Legyenek A, B, C tetszőleges halmazok. Igazoljuk a következő állításokat.
(a) ha $A \subseteq C$ és $B \subseteq C$ akkor $A \cup B \subseteq C$
(b) ha $A \subseteq B$ és $A \subseteq C$ akkor $A \subseteq B \cap C$
(c) $A \cup (B \cap A) = A$

(a) A⊆C -> ha x∈A, akkor x∈C; B⊆C -> ha x∈B, akkor x∈C;
AUB = { x | x∈A v x∈B } => AUB = { x∈C | x∈A v x∈B } => AUB⊆C
(b) A⊆B -> ha x∈A, akkor x∈B; A⊆C -> ha x∈A, akkor x∈C;
B∩C = { x | x∈B ^ x∈C } => B∩C = { x∈A | x∈B ^ x∈C } => A⊆B∩C
(c) A U (B ∩ A) = { x | x∈A v x∈ (B ∩ A) } = { x | x∈A v (x∈B ^ x∈A) } =
= { x | (x∈A v x∈A) ^ (x∈A v x∈B) } = { x | x∈A ^ (x∈A v x∈B) } =
= { x | (x∈A ^ x∈A) v (x∈A ^ x∈B) } = { x | x∈A } = A

Legyen A és B nemüres halmazok. Igazolja a következő egyenlőségeket.
(a) $(A \setminus B) \cap B = \emptyset$
(b) $\left(A \cup \overline{B}\right) \cap \left(\overline{A} \cup \overline{B}\right) = \overline{B}$

(a) (A \ B) ∩ B = { x | x∈(A \ B) ∩ B } = { x | x∈(A \ B) ^ x∈B } =
= { x | (x∈A ^ x∉B) ^ x∈B } = { x | x∈A ^ x∉B ^ x∈B } = { }
(b)

$(A \cup \overline{B}) \cap (\overline{A} \cup \overline{B}) = \{ x \mid x \in (A \cup \overline{B}) \cap (\overline{A} \cup \overline{B}) \} =$
$= \{ x \mid (x \in A \vee x \notin B) \wedge (x \notin A \vee x \notin B) \} =$
$= \{ x \mid (x \in A \wedge x \notin A) \vee (x \in A \wedge x \notin B) \vee (x \notin B \wedge x \notin A) \vee (x \notin B \wedge x \notin B) \}$
$= \{ x \mid \{\} \vee (x \in A \setminus B) \vee \{\} \vee x \notin B \} = \overline{B}$

Legyenek A, B, C nemüres halmazok. Igazolja a következő egyenlőséget:
$(A \cup B) \times C = (A \times C) \cup (B \times C)$.

(A U B) × C = { (x,y) | x∈(A U B), y∈C } = { (x,y) | (x∈A v x∈B), y∈C } = { (x,y) | (x∈A ^ y∈C) v (x∈B ^ y∈C) } = { (x,y) | (x,y)∈(A × C) v (x,y)∈(B × C) } = (A × C) U (B × C)

Legyen az alaphalmaz U továbbá $A, B, C \subseteq U$ tetszőleges halmazok. Igazolja a következő egyenlőségeket.

(a) $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$

(b) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

(c) $A \setminus \ (A \setminus (B \setminus C)) = A \cap B \cap \overline{C}$

(a) $(A \cap B) \setminus C = \{ x \mid x\in(A \cap B) \setminus C \} = \{ x \mid (x\in A \wedge x\in B) \setminus C) \} =$

$= \{ x \mid (x\in A \wedge x\in B) \wedge x\notin C) \} = \{ x \mid x\in A \wedge x\in B \wedge x\notin C) \} =$

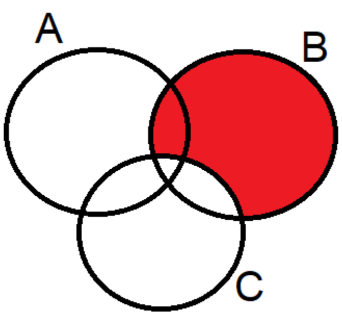
$= \{ x \mid (x\in A \wedge x\notin C) \wedge (x\in B \wedge x\notin C) \} = (A \setminus C) \cap (B \setminus C)$

(b) $A \setminus (B \cup C) = \{ x \mid x\in A \setminus (B \cup C) \} = \{ x \mid x\in A \wedge x\notin(B \cup C) \} =$

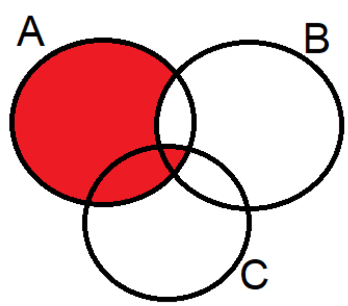
$= \{ x \mid x\in A \wedge (x\notin B \wedge x\notin C) \} = \{ x \mid (x\in A \wedge x\notin B) \wedge (x\notin A \wedge x\notin C) \} =$

$= (A \setminus B) \cap (A \setminus C)$

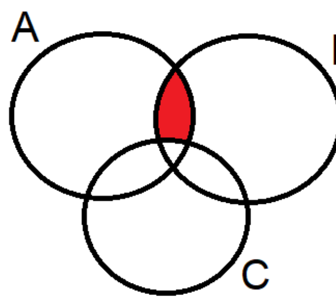
(c)



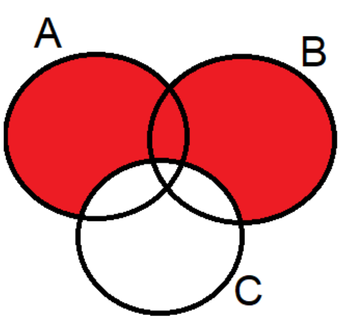
B \ C



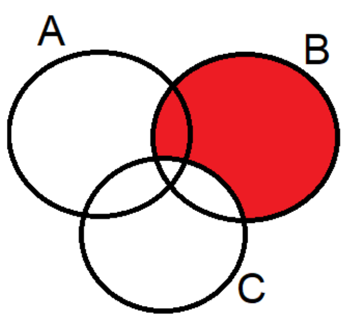
A \ (B \ C)



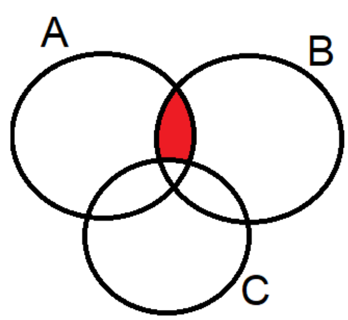
A \ (A \ (B \ C))



$\overline{C}$



$B \cap \overline{C}$

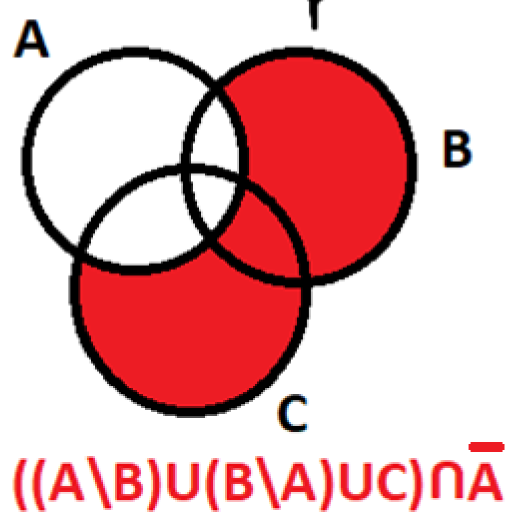


$A \cap B \cap \overline{C}$

Bizonyítsa be a következő összefüggést: $\overline{(\overline{A\cap B}\cup C)\cap\overline{A}}\cup\overline{B}\cup\overline{C} = A\cup\overline{B}\cup\overline{C}$.

$$\underbrace{\overline{(\overline{A\cap B}\cup C)\cap\overline{A}}}_{\stackrel{?}{=}A}\cup\overline{B}\cup\overline{C}=A\cup\overline{B}\cup\overline{C}$$

$$\overline{(\overline{A\cap B}\cup C)\cap\overline{A}} = \overline{\underbrace{((A\setminus B)\cup(B\setminus A)\cup C)\cap\overline{A}}}$$



$$=\overline{(B\setminus A)\cup(C\setminus A)}=A$$