Legyen az alaphalmaz $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, továbbá legyen $A = \{x \mid x \in \mathbb{N} \wedge 1 \leq x \leq 4\}$, $B = \{0, 2, 4, 8\}$, $C = \{\text{az egyjegyű prímszámok}\}$.

(a) Határozza meg a következő halmazokat:

$A \cap B$ $\quad B \cup C$ $\quad A \setminus C$ $\quad \overline{C}$

1.) $U = \{0,1,2,3,4,5,6,7,8,9\}$; $A = \{x \mid x \in \mathbb{N} \wedge 1 \leq x \leq 4\}$; $B = \{0,2,4,8\}$; $C = \{\text{egyjegyü prímszámok}\}$

- $A \cap B = \{x \in U \mid x \in A \wedge x \in B\} = \{2,4\}$

$B \cup C = \{x \in U \mid x \in B \vee x \in C\} = \{0,2,3,4,5,7,8\}$

$A \setminus C = \{x \in U \mid x \in A \wedge x \notin C\} = \{1,4\}$

$\overline{C} = \{x \mid x \notin C \wedge x \in U\} = \{0,1,4,6,8,9\}$

(b) Tekintsük az $X = \{A, B, C\}$ halmazrendszert. Határozza meg a következő halmazokat.

$\cap X$ $\quad \cup X$

- $X = \{A, B, C\}$

$\cap X = \{x \in U \mid x \in A \wedge x \in B \wedge x \in C\} = A \cap B \cap C = \{2\}$

$\cup X = \{x \in U \mid x \in A \vee x \in B \vee x \in C\} = A \cup B \cup C = \{0,1,2,3,4,5,7,8\}$

(c) Állapítsa meg a következő kijelentések logikai értékét, ha $Y = \{\{x \mid x \in U \text{ és } x \text{ páros}\}, \{x \mid x \in U \text{ és } x \text{ páratlan}\}\}$.

$4 \in B$	$A \subseteq B$	$\{\emptyset\} \subseteq X \cup Y$	$3 \in A \cap B$
$\{1,2\} \subseteq A$	$A \in X \cup Y$	$A \subseteq X \cup Y$	$C \cap \emptyset = \emptyset$
$2 \subseteq A$	$\{2\} \subseteq A$	$2 \in X \cup Y$	$\{2\} \in X \cap Y$

• $Y=\{\{x \mid x\in U \wedge 2\mid x\},\{x \mid x\in U \wedge 2\nmid x\}\}$

$4\in B$: IGAZ $A\subseteq B$: HAMIS $\{\emptyset\}\subseteq X\cup Y$: IGAZ

$3\in A\cap B$: HAMIS $\{1,2\}\subseteq A$: IGAZ $A\in X\cup Y$: HAMIS

$A\subseteq X\cup Y$: IGAZ $C\cap\emptyset=\emptyset$: IGAZ $2\subseteq A$: HAMIS

$\{2\}\subseteq A$: IGAZ $2\in X\cup Y$: IGAZ $\{2\}\in X\cup Y$: HAMIS

! elem $\in$ HALMAZ $\Leftrightarrow$ elem szerepel a HALMAZ-ban

! elem $\subseteq$ HALMAZ $\Leftrightarrow$ elem részhalmaza a HALMAZ-nak $\Rightarrow$ ↯

! $\{$elem$\}$ $\subseteq$ HALMAZ $\Leftrightarrow$ a $\{$elem$\}$ halmaz részhalmaza a HALMAZ-nak

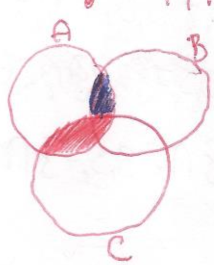
! $\{$elem$\}$ $\in$ HALMAZ $\Leftrightarrow$ a $\{$elem$\}$ halmaz szerepel a HALMAZ-ban

Keressünk olyan A, B, C halmazokat, melyekre egyszerre teljesülnek a következők:

$$A \cap B \neq \emptyset, \quad A \cap C = \emptyset, \quad (A \cap B) \setminus C = \emptyset.$$

2.) $A,B,C=?$ $A\cap B\neq\emptyset \wedge A\cap C=\emptyset \wedge (A\cap B)\setminus C=\emptyset$

↯



$(A\cap C=\emptyset \wedge A\cap B\neq\emptyset) \Rightarrow (A\cap B)\setminus C\neq\emptyset$, de

$(A\cap B)\setminus C=\emptyset \Rightarrow$ ↯

Tekintsük az $X = \{\{1,2,3\},\{2,3,4,5\},\{0,2,3,7\}\}$ halmazrendszert. Határozza meg a következő halmazokat:

(a) $\cap X$

(b) $X \cup \{\{3,5,7\},\{1\},\{2\}\}$

(c) $\cup(X \cup \{\{3,5,7\},\{1\},\{2\}\})$

(d) $\cap(X \cup \{\{3,5,7\},\{1\},\{2\}\})$

4.) $X=\{\{1,2,3\},\{2,3,4,5\},\{0,2,3,7\}\}$

$\cap X=\{1,2,3\}\cap\{2,3,4,5\}\cap\{0,2,3,7\}=\{2,3\}$

$X\cup\{\{3,5,7\},\{1\},\{2\}\}=\{\{1,2,3\},\{2,3,4,5\},\{0,2,3,7\},\{3,5,7\},\{1\},\{2\}\}$

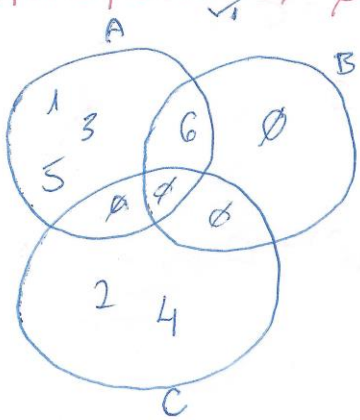
$\cup(X\cup\{\{3,5,7\},\{1\},\{2\}\})=\{1,2,3\}\cup\{2,3,4,5\}\cup\{0,2,3,7\}\cup\{3,5,7\}\cup\{1\}\cup\{2\}=\{0,1,2,3,4,5,7\}$

$\cap(X\cup\{\{3,5,7\},\{1\},\{2\}\})=\{1,2,3\}\cap\{2,3,4,5\}\cap\{0,2,3,7\}\cap\{3,5,7\}\cap\{1\}\cap\{2\}=\{\}$

Határozza meg az A, B, C halmazok elemeit, ha tudjuk, hogy $A \setminus B = \{1,3,5\}$, $A \cup B \cup C = \{1,2,3,4,5,6\}$, $(A \cap C) \cup (B \cap C) = \emptyset$, $C \setminus B = \{2,4\}$ és $(A \cap B) \setminus C = \{6\}$.

6.) $A\setminus B=\{1,3,5\}$ ✓2 ; $A\cup B\cup C=\{1,2,3,4,5,6\}$, ✓5

$(A\cap C)\cup(B\cap C)=\emptyset$ ✓1 ; $C\setminus B=\{2,4\}$ ✓3 ; $(A\cap B)\setminus C=\{6\}$ ✓4



$A=\{1,3,5,6\}$

$B=\{6\}$

$C=\{2,4\}$

Legyenek A, B, C tetszőleges halmazok, U az alaphalmaz, $A, B, C \subseteq U$. Igazoljuk a következő azonosságokat.

(a) $A \cup B = B \cup A$

(b) $(A \cup B) \cup C = A \cup (B \cup C)$

(c) $A \cap B = B \cap A$

(d) $(A \cap B) \cap C = A \cap (B \cap C)$

(e) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(f) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(g) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(h) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

(i) $A \cup \overline{A} = U$

(j) $A \cap \overline{A} = \emptyset$

(k) $\overline{\overline{A}} = A$

7.) $A, B, C \subseteq U$

• $A \cup B = B \cup A$, $A \cap B = B \cap A$

kommutatív

$A \cup B = \{x \mid x \in A \lor x \in B\} = B \cup A$

$A \cap B = \{x \mid x \in A \land x \in B\} = B \cap A$

• $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$,

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

disztributív

$A \cup (B \cap C) = \{x \mid x \in A \lor x \in (B \cap C)\} =$

$= \{x \mid x \in A \lor (x \in B \land x \in C)\} =$

$= \{x \mid (x \in A \lor x \in B) \land (x \in A \lor x \in C)\} =$

$= \{x \mid x \in (A \cup B) \land x \in (A \cup C)\} =$

$= (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = \{x \mid x \in A \land x \in (B \cup C)\} = \{x \mid (x \in A \land x \in B) \lor$
$\lor (x \in A \land x \in C)\} = \{x \mid x \in (A \cap B) \lor x \in (A \cap C)\} = (A \cap B) \cup (A \cap C)$

• $(A \cup B) \cup C = A \cup (B \cup C)$,

$(A \cap B) \cap C = A \cap (B \cap C)$

asszociatív

$A \cup B = \{x \mid x \in A \lor x \in B\}$

$(A \cup B) \cup C = \{x \mid x \in (A \cup B) \lor x \in C\} =$

$= \{x \mid x \in A \lor x \in B \lor x \in C\} =$

$= \{x \mid x \in A \lor x \in (B \cup C)\} =$

$= A \cup (B \cup C)$

$A \cap B = \{x \mid x \in A \land x \in B\}$

$(A \cap B) \cap C = \{x \mid x \in (A \cap B) \land x \in C\} =$

$= \{x \mid x \in A \land x \in B \land x \in C\} =$

$= \{x \mid x \in A \land x \in (B \cap C)\} =$

$= A \cap (B \cap C)$

• $\overline{A \cup B} = \overline{A} \cap \overline{B}$, $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A \cup B} = \{x \mid x \notin (A \cup B)\} = \{x \mid x \notin A \wedge x \notin B\} = \{x \mid x \in \overline{A} \wedge x \in \overline{B}\} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \{x \mid x \notin (A \cap B)\} = \{x \mid x \notin A \vee x \notin B\} = \{x \mid x \in \overline{A} \vee x \in \overline{B}\} = \overline{A} \cup \overline{B}$

• $A \cup \overline{A} = U$, $A \cap \overline{A} = \emptyset$, $\overline{\overline{A}} = A$

$A \cup \overline{A} = \{x \mid x \in A \vee x \in \overline{A}\} = \{x \mid x \in A \vee x \notin A\} = U$

$A \cap \overline{A} = \{x \mid x \in A \wedge x \in \overline{A}\} = \{x \mid x \in A \wedge x \notin A\} = \emptyset$

$\overline{\overline{A}} = \{x \mid x \in \overline{\overline{A}}\} = \{x \mid x \notin \overline{A}\} = \{x \mid x \in A\} = A$

Igazolja a következő azonosságokat.

(a) $A \triangle \emptyset = A$

(b) $A \triangle A = \emptyset$

(c) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

(d) $A \triangle (A \triangle B) = B$

8.) • $A \triangle \emptyset = A$

$A \triangle \emptyset = \underbrace{(A \cup \emptyset)}_{A} \setminus \underbrace{(A \cap \emptyset)}_{\emptyset} = \underbrace{(A \setminus \emptyset)}_{A} \cup \underbrace{(\emptyset \setminus A)}_{\emptyset} = A$

• $A \triangle A = \emptyset$

$A \triangle A = \underbrace{(A \cup A)}_{A} \setminus \underbrace{(A \cap A)}_{A} = \underbrace{(A \setminus A)}_{\emptyset} \cup \underbrace{(A \setminus A)}_{\emptyset} = \emptyset$

- $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

asszociatív

$A \triangle (B \triangle C) =$

$(A\cup((B\cup C)\setminus(B\cap C)))\setminus(A\cap((B\cup C)\setminus(B\cap C)))=(A\cup((B\setminus C)\cup((C\setminus B)))\setminus(A\cap((B\setminus C)\cup(C\setminus B)))=$
$=(A\setminus B\setminus C)\cup(B\setminus A\setminus C)\cup(C\setminus B\setminus A)$

$$(A \triangle B) \triangle C = (((A\cup B)\setminus(A\cap B))\cup C)\setminus(((A\cup B)\setminus(A\cap B))\cap C) =$$

$$=(((A\setminus B)\cup(B\setminus A))\cup C)\setminus(((A\setminus B)\cup(B\setminus A))\cap C) = \underline{\underline{(A\setminus B\setminus C)\cup(B\setminus A\setminus C)\cup(C\setminus B\setminus A)}}$$

- $A \triangle (A \triangle B) = B$

$$A \triangle (A \triangle B) = \underbrace{(A\setminus((A\setminus B)\cup(B\setminus A)))}_{A\cap B}\cup\underbrace{(((A\setminus B)\cup(B\setminus A))\setminus A)}_{B\setminus A} = (A\cap B)\cup(B\setminus A) = B$$