.22	6.6	6.6
	G8	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	15.1	5.27	1.38	6.6	6.6
	G9	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	0.4	0.00	0.00	15.0	17.62	8.85	6.6	6.6
15	G1	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	1.9	0.08	0.00	15.6	22.91	16.59	13.9	13.9
	G2	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	1.8	0.44	0.00	15.6	31.53	23.45	13.8	13.8
	G3	0.00	0.00	0.1	0.00	0.00	0.00	0.00	0.00	1.7	0.00	0.00	15.6	49.19	30.38	13.9	13.9
	G4	0.00	0.00	0.1	0.00	0.00	0.00	0.00	0.00	1.9	0.09	0.00	16.3	24.18	19.59	13.9	13.9
	G5	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	1.7	0.00	0.00	16.2	36.99	31.26	13.8	13.8
	G6	0.00	0.00	0.1	0.00	0.00	0.00	0.00	0.00	1.6	0.00	0.00	16.4	53.91	33.25	13.8	13.8
	G7	0.00	0.00	< 0.1	0.00	0.00	0.00	0.00	0.00	1.8	0.00	0.00	16.3	24.92	16.71	13.9	13.9
	G8	0.00	0.00	0.1	0.00	0.00	0.00	0.00	0.00	1.6	0.00	0.00	16.5	30.37	20.66	13.8	13.8
	G9	0.00	0.00	0.1	0.00	0.00	0.00	0.00	0.00	1.5	0.00	0.00	16.6	64.26	25.25	13.8	13.8

Table 8 continued

n	Group	ILS _{batch}				ILS _{DP}				BRKGA _{DP}				IH			
		Avg (%)	Min (%)	Time (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	
20	G1	0.00	0.00	0.2	0.23	0.00	0.00	0.00	0.00	6.2	0.29	0.00	23.6	40.62	34.56	23.8	
	G2	0.00	0.00	0.2	0.00	0.00	0.00	0.26	0.00	5.7	0.11	0.00	23.1	56.87	50.64	23.7	
	G3	0.00	0.00	0.3	0.00	0.00	0.00	0.00	0.00	5.6	0.00	0.00	23.2	70.14	59.39	23.9	
	G4	0.00	0.00	0.2	0.35	0.00	0.00	0.01	0.00	5.9	0.06	0.00	25.2	32.93	27.07	23.7	
	G5	0.00	0.00	0.2	0.00	0.00	0.00	0.04	0.00	6.0	0.14	0.00	26.3	46.24	38.60	23.7	
	G6	0.00	0.00	0.3	0.00	0.00	0.00	0.05	0.00	5.2	0.00	0.00	26.5	69.62	62.59	23.7	
	G7	0.00	0.00	0.2	0.42	0.00	0.00	0.00	0.00	5.4	0.04	0.00	25.9	36.58	31.10	23.7	
	G8	0.00	0.00	0.3	0.35	0.00	0.00	0.00	0.00	5.0	0.13	0.00	24.7	53.55	44.59	23.7	
	G9	0.00	0.00	0.3	0.00	0.00	0.00	0.00	0.00	5.0	0.30	0.00	24.0	72.83	69.92	23.7	

Table 8 continued

n	Group	ILS _{batch}				ILS _{DP}				BRKGAdP				IH			
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)		
25	G1	0.06	0.00	0.3	1.37	0.00	0.26	0.00	14.6	1.89	0.00	25.7	50.75	40.41	36.4		
	G2	0.00	0.00	0.4	2.48	0.00	0.32	0.00	14.4	0.28	0.00	25.7	59.58	52.31	36.5		
	G3	0.00	0.00	0.5	0.34	0.00	0.00	0.00	13.6	0.68	0.00	25.5	76.96	72.30	36.5		
	G4	0.00	0.00	0.5	0.78	0.00	0.02	0.00	14.5	0.18	0.00	27.8	38.56	32.79	36.3		
	G5	0.00	0.00	0.4	0.99	0.00	0.00	0.00	14.5	0.57	0.00	27.1	57.27	41.87	36.4		
	G6	0.00	0.00	0.6	0.07	0.00	0.04	0.00	12.1	0.12	0.00	30.7	80.06	76.46	36.4		
	G7	0.00	0.00	0.4	1.19	0.18	0.00	0.00	13.8	0.62	0.00	30.1	44.51	40.89	36.3		
	G8	0.00	0.00	0.6	2.03	0.00	0.12	0.00	13.2	0.34	0.00	30.8	57.68	54.31	36.4		
	G9	0.00	0.00	0.6	0.00	0.00	0.00	0.00	11.5	0.00	0.00	29.9	85.22	80.76	36.4		
30	G1	0.00	0.00	0.5	1.41	0.00	0.54	0.14	32.2	0.91	0.02	29.8	45.24	40.25	51.9		
	G2	0.01	0.00	0.8	2.91	2.26	0.71	0.13	30.1	1.29	0.00	28.8	64.60	57.07	51.9		
	G3	0.01	0.00	0.9	3.05	0.84	0.46	0.00	28.7	2.44	0.95	28.7	75.29	66.28	51.8		
	G4	0.00	0.00	0.7	0.97	0.00	0.10	0.00	32.6	0.54	0.12	32.8	42.30	38.59	51.7		
	G5	0.00	0.00	1.1	0.79	0.00	0.25	0.00	28.3	2.52	1.09	32.6	63.46	57.04	51.7		
	G6	0.00	0.00	1.2	1.62	0.00	0.24	0.00	24.1	2.98	1.08	34.1	83.19	76.36	51.8		
	G7	0.00	0.00	0.7	1.17	0.44	0.12	0.00	28.2	0.31	0.00	34.4	49.53	45.54	51.7		
	G8	0.01	0.00	0.9	1.16	0.04	0.59	0.05	26.6	1.29	0.27	34.6	55.37	42.75	51.7		
	G9	0.00	0.00	1.0	4.24	0.52	0.61	0.00	25.6	0.70	0.00	35.0	81.05	73.72	51.7		

Table 8 continued

<i>n</i>	Group	ILS _{batch}				ILS _{DP}				BRKGAdP				IH			
		Avg (%)	Min (%)	Time (s)	Avg (%)	Min (LB) (%)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)
35	G1	0.04	0.00	0.8	2.05	0.48	1.33	0.62	63.3	1.80	1.03	35.7	58.26	53.52	70.3		
	G2	0.14	0.00	0.9	1.37	0.00	1.05	0.00	60.9	2.37	0.06	35.1	68.55	63.08	70.5		
	G3	0.00	0.00	1.3	1.99	0.00	0.81	0.00	63.0	3.19	1.94	33.8	82.03	75.01	70.2		
	G4	0.05	0.00	1.6	0.91	0.02	0.12	0.01	73.5	1.02	0.20	36.7	48.45	38.94	69.9		
	G5	0.07	0.00	1.8	3.55	0.00	0.48	0.00	65.8	2.53	0.99	36.7	64.29	57.26	69.9		
	G6	0.00	0.00	2.0	2.33	0.61	0.70	0.00	50.9	5.35	1.14	37.8	85.24	78.26	69.8		
	G7	0.00	0.00	1.2	0.85	0.00	0.08	0.00	58.1	1.27	0.28	37.9	47.54	42.87	70.0		
	G8	0.00	0.00	1.7	1.68	0.36	0.40	0.05	64.5	1.74	0.36	37.3	71.07	64.21	70.0		
	G9	0.00	0.00	2.0	0.89	0.00	0.11	0.00	52.0	2.15	1.19	38.9	82.33	75.54	69.8		
50	G1	0.06	0.00	4.1	1.95	0.13	2.63	1.71	295.4	4.06	3.36	45.6	62.83	56.14	141.1		
	G2	0.15	0.00	4.5	2.38	0.35	3.61	1.61	297.4	8.26	6.83	46.4	69.05	64.63	141.0		
	G3	0.10	0.00	5.2	3.29	0.98	3.67	0.95	263.2	11.11	8.41	47.0	85.37	79.80	141.1		
	G4	0.06	0.00	5.6	0.76	0.51	0.55	0.34	374.0	2.59	1.53	52.1	55.38	50.03	140.6		
	G5	0.07	0.00	6.7	1.45	0.30	1.25	0.20	331.2	5.26	3.74	51.9	69.87	61.86	140.6		
	G6	0.00	0.00	8.0	2.73	0.81	1.01	0.14	232.2	7.82	3.77	48.9	87.71	85.47	140.7		
	G7	0.00	0.00	5.6	0.86	0.00	0.70	0.03	283.5	2.46	1.27	49.4	55.40	48.56	140.7		
	G8	0.04	0.00	5.9	1.48	0.00	1.96	0.91	277.2	6.60	5.08	51.9	71.34	65.89	140.7		
	G9	0.16	0.00	6.7	9.08	2.90	3.14	0.89	218.9	10.33	8.28	51.9	89.45	85.30	140.8		

Table 8 continued

n	Group	ILS _{batch}				ILS _{pp}				BRKG _{App}				IH			
		Avg (%)	Min (%)	Time (s)	Avg (LB) (%)	Min (LB) (%)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)	Avg (%)	Min (%)	Time (s)		
75	G1	0.53	0.31	16.7	2.14	1.83	3.88	3.00	2146.7	7.53	6.56	66.2	58.79	56.07	317.2		
	G2	0.82	0.05	20.6	4.16	2.54	7.00	5.86	1940.1	14.65	11.68	61.1	76.18	66.82	317.5		
	G3	1.86	0.31	26.4	6.55	1.95	8.31	5.89	1515.8	25.28	21.02	60.4	90.52	88.28	317.4		
	G4	0.05	0.00	26.6	0.36	0.20	0.99	0.24	2805.7	6.11	5.70	66.2	58.78	56.55	315.8		
	G5	0.08	0.00	35.7	1.07	0.93	4.79	2.67	2131.7	12.74	11.32	67.0	75.39	70.87	316.1		
	G6	0.23	0.00	41.9	3.74	0.75	5.40	3.88	1540.4	23.06	19.03	67.7	90.76	86.94	316.0		
	G7	0.06	0.01	26.2	0.87	0.55	1.76	1.39	1883.0	5.36	3.86	78.0	59.67	55.29	316.4		
	G8	0.21	0.01	37.7	1.68	0.67	3.99	2.54	1528.8	12.55	9.18	80.5	78.34	74.35	316.2		
	G9	0.07	0.00	40.1	3.74	2.94	2.68	0.77	1263.5	18.88	12.24	75.0	91.11	90.15	316.4		
100	G1	0.54	0.25	54.7	–	–	5.91	4.82	6413.2	10.94	10.00	89.8	64.19	61.53	571.0		
	G2	1.00	0.59	68.5	–	–	9.20	4.14	5750.7	19.24	15.68	89.2	76.52	72.69	571.1		
	G3	1.05	0.50	86.5	–	–	12.07	7.59	4676.2	36.41	31.63	89.5	90.98	89.35	570.7		
	G4	0.09	0.03	95.9	0.53	0.33	1.56	0.85	7200.1	8.31	5.91	97.3	60.00	57.58	568.7		
	G5	0.41	0.10	128.0	1.86	0.54	4.85	2.98	7026.9	17.52	15.22	99.9	77.73	73.69	568.8		
	G6	0.23	0.09	140.2	6.30	3.90	7.20	5.16	4625.8	37.99	33.05	99.6	92.17	88.54	568.8		
	G7	0.25	0.03	100.5	1.09	0.73	2.61	1.30	5732.1	7.73	5.79	104.5	62.54	59.73	569.8		
	G8	0.70	0.07	106.0	–	–	5.67	3.62	4807.3	18.69	13.45	103.1	81.36	78.94	569.1		
	G9	0.45	0.00	133.1	–	–	4.25	0.55	3691.0	40.19	33.17	99.4	93.95	90.77	569.1		
	Mean	0.12	0.03	15.63			1.49	0.80	867.30	5.22	3.92	40.88	57.04	50.80	136.60		

Table 9 Number of best known solutions found by each method for $Q = 20$

$ J $	Method			
	ILS _{batch}	ILS _{DP}	BRKGA _{DP}	IH
10	45	45	45	28
15	45	45	45	0
20	45	45	45	0
25	45	44	42	0
30	45	44	37	0
35	45	41	30	0
50	44	17	1	0
75	35	0	0	0
100	19	0	0	0
Total	368	281	245	28

Table 10 Number of best known solutions found by each method for $Q = 40$

$ J $	Method			
	ILS _{batch}	ILS _{DP}	BRKGA _{DP}	IH
10	45	45	45	38
15	45	45	45	0
20	45	45	45	0
25	45	45	44	0
30	45	39	30	0
35	45	41	24	0
50	44	13	0	0
75	34	0	0	0
100	26	0	0	0
Total	374	273	233	38

the average gap tends to be larger for groups G1, G2, G3, where the average job size is smaller. Also, note that the number of singletons increases with the job size, as can be observed in Fig. 7d. This result strengthens the hypothesis that groups with smaller average job size (i.e., G1, G2, and G3) are more difficult since the batching decisions seem to become challenging as the job sizes decrease, leading to a few number of singletons. On the other hand, the key difference between the instances whose groups have the same job size distribution is the release date values, which assume increasing values according to the group. In other words, G1, G4, and G7 are the groups with smaller release date values, whereas G3, G6, and G9 are the groups with larger release

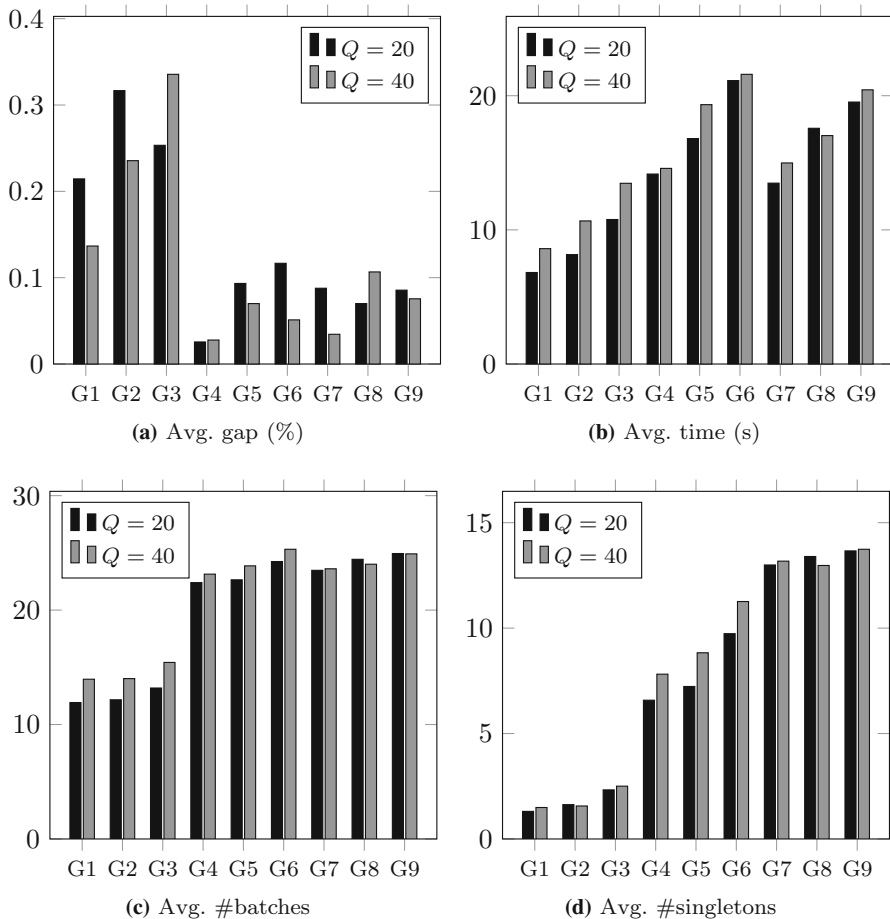


Fig. 7 Results of ILS_{batch} by group. Four distinct metrics are considered for analysis

date values. Figure 7b suggests that the CPU time is likely to increase with the job size and release date values. This can be explained by the average number of batches that also follows the same behavior (see Fig. 7c), thus directly affecting the runtime performance because the smaller the number of batches, the faster the local search and vice-versa. Indeed, we can see a clear correlation between Fig. 7b and c.

5 Concluding remarks

In this paper, we addressed the single machine total weighted tardiness batch scheduling problem (TWTBSP). In order to solve the problem, we designed a metaheuristic algorithm based on iterated local search (ILS) that is composed of five neighborhood structures and two perturbation mechanisms. Also, three alternative dynamic programming (DP)-based procedures were implemented to serve as a reference for

measuring the efficiency of the proposed algorithm. One of them is a heuristic developed by Wang (2011), whereas the other two are based on biased random key genetic algorithm (BRKGA) and ILS, respectively. Moreover, we showed that the problem of finding the optimal TWT batching given a fixed sequence of jobs is \mathcal{NP} -hard and provided an exact pseudo-polynomial DP for the problem. However, the existing non-exact DP procedure scales much better than the exact one without degrading the solution quality, and it was therefore used in the DP-based heuristics.

Extensive computational experiments on newly proposed benchmark instances showed that the results obtained by our heuristic, called ILS_{batch} , outperformed the other ones both in solution quality (by a far margin) and CPU time. The results suggest that, despite its convenience and popular use in the batch scheduling literature, DP-based procedures may not efficiently scale to larger instances. Furthermore, ILS_{batch} was compared to lower bounds made available by Pessoa et al. (2020), where it was observed that our algorithm was capable of finding all the known optima and producing solutions with relatively small gaps, even for instances with up to 100 jobs. Finally, it was also shown that the instances tend to become difficult as the average job size decreases and that the CPU time increases as the average job size and release date values increase. The first happens because the smaller the average job size, the smaller the number of batches (thus fewer singletons), and therefore the batching decisions become more intricate. The second occurs because the average number of batches increases with the average job size and thus the local search phase becomes more time consuming, as the complexity of some of the neighborhoods increases with the number of batches.

Future research includes the development of a hybrid approach combining the advantages of ILS_{batch} with an exact method. For example, one can use the information found during the heuristic search to feed an integer programming-based algorithm to find improved solutions. Finally, one can also extend the proposed algorithm to solve related problems, such as the one with incompatible job families (Balasubramanian et al. 2004; Perez et al. 2005).

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Appendix A: TWTBSP with fixed sequence of jobs

SINGLE MACHINE TOTAL WEIGHTED TARDINESS BATCH SCHEDULING WITH FIXED SEQUENCE OF JOBS (TWTBSP_{SEQ})

Instance: A sequence of jobs $\pi = (\pi_{[1]}, \pi_{[2]}, \dots, \pi_{[n]})$

Objective: Find a batching $\mathcal{B} = (B_1, B_2, \dots, B_m)$ which minimizes the TWT and is feasible for the TWTBSP, such that $\forall i, j \in \{1, 2, \dots, n\}, i < j$, the batch associated with $\pi_{[i]}$ must be processed before the one associated with $\pi_{[j]}$, unless they have been assigned to the same batch.

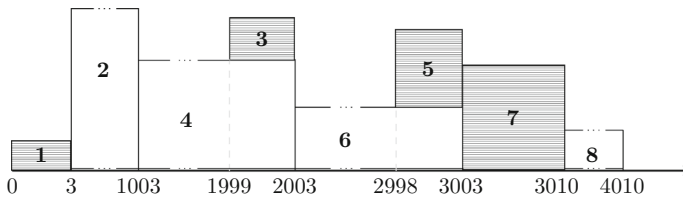


Fig. 8 A certificate for a $\text{TWTBSP}_{\text{seq}}$ instance built from a SSP instance with $S = \{3, 4, 5, 7\}$, and $V = 10$

The decision version of $\text{TWTBSP}_{\text{seq}}$ (or $\text{TWTBSP}_{\text{seq}}$ for short) includes the maximum weighted tardiness TWT_{max} as an additional data, and asks whether a feasible $\text{TWTBSP}_{\text{seq}}$ batching with $\text{TWT} \leq \text{TWT}_{\text{max}}$ exists or not. The next proposition proves the hardness of $\text{TWTBSP}_{\text{seq}}$ by showing that its decision version is \mathcal{NP} -Complete.

Proposition 1 *The decision version of $\text{TWTBSP}_{\text{seq}}$ is \mathcal{NP} -Complete, even if $r_j = 0$, for $j = 1, \dots, n$.*

Proof Clearly the $\text{TWTBSP}_{\text{seq}}$ belongs to \mathcal{NP} (since a certificate for the positive answer is easily verified in polynomial time). It remains to prove its completeness, which we show through a reduction from the Subset Sum Problem (SSP).

Given a set $S = \{v_1, \dots, v_{\bar{n}}\}$ of positive integers and a target positive integer V , the SSP asks whether a subset S' of S whose elements sum exactly V exists or not. For the sake of simplicity, we assume without loss of generality in this proof that $\pi_{[j]} = j$, for $j = 1, \dots, n$. Let also M and M' be sufficiently large numbers whose values we define later, with $M > M'$, and $\bar{V} = \sum_{j=1}^{\bar{n}} v_j - V$.

For a given instance of the SSP, we show next how to build a corresponding instance for the $\text{TWTBSP}_{\text{seq}}$ such that $r_j = 0$, for $j = 1, \dots, n$:

- $n = 2\bar{n}$, $Q = \bar{n} + 1$, and $\text{TWT}_{\text{max}} = M\bar{V} + M'$,
- $p_{2j-1} = v_j$, $d_{2j-1} = (j-1)M$, $w_{2j-1} = v_j$, and $s_{2j-1} = j$, for $j = 1, \dots, \bar{n}$,
- $p_{2j} = M$, $d_{2j} = w_{2j} = 0$, and $s_{2j} = \bar{n} - j + 1$, for $j = 1, \dots, \bar{n} - 1$,
- $p_n = M$, $d_n = Mn + V$, $w_n = \text{TWT}_{\text{max}} + 1$, and $s_n = 1$.

Jobs with odd indices are referred to as the *short* ones and those with even indices as the *long* ones.

For example, consider a SSP instance with $S = \{3, 4, 5, 7\}$, and $V = 10$, where $v_1 = 3$ and $v_4 = 7$ compose a certificate for the positive answer. Let us also assume that $M = 1000$. In this case, the corresponding $\text{TWTBSP}_{\text{seq}}$ instance would have $p_1 = 3$, $p_3 = 4$, $p_5 = 5$, $p_7 = 7$, and the remaining processing times equal to 1000. Moreover, $s_1, s_2, s_3, s_4, s_5, s_6, s_7$, and s_8 would be equal to 1, 4, 2, 3, 3, 2, 4, and 1, respectively, and $Q = 5$. Figure 8 represents a certificate for the positive answer to this instance, where short jobs are shadowed. Note that the numbers v_1 and v_4 selected in the SSP certificate correspond to the short jobs that belong to unitary batches.

In general, given a certificate S' for the SSP instance, the batches of a certificate for the corresponding $\text{TWTBSP}_{\text{seq}}$ instance are constructed as follows. Start with

an empty \mathcal{B} . Then, for $j = 1, \dots, \bar{n}$, if $v_j \in S'$, add two unitary batches $\{2j - 1\}$ and $\{2j\}$ to \mathcal{B} . Otherwise, if $v_j \notin S'$, add a single batch $\{2j - 1, 2j\}$ to \mathcal{B} .

We prove now that (by setting appropriate values for M and M') this is a valid certificate for the $\text{TWTBSP}_{\text{seq}}$ instance. First, note that, since there are exactly \bar{n} long jobs and the processing times of all short jobs that are alone in their batches sum up to V , the completion time of job n is exactly $d_n = M\bar{n} + V$. Moreover, for all j

such that $v_j \in S'$, its completion time is exactly $(j - 1)M + \sum_{\substack{k=1 \\ v_k \in S'}}^j v_k$, contributing with

$v_j \sum_{\substack{k=1 \\ v_k \in S'}}^j v_k$ to the TWT. Furthermore, for all j such that $v_j \in S \setminus S'$, its completion time

is exactly $jM + \sum_{\substack{k=1 \\ v_k \in S'}}^{j-1} v_k$, contributing with $v_j \left(M + \sum_{\substack{k=1 \\ v_k \in S'}}^{j-1} v_k \right)$ to the TWT. Then, by

setting $M' = \left(\sum_{k=1}^{\bar{n}} v_k \right)^2$, we make sure that the sum of all TWT terms that do not depend on M does not exceed M' . Clearly, the TWT terms that depend on M sum exactly $M\bar{V}$, which results in a TWT not greater than TWT_{\max} .

It remains to prove that a valid certificate for the $\text{TWTBSP}_{\text{seq}}$ instance also implies a positive answer to the SSP instance. To see this, note that the jobs $2j$ and $2j + 1$, for $j = 1, \dots, \bar{n} - 1$, cannot belong to the same batch due to the capacity constraints. Thus, it only remains the choices of whether or not to join the jobs $2j - 1$ and $2j$ into a single batch, for $j = 1, \dots, \bar{n}$.

Now, consider that set S' built as follows: for all $j \in \{1, \dots, \bar{n}\}$ such that jobs $2j - 1$ and $2j$ do not form a batch, add v_j to S' . We argue that $\sum_{j \in S'} v_j = V$. First, if we had $\sum_{j \in S'} v_j > V$, this would lead to a completion time greater than d_n for job n , forcing the TWT to be at least $\text{TWT}_{\max} + 1$. Moreover, if we had $\sum_{v \in S'} v < V$, and thus $\sum_{v \in S \setminus S'} v > \bar{V}$, the contribution of the short jobs to the TWT would not be smaller than $M(\bar{V} + 1)$ which exceeds TWT_{\max} , assuming that $M > M'$.

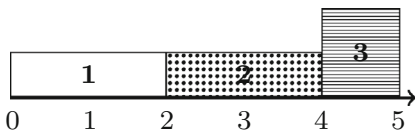
To complete this proof, we observe that setting $M = M' + 1$ satisfies all the assumptions made so far on the value of M . \square

Appendix B : DP proposed by Chou and Wang (2008)

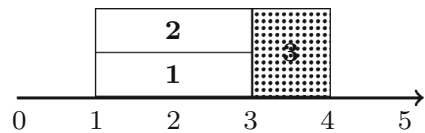
Given a sequence of jobs $\pi = (\pi_{[1]}, \pi_{[2]}, \dots, \pi_{[k]}, \dots, \pi_{[n]})$, we define $\pi^k = (\pi_{[1]}, \pi_{[2]}, \dots, \pi_{[k]})$ as a subset of π containing the first k jobs, and $\pi_j^k = (\pi_{[k-j+1]}, \dots, \pi_{[k]})$ as a subset of π^k containing the j jobs from position $k - j + 1$ to position k . The minimum TWT for π^k is given by $\text{TWT}(\pi^k) = \min_{1 \leq j \leq k} \{f(\pi^k, j)\}$

Job	p_j	d_j	s_j	w_j	r_j
1	2	2	1	1	0
2	2	4	1	1	1
3	1	4	2	2	0
$Q = 2$					

(a) Instance



(b) DP solution with TWT=2



(c) Optimal solution with TWT=1

Fig. 9 Counterexample showing that the DP algorithm by Chou and Wang (2008) is not exact for the TWTBSP_{seq}. The input is the job sequence $\pi = (1, 2, 3)$

such that

$$f(\pi^k, j) = \begin{cases} TWT(\pi^{k-j}) + \sum_{i \in \pi_j^k} w_i \mathcal{T}_{ij}, & \text{if } \sum_{i \in \pi_j^k} q_i \leq Q \\ \infty, & \text{otherwise} \end{cases}$$

where $\mathcal{T}_{ij} = \max(0, g(\pi^k, j) - d_i)$. Let $\gamma(\pi^k) = \arg \min_{1 \leq j \leq k} \{f(\pi^k, j)\}$ be the index which defines the last batch of π^k as $(\pi_{[k-\gamma(\pi^k)+1]}, \dots, \pi_{[k]})$. The recursive function $g(\pi^k, j) = \max_{i \in \pi_j^k} \{g(\pi^{k-j}, \gamma(\pi^{k-j})), r_i\} + \max_{i \in \pi_j^k} p_i$ returns the completion time of the last batch of π^k assuming that it is defined as $(\pi_{[j]}, \dots, \pi_{[k]})$. In addition, we have $\pi^0 = \emptyset$, $\gamma(\emptyset) = \emptyset$, $g(\emptyset, \emptyset) = 0$, and $TWT(\emptyset) = 0$. To recover the solution, it is necessary to build the batches from right to left using γ . For example, if $l_1 = \gamma(\pi^n)$, the last batch is $(\pi_{[n-l_1+1]}, \dots, \pi_{[n]})$. Given that $l_2 = l_1 + \gamma(\pi^{n-l_1})$, the last but one batch is $(\pi_{[n-l_2+1]}, \dots, \pi_{[n-l_1]})$. The process of finding $l_k = l_{k-1} + \gamma(\pi^{n-l_{k-1}})$ is repeated until the first batch is built.

Figure 9 shows a 3-job instance for which the DP algorithm fails to find the optimal batching over the sequence $\pi = (1, 2, 3)$. This is due to the fact that solving a subproblem π^k optimally ($k < n$) can be seen as a greedy decision because it does not guarantee that the completion time of the last batch of π^k is minimal. In the example, the DP procedure places jobs 1 and 2 in different batches, where $TWT(\pi^2) = 0$ and $g(\pi^2) = 4$. Thus, $TWT(\pi^3) = 2$ because the job 3 is forced to have weighted tardiness of 2 (the solution where 1 and 2 are in the same batch with completion time 3 is not evaluated). Table 11 shows the complete execution of the algorithm. Let $l_1 = \gamma(\pi^3) = 1$, we have that the third batch is $(\pi_{[3-l_1+1]}, \dots, \pi_{[3]}) = (\pi_{[3-1+1]}, \dots, \pi_{[3]}) = (\pi_{[3]})$; let $l_2 = l_1 + \gamma(\pi^{3-l_1}) = 1 + \gamma(\pi^2) = 2$, the second batch is $(\pi_{[3-l_2+1]}, \dots, \pi_{[3-l_1]}) = (\pi_{[3-2+1]}, \dots, \pi_{[2]}) = (\pi_{[2]})$; and let $l_3 = l_2 + \gamma(\pi^{3-l_2}) = 2 + \gamma(\pi^1) = 3$, the first batch is $(\pi_{[3-l_3+1]}, \dots, \pi_{[3-l_2]}) = (\pi_{[3-3+1]}, \dots, \pi_{[1]}) = (\pi_{[1]})$.

Table 11 Execution of the DP algorithm by Chou and Wang (2008) over the instance in Fig. 9

k	π^k	j	$f(\pi^k, j)$	$g(\pi^k, j)$	$\gamma(\pi^k)$	$TWT(\pi^k)$
1	(1)	1	$TWT(\pi^0) + w_1\mathcal{I}_{11} =$ $= 0 + \max(0, g(\pi^1, 1) - d_1) =$ $= \max(0, 0) = 0$	$\max(g(\pi^0, \gamma(\pi^0)), r_1)$ $+ p_1 = r_1 + p_1 = 2$	1	0
2	(1, 2)	1	$TWT(\pi^1) + w_2\mathcal{I}_{21} =$ $= 0 + \max(0, g(\pi^2, 1) - d_2) =$ $= \max(0, 4 - 4) = 0$	$\max(g(\pi^1, \gamma(\pi^1)), r_2)$ $+ p_2$ $= \max(2, 0) + 2 = 4$	1	0
		2	$TWT(\pi^0) + w_1\mathcal{I}_{12} + w_2\mathcal{I}_{22} =$ $= 0 + \max(0, g(\pi^2, 2) - d_1) +$ $\max(0, g(\pi^2, 2) - d_2) = 1$	$\max(g(\emptyset, \emptyset), r_2) +$ $\max(p_1, p_2) =$ $= 1 + 2 = 3$		
3	(1, 2, 3)	1	$TWT(\pi^2) + w_3\mathcal{I}_{31} =$ $= 0 + 2 \max(0, g(\pi^3, 1) - d_3) =$ $= 2 \max(0, 5 - 4) = 2$	$\max(g(\pi^2, \gamma(\pi^2)), r_3)$ $+ p_3 =$ $= \max(4, 0) + 1 = 5$	1	2
		2	$\infty (\sum_{i \in \pi_j^k} q_i > Q)$	–		
		3	$\infty (\sum_{i \in \pi_j^k} q_i > Q)$	–		

Appendix C : Exact DP algorithm for the TWTBSP_{seq}

In this appendix, we describe an exact dynamic programming approach for TWTBSP_{seq}. Given a sequence of jobs $\pi = (\pi_{[1]}, \pi_{[2]}, \dots, \pi_{[k]}, \dots, \pi_{[n]})$, the algorithm determines the values $TWT_{i,j}^r$ for $1 \leq i < j \leq n$ and $r \geq 0$. Each value $TWT_{i,j}^r$ is defined as the total weighted tardiness found by the DP approach for TWTBSP_{seq} on an instance $\{\pi_{[i]}, \dots, \pi_{[j]}\} \subseteq \pi$ starting with a release date r . If $TWT_{i',j}^{r'}$ is known for all $i < i'$ and $r < r'$, then we can consider each possible initial batch B' containing jobs $\{\pi_{[i]}, \dots, \pi_{[k]}\}$ and compute the corresponding solution $TWT_{i,j}^r$ using the following recursive formula:

$$TWT_{i,j}^r = \min_{i \leq k \leq j} \left(w(i, k) + TWT_{k+1,j}^{C_{i,k}^r} \right) \quad (1)$$

where $C_{i,k}^r$ is the completion time of the batch B' with release date r and

$$w(i, k) = \begin{cases} \sum_{\rho=i}^k w_{\rho} T_{\rho} & \text{if } \sum_{\rho=i}^k s_{\rho} \leq Q \\ w(i, k) = \infty & \text{otherwise.} \end{cases}$$

Note that if B' is a feasible batch, then $w(i, j)$ represents the weighted tardiness of B' , and when $k = j$, the procedure sets $TWT_{k+1,j}^{C_{i,k}^r} = 0$.

Proposition 2 *Given sequence of jobs $\pi = (\pi_{[i]}, \pi_{[i+1]}, \dots, \pi_{[j]})$. The DP algorithm finds an optimal batch partitioning for the TWTBSP in $\mathcal{O}(n^3 \max(r_{\max}, p_{\text{sum}}))$ pseudo-polynomial time, where r_{\max} is the maximum release date for all jobs in π and $p_{\text{sum}} = \sum_{k=i}^j p_k$.*

Proof The correctness of the algorithm follows directly by induction on $j - i$. When $j - i = 1$, we have only one job in a single batch, and thus $TWT_{i,j}^r = w(i, j) = OPT(i, j, r)$. Now consider arbitrary values of i, j and r , and suppose the statement is true for all pair (i', j') with $j' - i' < j - i$.

Let $\mathcal{B} = \{B_1, \dots, B_{\eta}\}$ be an optimal solution with total weighted tardiness $OPT(i, j, r)$ and initial batch $B_1 = \{\pi_{[i]}, \dots, \pi_{[\ell]}\}$ with completion time $C_{i,\ell}^r$.

The algorithm uses recurrence (1) to compute $TWT_{i,j}^r$ by minimizing

$$TWT_{i,j}^r = \min_{i \leq k \leq j} \left(w(i, k) + TWT_{k+1,j}^{C_{i,k}^r} \right) \leq \left(w(i, \ell) + TWT_{\ell+1,j}^{C_{i,\ell}^r} \right)$$

Since $j - (\ell + 1) < j - i$, and using the induction hypothesis, we know that $TWT_{\ell+1,j}^{C_{i,\ell}^r} = OPT(\ell + 1, j, C_{i,\ell}^r)$. Therefore, we have that $TWT_{i,j}^r \leq w(i, \ell) + OPT(\ell + 1, j, C_{i,\ell}^r) = OPT(i, j, r)$, and thus $TWT_{i,j}^r$ is an optimal value.

Let $r_{\max} = \max_{k=1}^j r_k$ and $p_{\text{sum}} = \sum_{k=i}^j p_k$. The proposed dynamic programming can be solved in $\mathcal{O}(n^3 \max(r_{\max}, p_{\text{sum}}))$. \square

It is obvious that by removing the first batch B_1 from the optimal partitioning, the remaining solution set must be an optimal solution to the subproblem defined by release date $r = C(B_1)$ and jobs set $\{\pi_{[|B_1|+1]}, \dots, \pi_{[n]}\}$. Any other choice may increase the optimal solution value. Hence, $\text{TWTBSP}_{\text{seq}}$ has the property of an optimal substructure needed for applying a dynamic programming approach.

Appendix D : Detailed results obtained by $\text{ILS}_{\text{batch}}$

This appendix reports the detailed results found by $\text{ILS}_{\text{batch}}$ on each individual instance. More precisely, we report the minimum (Min) and average (Avg) TWT values among the 10 runs, as well as the CPU time in seconds. Moreover, we also report the best known solution (BKS) found after performing a long run of the algorithm, in which a **time limit of 1 h was imposed as a stopping criterion**, instead of the maximum number of restarts. Finally, we provide the lower bounds (LBs) made available in Pessoa et al. (2020). Bold values indicate that the optimal solution was found, whereas underlined values indicate that the BKS is better than the minimum value found during the regular runs (Tables 12 and 13).

Table 12 Detailed results of ILS_{batch} with $Q = 20$

Instance	LB	Min	Avg	Time (s)	BKS
Q20_10I_G1_1	404.00	404	404.00	0.04	404
Q20_10I_G1_2	186.00	186	186.00	0.02	186
Q20_10I_G1_3	209.00	209	209.00	0.03	209
Q20_10I_G1_4	186.00	186	186.00	0.02	186
Q20_10I_G1_5	448.00	448	448.00	0.03	448
Q20_10I_G2_1	382.00	382	382.00	0.04	382
Q20_10I_G2_2	384.00	384	384.00	0.06	384
Q20_10I_G2_3	486.00	486	486.00	0.04	486
Q20_10I_G2_4	545.00	545	545.00	0.05	545
Q20_10I_G2_5	207.00	207	207.00	0.02	207
Q20_10I_G3_1	249.00	249	249.00	0.04	249
Q20_10I_G3_2	66.00	66	66.00	0.04	66
Q20_10I_G3_3	37.00	37	37.00	0.04	37
Q20_10I_G3_4	177.00	177	177.00	0.03	177
Q20_10I_G3_5	87.00	87	87.00	0.04	87
Q20_10I_G4_1	481.00	481	481.00	0.04	481
Q20_10I_G4_2	1432.00	1432	1432.00	0.03	1432
Q20_10I_G4_3	257.00	257	257.00	0.02	257
Q20_10I_G4_4	368.00	368	368.00	0.03	368
Q20_10I_G4_5	330.00	330	330.00	0.04	330

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_10I_G5_1	308.00	308	308.00	0.04	308
Q20_10I_G5_2	463.00	463	463.00	0.05	463
Q20_10I_G5_3	389.00	389	389.00	0.04	389
Q20_10I_G5_4	476.00	476	476.00	0.03	476
Q20_10I_G5_5	386.00	386	386.00	0.03	386
Q20_10I_G6_1	850.00	850	850.00	0.03	850
Q20_10I_G6_2	289.00	289	289.00	0.05	289
Q20_10I_G6_3	271.00	271	271.00	0.05	271
Q20_10I_G6_4	126.00	126	126.00	0.04	126
Q20_10I_G6_5	302.00	302	302.00	0.03	302
Q20_10I_G7_1	1234.00	1234	1234.00	0.03	1234
Q20_10I_G7_2	422.00	422	422.00	0.04	422
Q20_10I_G7_3	401.00	401	401.00	0.04	401
Q20_10I_G7_4	882.00	882	882.00	0.02	882
Q20_10I_G7_5	621.00	621	621.00	0.04	621
Q20_10I_G8_1	366.00	366	366.00	0.03	366
Q20_10I_G8_2	125.00	125	125.00	0.03	125
Q20_10I_G8_3	531.00	531	531.00	0.03	531
Q20_10I_G8_4	281.00	281	281.00	0.05	281

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_10I_G8_5	376.00	376	376.00	0.03	376
Q20_10I_G9_1	504.00	504	504.00	0.04	504
Q20_10I_G9_2	526.00	526	526.00	0.05	526
Q20_10I_G9_3	653.00	653	653.00	0.03	653
Q20_10I_G9_4	373.00	373	373.00	0.03	373
Q20_10I_G9_5	237.00	237	237.00	0.04	237
Q20_15I_G1_1	774.00	774	774.00	0.08	774
Q20_15I_G1_2	785.00	785	785.00	0.08	785
Q20_15I_G1_3	417.00	417	417.00	0.09	417
Q20_15I_G1_4	1236.00	1236	1236.00	0.08	1236
Q20_15I_G1_5	764.00	764	764.00	0.08	764
Q20_15I_G2_1	408.00	408	408.00	0.11	408
Q20_15I_G2_2	554.00	554	554.00	0.11	554
Q20_15I_G2_3	575.00	575	575.00	0.08	575
Q20_15I_G2_4	262.00	262	262.00	0.09	262
Q20_15I_G2_5	549.00	549	549.00	0.08	549
Q20_15I_G3_1	406.00	406	406.00	0.13	406
Q20_15I_G3_2	390.00	390	390.00	0.14	390
Q20_15I_G3_3	443.00	443	443.00	0.09	443

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_15J_G3_4	781.00	781	781.00	0.11	781
Q20_15J_G3_5	592.00	592	592.00	0.08	592
Q20_15J_G4_1	1567.00	1567	1567.00	0.09	1567
Q20_15J_G4_2	1714.00	1714	1714.00	0.11	1714
Q20_15J_G4_3	1164.00	1164	1164.00	0.11	1164
Q20_15J_G4_4	1685.00	1685	1685.00	0.13	1685
Q20_15J_G4_5	2448.00	2448	2448.00	0.12	2448
Q20_15J_G5_1	872.00	872	872.00	0.13	872
Q20_15J_G5_2	1564.00	1564	1564.00	0.11	1564
Q20_15J_G5_3	1268.00	1268	1268.00	0.13	1268
Q20_15J_G5_4	440.00	440	440.00	0.12	440
Q20_15J_G5_5	1805.00	1805	1805.00	0.10	1805
Q20_15J_G6_1	586.00	586	586.00	0.13	586
Q20_15J_G6_2	763.00	763	763.00	0.14	763
Q20_15J_G6_3	707.00	707	707.00	0.11	707
Q20_15J_G6_4	692.00	692	692.00	0.13	692
Q20_15J_G6_5	598.00	598	598.00	0.10	598

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_15I_G7_1	1847.00	1847	1847.00	0.10	1847
Q20_15I_G7_2	1841.00	1841	1841.00	0.12	1841
Q20_15I_G7_3	1742.00	1742	1742.00	0.09	1742
Q20_15I_G7_4	1450.00	1450	1450.00	0.11	1450
Q20_15I_G7_5	1949.00	1949	1949.00	0.10	1949
Q20_15I_G8_1	887.00	887	887.00	0.09	887
Q20_15I_G8_2	2219.00	2219	2219.00	0.14	2219
Q20_15I_G8_3	898.00	898	898.00	0.12	898
Q20_15I_G8_4	1317.00	1317	1317.00	0.11	1317
Q20_15I_G8_5	1402.00	1402	1402.00	0.11	1402
Q20_15I_G9_1	275.00	275	275.00	0.10	275
Q20_15I_G9_2	346.00	346	346.00	0.10	346
Q20_15I_G9_3	907.00	907	907.00	0.10	907
Q20_15I_G9_4	636.00	636	636.00	0.11	636
Q20_15I_G9_5	1110.00	1110	1110.00	0.11	1110
Q20_20I_G1_1	1547.00	1547	1547.00	0.17	1547
Q20_20I_G1_2	2268.00	2268	2268.00	0.19	2268
Q20_20I_G1_3	1499.00	1499	1499.00	0.15	1499

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_20J_G1_4	1532.00	1532	1532.00	0.17	1532
Q20_20J_G1_5	1164.00	1164	1164.00	0.14	1164
Q20_20J_G2_1	1078.00	1078	1078.00	0.20	1078
Q20_20J_G2_2	652.00	652	652.00	0.24	652
Q20_20J_G2_3	977.00	977	977.00	0.18	977
Q20_20J_G2_4	949.00	949	949.00	0.20	949
Q20_20J_G2_5	1868.35	1915	1915.00	0.17	1915
Q20_20J_G3_1	416.00	416	416.00	0.35	416
Q20_20J_G3_2	273.00	273	273.00	0.29	273
Q20_20J_G3_3	907.00	907	907.00	0.25	907
Q20_20J_G3_4	585.00	585	585.00	0.23	585
Q20_20J_G3_5	1120.00	1120	1120.00	0.21	1120
Q20_20J_G4_1	3965.41	4017	4017.00	0.22	4017
Q20_20J_G4_2	5664.00	5664	5664.00	0.29	5664

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_20I_G4_3	3601.47	3645	3645.00	0.24	3645
Q20_20I_G4_4	4095.00	4095	4095.00	0.17	4095
Q20_20I_G4_5	4322.55	4369	4369.00	0.25	4369
Q20_20I_G5_1	1079.44	1089	1089.00	0.25	1089
Q20_20I_G5_2	2006.78	2051	2051.00	0.30	2051
Q20_20I_G5_3	2920.00	2920	2920.00	0.29	2920
Q20_20I_G5_4	2202.65	2214	2214.00	0.23	2214
Q20_20I_G5_5	999.00	999	999.00	0.24	999
Q20_20I_G6_1	1515.00	1515	1515.00	0.34	1515
Q20_20I_G6_2	904.00	904	904.00	0.26	904
Q20_20I_G6_3	1615.00	1615	1615.00	0.24	1615
Q20_20I_G6_4	450.00	450	450.00	0.33	450
Q20_20I_G6_5	508.00	508	508.00	0.31	508
Q20_20I_G7_1	4715.83	4733	4733.00	0.23	4733
Q20_20I_G7_2	2576.00	2576	2576.00	0.21	2576
Q20_20I_G7_3	2705.82	2808	2808.00	0.19	2808
Q20_20I_G7_4	1357.00	1357	1357.00	0.26	1357
Q20_20I_G7_5	3985.00	3985	3985.00	0.21	3985
Q20_20I_G8_1	2259.83	2324	2324.00	0.26	2324
Q20_20I_G8_2	1879.00	1879	1879.00	0.27	1879

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_20I_G8_3	3293.00	3293	3293.00	0.34	3293
Q20_20I_G8_4	2059.85	2062	2062.00	0.21	2062
Q20_20I_G8_5	1388.00	1388	1388.00	0.31	1388
Q20_20I_G9_1	1752.00	1752	1752.00	0.23	1752
Q20_20I_G9_2	703.00	703	703.00	0.29	703
Q20_20I_G9_3	708.00	708	708.00	0.28	708
Q20_20I_G9_4	427.00	427	427.00	0.28	427
Q20_20I_G9_5	1244.00	1244	1244.00	0.27	1244
Q20_25I_G1_1	3120.45	3134	3134.00	0.37	3134
Q20_25I_G1_2	1852.00	1852	1852.00	0.35	1852
Q20_25I_G1_3	3642.54	3734	3738.70	0.34	3734
Q20_25I_G1_4	1905.48	1939	1939.10	0.30	1939
Q20_25I_G1_5	2628.47	2706	2706.00	0.30	2706
Q20_25I_G2_1	2442.00	2442	2442.00	0.43	2442
Q20_25I_G2_2	1408.00	1408	1408.00	0.34	1408
Q20_25I_G2_3	502.00	502	502.00	0.41	502
Q20_25I_G2_4	1625.50	1647	1647.00	0.29	1647
Q20_25I_G2_5	2033.39	2068	2068.00	0.33	2068
Q20_25I_G3_1	1216.00	1216	1216.00	0.36	1216
Q20_25I_G3_2	604.00	604	604.00	0.36	604

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_25J_G3_3	442.00	442	442.00	0.60	442
Q20_25J_G3_4	931.00	931	931.00	0.55	931
Q20_25J_G3_5	1562.12	1569	1569.00	0.68	1569
Q20_25J_G4_1	4612.01	4706	4706.00	0.45	4706
Q20_25J_G4_2	4394.92	4456	4456.00	0.41	4456
Q20_25J_G4_3	4751.00	4758	4758.00	0.54	4758
Q20_25J_G4_4	4687.00	4687	4687.00	0.45	4687
Q20_25J_G4_5	5132.00	5132	5132.00	0.58	5132
Q20_25J_G5_1	3462.85	3529	3529.00	0.55	3529
Q20_25J_G5_2	1927.60	1999	1999.00	0.71	1999
Q20_25J_G5_3	1185.06	1253	1253.00	0.86	1253
Q20_25J_G5_4	4499.00	4499	4499.00	0.49	4499
Q20_25J_G5_5	1399.00	1399	1399.00	0.53	1399
Q20_25J_G6_1	1249.00	1249	1249.00	0.74	1249
Q20_25J_G6_2	913.50	960	960.00	0.60	960
Q20_25J_G6_3	1214.96	1223	1223.00	0.57	1223
Q20_25J_G6_4	1585.00	1585	1585.00	0.71	1585
Q20_25J_G6_5	3327.00	3327	3327.00	0.37	3327
Q20_25J_G7_1	5641.00	5641	5641.00	0.55	5641
Q20_25J_G7_2	5592.00	5592	5592.00	0.38	5592

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_25I_G7_3	5835.00	5835	5835.00	0.49	5835
Q20_25I_G7_4	5043.00	5043	5043.00	0.54	5043
Q20_25I_G7_5	5896.27	5968	5973.10	0.44	5968
Q20_25I_G8_1	2662.16	2675	2675.00	0.65	2675
Q20_25I_G8_2	2523.00	2523	2523.00	0.40	2523
Q20_25I_G8_3	4459.00	4459	4459.00	0.48	4459
Q20_25I_G8_4	1582.88	1730	1730.00	0.59	1730
Q20_25I_G8_5	2111.88	2195	2195.00	0.54	2195
Q20_25I_G9_1	1933.87	1937	1937.00	0.54	1937
Q20_25I_G9_2	2128.00	2128	2128.00	0.53	2128
Q20_25I_G9_3	1670.00	1670	1670.00	0.50	1670
Q20_25I_G9_4	4842.88	4933	4933.00	0.66	4933
Q20_25I_G9_5	2355.43	2391	2391.00	0.67	2391
Q20_30I_G1_1	2271.77	2327	2327.00	0.55	2327
Q20_30I_G1_2	2311.12	2332	2332.00	0.60	2332
Q20_30I_G1_3	3744.52	3819	3819.60	0.57	3819
Q20_30I_G1_4	3597.13	3691	3691.00	0.57	3691

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_30I_G1_5	3920.55	3969	3969.00	0.71	3969
Q20_30I_G2_1	2855.00	2894	2894.00	0.51	2894
Q20_30I_G2_2	3704.98	3818	3818.00	0.71	3818
Q20_30I_G2_3	1608.83	1629	1629.00	0.52	1629
Q20_30I_G2_4	3645.70	3765	3769.00	0.58	3765
Q20_30I_G2_5	2418.60	2466	2466.00	0.69	2466
Q20_30I_G3_1	1948.87	1995	1995.00	0.77	1995
Q20_30I_G3_2	1643.00	1643	1643.00	0.90	1643
Q20_30I_G3_3	808.00	808	808.00	0.68	808
Q20_30I_G3_4	2544.21	2651	2661.20	0.51	2651
Q20_30I_G3_5	2022.00	2022	2022.00	0.71	2022
Q20_30I_G4_1	7739.45	7775	7775.00	0.79	7775
Q20_30I_G4_2	5782.81	5836	5836.00	0.89	5836
Q20_30I_G4_3	7228.12	7312	7312.00	0.83	7312
Q20_30I_G4_4	4365.64	4481	4481.00	0.88	4481
Q20_30I_G4_5	9453.54	9462	9462.00	0.96	9462
Q20_30I_G5_1	5090.00	5090	5090.00	0.82	5090

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_30J_G5_2	7796.99	7817	7817.00	0.90	7817
Q20_30J_G5_3	3218.93	3395	3395.00	0.93	3395
Q20_30J_G5_4	3942.97	4070	4070.00	0.91	4070
Q20_30J_G5_5	4230.00	4230	4230.00	1.03	4230
Q20_30J_G6_1	2621.35	2790	2790.00	1.00	2790
Q20_30J_G6_2	1444.00	1444	1444.00	1.09	1444
Q20_30J_G6_3	2290.00	2290	2290.00	1.10	2290
Q20_30J_G6_4	1439.11	1476	1476.00	1.57	1476
Q20_30J_G6_5	2395.80	2516	2516.00	1.39	2516
Q20_30J_G7_1	6036.27	6107	6107.00	0.66	6107
Q20_30J_G7_2	7355.29	7497	7497.00	0.74	7497
Q20_30J_G7_3	6980.23	7052	7052.00	0.69	7052
Q20_30J_G7_4	5457.00	5457	5457.00	0.72	5457
Q20_30J_G7_5	7509.09	7774	7774.00	0.86	7774
Q20_30J_G8_1	3064.14	3266	3266.00	1.11	3266
Q20_30J_G8_2	4459.86	4528	4528.00	1.10	4528
Q20_30J_G8_3	4844.22	4911	4911.00	1.21	4911
Q20_30J_G8_4	4995.50	5019	5019.00	1.02	5019
Q20_30J_G8_5	5398.00	5398	5398.00	0.97	5398
Q20_30J_G9_1	4082.00	4082	4082.00	1.29	4082

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_30I_G9_2	576.00	576	576.00	0.85	576
Q20_30I_G9_3	1572.00	1572	1572.00	1.06	1572
Q20_30I_G9_4	5016.65	5171	5172.20	0.97	5171
Q20_30I_G9_5	2952.41	2980	2980.00	0.83	2980
Q20_35I_G1_1	3938.30	4070	4070.00	0.85	4070
Q20_35I_G1_2	5216.53	5360	5360.00	0.82	5360
Q20_35I_G1_3	3517.23	3609	3609.00	0.71	3609
Q20_35I_G1_4	4218.33	4232	4249.20	0.60	4232
Q20_35I_G1_5	4674.58	4761	4761.00	0.76	4761
Q20_35I_G2_1	4315.00	4315	4315.00	0.94	4315
Q20_35I_G2_2	1850.10	1898	1898.00	0.91	1898
Q20_35I_G2_3	1879.01	1925	1925.00	1.36	1925
Q20_35I_G2_4	2094.92	2131	2131.00	0.78	2131
Q20_35I_G2_5	1382.50	1464	1464.00	1.20	1464
Q20_35I_G3_1	1912.00	1912	1912.00	1.13	1912
Q20_35I_G3_2	1540.95	1609	1614.10	1.34	1609
Q20_35I_G3_3	2351.35	2466	2466.00	1.53	2466
Q20_35I_G3_4	1854.31	1926	1926.00	1.25	1926
Q20_35I_G3_5	1463.77	1507	1507.00	1.30	1507

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_35J_G4_1	6724.78	6755	6755.00	1.55	6755
Q20_35J_G4_2	11516.64	11653	11653.00	1.28	11653
Q20_35J_G4_3	11692.16	12017	12017.00	1.10	12017
Q20_35J_G4_4	8033.12	8066	8066.00	1.89	8066
Q20_35J_G4_5	7553.38	7595	7595.00	1.18	7595
Q20_35J_G5_1	7316.81	7449	7449.00	2.57	7449
Q20_35J_G5_2	6853.34	6897	6897.00	1.85	6897
Q20_35J_G5_3	5621.89	5837	5837.00	1.73	5837
Q20_35J_G5_4	8311.50	8510	8510.00	1.65	8510
Q20_35J_G5_5	3541.09	3637	3637.00	1.94	3637
Q20_35J_G6_1	2979.66	3048	3048.00	2.35	3048
Q20_35J_G6_2	3540.00	3540	3540.00	1.99	3540
Q20_35J_G6_3	2824.37	2833	2833.00	1.69	2833
Q20_35J_G6_4	1688.00	1688	1688.00	2.07	1688
Q20_35J_G6_5	2052.18	2174	2174.00	2.39	2174

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_35J_G7_1	14032.21	14074	14074.00	1.22	14074
Q20_35J_G7_2	5803.85	5914	5914.00	1.05	5914
Q20_35J_G7_3	9162.08	9238	9263.60	1.83	9238
Q20_35J_G7_4	9816.21	9850	9850.00	1.20	9850
Q20_35J_G7_5	8487.00	8487	8487.00	1.05	8487
Q20_35J_G8_1	4589.93	4726	4726.00	1.70	4726
Q20_35J_G8_2	7124.79	7285	7285.00	1.81	7285
Q20_35J_G8_3	9885.65	9919	9919.00	1.76	9919
Q20_35J_G8_4	5928.15	5945	5945.00	1.58	5945
Q20_35J_G8_5	4813.24	4923	4923.00	1.51	4923
Q20_35J_G9_1	1966.93	2270	2270.00	2.01	2270
Q20_35J_G9_2	755.00	755	755.00	1.97	755
Q20_35J_G9_3	10,048.54	10,175	10,175.00	2.77	10,175
Q20_35J_G9_4	1773.73	1868	1868.00	1.92	1868

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_35I_G9_5	3461.78	3555	3555.00	2.09	3555
Q20_50I_G1_1	9265.91	9346	9349.60	3.13	9346
Q20_50I_G1_2	8322.07	8742	8744.70	3.00	8742
Q20_50I_G1_3	10,639.50	10,811	10,873.70	3.18	10,811
Q20_50I_G1_4	9059.63	9218	9241.60	3.12	9218
Q20_50I_G1_5	7537.95	7568	7604.60	3.03	7568
Q20_50I_G2_1	5411.79	5548	5551.30	3.13	5548
Q20_50I_G2_2	5810.84	6127	6151.40	4.35	<u>6097</u>
Q20_50I_G2_3	7873.95	8059	8059.00	3.84	8059
Q20_50I_G2_4	5716.57	5978	5979.50	3.73	5978
Q20_50I_G2_5	7703.91	7843	7895.80	3.10	7843
Q20_50I_G3_1	2012.23	2179	2179.00	4.60	2179
Q20_50I_G3_2	2948.04	3283	3283.00	5.11	3283
Q20_50I_G3_3	2124.59	2179	2179.00	4.20	2179
Q20_50I_G3_4	3038.06	3204	3207.90	5.34	3204

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_50I_G3_5	4174.54	4329	4333.60	4.43	4329
Q20_50I_G4_1	14,424.91	14,498	14,498.00	4.68	14,498
Q20_50I_G4_2	25,677.00	25,677	25,677.00	5.93	25,677
Q20_50I_G4_3	19,041.69	19,051	19,051.00	6.16	19,051
Q20_50I_G4_4	16,485.77	16,733	16,735.50	5.50	16,733
Q20_50I_G4_5	19,402.73	19,799	19,799.00	6.95	19,799
Q20_50I_G5_1	14,392.37	14,589	14,589.00	6.97	14,589
Q20_50I_G5_2	12,165.83	12,308	12,319.20	6.26	12,308
Q20_50I_G5_3	8137.89	8312	8312.00	5.55	8312
Q20_50I_G5_4	13,764.82	14,067	14,079.60	8.03	14,067
Q20_50I_G5_5	11,400.01	11,475	11,475.00	6.34	11,475
Q20_50I_G6_1	5322.65	5557	5558.00	7.47	5557
Q20_50I_G6_2	4424.02	4688	4688.00	7.50	4688
Q20_50I_G6_3	4666.45	4954	4954.00	7.67	4954
Q20_50I_G6_4	7728.42	8108	8108.00	8.29	8108
Q20_50I_G6_5	3097.16	3217	3217.00	7.26	3217
Q20_50I_G7_1	16,801.30	16,873	16,874.40	5.91	16,873
Q20_50I_G7_2	15,149.01	15,376	15,408.20	6.48	15,376
Q20_50I_G7_3	25,315.83	25,350	25,350.00	5.40	25,350
Q20_50I_G7_4	11,275.05	11,530	11,530.00	5.52	11,530

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_50J_G7_5	23,685.90	23,777	23,777.00	5.65	23,777
Q20_50J_G8_1	10,704.01	11,043	11,043.00	7.66	11,043
Q20_50J_G8_2	11,661.24	11,790	11,790.00	5.01	11,790
Q20_50J_G8_3	11,119.40	11,372	11,372.00	6.62	11,372
Q20_50J_G8_4	10,672.38	10,988	10,990.10	6.49	10,988
Q20_50J_G8_5	12,568.27	12,684	12,722.30	6.14	12,684
Q20_50J_G9_1	3081.25	3188	3188.00	7.15	3188
Q20_50J_G9_2	6266.25	6474	6530.20	8.13	6474
Q20_50J_G9_3	6977.10	7087	7113.40	8.00	7087
Q20_50J_G9_4	6065.58	6117	6117.00	10.51	6117
Q20_50J_G9_5	1890.05	1952	1952.90	7.94	1952
Q20_75J_G1_1	21,006.61	21,284	21,317.50	17.88	21,267
Q20_75J_G1_2	24,809.55	25,285	25,332.80	12.30	25,201
Q20_75J_G1_3	20,346.40	20,702	20,777.30	12.93	20,668
Q20_75J_G1_4	25,821.72	26,327	26,464.60	15.95	26,210
Q20_75J_G1_5	19,665.88	20,129	20,232.40	13.71	20,129
Q20_75J_G2_1	12,436.07	13,012	13,093.00	16.47	12,911
Q20_75J_G2_2	12,785.29	13,339	13,420.20	15.55	13,339

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_75I_G2_3	10,012.05	10,373	10,514.60	15.91	10,373
Q20_75I_G2_4	10,811.24	11,369	11,416.30	14.16	11,369
Q20_75I_G2_5	8604.05	9174	9268.50	19.33	9174
Q20_75I_G3_1	4835.50	5214	5214.00	22.05	5214
Q20_75I_G3_2	3546.02	3684	3685.50	23.69	3684
Q20_75I_G3_3	6324.97	6827	6924.30	21.06	6827
Q20_75I_G3_4	8576.14	9152	9223.20	25.17	9152
Q20_75I_G3_5	8218.97	8633	8660.00	20.69	8633
Q20_75I_G4_1	41,151.80	41,213	41,213.00	29.69	41,213
Q20_75I_G4_2	51,809.91	52,242	52,310.10	29.86	52,242
Q20_75I_G4_3	54,095.38	54,298	54,298.00	30.13	54,298

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_75J_G4_4	52,083.60	52,318	52,335.40	29.50	52,318
Q20_75J_G4_5	52,130.15	52,478	52,550.80	26.73	52,464
Q20_75J_G5_1	34,092.92	34,361	34,404.80	36.53	34,361
Q20_75J_G5_2	22,518.71	23,019	23,105.10	29.36	23,019
Q20_75J_G5_3	26,024.72	26,275	26,275.00	40.31	26,275
Q20_75J_G5_4	19,665.02	20,172	20,278.80	35.70	20,172
Q20_75J_G5_5	21,033.18	21,317	21,345.40	36.74	21,317
Q20_75J_G6_1	16,790.88	17,267	17,311.00	47.18	17,260
Q20_75J_G6_2	8500.12	8816	8816.00	43.98	8816
Q20_75J_G6_3	9749.39	9864	9864.00	40.81	9864
Q20_75J_G6_4	6685.74	7201	7204.80	33.46	7201
Q20_75J_G6_5	5365.48	5736	5808.80	45.32	5736
Q20_75J_G7_1	40,197.51	40,482	40,637.90	26.87	40,482
Q20_75J_G7_2	43,122.41	43,432	43,489.50	36.45	43,432

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_75J_G7_3	30,333.38	30,948	31,027.70	24.36	30,858
Q20_75J_G7_4	42,021.37	42,611	42,720.30	23.55	42,611
Q20_75J_G7_5	36,871.77	37,387	37,418.80	20.26	37,337
Q20_75J_G8_1	28,561.78	29,340	29,429.20	34.85	29,340
Q20_75J_G8_2	17,224.53	17,614	17,623.70	32.64	17,614
Q20_75J_G8_3	35,597.16	36,518	36,581.00	35.82	36,414
Q20_75J_G8_4	26,034.53	26,407	26,440.90	36.60	26,407
Q20_75J_G8_5	24,773.59	25,416	25,434.40	31.72	25,416
Q20_75J_G9_1	6908.53	7028	7028.00	35.88	7028
Q20_75J_G9_2	5212.60	5616	5616.00	39.60	5616
Q20_75J_G9_3	10,221.89	10,710	10,771.70	34.96	10,710
Q20_75J_G9_4	8544.75	8934	8962.20	36.72	8934
Q20_75J_G9_5	9894.28	10,211	10,211.00	34.54	10,211
Q20_100J_G1_1	35,205.10	36,109	36,278.60	40.37	35,930
Q20_100J_G1_2	39,092.30	40,318	40,612.50	40.01	39,985
Q20_100J_G1_3	40,960.58	41,823	41,947.40	47.90	41,519
Q20_100J_G1_4	37,365.52	38,459	38,556.20	38.00	38,254
Q20_100J_G1_5	34,809.20	35,731	35,903.00	42.85	35,663
Q20_100J_G2_1	29,342.03	30,461	30,587.70	53.20	30,453

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_100J_G2_2	23,717.58	24,749	25,312.90	52.75	<u>24,639</u>
Q20_100J_G2_3	21,804.13	22,584	22,902.20	52.24	<u>22,477</u>
Q20_100J_G2_4	23,276.94	24,671	25,049.40	50.76	24,671
Q20_100J_G2_5	24,742.71	25,921	26,182.20	47.12	<u>25,818</u>
Q20_100J_G3_1	7449.83	8510	8573.20	68.84	<u>8510</u>
Q20_100J_G3_2	4524.81	5107	5127.50	59.08	<u>5061</u>
Q20_100J_G3_3	11,635.56	12,902	13,263.10	59.21	<u>12,871</u>
Q20_100J_G3_4	6644.86	7481	7514.80	69.93	<u>7438</u>
Q20_100J_G3_5	10,561.57	11,999	12,147.30	76.91	<u>11,910</u>
Q20_100J_G4_1	94,699.32	95,150	95,157.00	87.46	<u>95,150</u>
Q20_100J_G4_2	65,523.59	66,298	66,439.30	83.84	<u>66,235</u>
Q20_100J_G4_3	69,671.64	70,065	70,149.60	91.51	<u>70,037</u>
Q20_100J_G4_4	81,881.55	82,578	82,714.40	93.42	<u>82,578</u>
Q20_100J_G4_5	82,965.40	83,416	83,529.60	90.41	<u>83,416</u>
Q20_100J_G5_1	46,313.53	47,466	47,555.60	93.12	<u>47,434</u>
Q20_100J_G5_2	31,651.30	32,600	32,961.30	121.42	<u>32,600</u>
Q20_100J_G5_3	51,635.75	52,786	52,933.40	109.51	<u>52,786</u>
Q20_100J_G5_4	69,372.41	70,025	70,335.70	98.78	<u>70,025</u>
Q20_100J_G5_5	37,169.67	38,135	38,348.80	102.74	<u>38,043</u>
Q20_100J_G6_1	13,005.06	13,788	13,848.60	122.34	<u>13,788</u>

Table 12 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q20_100J_G6_2	17,459.60	18,514	18,646.00	146.67	18,446
Q20_100J_G6_3	15,609.43	17,071	17,168.20	118.92	17,044
Q20_100J_G6_4	8873.67	9374	9436.40	135.80	9374
Q20_100J_G6_5	15,666.43	16,485	16,537.10	156.52	16,408
Q20_100J_G7_1	72,195.10	72,602	72,806.60	103.34	72,602
Q20_100J_G7_2	53,706.02	54,496	54,745.80	74.62	54,387
Q20_100J_G7_3	98,158.55	98,760	98,965.30	86.66	98,760
Q20_100J_G7_4	82,930.39	83,776	83,944.70	77.15	83,706
Q20_100J_G7_5	78,394.74	79,203	79,406.00	90.13	79,099
Q20_100J_G8_1	54,473.10	55,451	55,593.90	128.55	55,451
Q20_100J_G8_2	57,422.98	58,513	58,700.70	135.24	58,513
Q20_100J_G8_3	27,337.01	28,079	28,168.60	102.65	28,079
Q20_100J_G8_4	44,579.69	45,507	45,846.90	108.55	45,478
Q20_100J_G8_5	34,191.92	35,205	35,253.00	93.30	35,204
Q20_100J_G9_1	9109.82	10,054	10,175.10	122.22	10,054
Q20_100J_G9_2	22,536.43	23,794	23,796.40	146.15	23,786
Q20_100J_G9_3	11,880.43	13,196	13,255.50	122.17	13,196
Q20_100J_G9_4	9168.29	9696	9700.80	117.56	9696
Q20_100J_G9_5	10,893.26	11,712	11,712.00	126.59	11,712

Table 13 Detailed results of ILS_{batch} with $Q = 40$

Instance	LB	Min	Avg	Time (s)	BKS
Q40_10I_G1_1	754.00	754	754.00	0.03	754
Q40_10I_G1_2	351.00	351	351.00	0.03	351
Q40_10I_G1_3	287.00	287	287.00	0.02	287
Q40_10I_G1_4	428.00	428	428.00	0.02	428
Q40_10I_G1_5	235.00	235	235.00	0.03	235
Q40_10I_G2_1	228.00	228	228.00	0.04	228
Q40_10I_G2_2	109.00	109	109.00	0.03	109
Q40_10I_G2_3	493.00	493	493.00	0.04	493
Q40_10I_G2_4	230.00	230	230.00	0.03	230
Q40_10I_G2_5	374.00	374	374.00	0.04	374
Q40_10I_G3_1	159.00	159	159.00	0.04	159
Q40_10I_G3_2	126.00	126	126.00	0.06	126
Q40_10I_G3_3	50.00	50	50.00	0.02	50
Q40_10I_G3_4	323.00	323	323.00	0.06	323
Q40_10I_G3_5	185.00	185	185.00	0.06	185
Q40_10I_G4_1	174.00	174	174.00	0.03	174
Q40_10I_G4_2	248.00	248	248.00	0.03	248
Q40_10I_G4_3	241.00	241	241.00	0.03	241
Q40_10I_G4_4	297.00	297	297.00	0.03	297

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_10I_G4_5	848.00	848	848.00	0.03	848
Q40_10I_G5_1	317.00	317	317.00	0.03	317
Q40_10I_G5_2	576.00	576	576.00	0.03	576
Q40_10I_G5_3	885.00	885	885.00	0.04	885
Q40_10I_G5_4	956.00	956	956.00	0.04	956
Q40_10I_G5_5	282.00	282	282.00	0.04	282
Q40_10I_G6_1	355.00	355	355.00	0.04	355
Q40_10I_G6_2	453.00	453	453.00	0.05	453
Q40_10I_G6_3	370.00	370	370.00	0.05	370
Q40_10I_G6_4	213.00	213	213.00	0.04	213
Q40_10I_G6_5	337.00	337	337.00	0.03	337
Q40_10I_G7_1	990.00	990	990.00	0.04	990
Q40_10I_G7_2	629.00	629	629.00	0.02	629
Q40_10I_G7_3	1402.00	1402	1402.00	0.02	1402
Q40_10I_G7_4	280.00	280	280.00	0.02	280
Q40_10I_G8_5	433.00	433	433.00	0.03	433
Q40_10I_G9_1	171.00	171	171.00	0.02	171
Q40_10I_G9_2	153.00	153	153.00	0.02	153
Q40_10I_G9_3	474.00	474	474.00	0.03	474
Q40_10I_G9_4	261.00	261	261.00	0.03	261

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_10J_G7_5	795.00	795	795.00	0.02	795
Q40_10J_G8_1	504.00	504	504.00	0.02	504
Q40_10J_G8_2	546.00	546	546.00	0.02	546
Q40_10J_G8_3	389.00	389	389.00	0.04	389
Q40_10J_G8_4	656.00	656	656.00	0.02	656
Q40_10J_G9_5	371.00	371	371.00	0.05	371
Q40_15J_G1_1	1388.00	1388	1388.00	0.07	1388
Q40_15J_G1_2	898.00	898	898.00	0.07	898
Q40_15J_G1_3	1199.00	1199	1199.00	0.11	1199
Q40_15J_G1_4	933.00	933	933.00	0.09	933
Q40_15J_G1_5	693.00	693	693.00	0.08	693
Q40_15J_G2_1	769.00	769	769.00	0.10	769
Q40_15J_G2_2	351.00	351	351.00	0.06	351
Q40_15J_G2_3	858.00	858	858.00	0.11	858
Q40_15J_G2_4	517.00	517	517.00	0.08	517
Q40_15J_G2_5	1205.00	1205	1205.00	0.10	1205
Q40_15J_G3_1	912.00	912	912.00	0.13	912
Q40_15J_G3_2	1247.00	1247	1247.00	0.12	1247

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_15I_G3_3	366.00	366	366.00	0.12	366
Q40_15I_G3_4	91.00	91	91.00	0.16	91
Q40_15I_G3_5	364.00	364	364.00	0.11	364
Q40_15I_G4_1	2555.00	2555	2555.00	0.13	2555
Q40_15I_G4_2	3719.00	3719	3719.00	0.11	3719
Q40_15I_G4_3	2009.00	2009	2009.00	0.10	2009
Q40_15I_G4_4	1025.00	1025	1025.00	0.10	1025
Q40_15I_G4_5	1730.00	1730	1730.00	0.08	1730
Q40_15I_G5_1	1196.00	1196	1196.00	0.09	1196
Q40_15I_G5_2	1337.00	1337	1337.00	0.08	1337
Q40_15I_G5_3	923.00	923	923.00	0.09	923
Q40_15I_G5_4	937.00	937	937.00	0.11	937
Q40_15I_G5_5	899.00	899	899.00	0.08	899
Q40_15I_G6_1	329.00	329	329.00	0.14	329
Q40_15I_G6_2	366.00	366	366.00	0.11	366
Q40_15I_G6_3	1895.00	1895	1895.00	0.13	1895
Q40_15I_G6_4	693.00	693	693.00	0.11	693
Q40_15I_G6_5	415.00	415	415.00	0.12	415
Q40_15I_G7_1	2007.00	2007	2007.00	0.10	2007
Q40_15I_G7_2	1303.00	1303	1303.00	0.09	1303

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_15J_G7_3	1009.00	1009	1009.00	0.10	1009
Q40_15J_G7_4	1048.00	1048	1048.00	0.12	1048
Q40_15J_G7_5	1428.00	1428	1428.00	0.07	1428
Q40_15J_G8_1	1669.00	1669	1669.00	0.09	1669
Q40_15J_G8_2	1613.00	1613	1613.00	0.12	1613
Q40_15J_G8_3	1800.00	1800	1800.00	0.13	1800
Q40_15J_G8_4	1282.00	1282	1282.00	0.09	1282
Q40_15J_G8_5	1487.00	1487	1487.00	0.08	1487
Q40_15J_G9_1	106.00	106	106.00	0.11	106
Q40_15J_G9_2	832.00	832	832.00	0.12	832
Q40_15J_G9_3	419.00	419	419.00	0.09	419
Q40_15J_G9_4	2203.00	2203	2203.00	0.10	2203
Q40_15J_G9_5	327.00	327	327.00	0.14	327
Q40_20J_G1_1	1308.67	1324	1324.00	0.21	1324
Q40_20J_G1_2	1885.00	1885	1885.00	0.16	1885
Q40_20J_G1_3	1170.00	1170	1170.00	0.16	1170
Q40_20J_G1_4	1935.00	1935	1935.00	0.15	1935
Q40_20J_G1_5	1898.00	1898	1898.00	0.25	1898
Q40_20J_G2_1	597.00	597	597.00	0.21	597

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_20I_G2_2	1446.00	1446	1446.00	0.20	1446
Q40_20I_G2_3	1036.00	1036	1036.00	0.24	1036
Q40_20I_G2_4	935.00	935	935.00	0.24	935
Q40_20I_G2_5	1236.00	1236	1236.00	0.17	1236
Q40_20I_G3_1	391.00	391	391.00	0.31	391
Q40_20I_G3_2	608.00	608	608.00	0.34	608
Q40_20I_G3_3	684.00	684	684.00	0.29	684
Q40_20I_G3_4	1326.00	1326	1326.00	0.37	1326
Q40_20I_G3_5	645.00	645	645.00	0.29	645
Q40_20I_G4_1	3446.00	3446	3446.00	0.26	3446
Q40_20I_G4_2	2710.91	2734	2734.00	0.25	2734
Q40_20I_G4_3	2834.00	2834	2834.00	0.31	2834
Q40_20I_G4_4	3802.00	3802	3802.00	0.23	3802
Q40_20I_G4_5	4499.90	4541	4541.00	0.20	4541
Q40_20I_G5_1	1931.00	1931	1931.00	0.25	1931
Q40_20I_G5_2	2395.00	2395	2395.00	0.27	2395

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_20I_G5_3	4309.00	4309	4309.00	0.20	4309
Q40_20I_G5_4	2045.00	2045	2045.00	0.24	2045
Q40_20I_G5_5	2093.00	2093	2093.00	0.21	2093
Q40_20I_G6_1	1088.00	1088	1088.00	0.22	1088
Q40_20I_G6_2	1368.00	1368	1368.00	0.26	1368
Q40_20I_G6_3	1793.00	1793	1793.00	0.28	1793
Q40_20I_G6_4	828.00	828	828.00	0.29	828
Q40_20I_G6_5	2044.00	2044	2044.00	0.22	2044
Q40_20I_G7_1	1798.00	1798	1798.00	0.16	1798
Q40_20I_G7_2	2540.00	2540	2540.00	0.18	2540
Q40_20I_G7_3	3026.00	3026	3026.00	0.20	3026
Q40_20I_G7_4	4320.07	4413	4413.00	0.25	4413
Q40_20I_G7_5	4302.00	4302	4302.00	0.21	4302
Q40_20I_G8_1	1250.00	1250	1250.00	0.25	1250
Q40_20I_G8_2	2226.86	2267	2267.00	0.23	2267
Q40_20I_G8_3	1792.00	1792	1792.00	0.26	1792
Q40_20I_G8_4	1572.00	1572	1572.00	0.23	1572
Q40_20I_G8_5	1569.00	1569	1569.00	0.29	1569
Q40_20I_G9_1	1562.00	1562	1562.00	0.23	1562

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_20J_G9_2	1266.00	1266	1266.00	0.28	1266
Q40_20J_G9_3	535.00	535	535.00	0.36	535
Q40_20J_G9_4	909.00	909	909.00	0.26	909
Q40_20J_G9_5	1097.00	1097	1097.00	0.28	1097
Q40_25J_G1_1	3422.00	3422	3422.00	0.32	3422
Q40_25J_G1_2	1972.48	2015	2021.40	0.34	2015
Q40_25J_G1_3	2058.97	2083	2083.00	0.30	2083
Q40_25J_G1_4	2873.65	2922	2922.00	0.33	2922
Q40_25J_G1_5	2096.16	2131	2131.00	0.36	2131
Q40_25J_G2_1	1681.19	1704	1704.00	0.44	1704
Q40_25J_G2_2	1287.00	1378	1378.00	0.32	1378
Q40_25J_G2_3	1535.80	1584	1584.00	0.37	1584
Q40_25J_G2_4	2369.00	2369	2369.00	0.36	2369
Q40_25J_G2_5	1917.87	1945	1945.00	0.34	1945
Q40_25J_G3_1	947.53	948	948.00	0.56	948
Q40_25J_G3_2	1230.12	1238	1238.00	0.39	1238
Q40_25J_G3_3	971.24	979	979.00	0.54	979
Q40_25J_G3_4	476.00	476	476.00	0.59	476
Q40_25J_G3_5	1092.61	1095	1095.00	0.42	1095
Q40_25J_G4_1	5640.00	5640	5640.00	0.41	5640

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_25I_G4_2	3916.00	3916	3916.00	0.52	3916
Q40_25I_G4_3	5613.67	5752	5752.00	0.36	5752
Q40_25I_G4_4	5732.50	5802	5802.00	0.50	5802
Q40_25I_G4_5	6456.67	6477	6477.00	0.50	6477
Q40_25I_G5_1	1821.00	1821	1821.00	0.44	1821
Q40_25I_G5_2	3676.00	3676	3676.00	0.41	3676
Q40_25I_G5_3	4428.00	4440	4440.00	0.38	4440
Q40_25I_G5_4	6856.00	6856	6856.00	0.44	6856
Q40_25I_G5_5	2057.06	2158	2158.00	0.39	2158
Q40_25I_G6_1	1154.00	1154	1154.00	0.52	1154
Q40_25I_G6_2	2008.00	2008	2008.00	0.68	2008
Q40_25I_G6_3	2069.90	2077	2077.00	0.62	2077
Q40_25I_G6_4	1303.00	1303	1303.00	0.56	1303
Q40_25I_G6_5	1264.00	1264	1264.00	0.70	1264
Q40_25I_G7_1	4318.00	4326	4326.00	0.56	4326
Q40_25I_G7_2	5287.00	5302	5302.00	0.35	5302
Q40_25I_G7_3	6318.02	6393	6393.00	0.45	6393
Q40_25I_G7_4	3289.58	3408	3408.00	0.37	3408

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_25J_G7_5	3753.14	3784	3784.00	0.51	3784
Q40_25J_G8_1	3867.31	3906	3906.00	0.56	3906
Q40_25J_G8_2	2474.67	2498	2498.00	0.40	2498
Q40_25J_G8_3	3283.00	3283	3283.00	0.73	3283
Q40_25J_G8_4	4753.00	4870	4870.00	0.51	4870
Q40_25J_G8_5	2821.70	2996	2996.00	0.65	2996
Q40_25J_G9_1	1387.00	1387	1387.00	0.57	1387
Q40_25J_G9_2	1579.00	1579	1579.00	0.63	1579
Q40_25J_G9_3	825.00	825	825.00	0.55	825
Q40_25J_G9_4	472.00	472	472.00	0.67	472
Q40_25J_G9_5	740.00	740	740.00	0.58	740
Q40_30J_G1_1	5986.89	6133	6133.00	0.67	6133
Q40_30J_G1_2	5294.63	5498	5498.00	0.53	5498

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_30J_G1_3	4068.00	4068	4068.00	0.48	4068
Q40_30J_G1_4	4738.74	4785	4785.00	0.65	4785
Q40_30J_G1_5	3949.00	3949	3949.00	0.41	3949
Q40_30J_G2_1	2475.79	2574	2574.00	0.78	2574
Q40_30J_G2_2	3429.65	3515	3516.90	0.92	3515
Q40_30J_G2_3	2609.53	2670	2670.00	0.58	2670
Q40_30J_G2_4	2110.63	2167	2167.00	0.85	2167
Q40_30J_G2_5	2383.49	2467	2467.00	0.78	2467
Q40_30J_G3_1	982.96	1045	1045.00	0.85	1045
Q40_30J_G3_2	2232.48	2325	2326.00	1.08	2325
Q40_30J_G3_3	3073.62	3138	3138.00	0.98	3138
Q40_30J_G3_4	1471.83	1508	1508.00	0.85	1508
Q40_30J_G3_5	2103.22	2121	2121.00	0.91	2121
Q40_30J_G4_1	9104.64	9118	9118.00	0.73	9118
Q40_30J_G4_2	7485.63	7812	7812.00	0.77	7812
Q40_30J_G4_3	7169.76	7196	7196.00	0.78	7196
Q40_30J_G4_4	7680.00	7680	7680.00	0.63	7680
Q40_30J_G4_5	10,097.80	10,112	10,112.00	0.79	10,112

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_30I_G5_1	4881.00	4881	4881.00	1.17	4881
Q40_30I_G5_2	4666.62	4686	4686.00	1.36	4686
Q40_30I_G5_3	3890.24	3979	3979.00	0.89	3979
Q40_30I_G5_4	6078.50	6159	6159.00	1.25	6159
Q40_30I_G5_5	5620.00	5620	5620.00	1.06	5620
Q40_30I_G6_1	2595.80	2762	2762.00	1.34	2762
Q40_30I_G6_2	2652.00	2652	2652.00	1.22	2652
Q40_30I_G6_3	1064.92	1074	1074.00	1.08	1074
Q40_30I_G6_4	1429.00	1429	1429.00	1.06	1429
Q40_30I_G6_5	2060.62	2086	2086.00	1.34	2086
Q40_30I_G7_1	7038.97	7119	7119.00	0.69	7119
Q40_30I_G7_2	4901.03	5045	5045.00	0.72	5045
Q40_30I_G7_3	6632.34	6689	6689.00	0.87	6689
Q40_30I_G7_4	5503.76	5528	5528.00	0.73	5528
Q40_30I_G7_5	4715.00	4742	4742.00	0.62	4742

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_30J_G8_1	4489.00	4489	4491.00	0.97	4489
Q40_30J_G8_2	3925.77	4018	4018.00	0.89	4018
Q40_30J_G8_3	10,042.50	10,158	10,158.00	0.83	10,158
Q40_30J_G8_4	5065.29	5091	5091.00	0.99	5091
Q40_30J_G8_5	6221.31	6337	6337.00	0.80	6337
Q40_30J_G9_1	2652.10	2666	2666.00	0.77	2666
Q40_30J_G9_2	1665.88	1795	1795.00	1.07	1795
Q40_30J_G9_3	1271.00	1335	1335.00	1.15	1335
Q40_30J_G9_4	2521.41	2722	2722.00	0.91	2722
Q40_30J_G9_5	2555.88	2590	2590.00	1.26	2590
Q40_35J_G1_1	3236.48	3302	3302.00	0.81	3302
Q40_35J_G1_2	3938.16	4068	4069.00	0.84	4068
Q40_35J_G1_3	5706.61	5833	5833.00	0.85	5833
Q40_35J_G1_4	3593.97	3677	3683.20	0.81	3677
Q40_35J_G1_5	4351.00	4372	4372.00	0.78	4372
Q40_35J_G2_1	2118.00	2118	2118.00	0.87	2118
Q40_35J_G2_2	3833.21	3962	3989.00	1.05	3962
Q40_35J_G2_3	2441.79	2446	2446.00	1.12	2446
Q40_35J_G2_4	1851.45	1895	1895.00	0.88	1895
Q40_35J_G2_5	3012.14	3026	3026.00	0.77	3026

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_35J_G3_1	2413.10	2558	2558.00	1.14	2558
Q40_35J_G3_2	1814.04	1832	1832.00	1.61	1832
Q40_35J_G3_3	2035.76	2094	2094.00	1.38	2094
Q40_35J_G3_4	1383.00	1383	1383.00	1.25	1383
Q40_35J_G3_5	2430.18	2443	2443.00	1.04	2443
Q40_35J_G4_1	7432.25	7434	7434.00	1.17	7434
Q40_35J_G4_2	6864.34	6973	6990.90	1.55	6973
Q40_35J_G4_3	7443.55	7499	7499.00	2.00	7499
Q40_35J_G4_4	14,458.45	14,655	14,655.00	1.48	14,655
Q40_35J_G4_5	11,870.34	11,946	11,946.00	1.68	11,946
Q40_35J_G5_1	9108.26	9243	9243.00	2.28	9243
Q40_35J_G5_2	5405.36	5725	5745.00	1.94	5725
Q40_35J_G5_3	5616.00	5616	5616.00	1.24	5616
Q40_35J_G5_4	8968.19	9224	9224.00	1.74	9224
Q40_35J_G5_5	5901.34	6388	6388.00	1.59	6388
Q40_35J_G6_1	2322.93	2351	2351.00	2.49	2351
Q40_35J_G6_2	2409.15	2465	2465.00	1.91	2465
Q40_35J_G6_3	2990.33	3197	3197.00	1.81	3197
Q40_35J_G6_4	1475.88	1485	1485.00	2.34	1485
Q40_35J_G6_5	4903.30	4958	4958.00	1.54	4958

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_35J_G7_1	10,440.00	10,440	10,440.00	1.33	10,440
Q40_35J_G7_2	12,627.42	12,643	12,643.00	1.13	12,643
Q40_35J_G7_3	12,897.52	13,146	13,146.00	1.18	13,146
Q40_35J_G7_4	9444.74	9661	9661.00	1.54	9661
Q40_35J_G7_5	7792.00	7792	7792.00	0.89	7792
Q40_35J_G8_1	5468.24	5488	5488.00	1.34	5488
Q40_35J_G8_2	5633.46	5681	5681.00	1.96	5681
Q40_35J_G8_3	7376.22	7566	7566.00	1.55	7566
Q40_35J_G8_4	2639.94	2675	2675.00	1.74	2675
Q40_35J_G8_5	5117.39	5295	5295.60	1.90	5295
Q40_35J_G9_1	2247.71	2265	2265.00	2.06	2265
Q40_35J_G9_2	3597.15	3663	3663.00	1.75	3663
Q40_35J_G9_3	2205.00	2205	2205.00	2.01	2205
Q40_35J_G9_4	6301.65	6399	6399.00	2.05	6399
Q40_35J_G9_5	5328.67	5349	5349.00	1.97	5349
Q40_50J_G1_1	14,410.94	14,525	14,540.50	3.78	14,525
Q40_50J_G1_2	8280.14	8499	8511.40	3.67	8499
Q40_50J_G1_3	8698.47	8710	8710.00	4.52	8710

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_50I_G1_4	10,395.25	10,784	10,784.00	3.61	10,784
Q40_50I_G1_5	11,059.75	11,329	11,334.20	4.87	11,329
Q40_50I_G2_1	10,446.85	10,484	10,484.00	4.16	10,484
Q40_50I_G2_2	10,578.43	10,853	10,865.40	4.23	10,853
Q40_50I_G2_3	5071.90	5434	5453.00	5.55	<u>5419</u>
Q40_50I_G2_4	11,135.62	11,180	11,180.00	3.93	11,180
Q40_50I_G2_5	10,290.48	10,448	10,448.00	4.86	10,448
Q40_50I_G3_1	5833.38	6025	6057.00	5.31	6025
Q40_50I_G3_2	2400.82	2451	2451.00	5.41	2451
Q40_50I_G3_3	4281.56	4324	4324.00	5.11	4324
Q40_50I_G3_4	3250.95	3452	3452.00	4.93	3452
Q40_50I_G3_5	5609.98	5837	5837.00	5.23	5837
Q40_50I_G4_1	14,605.43	14,692	14,717.60	5.49	14,692
Q40_50I_G4_2	18,192.93	18,287	18,287.00	4.81	18,287
Q40_50I_G4_3	20,605.27	20,903	20,910.00	6.40	20,903
Q40_50I_G4_4	22,971.60	23,093	23,093.00	6.54	23,093

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_50I_G4_5	24,063.57	24,165	24,190.00	4.77	24,165
Q40_50I_G5_1	20,666.70	20,931	20,944.00	6.42	20,931
Q40_50I_G5_2	12,297.78	12,524	12,543.00	6.70	12,524
Q40_50I_G5_3	9019.91	9284	9284.00	6.51	9284
Q40_50I_G5_4	11,306.79	11,326	11,341.20	7.60	11,326
Q40_50I_G5_5	18,352.25	18,501	18,501.00	6.33	18,501
Q40_50I_G6_1	5021.03	5097	5097.00	6.88	5097
Q40_50I_G6_2	6135.38	6262	6262.00	7.32	6262
Q40_50I_G6_3	4994.12	5296	5296.00	7.99	5296
Q40_50I_G6_4	6021.53	6246	6246.60	8.92	6246
Q40_50I_G6_5	7116.81	7175	7175.00	8.90	7175
Q40_50I_G7_1	17,911.03	18,019	18,019.00	6.56	18,019
Q40_50I_G7_2	22,211.55	22,535	22,535.00	5.25	22,535
Q40_50I_G7_3	24,433.00	24,433	24,433.00	5.30	24,433
Q40_50I_G7_4	20,237.79	20,449	20,449.00	5.89	20,449
Q40_50I_G7_5	13,226.42	13,390	13,390.00	4.87	13,390

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_50J_G8_1	14,109.55	14,382	14,403.60	5.56	14,382
Q40_50J_G8_2	10,387.36	10,542	10,542.00	4.75	10,542
Q40_50J_G8_3	11,870.00	11,870	11,870.00	5.51	11,870
Q40_50J_G8_4	16,124.11	16,458	16,467.10	6.64	16,458
Q40_50J_G8_5	11,259.84	11,467	11,467.00	7.27	11,467
Q40_50J_G9_1	5423.22	5916	5947.20	6.80	5916
Q40_50J_G9_2	5961.71	6301	6301.00	5.73	6301
Q40_50J_G9_3	2179.73	2645	2647.10	7.20	2645
Q40_50J_G9_4	5862.69	6027	6037.70	7.07	6027
Q40_50J_G9_5	3660.05	4096	4097.60	6.61	4096
Q40_75J_G1_1	30,049.34	30,527	30,637.80	17.23	<u>30,452</u>
Q40_75J_G1_2	32,982.90	33,499	33,598.60	15.32	<u>33,418</u>
Q40_75J_G1_3	30,418.73	30,893	31,020.30	15.57	<u>30,826</u>
Q40_75J_G1_4	28,324.26	29,019	29,109.60	19.20	29,019
Q40_75J_G1_5	27,558.80	28,042	28,209.10	16.10	28,042
Q40_75J_G2_1	25,842.55	26,470	26,581.90	19.25	<u>26,251</u>
Q40_75J_G2_2	13,302.13	13,642	13,648.40	19.73	<u>13,642</u>
Q40_75J_G2_3	15,969.72	16,643	16,749.50	22.07	<u>16,587</u>
Q40_75J_G2_4	10,556.54	11,155	11,242.70	20.71	<u>11,155</u>
Q40_75J_G2_5	14,900.19	15,523	15,640.10	21.17	<u>15,472</u>

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_75J_G3_1	7369.94	7814	8181.50	27.04	<u>7691</u>
Q40_75J_G3_2	7263.16	7597	7622.70	25.95	7597
Q40_75J_G3_3	6421.98	6976	7118.40	26.61	6976
Q40_75J_G3_4	5801.39	6183	6202.30	24.73	6183
Q40_75J_G3_5	9733.85	9855	9928.50	27.47	9855
Q40_75J_G4_1	40,135.43	40,225	40,241.00	27.45	40,225
Q40_75J_G4_2	47,976.01	48,177	48,177.00	27.55	48,177
Q40_75J_G4_3	41,313.83	41,390	41,396.00	27.19	41,390
Q40_75J_G4_4	52,103.01	52,217	52,310.50	25.83	52,217
Q40_75J_G4_5	46,359.91	46,605	46,607.30	24.94	46,605
Q40_75J_G5_1	25,362.66	25,676	25,683.10	41.08	25,676
Q40_75J_G5_2	34,122.89	34,465	34,488.10	30.67	<u>34,457</u>
Q40_75J_G5_3	36,176.96	36,416	36,516.30	37.52	36,416
Q40_75J_G5_4	23,283.05	23,501	23,501.00	33.15	23,501

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_75I_G5_5	23,799.69	24,088	24,088.00	35.84	24,088
Q40_75I_G6_1	5770.82	6095	6095.00	36.22	6095
Q40_75I_G6_2	9883.89	10,353	10,353.00	42.42	10,353
Q40_75I_G6_3	13,389.65	14,300	14,344.10	39.14	14,182
Q40_75I_G6_4	18,595.65	18,867	18,867.00	39.63	18,867
Q40_75I_G6_5	15,438.88	15,556	15,556.00	51.84	15,556
Q40_75I_G7_1	41,112.83	41,511	41,516.10	29.81	41,511
Q40_75I_G7_2	57,219.47	57,877	57,925.80	28.31	57,877
Q40_75I_G7_3	44,119.15	44,538	44,570.70	24.59	44,538
Q40_75I_G7_4	46,232.29	46,482	46,489.20	25.37	46,482
Q40_75I_G7_5	41,084.99	41,298	41,327.20	23.02	41,273
Q40_75I_G8_1	26,349.07	26,625	26,639.50	36.62	26,625
Q40_75I_G8_2	23,281.04	23,717	23,738.50	36.22	23,717
Q40_75I_G8_3	33,268.86	33,489	33,492.60	51.90	33,489
Q40_75I_G8_4	20,524.87	21,139	21,210.30	32.60	21,125

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_75J_G8_5	31,221.03	31,536	31,697.60	31.10	31,536
Q40_75J_G9_1	11,043.59	11,465	11,480.20	46.79	11,465
Q40_75J_G9_2	12,616.98	12,999	12,999.00	37.21	12,999
Q40_75J_G9_3	12,640.60	13,076	13,076.00	39.96	13,076
Q40_75J_G9_4	11,739.87	12,404	12,430.80	39.36	12,404
Q40_75J_G9_5	7154.89	7379	7380.90	37.28	7379
Q40_100J_G1_1	77.99	45,831	45,953.10	49.95	45,829
Q40_100J_G1_2	170.28	57,383	57,565.60	54.38	57,067
Q40_100J_G1_3	610.74	45,965	46,092.30	52.53	45,820
Q40_100J_G1_4	38,205.44	38,786	38,834.10	56.12	38,738
Q40_100J_G1_5	626.15	59,648	59,833.50	60.44	59,385
Q40_100J_G2_1	126.44	42,378	42,544.90	71.95	42,175
Q40_100J_G2_2	235.41	37,100	37,315.00	60.40	37,093
Q40_100J_G2_3	217.98	34,484	35,077.80	78.89	34,370

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_100J_G2_4	56.42	28,677	28,870.90	69.05	<u>28,676</u>
Q40_100J_G2_5	216.06	33,849	34,113.60	62.01	<u>33,820</u>
Q40_100J_G3_1	440.28	19,591	19,714.90	80.99	<u>19,512</u>
Q40_100J_G3_2	300.40	15,601	15,691.30	99.47	<u>15,601</u>
Q40_100J_G3_3	276.54	17,322	17,542.80	88.29	<u>17,242</u>
Q40_100J_G3_4	420.95	10,827	10,882.10	82.40	<u>10,827</u>
Q40_100J_G3_5	254.20	14,112	14,321.30	81.44	<u>14,112</u>
Q40_100J_G4_1	95,694.62	96,013	96,044.20	101.29	<u>96,013</u>
Q40_100J_G4_2	80,696.38	80,883	80,962.00	87.52	<u>80,883</u>
Q40_100J_G4_3	85,853.45	86,275	86,424.80	111.06	<u>86,275</u>
Q40_100J_G4_4	84,622.12	85,089	85,150.30	93.46	<u>85,089</u>
Q40_100J_G4_5	95,385.83	95,961	96,038.10	85.99	<u>95,961</u>
Q40_100J_G5_1	61,353.79	61,698	61,760.10	104.66	<u>61,698</u>
Q40_100J_G5_2	53,238.81	54,039	54,141.50	123.20	<u>53,968</u>
Q40_100J_G5_3	47,573.43	48,431	48,573.00	121.53	<u>48,431</u>
Q40_100J_G5_4	56,602.82	56,829	56,912.30	149.38	<u>56,829</u>
Q40_100J_G5_5	45,414.25	46,914	47,482.80	141.03	<u>46,914</u>
Q40_100J_G6_1	28,481.90	29,549	29,638.30	156.44	<u>29,549</u>

Table 13 continued

Instance	LB	Min	Avg	Time (s)	BKS
Q40_100J_G6_2	14,364.25	15,511	15,526.20	117.82	15,511
Q40_100J_G6_3	30,386.06	31,579	31,722.90	131.74	31,579
Q40_100J_G6_4	9202.30	10,339	10,358.00	129.65	10,339
Q40_100J_G6_5	18,256.67	19,147	19,165.00	165.29	19,147
Q40_100J_G7_1	89,822.18	90,452	90,484.20	103.96	90,452
Q40_100J_G7_2	59,880.09	60,536	60,714.40	96.80	60,377
Q40_100J_G7_3	86,484.81	87,341	87,370.70	109.89	87,341
Q40_100J_G7_4	74,908.15	75,441	75,690.90	92.40	75,426
Q40_100J_G7_5	68,383.69	69,176	69,272.50	99.24	69,084
Q40_100J_G8_1	26,097.73	27,559	27,578.00	113.81	27,559
Q40_100J_G8_2	49,530.86	50,868	51,002.80	115.92	50,868
Q40_100J_G8_3	1114.98	38,922	39,336.30	98.82	38,922
Q40_100J_G8_4	552.60	41,660	41,836.60	92.45	41,436
Q40_100J_G8_5	599.01	43,238	43,741.90	109.06	43,238
Q40_100J_G9_1	600.07	17,952	18,022.40	126.85	17,952
Q40_100J_G9_2	15,518.21	16,724	16,829.00	159.60	16,724
Q40_100J_G9_3	1137.03	24,371	24,472.40	127.36	24,364
Q40_100J_G9_4	363.94	4169	4169.00	119.67	4169
Q40_100J_G9_5	575.10	16,213	16,330.00	131.78	16,197

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