PROBLEM A - PATRILINEAL CLANS

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Problem Statement

- Bottleneck in genetic diversity among Y-chromosomes occured from 7000 BCE to 5000 BCE
- Competition among patrilineal clans as a potential reason
- Previous model assumes uniform group of females

Question: How do we model population dynamics of different patrilineal clans in conflict while taking into account females associated with the clans?

Model: Assumptions

- The populations tend toward a carrying capacity and a 50:50 gender ratio
- Growth rates are proportional to clan sizes
- Females have some preference to stay in their own clan or to seek better mating opportunities
- Males die from conflict at rates proportional to the size of the male populations in both clans
- Same population dynamics, carrying capacity, and conflict morbidity rates across clans

Simple Population Model

$$\frac{dF_1}{dt} = \overbrace{r_1 F_1 M_1 \left(1 - \frac{F_1}{K - M_1}\right)}^{\text{Logistic growth}} - \overbrace{r_2 F_1^2}^{\text{Turnover rate}}$$

$$\frac{dM_1}{dt} = r_1 F_1 M_1 \left(1 - \frac{M_1}{K - F_1}\right) - r_2 M_1^2$$

growth rate r_1 and turnover rate r_2

carrying capacity K

Updated Conflict Model

$$\frac{dF_1}{dt} = \overbrace{r_1 F_1 M_1 \left(1 - \frac{F_1}{K - M_1}\right)}^{\text{Logistic growth}} - \overbrace{r_2 F_1^2}^{\text{Turnover rate}} - \overbrace{c_1 F_1 M_2 + c_2 F_2 M_1}^{\text{Intermarriage}}$$

$$\frac{dM_1}{dt} = r_1 F_1 M_1 \left(1 - \frac{M_1}{K - F_1}\right) - r_2 M_1^2 - \underbrace{q\, M_1 M_2}_{\text{Conflict morbidity}}$$

growth rate r_1 and turnover rate r_2

carrying capacity K

conflict morbidity rate q

Intermarriage rates c_1 and c_2

Complete Model: Two Tribes

Logistic growth | Turnover rate | Intermarriage

$$\frac{dF_1}{dt} = r_1 F_1 M_1 \left(1 - \frac{F_1}{K - M_1}\right) - r_2 F_1^2 - c_1 F_1 M_2 + c_2 F_2 M_1$$

$$\frac{dF_2}{dt} = r_1 F_2 M_2 \left(1 - \frac{F_2}{K - M_2}\right) - r_2 F_2^2 - c_2 F_2 M_1 + c_1 F_1 M_2$$

Conflict morbidity

$$\frac{dM_1}{dt} = r_1 F_1 M_1 \left(1 - \frac{M_1}{K - F_1}\right) - r_2 M_1^2 - q\, M_1 M_2$$

$$\frac{dM_2}{dt} = r_1 F_2 M_2 \left(1 - \frac{M_2}{K - F_2}\right) - r_2 M_2^2 - q\, M_1 M_2$$

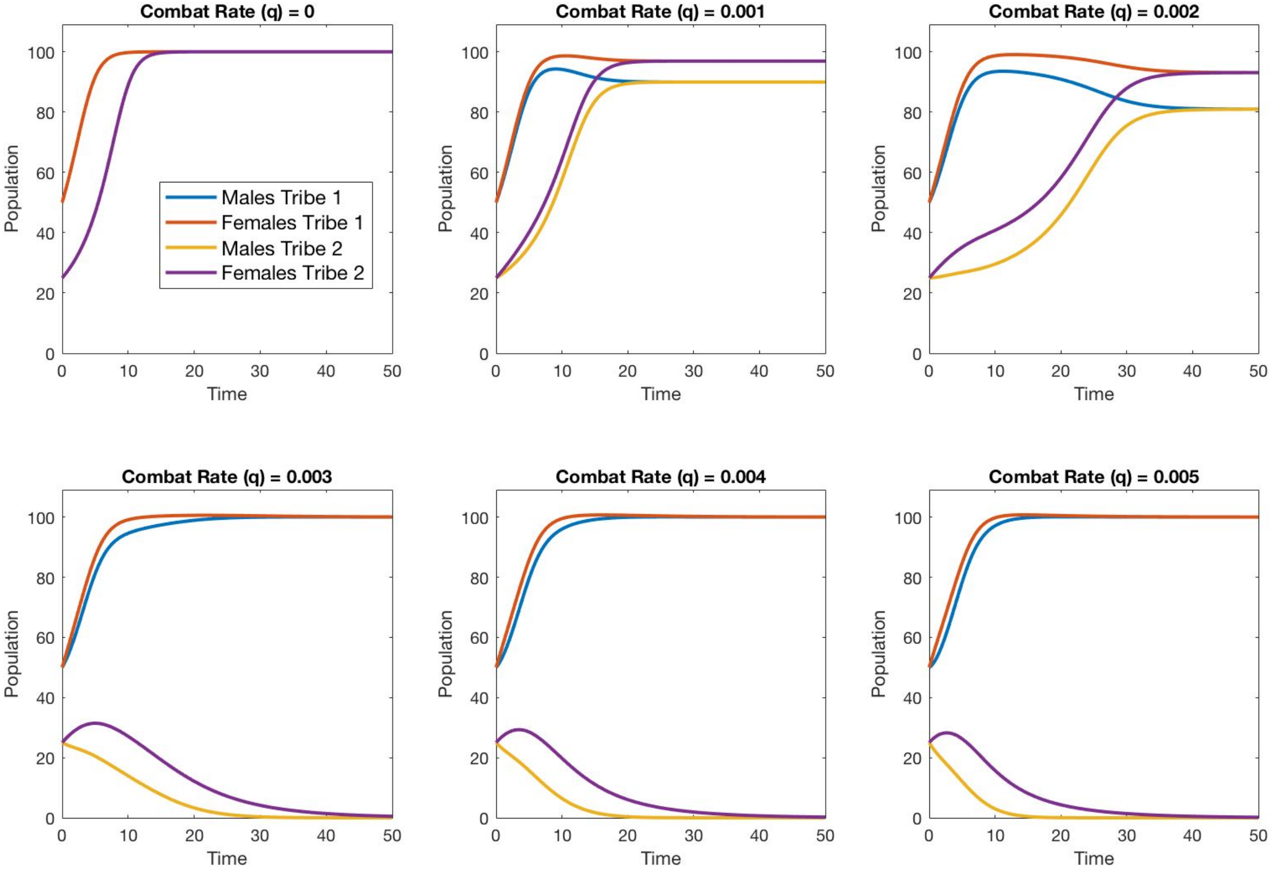
growth rate r_1 and turnover rate r_2

carrying capacity K

conflict morbidity rate q

Intermarriage rates c_1 and c_2

Solution & Analysis: Conflict



Solution curves with varying conflict morbidity rate (q)

And constant:

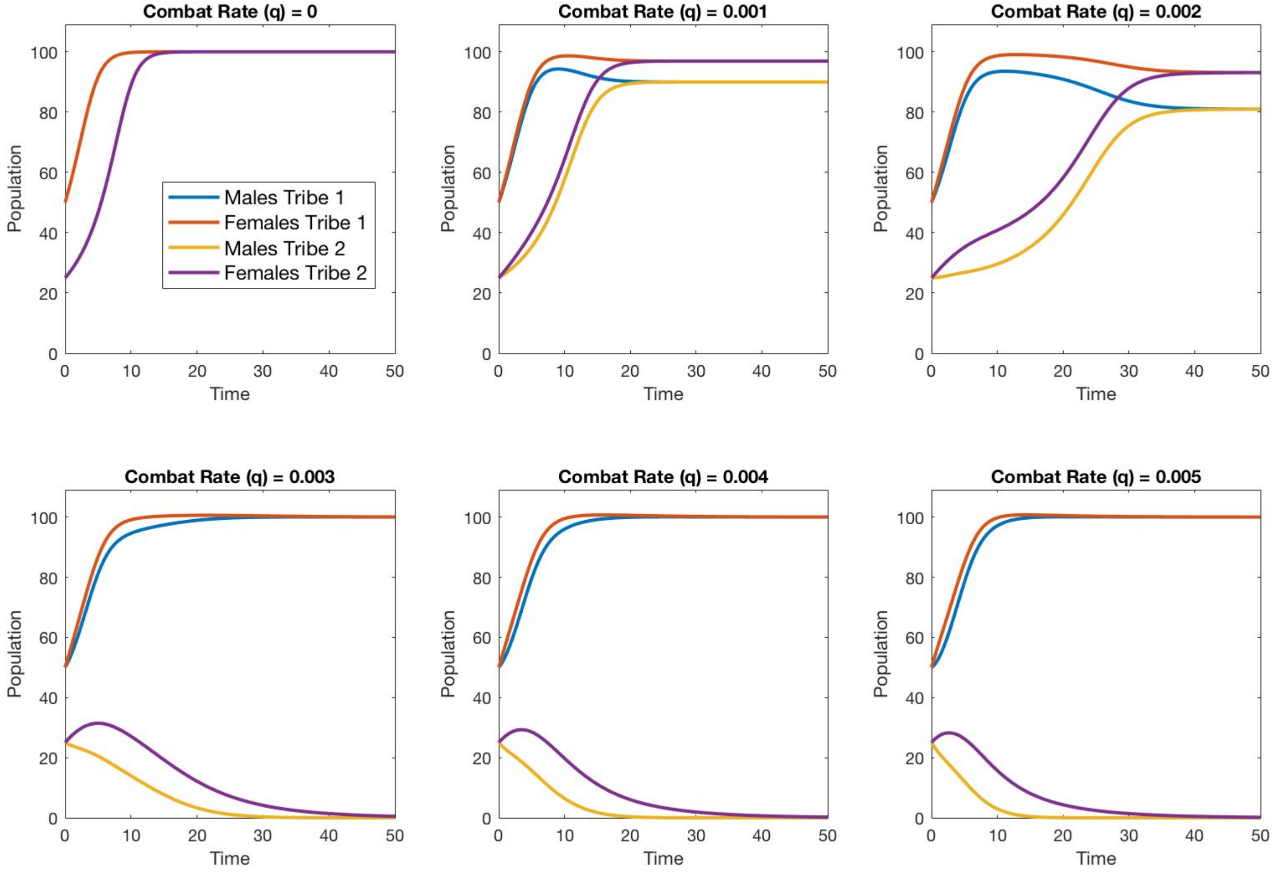
$r_1 = 0.01$

$r_2 = 0.0005$

$k = 300$

$c_1 = c_2 = 0.001$

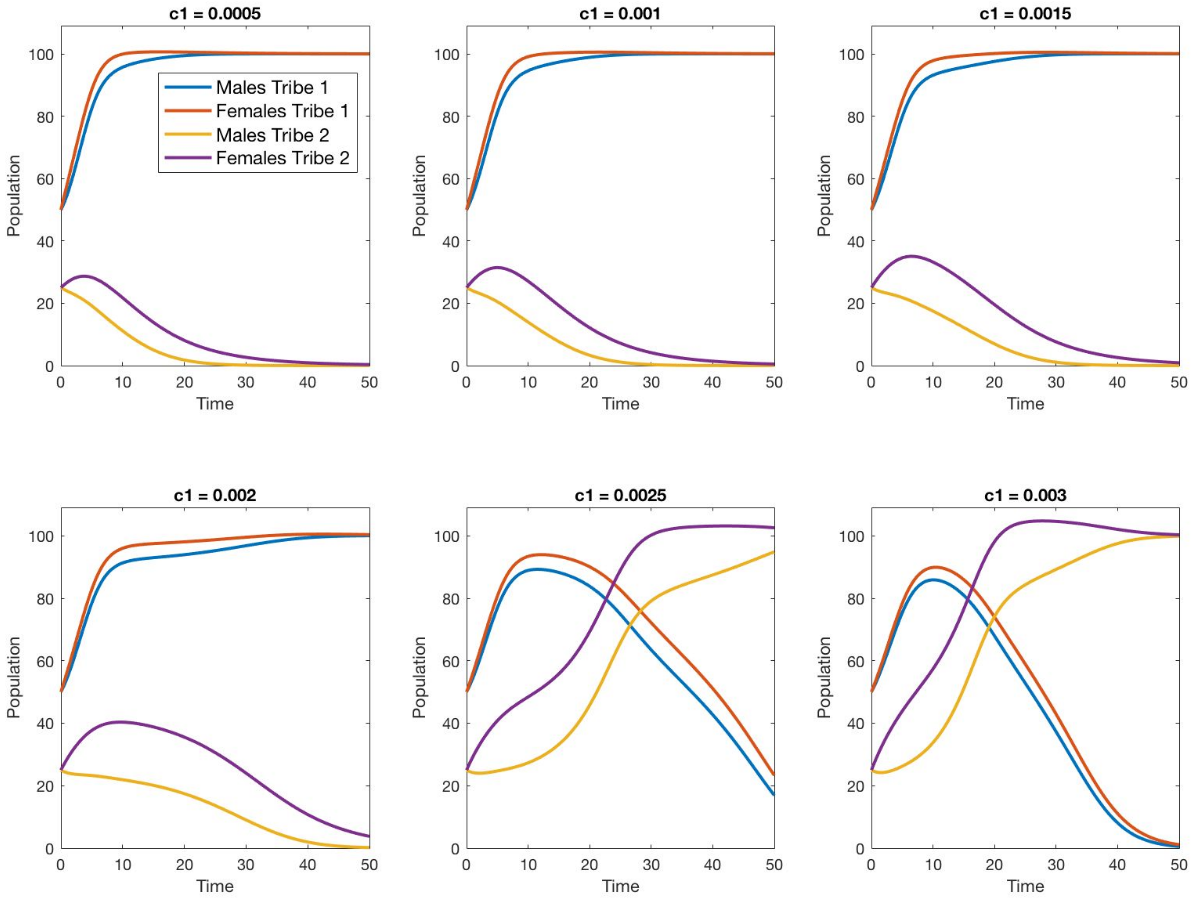
Solution & Analysis: Conflict



Our model does not show that male populations will always go extinct

Our model requires conflict morbidity rates to be sufficiently high to create genetic bottleneck

Solution & Analysis: Intermarriage



Changing the rate of intermarriage between F1 and M2 (c1)

Intermarriage between F2 and M1 is constant

Keeping other parameters constant

Limitations & Extensions

- Simplification of population dynamics
  - Groups could split apart or combine, etc.
- Potential variation in parameters between clans
  - Different conflict morbidity rates according to technology
  - Different growth rates and carrying capacities according to environment
- Analysis of model for more than one clan:

Logistic growth

Turnover rate

Intermarriage

$$\frac{dF_i}{dt} = \boxed{r_1 F_i M_i \left(1 - \frac{F_i}{K - M_i}\right)} - \boxed{r_2 F_i^2} - \boxed{\sum_{k=1,\, k \neq i}^{n} c_i F_i M_k + \sum_{k=1,\, k \neq i}^{n} c_k F_k M_i}$$

$$\frac{dM_i}{dt} = \boxed{r_1 F_i M_i \left(1 - \frac{M_i}{K_i - F_i}\right)} - \boxed{r_2 M_1^2} - \boxed{\sum_{k=1}^{n} q\, M_i M_k}$$

Conflict morbidity

Additional Issue - Increase in Mobility

Intermarriage: Female preference for staying or leaving clans

↑ Mobility → ↑ intermarriage

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Conflict morbidity:

↑ Mobility ➤ ↓ conflict morbidity if avoidant

↑ Mobility ➤ ↑ conflict morbidity if imperialist

↓ Mobility ➤ ↓ conflict morbidity - less interaction between groups

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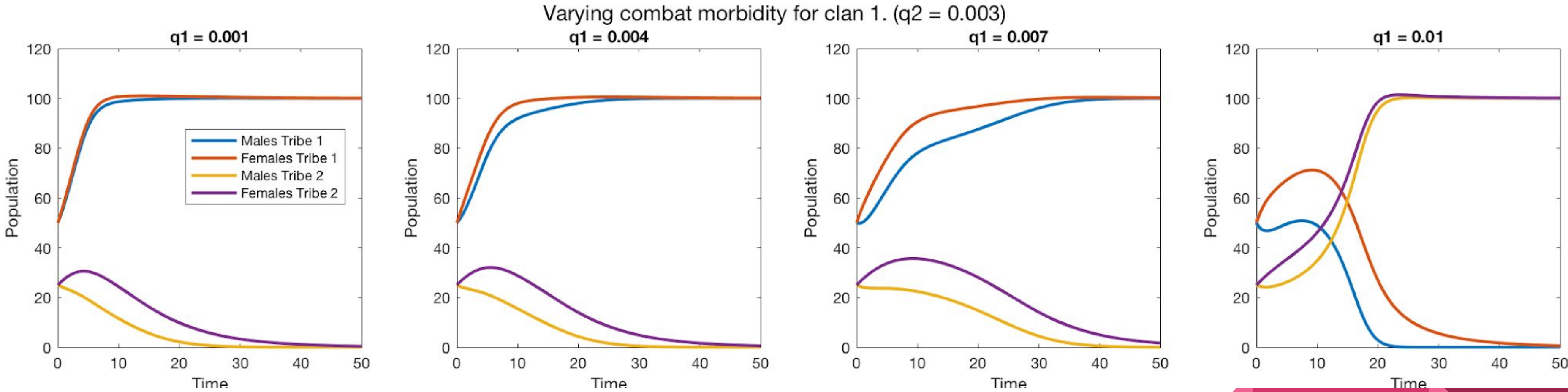
Carrying capacity:

↑ Mobility ➤ ↑ carrying capacity

Groups reaching capacity could move to find new resources

Additional Issue - Different Technologies

The smaller clan has domesticated horses; this gives them an advantage during conflict



Keeping clan 2's morbidity the same, increasing clan 1's morbidity rate - analogous to clan 2 having horses

Summary

- Developed a model for patrilineal clans during conflicts for both males and females population
- Extinction happens when conflict morbidity rate is high enough
- Intermarriage can alter the long-term outcomes of intense conflict
- The model can be extended to describe more than 2 clans

References

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Zeng, T. C., Aw, A. J., & Feldman, M. W. (2018). Cultural hitchhiking and competition between patrilineal kin groups explain the post-Neolithic Y-chromosome bottleneck. *Nature Communications*, *9*(1), 2077. https://doi.org/10.1038/s41467-018-04375-6