

EXERCISE SOLUTIONS FOR
LOGIC
BY PAUL TOMASSI

D. G. O.

Chapter One: How to Think Logically

EXERCISE 1.1

- 1 A Not an argument. There are no premises and there is no conclusion.
 B Argument. First two sentences are premises, and the "so" in the third sentence indicates it is a conclusion.
 C Argument. First two sentences are premises, and the "therefore" in the third sentence indicates it is a conclusion.
 D Not an argument. There are no premises and there is no conclusion.

2 A **Premises**

Professor Plum was in the drawing room and Miss Scarlet was in the conservatory. If Professor Plum was in the drawing room and the murder weapon was found in the drawing room then Professor Plum is in big trouble.

Conclusion

So, if the murder weapon was found in the drawing room then Professor Plum really is in big trouble

Reasons

The conclusion follows from the premises, and the "so" word in the last sentence indicates a conclusion.

B **Premises**

All human beings are mortal. After all, he is a human being.

Conclusion

So, it stands to reason that Socrates is mortal.

Reasons

The conclusion follows from the premises, and the "so" word in the second sentence indicates a conclusion.

C **Premises**

Very few elephants can fly. Very few elephants are pink. For fewer pink elephants than ordinary elephants can actually fly.

Conclusion

So, the pink flying elephant is truly a rare creature.

Reasons

The conclusion follows from the premises, and these sentences are in the form of an argument with the words "so" and "for".

D **Premises**

For the murderer used the knife and Professor Plum had the knife. And the murder was committed in the hall and Professor Plum was certainly in the hall earlier.

Conclusion

Professor Plum was obviously the murderer in this instance.

Reasons

The conclusion follows from the premises not deductively, but inductively. These sentences are in the form of an argument with the word "for".

- 3 A An argument is valid if it is impossible that it's premises be true and its conclusion false. It follows, that if it is not a valid argument, then it is possible, that it's premises be true, and its conclusion false. Also, it is a valid argument only if it is impossible that it's premises be true and its conclusion false. It follows, that if it is possible that it's premises be true and its conclusion false, then it is an invalid argument. All in all, an argument is invalid if and only if it is possible that it's premises be true and its conclusion false.
- B Yes, a valid argument can have false conclusion if and only if it has a false premise. In that case, the premise is false, which means that it is impossible that it's premises are true, and it's conclusion false.
- C No, a for a valid argument it is not possible (impossible) to have true premises and false conclusion.
- D No. It is impossible that a true premise be true, and a true conclusion to be false, therefore, it is a valid argument.
- E No. A sound argument is both valid and has true premises.
- F Yes. A sound argument is valid and has true premises. It is valid, so it is not true, that it's premises are true and it's conclusion false. The premises are true, therefore this last sentence can only be true if the conclusion is true.
- G An argument form is valid if and only if every substitution-instance of that form is valid.
- H An argument form is invalid if and only if there is any substitution-instance of that form that is invalid.
- 4 A Valid, not sound.
- B Invalid. We do not know whether "better than" in this context is transitive.
- C Invalid.
- D Invalid.
- 5 (i) C If p then q . q . Therefore, p . D If p then q . Not p . Therefore, not q .
- (ii) Yes, because both premises are true, but both conclusions are not true.
- 6 If I clean my room, then it is not the case, that there is dirt on the floor. It is not the case, that I clean my room. Therefore, there is no dirt on the floor.
- 7 By the definition given in this chapter, sentential variable is a variable, whose value is a well-formed sentence. Both 1 and 2 are one whole sentence which cannot be divided. The whole argument then is: p therefore, q .
- (i) No, it is not a valid form. Counterexample is where p is "snow is white", q is "there are pink elephants". Here p is true, and q is not, then it is not true, that p is true and q is false.
- (ii) Yes, intuitively (or modally) it is a valid argument, because it is not true, that the premise be true and the conclusion be false.

Chapter Two:

How to Prove that You Can Argue Logically #1

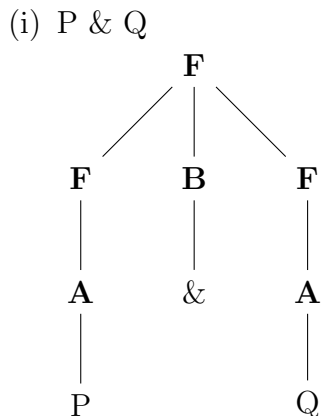
EXERCISE 2.1

- 1 The main connective in each case are

| | |
|--------------------|---------------------|
| (i) conjunction, | (vi) disjunction, |
| (ii) negation, | (vii) conditional, |
| (iii) conjunction, | (viii) conjunction, |
| (iv) conjunction, | (ix) biconditional, |
| (v) disjunction, | (x) biconditional. |
- 2 For (ii) and (x) it's the whole formula. For (iii) it's $\sim P$ and $\sim Q$.
For (iv) it's $\sim(P \ \& \ Q)$ and $\sim Q$.
- 3 The key is P: Blind Lemon Jefferson is the only bluesman.
Q: Dr Strangely Strange is a bluesman.
R: Mr Oddly Normal is a bluesman.
S: Blind Lemon Jefferson is a milkman.
T: Blind Lemon Jefferson is a bluesman.
U: Blind Lemon recorded it.
V: It's a blues album.
The sentences translated to PL are

- (i) $\sim P$,
- (ii) $\sim Q \ \& \ \sim R$,
- (iii) $S \rightarrow \sim T$,
- (iv) $\sim(U \rightarrow \sim V)$,
- (v) $S \vee T$,
- (vi) $\sim(S \vee T)$.

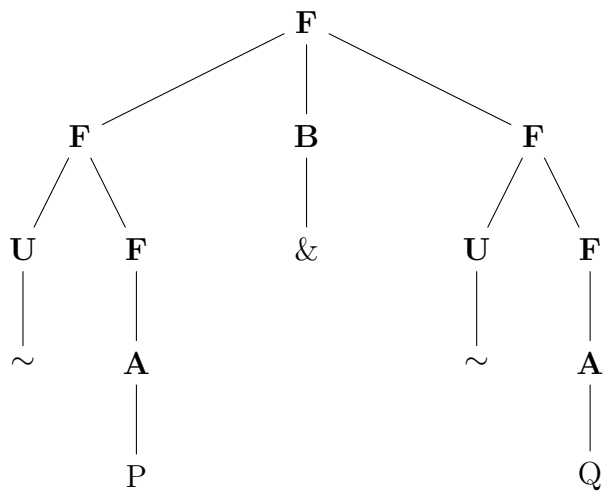
- 4 The trees are the followings.



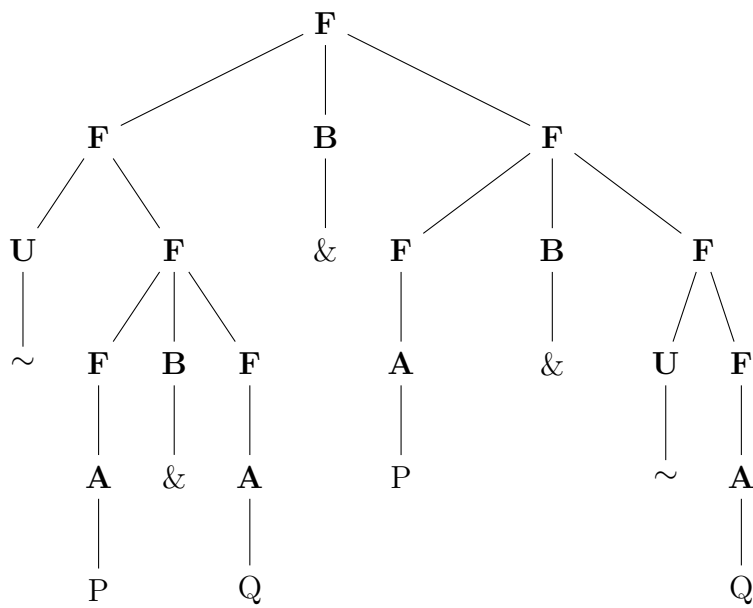
(ii) $\sim(P \ \& \ Q)$



(iii) $\sim P \ \& \ \sim Q$



(iv) $\sim(P \ \& \ Q) \ \& \ (P \ \& \ \sim Q)$



(v) $P \vee Q$



(vi) $(P \& Q) \vee (Q \& R)$



(vii) $P \rightarrow Q$



(viii) $(P \rightarrow Q) \& (Q \rightarrow R)$



(ix) $(P \& Q) \leftrightarrow (Q \& P)$



(x) $\sim((P \& Q) \leftrightarrow (Q \& P))$



EXERCISE 2.2

1 $P \vee Q, \sim R \rightarrow Q, \sim R : \sim P$

- (i) P: Big Bill Broonzy is a Delta bluesman.
- (ii) Q: Big Bill Broonzy is a Chicago bluesman.
- (iii) R: Big Bill Broonzy was born in Mississippi.

This is an invalid sequent.

2 $\sim(P \ \& \ Q), P : Q$

- (i) P: Etta James was an angel.
- (ii) Q: Robert Johnson sold his soul to the devil.

This is a valid sequent.

3 $\sim P \rightarrow \sim Q, \sim(P \ \& \ \sim R), R : Q$

- (i) P: There's light on in the Venue.
- (ii) Q: The band are on stage already.
- (iii) R: It's going to be a great night.

This is an invalid sequent.

4 $P \vee Q, P \rightarrow \sim R, Q \rightarrow \sim R : \sim R$

- (i) P: There's a punk rock band playing at the Venue tonight.
- (ii) Q: The music is strictly classical.
- (iii) R: There will be no blues at the Venue tonight.

This is a valid sequent.

5 $(P \ \& \ Q) \rightarrow \sim R : R \rightarrow (P \rightarrow Q)$

- (i) P: There's a band on stage.
- (ii) Q: The music is groovy.
- (iii) R: It is the Nasal Flute Orchestra.

This is an invalid sequent.

EXERCISE 2.3

1 Dependencies, line number, formula, rule which was used in the line.

2 The proofs are the followings.

1. $P, Q : P \& Q$

| | | | |
|--------|----|----------|---------|
| {1} | 1. | P | Premise |
| {2} | 2. | Q | Premise |
| {1, 2} | 3. | $P \& Q$ | 1, 2 &I |

2. $P, Q, R : (P \& Q) \& R$

| | | | |
|-----------|----|-----------------|---------|
| {1} | 1. | P | Premise |
| {2} | 2. | Q | Premise |
| {3} | 3. | R | Premise |
| {1, 2} | 4. | $P \& Q$ | 1, 2 &I |
| {1, 2, 3} | 5. | $(P \& Q) \& R$ | 3, 4 &I |

3. $P, Q, R, S : (P \& Q) \& (R \& S)$

| | | | |
|--------------|----|------------------------|---------|
| {1} | 1. | P | Premise |
| {2} | 2. | Q | Premise |
| {3} | 3. | R | Premise |
| {4} | 4. | S | Premise |
| {1, 2} | 5. | $P \& Q$ | 1, 2 &I |
| {3, 4} | 6. | $R \& S$ | 3, 4 &I |
| {1, 2, 3, 4} | 7. | $(P \& Q) \& (R \& S)$ | 5, 6 &I |

4. $P \& Q : P$

| | | | |
|-----|----|------------|---------|
| {1} | 1. | $(P \& Q)$ | Premise |
| {1} | 2. | P | 1 &E |

5. $P \& Q : Q$

| | | | |
|-----|----|------------|---------|
| {1} | 1. | $(P \& Q)$ | Premise |
| {1} | 2. | Q | 1 &E |

6. $(P \& Q) \& R : P$

| | | | |
|-----|----|-----------------|---------|
| {1} | 1. | $(P \& Q) \& R$ | Premise |
| {1} | 2. | $P \& Q$ | 1 &E |
| {1} | 3. | P | 2 &E |

7. $(P \& Q) \& (R \& S) : P$

| | | | |
|-----|----|------------------------|---------|
| {1} | 1. | $(P \& Q) \& (R \& S)$ | Premise |
| {1} | 2. | $P \& Q$ | 1 &E |
| {1} | 3. | P | 2 &E |

8. $(Q \& R), P : (P \& Q) \& R$

| | | | |
|--------|----|-----------------|---------|
| {1} | 1. | $Q \& R$ | Premise |
| {2} | 2. | P | Premise |
| {1} | 3. | Q | 1 &E |
| {1} | 4. | R | 1 &E |
| {1, 2} | 5. | $P \& Q$ | 2, 3 &I |
| {1, 2} | 6. | $(P \& Q) \& R$ | 4, 5 &I |

EXERCISE 2.4

1 The proofs are the followings.

1. $P \rightarrow Q, P : Q$

| | | | |
|--------|----|-------------------|---------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | P | Premise |
| {1, 2} | 3. | Q | 1, 2 MP |

2. $P \rightarrow (P \rightarrow Q), P : Q$

| | | | |
|--------|----|-----------------------------------|---------|
| {1} | 1. | $P \rightarrow (P \rightarrow Q)$ | Premise |
| {2} | 2. | P | Premise |
| {1, 2} | 3. | $P \rightarrow Q$ | 1, 2 MP |
| {1, 2} | 4. | Q | 2, 3 MP |

3. $P \rightarrow (P \& Q), P : Q$

| | | | |
|--------|----|--------------------------|---------|
| {1} | 1. | $P \rightarrow (P \& Q)$ | Premise |
| {2} | 2. | P | Premise |
| {1, 2} | 3. | $P \& Q$ | 1, 2 MP |
| {1, 2} | 4. | Q | 3 &E |

4. $P \rightarrow (Q \rightarrow R), P \rightarrow Q, P : R$

| | | | |
|-----------|----|-----------------------------------|---------|
| {1} | 1. | $P \rightarrow (Q \rightarrow R)$ | Premise |
| {2} | 2. | $P \rightarrow Q$ | Premise |
| {3} | 3. | P | Premise |
| {1, 3} | 4. | $Q \rightarrow R$ | 1, 3 MP |
| {2, 3} | 5. | Q | 2, 3 MP |
| {1, 2, 3} | 6. | R | 4, 5 MP |

2 The proofs are the followings.

(i) $P \rightarrow (Q \& R), P \& Q : P \& R$

| | | | |
|--------|----|--------------------------|---------|
| {1} | 1. | $P \rightarrow (Q \& R)$ | Premise |
| {2} | 2. | $P \& Q$ | Premise |
| {2} | 3. | P | 2 &E |
| {1, 2} | 4. | $Q \& R$ | 1, 3 MP |
| {1, 2} | 5. | R | 4 &E |
| {1, 2} | 6. | $P \& R$ | 3, 5 &I |

(ii) $(P \rightarrow Q) \rightarrow (R \rightarrow S), P \rightarrow Q, P \& R : S$

| | | | |
|-----------|----|---|---------|
| {1} | 1. | $(P \rightarrow Q) \rightarrow (R \rightarrow S)$ | Premise |
| {2} | 2. | $P \rightarrow Q$ | Premise |
| {3} | 3. | $P \& R$ | Premise |
| {1, 2} | 4. | $R \rightarrow S$ | 1, 2 MP |
| {3} | 5. | R | 3 &E |
| {1, 2, 3} | 6. | S | 4, 5 MP |

(iii) $P, P \rightarrow Q : P \& Q$

| | | | |
|--------|----|-------------------|---------|
| {1} | 1. | P | Premise |
| {2} | 2. | $P \rightarrow Q$ | Premise |
| {1, 2} | 3. | Q | 1, 2 MP |
| {1, 2} | 4. | $P \& Q$ | 1, 3 &I |

(iv) $P, P \rightarrow Q, P \rightarrow (Q \rightarrow R) : P \& R$

| | | | |
|-----------|----|-----------------------------------|---------|
| {1} | 1. | P | Premise |
| {2} | 2. | $P \rightarrow Q$ | Premise |
| {3} | 3. | $P \rightarrow (Q \rightarrow R)$ | Premise |
| {1, 2} | 4. | Q | 1, 2 MP |
| {1, 3} | 5. | $Q \rightarrow R$ | 1, 3 MP |
| {1, 2, 3} | 6. | R | 4, 5 MP |
| {1, 2, 3} | 7. | $P \& R$ | 1, 6 &I |

(v) $P, P \rightarrow Q, P \rightarrow (Q \rightarrow R), R \rightarrow S : (P \& Q) \& (R \& S)$

| | | | |
|-----------|-----|-----------------------------------|----------|
| {1} | 1. | P | Premise |
| {2} | 2. | $P \rightarrow Q$ | Premise |
| {3} | 3. | $P \rightarrow (Q \rightarrow R)$ | Premise |
| {4} | 4. | $R \rightarrow S$ | Premise |
| {1, 2} | 5. | Q | 1, 2 MP |
| {1, 3} | 6. | $Q \rightarrow R$ | 1, 3 MP |
| {1, 2, 3} | 7. | R | 5, 6 MP |
| {1, 2, 3} | 8. | S | 4, 7 MP |
| {1, 2} | 9. | $P \& Q$ | 1, 5 &I |
| {1, 2, 3} | 10. | $R \& S$ | 7, 8 &I |
| {1, 2, 3} | 11. | $(P \& Q) \& (R \& S)$ | 9, 10 &I |

EXERCISE 2.5

1 The proofs are the followings.

1. $P \rightarrow (Q \& R) : P \rightarrow Q$

| | | | |
|--------|----|--------------------------|-------------------|
| {1} | 1. | $P \rightarrow (Q \& R)$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {1, 2} | 3. | $Q \& R$ | 1, 2 MP |
| {1, 2} | 4. | Q | 3 &E |
| {1} | 5. | $P \rightarrow Q$ | 2, 4 CP |

2. $(P \& Q) \rightarrow R, P : Q \rightarrow R$

| | | | |
|-----------|----|--------------------------|-------------------|
| {1} | 1. | $(P \& Q) \rightarrow R$ | Premise |
| {2} | 2. | P | Premise |
| {3} | 3. | Q | Assumption for CP |
| {2, 3} | 4. | $P \& Q$ | 2, 3 &I |
| {1, 2, 3} | 5. | R | 1, 4 MP |
| {1, 2} | 6. | $Q \rightarrow R$ | 3, 5 CP |

3. $(P \& Q), (P \& R) \rightarrow S : R \rightarrow S$

| | | | |
|-----------|----|--------------------------|-------------------|
| {1} | 1. | $P \& Q$ | Premise |
| {2} | 2. | $(P \& R) \rightarrow S$ | Premise |
| {3} | 3. | R | Assumption for CP |
| {1} | 4. | P | 1 &E |
| {1, 3} | 5. | $P \& R$ | 3, 4 &I |
| {1, 2, 3} | 6. | S | 2, 5 MP |
| {1, 2} | 7. | $R \rightarrow S$ | 3, 6 CP |

4. $(P \& Q) \rightarrow R : P \rightarrow (Q \rightarrow R)$

| | | | |
|-----------|----|-----------------------------------|-------------------|
| {1} | 1. | $(P \& Q) \rightarrow R$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | Q | Assumption for CP |
| {2, 3} | 4. | $P \& Q$ | 2, 3 &I |
| {1, 2, 3} | 5. | R | 1, 4 MP |
| {1, 2} | 6. | $Q \rightarrow R$ | 3, 5 CP |
| {1} | 7. | $P \rightarrow (Q \rightarrow R)$ | 2, 6 CP |

5. $P \rightarrow Q : (P \& R) \rightarrow (R \& Q)$

| | | | |
|--------|----|---------------------------------|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $P \& R$ | Assumption for CP |
| {2} | 3. | P | 2 &E |
| {1, 2} | 4. | Q | 1, 3 MP |
| {2} | 5. | R | 2 &E |
| {1, 2} | 6. | $R \& Q$ | 4, 5 &I |
| {1} | 7. | $(P \& R) \rightarrow (R \& Q)$ | 2, 6 CP |

6. $P \rightarrow Q : (Q \rightarrow R) \rightarrow (P \rightarrow R)$

| | | | |
|-----------|----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $Q \rightarrow R$ | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2, 3} | 5. | R | 2, 4 MP |
| {1, 2} | 6. | $P \rightarrow R$ | 3, 5 CP |
| {1} | 7. | $(Q \rightarrow R) \rightarrow (P \rightarrow R)$ | 2, 6 CP |

7. $R \rightarrow P, Q \rightarrow S : (P \rightarrow Q) \rightarrow (R \rightarrow S)$

| | | | |
|--------------|----|---|-------------------|
| {1} | 1. | $R \rightarrow P$ | Premise |
| {2} | 2. | $Q \rightarrow S$ | Premise |
| {3} | 3. | $P \rightarrow Q$ | Assumption for CP |
| {4} | 4. | R | Assumption for CP |
| {1, 4} | 5. | P | 1, 4 MP |
| {1, 3, 4} | 6. | Q | 3, 5 MP |
| {1, 2, 3, 4} | 7. | S | 2, 6 MP |
| {1, 2, 3} | 8. | $R \rightarrow S$ | 4, 7 CP |
| {1, 2} | 9. | $(P \rightarrow Q) \rightarrow (R \rightarrow S)$ | 3, 8 CP |

8. $P \rightarrow Q : (P \rightarrow R) \rightarrow (P \rightarrow (Q \& R))$

| | | | |
|-----------|----|--|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $P \rightarrow R$ | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2} | 5. | R | 2, 3 MP |
| {1, 2, 3} | 6. | $Q \& R$ | 4, 5 &I |
| {1, 2} | 7. | $P \rightarrow (Q \& R)$ | 3, 6 CP |
| {1} | 8. | $(P \rightarrow R) \rightarrow (P \rightarrow (Q \& R))$ | 2, 7 CP |

9. $P \rightarrow (Q \rightarrow R) : (S \rightarrow Q) \rightarrow (P \rightarrow (S \rightarrow R))$

| | | | |
|--------------|-----|---|-------------------|
| {1} | 1. | $P \rightarrow (Q \rightarrow R)$ | Premise |
| {2} | 2. | $S \rightarrow Q$ | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {4} | 4. | S | Assumption for CP |
| {2, 4} | 5. | Q | 2, 4 MP |
| {1, 3} | 6. | $Q \rightarrow R$ | 1, 3 MP |
| {1, 2, 3, 4} | 7. | R | 5, 6 MP |
| {1, 2, 3} | 8. | $S \rightarrow R$ | 4, 7 CP |
| {1, 2} | 9. | $P \rightarrow (S \rightarrow R)$ | 3, 8 CP |
| {1} | 10. | $(S \rightarrow Q) \rightarrow (P \rightarrow (S \rightarrow R))$ | 2, 9 CP |

10. $P \rightarrow Q : ((R \& Q) \rightarrow S) \rightarrow ((R \& P) \rightarrow S)$

| | | | |
|-----------|-----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $(R \& Q) \rightarrow S$ | Assumption for CP |
| {3} | 3. | $R \& P$ | Assumption for CP |
| {3} | 4. | P | 3 &E |
| {3} | 5. | R | 3 &E |
| {1, 3} | 6. | Q | 1, 4 MP |
| {1, 3} | 7. | $R \& Q$ | 5, 6 &I |
| {1, 2, 3} | 8. | S | 2, 7 MP |
| {1, 2} | 9. | $(R \& P) \rightarrow S$ | 3, 8 CP |
| {1} | 10. | $((R \& Q) \rightarrow S) \rightarrow ((R \& P) \rightarrow S)$ | 2, 9 CP |

EXERCISE 2.6

1 The proofs are the followings.

1. : $((P \rightarrow P) \rightarrow Q) \rightarrow Q$

| | | | |
|-----|----|---|-------------------|
| {1} | 1. | P | Assumption for CP |
| — | 2. | $P \rightarrow P$ | 1, 1 CP |
| {3} | 3. | $(P \rightarrow P) \rightarrow Q$ | Assumption for CP |
| {3} | 4. | Q | 2, 3 MP |
| — | 5. | $((P \rightarrow P) \rightarrow Q) \rightarrow Q$ | 3, 4 CP |

2. : $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$

| | | | |
|-----------|----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Assumption for CP |
| {2} | 2. | $Q \rightarrow R$ | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2, 3} | 5. | R | 2, 4 MP |
| {1, 2} | 6. | $P \rightarrow R$ | 3, 5 CP |
| {1} | 7. | $(Q \rightarrow R) \rightarrow (P \rightarrow R)$ | 2, 6 CP |
| — | 8. | $(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$ | 1, 7 CP |

3. : $(Q \rightarrow R) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

| | | | |
|-----------|----|---|-------------------|
| {1} | 1. | $Q \rightarrow R$ | Assumption for CP |
| {2} | 2. | $P \rightarrow Q$ | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {2, 3} | 4. | Q | 2, 3 MP |
| {1, 2, 3} | 5. | R | 1, 4 MP |
| {1, 2} | 6. | $P \rightarrow R$ | 3, 5 CP |
| {1} | 7. | $(P \rightarrow Q) \rightarrow (P \rightarrow R)$ | 2, 6 CP |
| — | 8. | $(Q \rightarrow R) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$ | 1, 7 CP |

4. : $P \rightarrow (Q \rightarrow (P \& Q))$

| | | | |
|--------|----|--|-------------------|
| {1} | 1. | P | Assumption for CP |
| {2} | 2. | Q | Assumption for CP |
| {1, 2} | 3. | $P \& Q$ | 1, 2 &I |
| {1} | 4. | $Q \rightarrow (P \& Q)$ | 2, 3 CP |
| — | 5. | $P \rightarrow (Q \rightarrow (P \& Q))$ | 1, 4 CP |

EXERCISE 2.7

1 The proofs are the followings.

1. $P, P \leftrightarrow Q : Q$

| | | | |
|--------|----|--|---------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $P \leftrightarrow Q$ | Premise |
| {2} | 3. | $(P \rightarrow Q) \& (Q \rightarrow P)$ | \leftrightarrow E |
| {2} | 4. | $P \rightarrow Q$ | 3 &E |
| {1, 2} | 5. | Q | 1, 4 MP |

2. $P \& (P \leftrightarrow Q) : P \& Q$

| | | | |
|-----|----|--|-----------------------|
| {1} | 1. | $P \& (P \leftrightarrow Q)$ | Premise |
| {1} | 2. | P | 1 &E |
| {1} | 3. | $P \leftrightarrow Q$ | 1 &E |
| {1} | 4. | $(P \rightarrow Q) \& (Q \rightarrow P)$ | 3 \leftrightarrow E |
| {1} | 5. | $P \rightarrow Q$ | 4 &E |
| {1} | 6. | Q | 2, 5 MP |
| {1} | 7. | $P \& Q$ | 2, 6 &I |

3. $(P \& Q) \leftrightarrow P : P \rightarrow Q$

| | | | |
|--------|----|--|-----------------------|
| {1} | 1. | $(P \& Q) \leftrightarrow P$ | Premise |
| {1} | 2. | $((P \& Q) \rightarrow P) \& (P \rightarrow (P \& Q))$ | 1 \leftrightarrow E |
| {1} | 3. | $P \rightarrow (P \& Q)$ | 2 &E |
| {4} | 4. | P | Assumption for CP |
| {1, 4} | 5. | $P \& Q$ | 3, 4 MP |
| {1, 4} | 6. | Q | 5 &E |
| {1} | 7. | $P \rightarrow Q$ | 4, 6 CP |

4. $P \rightarrow Q : (Q \rightarrow P) \rightarrow (P \leftrightarrow Q)$

| | | | |
|--------|----|---|--------------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $Q \rightarrow P$ | Assumption for CP |
| {1, 2} | 3. | $Q \leftrightarrow P$ | 1, 2 \leftrightarrow I |
| {1} | 4. | $(Q \rightarrow P) \rightarrow (Q \leftrightarrow P)$ | 2, 3 CP |

5. $P \rightarrow (Q \leftrightarrow R) : (P \& Q) \rightarrow R$

| | | | |
|--------|----|--|-----------------------|
| {1} | 1. | $P \rightarrow (Q \leftrightarrow R)$ | Premise |
| {2} | 2. | $P \& Q$ | Assumption for CP |
| {2} | 3. | P | 2 &E |
| {2} | 4. | Q | 2 &E |
| {1, 2} | 5. | $Q \leftrightarrow R$ | 1, 3 MP |
| {1, 2} | 6. | $(Q \rightarrow R) \& (R \rightarrow Q)$ | 5 \leftrightarrow E |
| {1, 2} | 7. | $Q \rightarrow R$ | 6 &E |
| {1, 2} | 8. | R | 4, 7 MP |
| {1} | 9. | $(P \& Q) \rightarrow R$ | 2, 8 CP |

6. $P \leftrightarrow Q, Q \leftrightarrow R : P \leftrightarrow R$

| | | | |
|------------|-----|--|----------------------------|
| {1} | 1. | $P \leftrightarrow Q$ | Premise |
| {2} | 2. | $Q \leftrightarrow R$ | Premise |
| {1} | 3. | $(P \rightarrow Q) \& (Q \rightarrow P)$ | 1 \leftrightarrow E |
| {1} | 4. | $P \rightarrow Q$ | 3 &E |
| {1} | 5. | $Q \rightarrow P$ | 3 &E |
| {2} | 6. | $(Q \rightarrow R) \& (R \rightarrow Q)$ | 2 \leftrightarrow E |
| {2} | 7. | $Q \rightarrow R$ | 6 &E |
| {2} | 8. | $R \rightarrow Q$ | 6 &E |
| {9} | 9. | P | Assumption for CP |
| {1, 9} | 10. | Q | 4, 9 MP |
| {1, 2, 9} | 11. | R | 7, 10 MP |
| {1, 2} | 12. | $P \rightarrow R$ | 9, 11 CP |
| {13} | 13. | R | Assumption for CP |
| {2, 13} | 14. | Q | 8, 13 MP |
| {1, 2, 13} | 15. | P | 5, 14 MP |
| {1, 2} | 16. | $R \rightarrow P$ | 13, 15 CP |
| {1, 2} | 17. | $P \leftrightarrow R$ | 12, 16 \leftrightarrow I |

EXERCISE 3.1

1 $P, P \rightarrow (Q \rightarrow R), \sim R : \sim Q$

| | | | |
|-----------|----|-----------------------------------|---------|
| {1} | 1. | P | Premise |
| {2} | 2. | $P \rightarrow (Q \rightarrow R)$ | Premise |
| {3} | 3. | $\sim R$ | Premise |
| {1, 2} | 4. | $Q \rightarrow R$ | 1, 2 MP |
| {1, 2, 3} | 5. | $\sim Q$ | 3, 4 MT |

EXERCISE 3.2

1 The proofs are the followings.

1. $\sim\sim(P \ \& \ Q) : \sim\sim(Q \ \& \ P)$

| | | | |
|-----|----|------------------------|---------|
| {1} | 1. | $\sim\sim(P \ \& \ Q)$ | Premise |
| {1} | 2. | $P \ \& \ Q$ | 1 DNE |
| {1} | 3. | P | 2 &E |
| {1} | 4. | Q | 2 &E |
| {1} | 5. | $Q \ \& \ P$ | 3, 4 &I |
| {1} | 6. | $\sim\sim(Q \ \& \ P)$ | 5 DNI |

2. $\sim P \rightarrow \sim Q : Q \rightarrow P$

| | | | |
|--------|----|-----------------------------|-------------------|
| {1} | 1. | $\sim P \rightarrow \sim Q$ | Premise |
| {2} | 2. | Q | Assumption for CP |
| {2} | 3. | $\sim\sim Q$ | 2 DNI |
| {1, 2} | 4. | $\sim\sim P$ | 1, 3 MT |
| {1, 2} | 5. | P | 4 DNE |
| {1} | 6. | $Q \rightarrow P$ | 2, 5 CP |

3. $:(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$

| | | | |
|--------|----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Assumption for CP |
| {2} | 2. | $\sim Q$ | Assumption for CP |
| {1, 2} | 3. | $\sim P$ | 1, 2 MT |
| {1} | 4. | $\sim Q \rightarrow \sim P$ | 2, 3 CP |
| – | 5. | $(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$ | 1, 4 CP |

4. $Q \rightarrow R : (\sim Q \rightarrow \sim P) \rightarrow (P \rightarrow R)$

| | | | |
|-----------|----|---|-------------------|
| {1} | 1. | $Q \rightarrow R$ | Premise |
| {2} | 2. | $\sim Q \rightarrow \sim P$ | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {3} | 4. | $\sim\sim P$ | 3 DNI |
| {2, 3} | 5. | $\sim\sim Q$ | 2, 4 MT |
| {2, 3} | 6. | Q | 5 DNE |
| {1, 2, 3} | 7. | R | 1, 6 MP |
| {1, 2} | 8. | $P \rightarrow R$ | 3, 7 CP |
| {1} | 9. | $(\sim Q \rightarrow \sim P) \rightarrow (P \rightarrow R)$ | 2, 8 CP |

5. $(P \& Q) \rightarrow \sim R : R \rightarrow (P \rightarrow \sim Q)$

| | | | |
|-----------|-----|--|-------------------|
| {1} | 1. | $(P \& Q) \rightarrow \sim R$ | Premise |
| {2} | 2. | R | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {4} | 4. | Q | Assumption for CP |
| {2} | 5. | $\sim\sim R$ | 2 DNI |
| {1, 2} | 6. | $\sim(P \& Q)$ | 1, 5 MT |
| {3, 4} | 7. | $P \& Q$ | 3, 4 &I |
| {3} | 8. | $Q \rightarrow (P \& Q)$ | 4, 7 CP |
| {1, 2, 3} | 9. | $\sim Q$ | 6, 8 MT |
| {1, 2} | 10. | $P \rightarrow \sim Q$ | 3, 9 CP |
| {1} | 11. | $R \rightarrow (P \rightarrow \sim Q)$ | 2, 10 CP |

6. $P : [(\sim(Q \rightarrow R) \rightarrow \sim P)] \rightarrow [(\sim R \rightarrow \sim Q)]$

| | | | |
|-----------|----|--|-------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $\sim(Q \rightarrow R) \rightarrow \sim P$ | Assumption for CP |
| {1} | 3. | $\sim\sim P$ | 1 DNI |
| {1, 2} | 4. | $\sim\sim(Q \rightarrow R)$ | 2, 3 MT |
| {1, 2} | 5. | $Q \rightarrow R$ | 4 DNE |
| {6} | 6. | $\sim R$ | Assumption for CP |
| {1, 2, 6} | 7. | $\sim Q$ | 5, 6 MT |
| {1, 2} | 8. | $\sim R \rightarrow \sim Q$ | 6, 7 |
| {1} | 9. | $(\sim(Q \rightarrow R) \rightarrow \sim P) \rightarrow (\sim R \rightarrow \sim Q)$ | 2, 8 CP |

7. $P, \sim Q : \sim(P \rightarrow Q)$

| | | | |
|--------|----|-----------------------------------|-------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $\sim Q$ | Premise |
| {3} | 3. | $P \rightarrow Q$ | Assumption for CP |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1} | 5. | $(P \rightarrow Q) \rightarrow Q$ | 3, 4 CP |
| {1, 2} | 6. | $\sim(P \rightarrow Q)$ | 2, 5 MT |

8. $P, \sim P : Q$

| | | | |
|--------|----|------------------------|-------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $\sim P$ | Premise |
| {3} | 3. | $\sim Q$ | Assumption for CP |
| {1, 3} | 4. | $P \& \sim Q$ | 1, 3 &I |
| {1, 3} | 5. | P | 4 &E |
| {1} | 6. | $\sim Q \rightarrow P$ | 3, 5 CP |
| {1, 2} | 7. | $\sim\sim Q$ | 2, 6 MT |
| {1, 2} | 8. | Q | 7 DNE |

9. : $\sim P \rightarrow (P \rightarrow Q)$

| | | | |
|--------|-----|--|-------------------|
| {1} | 1. | P | Assumption for CP |
| {2} | 2. | $\sim P$ | Assumption for CP |
| {3} | 3. | $\sim Q$ | Assumption for CP |
| {1, 3} | 4. | $P \ \& \ \sim Q$ | 1, 3 &I |
| {1, 3} | 5. | P | 4 &E |
| {1} | 6. | $\sim Q \rightarrow P$ | 3, 5 CP |
| {1, 2} | 7. | $\sim \sim Q$ | 2, 6 MT |
| {1, 2} | 8. | Q | 7 DNE |
| {2} | 9. | $P \rightarrow Q$ | 1, 8 CP |
| – | 10. | $\sim P \rightarrow (P \rightarrow Q)$ | 2, 9 CP |

10. $P \rightarrow \sim P : \sim P$

| | | | |
|--------|-----|--|-------------------|
| {1} | 1. | $P \rightarrow \sim P$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {1, 2} | 3. | $\sim P$ | 1, 2 MP |
| {4} | 4. | $P \rightarrow \sim P$ | Assumption for CP |
| {2, 4} | 5. | $P \ \& \ (P \rightarrow \sim P)$ | 2, 4 &I |
| {2, 4} | 6. | P | 5 &E |
| {2} | 7. | $(P \rightarrow \sim P) \rightarrow P$ | 4, 6 CP |
| {1, 2} | 8. | $\sim(P \rightarrow \sim P)$ | 3, 7 MT |
| {1} | 9. | $P \rightarrow \sim(P \rightarrow \sim P)$ | 2, 8 CP |
| {1} | 10. | $\sim(\sim(P \rightarrow \sim P))$ | 1 DNI |
| {1} | 11. | $\sim P$ | 9, 10 MT |

EXERCISE 3.3

1 $(P \vee Q) \rightarrow R \vdash (P \rightarrow R) \ \& \ (Q \rightarrow R)$

| | | | |
|--------|-----|--|-------------------------|
| {1} | 1. | $(P \vee Q) \rightarrow R$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {2} | 3. | $P \vee Q$ | 2 vI (right-hand) |
| {1, 2} | 4. | R | 1, 3 MP |
| {1} | 5. | $P \rightarrow R$ | 2, 4 CP |
| {6} | 6. | Q | Assumption for CP |
| {6} | 7. | $P \vee Q$ | 6 vI (left-hand) |
| {1, 6} | 8. | R | 1, 7 MP |
| {1} | 9. | $Q \rightarrow R$ | 6, 8 CP |
| {1} | 10. | $(P \rightarrow R) \ \& \ (Q \rightarrow R)$ | 5, 9 &I |

EXERCISE 3.4

1 They refer to (1) the disjunction, (2) first disjunct as an assumption, (3) the conclusion assuming the first disjunct, (4) the second disjunct as an assumption, (5) the same conclusion assuming the second disjunct.

2 The proofs are the followings.

1. $P \vee Q : (P \vee R) \vee (Q \vee R)$

| | | | |
|-----|----|------------------------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {2} | 3. | $P \vee R$ | 2 $\vee I$ |
| {2} | 4. | $(P \vee R) \vee (Q \vee R)$ | 3 $\vee I$ |
| {5} | 5. | Q | Assumption for $\vee E$ |
| {5} | 6. | $Q \vee R$ | 5 $\vee I$ |
| {5} | 7. | $(P \vee R) \vee (Q \vee R)$ | 6 $\vee I$ |
| {1} | 8. | $(P \vee R) \vee (Q \vee R)$ | 1, 2, 4, 5, 7 $\vee E$ |

2. $(P \& Q) \vee (P \& R) : P \& (Q \vee R)$

| | | | |
|-----|-----|--------------------------|-------------------------|
| {1} | 1. | $(P \& Q) \vee (P \& R)$ | Premise |
| {2} | 2. | $P \& Q$ | Assumption for $\vee E$ |
| {2} | 3. | P | 2 $\& E$ |
| {2} | 4. | Q | 2 $\& E$ |
| {2} | 5. | $Q \vee R$ | 4 $\vee I$ |
| {2} | 6. | $P \& (Q \vee R)$ | 3, 5 $\& I$ |
| {7} | 7. | $P \& R$ | Assumption for $\vee E$ |
| {7} | 8. | P | 7 $\& E$ |
| {7} | 9. | R | 7 $\& E$ |
| {7} | 10. | $Q \vee R$ | 9 $\vee I$ |
| {7} | 11. | $P \& (Q \vee R)$ | 8, 10 $\& I$ |
| {1} | 12. | $P \& (Q \vee R)$ | 1, 2, 6, 7, 11 $\vee E$ |

3. $P \vee (P \& Q) : P$

| | | | |
|-----|----|-------------------|-------------------------|
| {1} | 1. | $P \vee (P \& Q)$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {3} | 3. | $P \& Q$ | Assumption for $\vee E$ |
| {3} | 4. | P | 4 $\& E$ |
| {1} | 5. | P | 1, 2, 2, 3, 4 $\vee E$ |

4. $P \vee P : P$

| | | | |
|-----|----|------------|-------------------------|
| {1} | 1. | $P \vee P$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {1} | 3. | $P \vee P$ | 1, 2, 2, 2, 2 $\vee E$ |

EXERCISE 3.5

1 $R \vee S, \sim Q \rightarrow \sim R, S \rightarrow Q : Q \vee P$

| | | | |
|-----------|-----|-----------------------------|-------------------------|
| {1} | 1. | $R \vee S$ | Premise |
| {2} | 2. | $\sim Q \rightarrow \sim R$ | Premise |
| {3} | 3. | $S \rightarrow Q$ | Premise |
| {4} | 4. | R | Assumption for $\vee E$ |
| {4} | 5. | $\sim \sim R$ | 4 DNI |
| {2, 4} | 6. | $\sim \sim Q$ | 2, 5 MT |
| {2, 4} | 7. | Q | 6 DNE |
| {9} | 9. | S | Assumption for $\vee E$ |
| {9, 3} | 10. | Q | 3, 9 MP |
| {1, 2, 3} | 11. | Q | 1, 2, 7, 9, 10 $\vee E$ |
| {1, 2, 3} | 12. | $Q \vee P$ | 11 $\vee I$ |

EXERCISE 3.6

1 The proofs are the followings.

1. $(P \vee Q) \& (P \vee R) : P \vee (Q \& R)$

| | | | |
|--------|-----|----------------------------|-------------------------|
| {1} | 1. | $(P \vee Q) \& (P \vee R)$ | Premise |
| {1} | 2. | $P \vee Q$ | 1 $\& E$ |
| {1} | 3. | $P \vee R$ | 1 $\& E$ |
| {4} | 4. | P | Assumption for $\vee E$ |
| {4} | 5. | $P \vee (Q \& R)$ | 4 $\vee I$ |
| {6} | 6. | Q | Assumption for $\vee E$ |
| {7} | 7. | R | Assumption for $\vee E$ |
| {6, 7} | 8. | $Q \& R$ | 6, 7 $\& I$ |
| {6, 7} | 9. | $P \vee (Q \& R)$ | 8 $\vee I$ |
| {1, 7} | 10. | $P \vee (Q \& R)$ | 2, 4, 5, 6, 9 $\vee E$ |
| {1} | 11. | $P \vee (Q \& R)$ | 3, 4, 5, 7, 10 $\vee E$ |

2. $P \vee (Q \vee R) : Q \vee (P \vee R)$

| | | | |
|-----|-----|---------------------|-------------------------|
| {1} | 1. | $P \vee (Q \vee R)$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {2} | 3. | $P \vee R$ | 2 $\vee I$ |
| {2} | 4. | $Q \vee (P \vee R)$ | 3 $\vee I$ |
| {5} | 5. | $Q \vee R$ | Assumption for $\vee E$ |
| {6} | 6. | Q | Assumption for $\vee E$ |
| {6} | 7. | $Q \vee (P \vee R)$ | 6 $\vee I$ |
| {8} | 8. | R | Assumption for $\vee E$ |
| {8} | 9. | $P \vee R$ | 8 $\vee I$ |
| {8} | 10. | $Q \vee (P \vee R)$ | 9 $\vee I$ |
| {5} | 11. | $Q \vee (P \vee R)$ | 5, 6, 7, 8, 10 $\vee E$ |
| {1} | 12. | $Q \vee (P \vee R)$ | 1, 2, 4, 5, 11 $\vee E$ |

EXERCISE 3.7

1 The proofs are the followings.

1. $P \rightarrow (Q \vee R), R \rightarrow S : P \rightarrow (Q \vee S)$

| | | | |
|-----------|-----|----------------------------|-------------------------|
| {1} | 1. | $P \rightarrow (Q \vee R)$ | Premise |
| {2} | 2. | $R \rightarrow S$ | Premise |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | $Q \vee R$ | 1, 3 MP |
| {5} | 5. | Q | Assumption for $\vee E$ |
| {5} | 6. | $Q \vee S$ | 5 $\vee I$ |
| {7} | 7. | R | Assumption for $\vee E$ |
| {2, 7} | 8. | S | 2, 7 MP |
| {2, 7} | 9. | $Q \vee S$ | 8 $\vee I$ |
| {1, 2, 3} | 10. | $Q \vee S$ | 4, 5, 6, 7, 9 $\vee E$ |
| {1, 2} | 11. | $P \rightarrow (Q \vee S)$ | 3, 10 CP |

2. $Q \rightarrow R : (P \vee Q) \rightarrow (P \vee R)$

| | | | |
|--------|----|-------------------------------------|-------------------------|
| {1} | 1. | $Q \rightarrow R$ | Premise |
| {2} | 2. | $P \vee Q$ | Assumption for CP |
| {3} | 3. | P | Assumption for $\vee E$ |
| {3} | 4. | $P \vee R$ | 3 $\vee I$ |
| {5} | 5. | Q | Assumption for $\vee E$ |
| {1, 5} | 6. | R | 1, 5 MP |
| {1, 5} | 7. | $P \vee R$ | 6 $\vee I$ |
| {1, 2} | 8. | $P \vee R$ | 2, 3, 4, 5, 7 $\vee E$ |
| {1} | 9. | $(P \vee Q) \rightarrow (P \vee R)$ | 2, 8 CP |

EXERCISE 3.8

1 The proofs are the followings.

1. $P \& (Q \vee R) : (P \& Q) \vee (P \& R)$

| | | | |
|--------|-----|--------------------------|-------------------------|
| {1} | 1. | $P \& (Q \vee R)$ | Premise |
| {1} | 2. | P | 1 &E |
| {1} | 3. | $Q \vee R$ | 1 &E |
| {4} | 4. | Q | Assumption for \vee E |
| {1, 4} | 5. | $P \& Q$ | 2, 4 &I |
| {1, 4} | 6. | $(P \& Q) \vee (P \& R)$ | 5 \vee I |
| {7} | 7. | R | Assumption for \vee E |
| {1, 7} | 8. | $P \& R$ | 2, 7 &I |
| {1, 7} | 9. | $(P \& Q) \vee (P \& R)$ | 8 \vee I |
| {1} | 10. | $(P \& Q) \vee (P \& R)$ | 3, 4, 6, 7, 9 \vee E |

2. $(P \vee Q) \rightarrow R : (P \rightarrow R) \& (Q \rightarrow R)$

| | | | |
|--------|-----|--|-------------------|
| {1} | 1. | $(P \vee Q) \rightarrow R$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {2} | 3. | $P \vee Q$ | 2 \vee I |
| {1, 2} | 4. | R | 1, 3 MP |
| {1} | 5. | $P \rightarrow R$ | 2, 4 CP |
| {6} | 6. | Q | Assumption for CP |
| {6} | 7. | $P \vee Q$ | 6 \vee I |
| {1, 6} | 8. | R | 1, 7 MP |
| {1} | 9. | $Q \rightarrow R$ | 6, 8 CP |
| {1} | 10. | $(P \rightarrow R) \& (Q \rightarrow R)$ | 5, 9 &I |

EXERCISE 3.9

1 The proofs are the followings.

1. $: \sim(P \& \sim P)$

| | | | |
|-----|----|---------------------|--------------------|
| {1} | 1. | $P \& \sim P$ | Assumption for RAA |
| – | 2. | $\sim(P \& \sim P)$ | 1, 1 RAA |

2. $P \rightarrow \sim P : \sim P$

| | | | |
|--------|----|------------------------|--------------------|
| {1} | 1. | $P \rightarrow \sim P$ | Premise |
| {2} | 2. | P | Assumption for RAA |
| {1, 2} | 3. | $\sim P$ | 1, 2 MP |
| {1, 2} | 4. | $P \& \sim P$ | 2, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |

3. $P \rightarrow Q, Q \rightarrow \sim P : \sim P$

| | | | |
|-----------|----|------------------------|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $Q \rightarrow \sim P$ | Premise |
| {3} | 3. | P | Assumption for RAA |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2, 3} | 5. | $\sim P$ | 2, 4 MP |
| {1, 2, 3} | 6. | $P \& \sim P$ | 3, 5 &I |
| {1, 2} | 7. | $\sim P$ | 3, 6 RAA |

4. $P \rightarrow Q, \sim P \rightarrow Q : Q$

| | | | |
|-----------|----|----------------------------|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $\sim P \rightarrow Q$ | Premise |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {1, 3} | 4. | $\sim P$ | 1, 3 MT |
| {2, 3} | 5. | $\sim \sim P$ | 2, 3 MT |
| {1, 2, 3} | 6. | $(\sim P) \& \sim(\sim P)$ | 4, 5 &I |
| {1, 2} | 7. | $\sim \sim Q$ | 3, 6 RAA |
| {1, 2} | 8. | Q | 7 DNE |

5. $\sim(P \vee Q) : \sim P$

| | | | |
|--------|----|--------------------------------|--------------------|
| {1} | 1. | $\sim(P \vee Q)$ | Premise |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee Q$ | 2 vI |
| {1, 2} | 4. | $(P \vee Q) \& \sim(P \vee Q)$ | 1, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |

6. $\sim(P \vee Q), R \rightarrow P : \sim R$

| | | | |
|-----------|----|--------------------------------|--------------------|
| {1} | 1. | $\sim(P \vee Q)$ | Premise |
| {2} | 2. | $R \rightarrow P$ | Premise |
| {3} | 3. | R | Assumption for RAA |
| {2, 3} | 4. | P | 2, 3 MP |
| {2, 3} | 5. | $P \vee Q$ | 4 vI |
| {1, 2, 3} | 6. | $(P \vee Q) \& \sim(P \vee Q)$ | 1, 5 &I |
| {1, 2} | 7. | $\sim R$ | 3, 6 RAA |

7. $(P \& Q) \rightarrow \sim R : R \rightarrow (P \rightarrow \sim Q)$

| | | | |
|--------------|-----|--|--------------------|
| {1} | 1. | $(P \& Q) \rightarrow \sim R$ | Premise |
| {2} | 2. | R | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {4} | 4. | Q | Assumption for RAA |
| {3, 4} | 5. | $P \& Q$ | 3, 4 &I |
| {1, 3, 4} | 6. | $\sim R$ | 1, 5 MP |
| {1, 2, 3, 4} | 7. | $R \& \sim R$ | 2, 6 &I |
| {1, 2, 3} | 8. | $\sim Q$ | 4, 7 RAA |
| {1, 2} | 9. | $P \rightarrow \sim Q$ | 3, 8 CP |
| {1} | 10. | $R \rightarrow (P \rightarrow \sim Q)$ | 2, 9 CP |

8. $P \rightarrow (Q \rightarrow (R \& \sim R)) : P \rightarrow \sim Q$

| | | | |
|-----------|----|---|--------------------|
| {1} | 1. | $P \rightarrow (Q \rightarrow (R \& \sim R))$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | Q | Assumption for RAA |
| {1, 2} | 4. | $Q \rightarrow (R \& \sim R)$ | 1, 2 MP |
| {1, 2, 3} | 5. | $R \& \sim R$ | 3, 4 MP |
| {1, 2} | 6. | $\sim Q$ | 3, 5 RAA |
| {1} | 7. | $P \rightarrow \sim Q$ | 2, 6 CP |

9. $\sim(P \& \sim Q) : P \rightarrow Q$

| | | | |
|-----------|----|--------------------------------------|--------------------|
| {1} | 1. | $\sim(P \& \sim Q)$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 4. | $P \& \sim Q$ | 2, 3 &I |
| {1, 2, 3} | 5. | $(P \& \sim Q) \& \sim(P \& \sim Q)$ | 1, 4 &I |
| {1, 2} | 6. | $\sim\sim Q$ | 3, 5 RAA |
| {1, 2} | 7. | Q | 6 DNE |
| {1} | 8. | $P \rightarrow Q$ | 2, 7 CP |

10. $P \rightarrow Q : \sim(P \& \sim Q)$

| | | | |
|--------|----|---------------------|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $P \& \sim Q$ | Assumption for RAA |
| {2} | 3. | P | 2 &E |
| {2} | 4. | $\sim Q$ | 2 &E |
| {1, 2} | 5. | Q | 1, 3 MP |
| {1, 2} | 6. | $Q \& \sim Q$ | 4, 5 &I |
| {1} | 7. | $\sim(P \& \sim Q)$ | 2, 6 RAA |

11. $\sim(P \rightarrow Q) : P \ \& \ \sim Q$

| | | | |
|-----------|-----|---|--------------------|
| {1} | 1. | $\sim(P \ \& \ \sim Q)$ | Assumption for CP |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 4. | $P \ \& \ \sim Q$ | 2, 3 &I |
| {1, 2, 3} | 5. | $(P \ \& \ \sim Q) \ \& \ \sim(P \ \& \ \sim Q)$ | 1, 4 &I |
| {1, 2} | 6. | $\sim\sim Q$ | 3, 5 RAA |
| {1, 2} | 7. | Q | 6 DNE |
| {1} | 8. | $P \rightarrow Q$ | 2, 7 CP |
| — | 9. | $(\sim(P \ \& \ \sim Q)) \rightarrow (P \rightarrow Q)$ | 1, 8 CP |
| {10} | 10. | $\sim(P \rightarrow Q)$ | Premise |
| {10} | 11. | $\sim\sim(P \ \& \ \sim Q)$ | 9, 10 MT |
| {10} | 12. | $P \ \& \ \sim Q$ | 11 DNE |

12. $P \rightarrow Q : (Q \rightarrow \sim P) \rightarrow \sim P$

| | | | |
|-----------|----|---|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $Q \rightarrow \sim P$ | Assumption for CP |
| {3} | 3. | P | Assumption for RAA |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2, 3} | 5. | $\sim P$ | 2, 4 MP |
| {1, 2, 3} | 6. | $P \ \& \ \sim P$ | 3, 5 &I |
| {1, 2} | 7. | $\sim P$ | 3, 6 RAA |
| {1} | 8. | $(Q \rightarrow \sim P) \rightarrow \sim P$ | 2, 7 CP |

13. $P \rightarrow R, Q \rightarrow \sim R : \sim(P \ \& \ Q)$

| | | | |
|-----------|----|------------------------|--------------------|
| {1} | 1. | $P \rightarrow R$ | Premise |
| {2} | 2. | $Q \rightarrow \sim R$ | Premise |
| {3} | 3. | $P \ \& \ Q$ | Assumption for RAA |
| {3} | 4. | P | 3 &E |
| {1, 3} | 5. | R | 1, 4 MP |
| {3} | 6. | Q | 3 &E |
| {2, 3} | 7. | $\sim R$ | 2, 6 MP |
| {1, 2, 3} | 8. | $R \ \& \ \sim R$ | 5, 7 &I |
| {1, 2} | 9. | $\sim(P \ \& \ Q)$ | 3, 8 RAA |

14. $\sim P : P \rightarrow Q$

| | | | |
|-----------|----|-------------------|--------------------|
| {1} | 1. | $\sim P$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 4. | $P \& \sim Q$ | 2, 3 &I |
| {2, 3} | 5. | P | 4 &E |
| {1, 2, 3} | 6. | $P \& \sim P$ | 1, 5 &I |
| {1, 2} | 7. | $\sim\sim Q$ | 3, 6 RAA |
| {1, 2} | 8. | Q | 7 DNE |
| {1} | 9. | $P \rightarrow Q$ | 2, 8 CP |

15. $P, \sim P : Q$

| | | | |
|-----------|----|---------------|--------------------|
| {1} | 1. | $\sim P$ | Premise |
| {2} | 2. | P | Premise |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 4. | $P \& \sim Q$ | 2, 3 &I |
| {2, 3} | 5. | P | 4 &E |
| {1, 2, 3} | 6. | $P \& \sim P$ | 1, 5 &I |
| {1, 2} | 7. | $\sim\sim Q$ | 3, 6 RAA |
| {1, 2} | 8. | Q | 7 DNE |

16. $: P \vee \sim P$

| | | | |
|-----|----|--|--------------------|
| {1} | 1. | $\sim(P \vee \sim P)$ | Assumption for RAA |
| {2} | 2. | P | Assumption for CP |
| {2} | 3. | $P \vee \sim P$ | 2 vI |
| — | 4. | $P \rightarrow (P \vee \sim P)$ | 2, 3 CP |
| {1} | 5. | $\sim P$ | 1, 4 MT |
| {1} | 6. | $P \vee \sim P$ | 5 vI |
| {1} | 7. | $(P \vee \sim P) \& (\sim(P \vee \sim P))$ | 1, 6 &I |
| — | 8. | $\sim\sim(P \vee \sim P)$ | 1, 7 RAA |
| — | 9. | $P \vee \sim P$ | 8 DNE |

17. $P \vee Q : \sim(\sim P \& \sim Q)$

| | | | |
|--------|-----|--------------------------|--------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $\sim P \& \sim Q$ | Assumption for RAA |
| {2} | 3. | $\sim P$ | 2 &E |
| {2} | 4. | $\sim Q$ | 2 &E |
| {5} | 5. | P | Assumption for vE |
| {2, 5} | 6. | $P \& \sim P$ | 3, 5 &I |
| {5} | 7. | $\sim(\sim P \& \sim Q)$ | 2, 6 RAA |
| {8} | 8. | Q | Assumption for vE |
| {2, 8} | 9. | $Q \& \sim Q$ | 4, 8 &I |
| {8} | 10. | $\sim(\sim P \& \sim Q)$ | 2, 9 RAA |
| {1} | 11. | $\sim(\sim P \& \sim Q)$ | 1, 5, 7, 8, 10 vE |

18. $\sim(P \vee Q) : \sim P \ \& \ \sim Q$

| | | | |
|--------|-----|--------------------------------------|--------------------|
| {1} | 1. | $\sim(P \vee Q)$ | Premise |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee Q$ | 2 vI |
| {1, 2} | 4. | $(P \vee Q) \ \& \ (\sim(P \vee Q))$ | 1, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| {6} | 6. | Q | Assumption for RAA |
| {6} | 7. | $P \vee Q$ | 6 vI |
| {1, 6} | 8. | $(P \vee Q) \ \& \ (\sim(P \vee Q))$ | 1, 7 &I |
| {1} | 9. | $\sim Q$ | 6, 8 RAA |
| {1} | 10. | $\sim P \ \& \ \sim Q$ | 5, 9 &I |

19. $\sim(\sim P \ \& \ \sim Q) : P \vee Q$

| | | | |
|---------|-----|--|--------------------|
| {1} | 1. | $\sim(P \vee Q)$ | Assumption for RAA |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee Q$ | 2 vI |
| {1, 2} | 4. | $(P \vee Q) \ \& \ (\sim(P \vee Q))$ | 1, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| {6} | 6. | Q | Assumption for RAA |
| {6} | 7. | $P \vee Q$ | 6 vI |
| {1, 6} | 8. | $(P \vee Q) \ \& \ (\sim(P \vee Q))$ | 1, 7 &I |
| {1} | 9. | $\sim Q$ | 6, 8 RAA |
| {1} | 10. | $\sim P \ \& \ \sim Q$ | 5, 9 &I |
| {11} | 11. | $\sim(\sim P \ \& \ \sim Q)$ | Premise |
| {1, 11} | 12. | $(\sim P \ \& \ \sim Q) \ \& \ (\sim(\sim P \ \& \ \sim Q))$ | 10, 11 &I |
| {11} | 13. | $\sim\sim(P \vee Q)$ | 1, 12 RAA |
| {11} | 14. | $P \vee Q$ | 13 DNE |

20. : $((P \rightarrow Q) \vee (Q \rightarrow R))$

| | | | |
|-------------|-----|--|--------------------|
| {1} | 1. | $\sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | Assumption for RAA |
| {2} | 2. | Q | Assumption for RAA |
| {3} | 3. | P | Assumption for CP |
| {2, 3} | 4. | $P \& Q$ | 2, 3 &I |
| {2, 3} | 5. | Q | 4 &E |
| {2} | 6. | $P \rightarrow Q$ | 3, 5 CP |
| {2} | 7. | $(P \rightarrow Q) \vee (Q \rightarrow R)$ | 6 vI |
| {1, 2} | 8. | $((P \rightarrow Q) \vee (Q \rightarrow R)) \& \sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | 1, 7 &I |
| {1} | 9. | $\sim Q$ | 1, 8 RAA |
| {10} | 10. | Q | Assumption for CP |
| {11} | 11. | $\sim R$ | Assumption for RAA |
| {10, 11} | 12. | $Q \& \sim R$ | 10, 11 &I |
| {10, 11} | 13. | Q | 12 &E |
| {1, 10, 11} | 14. | $Q \& \sim Q$ | 1, 13 &I |
| {1, 10} | 15. | $\sim\sim R$ | 11, 14 RAA |
| {1, 10} | 16. | R | 15 DNE |
| {1} | 17. | $Q \rightarrow R$ | 10, 16 CP |
| {1} | 18. | $(P \rightarrow Q) \vee (Q \rightarrow R)$ | 17 vI |
| {1} | 19. | $((P \rightarrow Q) \vee (Q \rightarrow R)) \& \sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | 1, 18 &I |
| — | 20. | $\sim\sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | 1, 19 RAA |
| — | 21. | $(P \rightarrow Q) \vee (Q \rightarrow R)$ | 20 DNE |

(Maybe the book says it can be done in 20 steps, because it uses only one Q assumption, and uses the assumption on line 2. for the assumption which is written in line 10. in my proof.)

REVISION EXERCISE I

1 The proofs are the followings.

1. $P \rightarrow Q : ((R \& Q) \rightarrow S) \rightarrow ((R \& P) \rightarrow S)$

| | | | |
|-----------|-----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $(R \& Q) \rightarrow S$ | Assumption for CP |
| {3} | 3. | $R \& P$ | Assumption for CP |
| {3} | 4. | R | 3 &E |
| {3} | 5. | P | 3 &E |
| {1, 3} | 6. | Q | 1, 5 MP |
| {1, 3} | 7. | $R \& Q$ | 4, 6 &I |
| {1, 2, 3} | 8. | S | 2, 7 MP |
| {1, 2} | 9. | $(R \& P) \rightarrow S$ | 3, 8 CP |
| {1} | 10. | $((R \& Q) \rightarrow S) \rightarrow ((R \& P) \rightarrow S)$ | 2, 9 CP |

2. $(P \& Q) \rightarrow \sim R : R \rightarrow (P \rightarrow \sim Q)$

| | | | |
|--------------|-----|--|--------------------|
| {1} | 1. | $(P \& Q) \rightarrow \sim R$ | Premise |
| {2} | 2. | R | Assumption for CP |
| {3} | 3. | P | Assumption for CP |
| {2} | 4. | $\sim \sim R$ | 2 DNI |
| {1, 2} | 5. | $\sim(P \& Q)$ | 1, 4 MT |
| {6} | 6. | Q | Assumption for RAA |
| {3, 6} | 7. | $P \& Q$ | 3, 6 &I |
| {1, 2, 3, 6} | 8. | $(P \& Q) \& \sim(P \& Q)$ | 5, 7 &I |
| {1, 2, 3} | 9. | $\sim Q$ | 6, 8 RAA |
| {1, 2} | 10. | $P \rightarrow \sim Q$ | 3, 9 CP |
| {1} | 11. | $R \rightarrow (P \rightarrow \sim Q)$ | 2, 10 CP |

3. $:(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$

| | | | |
|--------|----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Assumption for CP |
| {2} | 2. | $\sim Q$ | Assumption for CP |
| {1, 2} | 3. | $\sim P$ | 1, 2 MT |
| {1} | 4. | $\sim Q \rightarrow \sim P$ | 2, 3 CP |
| — | 5. | $(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$ | 1, 4 CP |

4. $P \vee Q : (P \vee R) \vee (Q \vee R)$

| | | | |
|-----|----|------------------------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {2} | 3. | $P \vee R$ | 2 $\vee I$ |
| {2} | 4. | $(P \vee R) \vee (Q \vee R)$ | 3 $\vee I$ |
| {5} | 5. | Q | Assumption for $\vee E$ |
| {5} | 6. | $Q \vee R$ | 5 $\vee I$ |
| {5} | 7. | $(P \vee R) \vee (Q \vee R)$ | 6 $\vee I$ |
| {1} | 8. | $(P \vee R) \vee (Q \vee R)$ | 1, 2, 4, 5, 7 $\vee E$ |

5. $P \rightarrow R, Q \rightarrow S : (P \vee Q) \rightarrow (R \vee S)$

| | | | |
|-----------|-----|-------------------------------------|-------------------------|
| {1} | 1. | $P \rightarrow R$ | Premise |
| {2} | 2. | $Q \rightarrow S$ | Premise |
| {3} | 3. | $P \vee Q$ | Assumption for CP |
| {4} | 4. | P | Assumption for $\vee E$ |
| {1, 4} | 5. | R | 1, 4 MP |
| {1, 4} | 6. | $R \vee S$ | 5 $\vee I$ |
| {7} | 7. | Q | Assumption for $\vee E$ |
| {2, 7} | 8. | S | 2, 7 MP |
| {2, 7} | 9. | $R \vee S$ | 8 $\vee I$ |
| {1, 2, 3} | 10. | $R \vee S$ | 3, 4, 6, 7, 9 $\vee E$ |
| {1, 2} | 11. | $(P \vee Q) \rightarrow (R \vee S)$ | 3, 10 CP |

6. $P \rightarrow (Q \vee R), Q \rightarrow R : P \rightarrow R$

| | | | |
|-----------|----|----------------------------|-------------------------|
| {1} | 1. | $P \rightarrow (Q \vee R)$ | Premise |
| {2} | 2. | $Q \rightarrow R$ | Premise |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | $Q \vee R$ | 1, 3 MP |
| {5} | 5. | Q | Assumption for $\vee E$ |
| {5, 2} | 6. | R | 2, 5 MP |
| {7} | 7. | R | Assumption for $\vee E$ |
| {1, 2, 3} | 8. | R | 4, 5, 6, 7, 7 $\vee E$ |
| {1, 2} | 9. | $P \rightarrow R$ | 3, 8 CP |

7. $(P \vee Q) \rightarrow R : (P \rightarrow R) \& (Q \rightarrow R)$

| | | | |
|--------|-----|--|-------------------|
| {1} | 1. | $(P \vee Q) \rightarrow R$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {2} | 3. | $P \vee Q$ | 2 vI |
| {1, 2} | 4. | R | 1, 3 MP |
| {1} | 5. | $P \rightarrow R$ | 2, 4 CP |
| {6} | 6. | Q | Assumption for CP |
| {6} | 7. | $P \vee Q$ | 6 vI |
| {1, 6} | 8. | R | 1, 7 CP |
| {1} | 9. | $Q \rightarrow R$ | 6, 8 CP |
| {1} | 10. | $(P \rightarrow R) \& (Q \rightarrow R)$ | 5, 9 &I |

8. $\sim(P \& \sim Q) : P \rightarrow Q$

| | | | |
|-----------|----|--------------------------------------|--------------------|
| {1} | 1. | $\sim(P \& \sim Q)$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 4. | $P \& \sim Q$ | 2, 3 &I |
| {1, 2, 3} | 5. | $(P \& \sim Q) \& \sim(P \& \sim Q)$ | 1, 4 &I |
| {1, 2} | 6. | $\sim\sim Q$ | 3, 5 RAA |
| {1, 2} | 7. | Q | 6 DNE |
| {1} | 8. | $P \rightarrow Q$ | 4, 7 CP |

9. $P \rightarrow (Q \leftrightarrow R) : (P \& Q) \rightarrow R$

| | | | |
|--------|----|--|-----------------------|
| {1} | 1. | $P \rightarrow (Q \leftrightarrow R)$ | Premise |
| {2} | 2. | $P \& Q$ | Assumption for CP |
| {2} | 3. | P | 2 &E |
| {2} | 4. | Q | 2 &E |
| {1, 2} | 5. | $Q \leftrightarrow R$ | 1, 3 MP |
| {1, 2} | 6. | $(Q \rightarrow R) \& (R \rightarrow Q)$ | 5 \leftrightarrow E |
| {1, 2} | 7. | $Q \rightarrow R$ | 7 &E |
| {1, 2} | 8. | R | 4, 7 MP |
| {1} | 9. | $(P \& Q) \rightarrow R$ | 2, 8 CP |

10. $\therefore \sim P \rightarrow (P \rightarrow Q)$

| | | | |
|-----------|-----|--|--------------------|
| {1} | 1. | $\sim P$ | Assumption for CP |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 4. | $P \& \sim Q$ | 2, 3 &I |
| {2, 3} | 5. | P | 4 &E |
| {1, 2, 3} | 6. | $P \& \sim P$ | 1, 5 &I |
| {1, 2} | 7. | $\sim\sim Q$ | 3, 6 RAA |
| {1, 2} | 8. | Q | 7 DNE |
| {1} | 9. | $P \rightarrow Q$ | 2, 8 CP |
| — | 10. | $\sim P \rightarrow (P \rightarrow Q)$ | 1, 9 CP |

REVISION EXERCISE II

1 The proofs are the followings.

1. $P \vee P : P$

| | | | |
|-----|----|------------|-------------------------|
| {1} | 1. | $P \vee P$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {1} | 3. | P | 1, 2, 2, 2, 2 $\vee E$ |

2. $P : (P \rightarrow Q) \rightarrow Q$

| | | | |
|--------|----|-----------------------------------|-------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $P \rightarrow Q$ | Assumption for CP |
| {1, 2} | 3. | Q | 1, 2 MP |
| {1} | 4. | $(P \rightarrow Q) \rightarrow Q$ | 2, 3 CP |

3. $P : (\sim(Q \rightarrow R) \rightarrow \sim P) \rightarrow ((\sim R \rightarrow \sim Q))$

| | | | |
|-----------|----|--|-------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $\sim(Q \rightarrow R) \rightarrow \sim P$ | Assumption for CP |
| {1} | 3. | $\sim\sim P$ | 1 DNI |
| {1, 2} | 4. | $\sim\sim(Q \rightarrow R)$ | 2, 3 MT |
| {1, 2} | 5. | $Q \rightarrow R$ | 4 DNE |
| {6} | 6. | $\sim R$ | Assumption for CP |
| {1, 2, 6} | 7. | $\sim Q$ | 5, 6 MT |
| {1, 2} | 8. | $\sim R \rightarrow \sim Q$ | 6, 7 CP |
| {1} | 9. | $(\sim(Q \rightarrow R) \rightarrow \sim P) \rightarrow (\sim R \rightarrow \sim Q)$ | 2, 8 CP |

4. $P \rightarrow (Q \vee R), R \rightarrow S : P \rightarrow (Q \vee S)$

| | | | |
|-----------|-----|----------------------------|-------------------------|
| {1} | 1. | $P \rightarrow (Q \vee R)$ | Premise |
| {2} | 2. | $R \rightarrow S$ | Premise |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | $Q \vee R$ | 1, 3 MP |
| {5} | 5. | Q | Assumption for $\vee E$ |
| {5} | 6. | $Q \vee S$ | 5 $\vee I$ |
| {7} | 7. | R | Assumption for $\vee E$ |
| {2, 7} | 8. | S | 2, 7 MP |
| {2, 7} | 9. | $Q \vee S$ | 8 $\vee I$ |
| {1, 2, 3} | 10. | $Q \vee S$ | 4, 5, 6, 7, 9 $\vee E$ |
| {1, 2} | 11. | $P \rightarrow (Q \vee S)$ | 3, 10 CP |

5. $\sim Q \rightarrow \sim R, R \vee S, S \rightarrow Q : Q \vee P$

| | | | |
|-----------|-----|-----------------------------|-------------------------|
| {1} | 1. | $\sim Q \rightarrow \sim R$ | Premise |
| {2} | 2. | $R \vee S$ | Premise |
| {3} | 3. | $S \rightarrow Q$ | Premise |
| {4} | 4. | R | Assumption for $\vee E$ |
| {4} | 5. | $\sim \sim R$ | 4 DNI |
| {1, 4} | 6. | $\sim \sim Q$ | 1, 5 MT |
| {1, 4} | 7. | Q | 6 DNE |
| {1, 4} | 8. | $Q \vee P$ | 7 $\vee I$ |
| {9} | 9. | S | Assumption for $\vee E$ |
| {3, 9} | 10. | Q | 3, 9 MP |
| {3, 9} | 11. | $Q \vee P$ | 10 $\vee I$ |
| {1, 2, 3} | 12. | $Q \vee P$ | 2, 4, 8, 9, 11 $\vee E$ |

6. $\sim(P \vee Q) : \sim P \ \& \ \sim Q$

| | | | |
|--------|-----|------------------------------------|--------------------|
| {1} | 1. | $\sim(P \vee Q)$ | Premise |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee Q$ | 2 $\vee I$ |
| {1, 2} | 4. | $(P \vee Q) \ \& \ \sim(P \vee Q)$ | 1, 3 $\&I$ |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| {6} | 6. | Q | Assumption for RAA |
| {6} | 7. | $P \vee Q$ | 6 $\vee I$ |
| {1, 6} | 8. | $(P \vee Q) \ \& \ \sim(P \vee Q)$ | 1, 7 $\&I$ |
| {1} | 9. | $\sim Q$ | 6, 8 RAA |
| {1} | 10. | $\sim P \ \& \ \sim Q$ | 5, 9 $\&I$ |

7. $\sim \sim(P \vee \sim Q) : (P \rightarrow \sim Q) \vee (\sim Q \rightarrow P)$

| | | | |
|---------|-----|--|-------------------------|
| {1} | 1. | $\sim \sim(P \vee \sim Q)$ | Premise |
| {1} | 2. | $P \vee \sim Q$ | 1 DNE |
| {3} | 3. | P | Assumption for $\vee E$ |
| {4} | 4. | $\sim Q$ | Assumption for CP |
| {3, 4} | 5. | $P \ \& \ \sim Q$ | 3, 4 $\&I$ |
| {3, 4} | 6. | P | 5 $\&E$ |
| {3} | 7. | $\sim Q \rightarrow P$ | 4, 6 CP |
| {3} | 8. | $(P \rightarrow \sim Q) \vee (\sim Q \rightarrow P)$ | 7 $\vee I$ |
| {9} | 9. | $\sim Q$ | Assumption for $\vee E$ |
| {10} | 10. | P | Assumption for CP |
| {9, 10} | 11. | $P \ \& \ \sim Q$ | 9, 10 $\&I$ |
| {9, 10} | 12. | $\sim Q$ | 11 $\&E$ |
| {9} | 13. | $P \rightarrow \sim Q$ | 10, 12 CP |
| {9} | 14. | $(P \rightarrow \sim Q) \vee (\sim Q \rightarrow P)$ | 13 $\vee I$ |
| {1} | 15. | $(P \rightarrow \sim Q) \vee (\sim Q \rightarrow P)$ | 2, 3, 8, 9, 14 $\vee E$ |

(Maybe the book says it can be done in 11 steps because it uses the same steps 3, 4, 5 instead of introducing steps 9, 10 11, which would shorten this proof to 12.)

8. $(P \vee Q) \leftrightarrow P : Q \rightarrow P$

| | | | |
|--------|----|--|-----------------------|
| {1} | 1. | $(P \vee Q) \leftrightarrow P$ | Premise |
| {1} | 2. | $((P \vee Q) \rightarrow P) \& (P \rightarrow (P \vee Q))$ | 1 \leftrightarrow E |
| {1} | 3. | $(P \vee Q) \rightarrow P$ | 2 &E |
| {4} | 4. | Q | Assumption for CP |
| {4} | 5. | $P \vee Q$ | 4 vI |
| {1, 4} | 6. | P | 3, 5 MP |
| {1} | 7. | $Q \rightarrow P$ | 4, 6 CP |

9. $(P \& Q) \vee (P \& R) : P \& (Q \vee R)$

| | | | |
|-----|-----|--------------------------|-------------------|
| {1} | 1. | $(P \& Q) \vee (P \& R)$ | Premise |
| {2} | 2. | $P \& Q$ | Assumption for vE |
| {2} | 3. | P | 2 &E |
| {2} | 4. | Q | 2 &E |
| {2} | 5. | $Q \vee R$ | 4 vI |
| {2} | 6. | $P \& (Q \vee R)$ | 3, 5 &I |
| {7} | 7. | $P \& R$ | Assumption for vE |
| {7} | 8. | P | 7 &E |
| {7} | 9. | R | 7 &E |
| {7} | 10. | $Q \vee R$ | 9 vI |
| {7} | 11. | $P \& (Q \vee R)$ | 8, 10 &I |
| {1} | 12. | $P \& (Q \vee R)$ | 1, 2, 6, 7, 11 vE |

10. $: P \vee \sim P$

| | | | |
|--------|----|--|--------------------|
| {1} | 1. | $\sim(P \vee \sim P)$ | Assumption for RAA |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee \sim P$ | 2 vI |
| {1, 2} | 4. | $(P \vee \sim P) \& \sim(P \vee \sim P)$ | 1, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| {1} | 6. | $P \vee \sim P$ | 5 vI |
| {1} | 7. | $(P \vee \sim P) \& \sim(P \vee \sim P)$ | 1, 6 &I |
| — | 8. | $\sim\sim(P \vee \sim P)$ | 1, 7 RAA |
| — | 9. | $P \vee \sim P$ | 8 DNE |

REVISION EXERCISE III

1 The proofs are the followings.

1. : $((P \rightarrow P) \rightarrow Q) \rightarrow Q$

| | | | |
|-----|----|---|-------------------|
| {1} | 1. | $(P \rightarrow P) \rightarrow Q$ | Assumption for CP |
| {2} | 2. | P | Assumption for CP |
| – | 3. | $P \rightarrow P$ | 2, 2 CP |
| {1} | 4. | Q | 1, 3 CP |
| – | 5. | $((P \rightarrow P) \rightarrow Q) \rightarrow Q$ | 1, 4 CP |

2. $\sim(P \rightarrow Q) : P \ \& \ \sim Q$

| | | | |
|-----------|-----|--|--------------------|
| {1} | 1. | $\sim(P \rightarrow Q)$ | Premise |
| {2} | 2. | $\sim(P \ \& \ \sim Q)$ | Assumption for RAA |
| {3} | 3. | P | Assumption for CP |
| {4} | 4. | $\sim Q$ | Assumption for RAA |
| {3, 4} | 5. | $P \ \& \ \sim Q$ | 3, 4 &I |
| {2, 3, 4} | 6. | $(P \ \& \ \sim Q) \ \& \ \sim(P \ \& \ \sim Q)$ | 2, 5 &I |
| {2, 3} | 7. | $\sim\sim Q$ | 4, 6 RAA |
| {2, 3} | 8. | Q | 7 DNE |
| {2} | 9. | $P \rightarrow Q$ | 3, 8 CP |
| {1, 2} | 10. | $(P \rightarrow Q) \ \& \ \sim(P \rightarrow Q)$ | 1, 9 &I |
| {1} | 11. | $\sim\sim(P \ \& \ \sim Q)$ | 2, 10 RAA |
| {1} | 12. | $P \ \& \ \sim Q$ | 11 DNE |

3. $(P \vee Q) \& (R \vee S) : ((P \& R) \vee (P \& S)) \vee ((Q \& R) \vee (Q \& S))$

| | | | |
|--------|-----|---|---------------------------|
| {1} | 1. | $(P \vee Q) \& (R \vee S)$ | Premise |
| {1} | 2. | $P \vee Q$ | 1 &E |
| {1} | 3. | $R \vee S$ | 1 &E |
| {4} | 4. | P | Assumption for \vee E |
| {5} | 5. | Q | Assumption for \vee E |
| {6} | 6. | R | Assumption for \vee E |
| {7} | 7. | S | Assumption for \vee E |
| {4, 6} | 8. | $P \& R$ | 4, 6 &I |
| {4, 6} | 9. | $(P \& R) \vee (P \& S)$ | 8 \vee I |
| {4, 7} | 10. | $P \& S$ | 4, 7 &I |
| {4, 7} | 11. | $(P \& R) \vee (P \& S)$ | 10 \vee I |
| {1, 4} | 12. | $(P \& R) \vee (P \& S)$ | 3, 6, 9, 7, 11 \vee E |
| {1, 4} | 13. | $((P \& R) \vee (P \& S))$ $\vee ((Q \& R) \vee (Q \& S))$ | 12 \vee I |
| {5, 6} | 14. | $Q \& R$ | 5, 6 &I |
| {5, 6} | 15. | $(Q \& R) \vee (Q \& S)$ | 14 \vee I |
| {5, 7} | 16. | $Q \& S$ | 5, 7 &I |
| {5, 7} | 17. | $(Q \& R) \vee (Q \& S)$ | 16 \vee I |
| {1, 5} | 18. | $(Q \& R) \vee (Q \& S)$ | 3, 6, 15, 7, 17 \vee E |
| {1, 5} | 20. | $((P \& R) \vee (P \& S))$ $\vee ((Q \& R) \vee (Q \& S))$ | 18 \vee I |
| {1} | 21. | $((P \& R) \vee (P \& S))$ $\vee ((Q \& R) \vee (Q \& S))$ | 2, 4, 13, 14, 20 \vee E |

4. $P \vee Q, \sim Q : P$

| | | | |
|-----------|-----|---------------------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $\sim Q$ | Premise |
| {3} | 3. | P | Assumption for \vee E |
| {4} | 4. | Q | Assumption for \vee E |
| {5} | 5. | $\sim P$ | Assumption for RAA |
| {2, 4} | 6. | $Q \& \sim Q$ | 2, 4 &I |
| {2, 4, 5} | 7. | $(Q \& \sim Q) \& \sim P$ | 5, 6 &I |
| {2, 4, 5} | 8. | $Q \& \sim Q$ | 7 &E |
| {2, 4} | 9. | $\sim \sim P$ | 5, 8 RAA |
| {2, 4} | 10. | P | 9 DNE |
| {1, 2} | 11. | P | 1, 3, 3, 4, 10 \vee E |

5. $P \vee Q, \sim P : Q$

| | | | |
|-----------|-----|---------------------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $\sim P$ | Premise |
| {3} | 3. | Q | Assumption for $\vee E$ |
| {4} | 4. | P | Assumption for $\vee E$ |
| {5} | 5. | $\sim Q$ | Assumption for RAA |
| {2, 4} | 6. | $P \& \sim P$ | 2, 4 &I |
| {2, 4, 5} | 7. | $(P \& \sim P) \& \sim Q$ | 5, 6 &I |
| {2, 4, 5} | 8. | $P \& \sim P$ | 7 &E |
| {2, 4} | 9. | $\sim \sim Q$ | 5, 8 RAA |
| {2, 4} | 10. | Q | 9 DNE |
| {1, 2} | 11. | Q | 1, 3, 3, 4, 10 $\vee E$ |

6. $: ((\sim P \rightarrow R) \& (\sim Q \rightarrow R)) \rightarrow (\sim(P \& Q) \rightarrow R)$

| | | | |
|--------|-----|---|-------------------|
| {1} | 1. | $(\sim P \rightarrow R) \& (\sim Q \rightarrow R)$ | Assumption for CP |
| {1} | 2. | $\sim P \rightarrow R$ | 1 &E |
| {1} | 3. | $\sim Q \rightarrow R$ | 1 &E |
| {4} | 4. | $\sim(P \& Q)$ | Assumption for CP |
| {5} | 5. | $\sim R$ | Assumption for CP |
| {1, 5} | 6. | $\sim \sim P$ | 2, 5 MT |
| {1, 5} | 7. | P | 6 DNE |
| {1, 5} | 8. | $\sim \sim Q$ | 3, 5 MT |
| {1, 5} | 9. | Q | 8 DNE |
| {1, 5} | 10. | $P \& Q$ | 7, 9 &I |
| {1} | 11. | $\sim R \rightarrow (P \& Q)$ | 5, 10 CP |
| {1, 4} | 12. | $\sim \sim R$ | 4, 11 MT |
| {1, 4} | 13. | R | 12 DNE |
| {1} | 14. | $\sim(P \& Q) \rightarrow R$ | 4, 13 CP |
| – | 15. | $((\sim P \rightarrow R) \& (\sim Q \rightarrow R)) \rightarrow (\sim(P \& Q) \rightarrow R)$ | 1, 14 CP |

7. $P \vee Q, P \vee R : P \vee (Q \& R)$

| | | | |
|--------|-----|-------------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $P \vee R$ | Premise |
| {3} | 3. | P | Assumption for $\vee E$ |
| {4} | 4. | Q | Assumption for $\vee E$ |
| {5} | 5. | R | Assumption for $\vee E$ |
| {3} | 6. | $P \vee (Q \& R)$ | 3 $\vee I$ |
| {4, 5} | 7. | $Q \& R$ | 4, 5 &I |
| {4, 5} | 8. | $P \vee (Q \& R)$ | 7 $\vee I$ |
| {1, 5} | 9. | $P \vee (Q \& R)$ | 1, 3, 6, 4, 8 $\vee E$ |
| {1} | 10. | $P \vee (Q \& R)$ | 2, 3, 6, 5, 9 $\vee E$ |

8. $P \leftrightarrow Q, Q \leftrightarrow R : P \leftrightarrow R$

| | | | |
|------------|-----|--|----------------------------|
| {1} | 1. | $P \leftrightarrow Q$ | Premise |
| {2} | 2. | $Q \leftrightarrow R$ | Premise |
| {1} | 3. | $(P \rightarrow Q) \& (Q \rightarrow P)$ | 1 \leftrightarrow E |
| {1} | 4. | $P \rightarrow Q$ | 3 &E |
| {1} | 5. | $Q \rightarrow P$ | 3 &E |
| {2} | 6. | $(Q \rightarrow R) \& (R \rightarrow Q)$ | 2 \leftrightarrow E |
| {2} | 7. | $Q \rightarrow R$ | 6 &E |
| {2} | 8. | $R \rightarrow Q$ | 6 &E |
| {9} | 9. | P | Assumption for CP |
| {1, 9} | 10. | Q | 4, 9 MP |
| {1, 2, 9} | 11. | R | 7, 10 MP |
| {1, 2} | 12. | $P \rightarrow R$ | 9, 11 CP |
| {13} | 13. | R | Assumption for CP |
| {2, 13} | 14. | Q | 8, 13 MP |
| {1, 2, 13} | 15. | P | 5, 14 MP |
| {1, 2} | 16. | $R \rightarrow P$ | 11, 15 CP |
| {1, 2} | 17. | $P \leftrightarrow R$ | 12, 16 \leftrightarrow I |

9. $: P \vee (P \rightarrow Q)$

| | | | |
|-----------|-----|--|--------------------|
| {1} | 1. | $\sim(P \vee (P \rightarrow Q))$ | Assumption for RAA |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee (P \rightarrow Q)$ | 2 vI |
| {1, 2} | 4. | $(P \vee (P \rightarrow Q)) \& \sim(P \vee (P \rightarrow Q))$ | 1, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| {2} | 6. | P | Assumption for CP |
| {7} | 7. | $\sim Q$ | Assumption for RAA |
| {1, 2} | 8. | $P \& \sim P$ | 6, 7 &I |
| {1, 2, 7} | 9. | $(P \& \sim P) \& \sim Q$ | 7, 8 &I |
| {1, 2, 7} | 10. | $P \& \sim P$ | 8 &E |
| {1, 2} | 11. | $\sim\sim Q$ | 7, 10 RAA |
| {1, 2} | 12. | Q | 11 DNE |
| {1} | 13. | $P \rightarrow Q$ | 6, 12 CP |
| {1} | 14. | $P \vee (P \rightarrow Q)$ | 13 vI |
| {1} | 15. | $(P \vee (P \rightarrow Q)) \& \sim(P \vee (P \rightarrow Q))$ | 1, 14 &I |
| — | 16. | $\sim\sim(P \vee (P \rightarrow Q))$ | 1, 15 RAA |
| — | 17. | $P \vee (P \rightarrow Q)$ | 16 DNE |

(Maybe the book says it can be done in 16 steps because it uses the same step for 2 and 6.)

10. : $((P \rightarrow Q) \vee (Q \rightarrow R))$

| | | | |
|-------------|-----|--|--------------------|
| {1} | 1. | $\sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | Assumption for RAA |
| {2} | 2. | Q | Assumption for RAA |
| {3} | 3. | P | Assumption for CP |
| {2, 3} | 4. | $P \& Q$ | 2, 3 &I |
| {2, 3} | 5. | Q | 4 &E |
| {2} | 6. | $P \rightarrow Q$ | 3, 5 CP |
| {2} | 7. | $(P \rightarrow Q) \vee (Q \rightarrow R)$ | 6 vI |
| {1, 2} | 8. | $((P \rightarrow Q) \vee (Q \rightarrow R)) \& \sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | 1, 7 &I |
| {1} | 9. | $\sim Q$ | 2, 8 RAA |
| {10} | 10. | Q | Assumption for CP |
| {11} | 11. | $\sim R$ | Assumption for RAA |
| {1, 10} | 12. | $Q \& \sim Q$ | 9, 10 &I |
| {1, 10, 11} | 13. | $(Q \& \sim Q) \& \sim R$ | 12, 13 &I |
| {1, 10, 11} | 14. | $Q \& \sim Q$ | 13 &E |
| {1, 10} | 15. | $\sim\sim R$ | 11, 14 RAA |
| {1, 10} | 16. | R | 15 DNE |
| {1} | 17. | $Q \rightarrow R$ | 10, 16 CP |
| {1} | 18. | $(P \rightarrow Q) \vee (Q \rightarrow R)$ | 17 vI |
| {1} | 19. | $((P \rightarrow Q) \vee (Q \rightarrow R)) \& \sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | 1, 18 &I |
| — | 20. | $\sim\sim((P \rightarrow Q) \vee (Q \rightarrow R))$ | 1, 19 RAA |
| — | 21. | $(P \rightarrow Q) \vee (Q \rightarrow R)$ | 20 DNE |

(Maybe the book says it can be done in 20 steps, because it uses only one Q assumption, and uses the assumption on line 2. for the assumption which is written in line 10. in my proof.)

REVISION EXERCISE IV

1 The proofs are the followings.

1. : $(P \vee Q) \rightarrow (Q \vee P)$

| | | | |
|-----|----|------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Assumption for CP |
| {2} | 2. | P | Assumption for $\vee E$ |
| {2} | 3. | $Q \vee P$ | 2 $\vee I$ |
| {4} | 4. | Q | Assumption for $\vee E$ |
| {4} | 5. | $Q \vee P$ | 4 $\vee I$ |
| — | 6. | $Q \vee P$ | 1, 2, 3, 4, 5 $\vee E$ |

2. : $\sim(P \vee Q) \rightarrow \sim P$

| | | | |
|--------|----|-------------------------------------|--------------------|
| {1} | 1. | $\sim(P \vee Q)$ | Assumption for CP |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee Q$ | 2 $\vee I$ |
| {1, 2} | 4. | $(P \vee Q) \& \sim(P \vee Q)$ | 1, 3 $\&I$ |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| — | 6. | $\sim(P \vee Q) \rightarrow \sim P$ | 1, 5 CP |

3. $\sim(P \& Q), P : \sim Q$

| | | | |
|-----------|----|----------------------------|--------------------|
| {1} | 1. | $\sim(P \& Q)$ | Premise |
| {2} | 2. | P | Premise |
| {3} | 3. | Q | Assumption for RAA |
| {2, 3} | 4. | $P \& Q$ | 2, 3 $\&I$ |
| {1, 2, 3} | 5. | $(P \& Q) \& \sim(P \& Q)$ | 1, 4 $\&I$ |
| {1, 2} | 6. | $\sim Q$ | 3, 5 RAA |

4. $\sim(P \& Q) : \sim P \vee \sim Q$

| | | | |
|-----------|-----|--|--------------------|
| {1} | 1. | $\sim(P \& Q)$ | Premise |
| {2} | 2. | $\sim(\sim P \vee \sim Q)$ | Assumption for RAA |
| {3} | 3. | $\sim P$ | Assumption for RAA |
| {3} | 4. | $\sim P \vee \sim Q$ | 3 $\vee I$ |
| {2, 3} | 5. | $(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$ | 2, 4 $\&I$ |
| {2} | 6. | $\sim\sim P$ | 3, 5 RAA |
| {2} | 7. | P | 6 DNE |
| {8} | 8. | Q | Assumption for RAA |
| {2, 8} | 9. | $P \& Q$ | 7, 8 $\&I$ |
| {1, 2, 8} | 10. | $(P \& Q) \& \sim(P \& Q)$ | 1, 9 $\&I$ |
| {1, 2} | 11. | $\sim Q$ | 8, 10 RAA |
| {1, 2} | 12. | $\sim P \vee \sim Q$ | 11 $\vee I$ |
| {1, 2} | 13. | $(\sim P \vee \sim Q) \& \sim(\sim P \vee \sim Q)$ | 2, 12 $\&I$ |
| {1} | 14. | $\sim\sim(\sim P \vee \sim Q)$ | 2, 13 RAA |
| {1} | 15. | $\sim P \vee \sim Q$ | 14 DNE |

5. $P \vee (Q \vee R) : (P \vee R) \vee Q$

| | | | |
|-----|-----|---------------------|-------------------------|
| {1} | 1. | $P \vee (Q \vee R)$ | Premise |
| {2} | 2. | P | Assumption for $\vee E$ |
| {2} | 3. | $P \vee R$ | 2 $\vee I$ |
| {2} | 4. | $(P \vee R) \vee Q$ | 3 $\vee I$ |
| {5} | 5. | $Q \vee R$ | Assumption for $\vee E$ |
| {6} | 6. | Q | Assumption for $\vee E$ |
| {6} | 7. | $(P \vee R) \vee Q$ | 6 $\vee I$ |
| {8} | 8. | R | Assumption for $\vee E$ |
| {8} | 9. | $P \vee R$ | 8 $\vee I$ |
| {8} | 10. | $(P \vee R) \vee Q$ | 9 $\vee I$ |
| {5} | 11. | $(P \vee R) \vee Q$ | 6, 7, 8, 10 $\vee E$ |
| {1} | 12. | $(P \vee R) \vee Q$ | 1, 2, 4, 5, 11 $\vee E$ |

6. $\sim P, \sim Q : \sim(P \vee Q)$

| | | | |
|-----------|-----|--------------------------------------|-------------------------|
| {1} | 1. | $\sim P$ | Premise |
| {2} | 2. | $\sim Q$ | Premise |
| {3} | 3. | $P \vee Q$ | Assumption for RAA |
| {4} | 4. | P | Assumption for $\vee E$ |
| {1, 4} | 5. | $P \& \sim P$ | 1, 4 $\& I$ |
| {6} | 6. | Q | Assumption for $\vee E$ |
| {2, 6} | 7. | $Q \& \sim Q$ | 2, 6 $\& I$ |
| {8} | 8. | $\sim(P \& \sim P)$ | Assumption for RAA |
| {2, 6, 8} | 9. | $(Q \& \sim Q) \& \sim(P \& \sim P)$ | 7, 8 $\& I$ |
| {2, 6, 8} | 10. | $Q \& \sim Q$ | 9 $\& E$ |
| {2, 6} | 11. | $\sim\sim(P \& \sim P)$ | 8, 10 RAA |
| {2, 6} | 12. | $P \& \sim P$ | 11 DNE |
| {1, 2, 3} | 13. | $P \& \sim P$ | 3, 4, 5, 6, 12 $\vee E$ |
| {1, 2} | 14. | $\sim(P \vee Q)$ | 3, 13 RAA |

7. $P \rightarrow (Q \vee R) : (P \rightarrow Q) \vee (P \rightarrow R)$

| | | | |
|-------------|-----|--|--------------------|
| {1} | 1. | $P \rightarrow (Q \vee R)$ | Premise |
| {2} | 2. | $\sim((P \rightarrow Q) \vee (P \rightarrow R))$ | Assumption for RAA |
| {3} | 3. | Q | Assumption for RAA |
| {4} | 4. | P | Assumption for CP |
| {3, 4} | 5. | $P \& Q$ | 3, 4 &I |
| {3, 4} | 6. | Q | 5 &E |
| {3} | 7. | $P \rightarrow Q$ | 4, 6 CP |
| {3} | 8. | $(P \rightarrow Q) \vee (P \rightarrow R)$ | 7 vI |
| {2, 3} | 9. | $((P \rightarrow Q) \vee (P \rightarrow R)) \& \sim((P \rightarrow Q) \vee (P \rightarrow R))$ | 2, 8 &I |
| {2} | 10. | $\sim Q$ | 3, 9 RAA |
| {1, 4} | 11. | $Q \vee R$ | 1, 4 MP |
| {12} | 12. | Q | Assumption for vE |
| {13} | 13. | $\sim R$ | Assumption for RAA |
| {2, 12} | 14. | $Q \& \sim Q$ | 10, 12 &I |
| {2, 12, 13} | 15. | $(Q \& \sim Q) \& \sim R$ | 13, 14 &I |
| {2, 12, 13} | 16. | $Q \& \sim Q$ | 15 &E |
| {2, 12} | 17. | $\sim\sim R$ | 13, 16 RAA |
| {2, 12} | 18. | R | 17 DNE |
| {19} | 19. | R | Assumption for vE |
| {1, 2, 4} | 20. | R | 11, 12, 18, 18 vE |
| {1, 2} | 22. | $P \rightarrow R$ | 4, 20 CP |
| {1, 2} | 23. | $(P \rightarrow Q) \vee (P \rightarrow R)$ | 22 vI |
| {1, 2} | 24. | $((P \rightarrow Q) \vee (P \rightarrow R)) \& \sim((P \rightarrow Q) \vee (P \rightarrow R))$ | 2, 23 &I |
| {1} | 25. | $\sim\sim((P \rightarrow Q) \vee (P \rightarrow R))$ | 2, 24 RAA |
| {1} | 26. | $(P \rightarrow Q) \vee (P \rightarrow R)$ | 25 DNE |

8. $P \rightarrow \sim Q, P \rightarrow \sim R, Q \vee R : P \rightarrow (\sim Q \vee R)$

| | | | |
|--------|----|---------------------------------|-------------------|
| {1} | 1. | $P \rightarrow \sim Q$ | Premise |
| {2} | 2. | $P \rightarrow \sim R$ | Premise |
| {3} | 3. | $Q \vee R$ | Premise |
| {4} | 4. | P | Assumption for CP |
| {1, 4} | 5. | $\sim Q$ | 1, 4 MP |
| {1, 4} | 6. | $\sim Q \vee R$ | 5 vI |
| {1} | 7. | $P \rightarrow (\sim Q \vee R)$ | 4, 6 CP |

9. $P \vee \sim Q, P \vee \sim R, Q \vee R : P$

| | | | |
|-----------|-----|---------------------------|--------------------------|
| {1} | 1. | $P \vee \sim Q$ | Premise |
| {2} | 2. | $P \vee \sim R$ | Premise |
| {3} | 3. | $Q \vee R$ | Premise |
| {4} | 4. | P | Assumption for $\vee E$ |
| {5} | 5. | $\sim Q$ | Assumption for $\vee E$ |
| {6} | 6. | $\sim R$ | Assumption for $\vee E$ |
| {7} | 7. | Q | Assumption for $\vee E$ |
| {8} | 8. | R | Assumption for $\vee E$ |
| {9} | 9. | $\sim P$ | Assumption for RAA |
| {5, 7} | 10. | $Q \& \sim Q$ | 5, 7 &I |
| {5, 7, 9} | 11. | $(Q \& \sim Q) \& \sim P$ | 9, 10 &I |
| {5, 7, 9} | 12. | $Q \& \sim Q$ | 11 &E |
| {5, 7} | 13. | $\sim \sim P$ | 9, 12 RAA |
| {5, 7} | 14. | P | 13 DNE |
| {8, 6} | 15. | $R \& \sim R$ | 8, 6 &I |
| {8, 6, 9} | 16. | $(R \& \sim R) \& \sim P$ | 9, 15 &I |
| {8, 6, 9} | 17. | $R \& \sim R$ | 16 &E |
| {8, 6} | 18. | $\sim \sim P$ | 9, 17 RAA |
| {8, 6} | 19. | P | 18 DNE |
| {3, 5, 6} | 20. | P | 3, 7, 14, 8, 19 $\vee E$ |
| {2, 3, 5} | 21. | P | 2, 4, 4, 6, 20 $\vee E$ |
| {1, 2, 3} | 22. | P | 1, 4, 4, 5, 21 $\vee E$ |

10. $(P \& Q) \rightarrow R : (P \rightarrow R) \vee (Q \rightarrow R)$

| | | | |
|--------------|-----|--|-----------------------|
| {1} | 1. | $(P \& Q) \rightarrow R$ | Premise |
| {2} | 2. | $\sim(Q \vee \sim Q)$ | Assumption for RAA |
| {3} | 3. | Q | Assumption for RAA |
| {3} | 4. | $Q \vee \sim Q$ | 3 vI |
| {2, 3} | 5. | $(Q \vee \sim Q) \& \sim(Q \vee \sim Q)$ | 2, 4 &I |
| {2} | 6. | $\sim Q$ | 3, 5 RAA |
| {2} | 7. | $Q \vee \sim Q$ | 6 vI |
| {2} | 8. | $(Q \vee \sim Q) \& \sim(Q \vee \sim Q)$ | 2, 7 &I |
| — | 9. | $\sim\sim(Q \vee \sim Q)$ | 2, 8 RAA |
| — | 10. | $Q \vee \sim Q$ | 9 DNE |
| {11} | 11. | Q | Assumption for vE |
| {12} | 12. | P | Assumption for CP |
| {11, 12} | 13. | $P \& Q$ | 11, 12 &I |
| {1, 11, 12} | 14. | R | 1, 13 MP |
| {1, 11} | 15. | $P \rightarrow R$ | 12, 14 CP |
| {1, 11} | 16. | $(P \rightarrow R) \vee (Q \rightarrow R)$ | 15 vI |
| {17} | 17. | $\sim Q$ | Assumption for vE |
| {18} | 18. | Q | Assumption for CP |
| {19} | 19. | $\sim R$ | Assumption for RAA |
| {17, 18} | 20. | $Q \& \sim Q$ | 17, 18 &I |
| {17, 18, 19} | 21. | $(Q \& \sim Q) \& \sim R$ | 19, 20 &I |
| {17, 18, 19} | 22. | $Q \& \sim Q$ | 21 &E |
| {17, 18} | 23. | $\sim\sim R$ | 19, 22 RAA |
| {17, 18} | 24. | R | 23 DNE |
| {17} | 25. | $Q \rightarrow R$ | 18, 24 CP |
| {17} | 26. | $(P \rightarrow R) \vee (Q \rightarrow R)$ | 25 vI |
| {1} | 27. | $(P \rightarrow R) \vee (Q \rightarrow R)$ | 10, 11, 16, 17, 26 vE |

EXERCISE 3.10

1 The proofs are the followings.

1. $P \vee Q, \sim P : Q$

| | | | |
|-----------|-----|---------------------------|-------------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $\sim P$ | Premise |
| {3} | 3. | P | Assumption for $\vee E$ |
| {4} | 4. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 5. | $P \& \sim P$ | 2, 3 $\&I$ |
| {2, 3, 4} | 6. | $(P \& \sim P) \& \sim Q$ | 4, 5 $\&I$ |
| {2, 3, 4} | 7. | $P \& \sim P$ | 6 $\&E$ |
| {2, 3} | 8. | $\sim \sim Q$ | 4, 7 RAA |
| {2, 3} | 9. | Q | 8 DNE |
| {10} | 10. | Q | Assumption for $\vee E$ |
| {1, 2} | 11. | Q | 1, 3, 9, 10 $\vee E$ |

2. $P \rightarrow Q, Q \rightarrow R : P \rightarrow R$

| | | | |
|-----------|----|-------------------|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $Q \rightarrow R$ | Premise |
| {3} | 3. | P | Assumption for CP |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2, 3} | 5. | R | 2, 4 MP |
| {1, 2} | 6. | $P \rightarrow R$ | 3, 5 CP |

3. $P \rightarrow Q, \sim P \rightarrow Q : Q$

| | | | |
|-----------|----|------------------------|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $\sim P \rightarrow Q$ | Premise |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {1, 3} | 4. | $\sim P$ | 1, 3 MT |
| {1, 2, 3} | 5. | Q | 2, 4 MP |
| {1, 2, 3} | 6. | $Q \& \sim Q$ | 3, 5 $\&I$ |
| {1, 2} | 7. | $\sim \sim Q$ | 3, 6 RAA |
| {1, 2} | 8. | Q | 7 DNE |

4. $P \rightarrow Q, P \rightarrow \sim Q : \sim P$

| | | | |
|-----------|----|------------------------|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $P \rightarrow \sim Q$ | Premise |
| {3} | 3. | P | Assumption for RAA |
| {1, 3} | 4. | Q | 1, 3 MP |
| {2, 3} | 5. | $\sim Q$ | 2, 3 MP |
| {1, 2, 3} | 6. | $Q \& \sim Q$ | 4, 5 $\&I$ |
| {1, 2} | 7. | $\sim P$ | 3, 6 RAA |

5. $\sim P \rightarrow P : P$

| | | | |
|--------|----|------------------------|--------------------|
| {1} | 1. | $\sim P \rightarrow P$ | Premise |
| {2} | 2. | $\sim P$ | Assumption for RAA |
| {1, 2} | 3. | P | 1, 2 MP |
| {1, 2} | 4. | $P \& \sim P$ | 2, 3 &I |
| {1} | 5. | $\sim\sim P$ | 2, 4 RAA |
| {1} | 6. | P | 5 DNE |

6. $P \rightarrow \sim P : \sim P$

| | | | |
|--------|----|------------------------|--------------------|
| {1} | 1. | $P \rightarrow \sim P$ | Premise |
| {2} | 2. | P | Assumption for RAA |
| {1, 2} | 3. | $\sim P$ | 1, 2 MP |
| {1, 2} | 4. | $P \& \sim P$ | 2, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |

Examination 1 in Formal Logic

1 The proofs are the followings.

1. $P \rightarrow Q : ((R \& Q) \rightarrow S) \rightarrow ((R \& P) \rightarrow S)$

| | | | |
|-----------|-----|---|-------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $(R \& Q) \rightarrow S$ | Assumption for CP |
| {3} | 3. | $R \& P$ | Assumption for CP |
| {3} | 4. | R | 3 &E |
| {3} | 5. | P | 3 &E |
| {1, 3} | 6. | Q | 1, 5 MP |
| {1, 3} | 7. | $R \& Q$ | 4, 6 &I |
| {1, 2, 3} | 8. | S | 2, 7 MP |
| {1, 2} | 9. | $(R \& P) \rightarrow S$ | 3, 8 CP |
| {1} | 10. | $((R \& Q) \rightarrow S) \rightarrow ((R \& P) \rightarrow S)$ | 2, 9 CP |

2. $P \vee Q : \sim(\sim P \& \sim Q)$

| | | | |
|--------|-----|--------------------------|--------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $\sim P \& \sim Q$ | Assumption for RAA |
| {2} | 3. | $\sim P$ | 2 &E |
| {2} | 4. | $\sim Q$ | 2 &E |
| {5} | 5. | P | Assumption for vE |
| {2, 5} | 6. | $P \& \sim P$ | 3, 5 &I |
| {5} | 7. | $\sim(\sim P \& \sim Q)$ | 2, 6 RAA |
| {8} | 8. | Q | Assumption for vE |
| {2, 8} | 9. | $Q \& \sim Q$ | 4, 8 &I |
| {8} | 10. | $\sim(\sim P \& \sim Q)$ | 2, 9 RAA |
| {1} | 11. | $\sim(\sim P \& \sim Q)$ | 1, 5, 7, 8, 10 vE |

3. $P \vee \sim P$

| | | | |
|--------|----|--|--------------------|
| {1} | 1. | $\sim(P \vee \sim P)$ | Assumption for RAA |
| {2} | 2. | P | Assumption for RAA |
| {2} | 3. | $P \vee \sim P$ | 2 vI |
| {1, 2} | 4. | $(P \vee \sim P) \ \& \ \sim(P \vee \sim P)$ | 1, 3 &I |
| {1} | 5. | $\sim P$ | 2, 4 RAA |
| {1} | 6. | $P \vee \sim P$ | 5 vI |
| {1} | 7. | $(P \vee \sim P) \ \& \ \sim(P \vee \sim P)$ | 1, 6 &I |
| – | 8. | $\sim\sim(P \vee \sim P)$ | 1, 7 RAA |
| – | 9. | $P \vee \sim P$ | 8 DNE |

2 The proofs are the followings.

1. $P \rightarrow Q : \sim P \vee Q$

| | | | |
|-----------|-----|--|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $\sim(\sim P \vee Q)$ | Assumption for RAA |
| {3} | 3. | P | Assumption for RAA |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 3} | 5. | $\sim P \vee Q$ | 4 vI |
| {1, 2, 3} | 6. | $(\sim P \vee Q) \ \& \ \sim(\sim P \vee Q)$ | 2, 5 &I |
| {1, 2} | 7. | $\sim P$ | 3, 6 RAA |
| {1, 2} | 8. | $\sim P \vee Q$ | 7 vI |
| {1, 2} | 9. | $(\sim P \vee Q) \ \& \ \sim(\sim P \vee Q)$ | 2, 8 &I |
| {1} | 10. | $\sim\sim(\sim P \vee Q)$ | 2, 9 RAA |
| {1} | 11. | $\sim P \vee Q$ | 10 DNE |

2. $\sim P \vee Q : P \rightarrow Q$

| | | | |
|-----------|-----|-----------------------------------|---------------------|
| {1} | 1. | $\sim P \vee Q$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {3} | 3. | $\sim P$ | Assumption for vE |
| {4} | 4. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 5. | $P \ \& \ \sim P$ | 2, 3 &I |
| {2, 3, 4} | 6. | $(P \ \& \ \sim P) \ \& \ \sim Q$ | 4, 5 &I |
| {2, 3, 4} | 7. | $P \ \& \ \sim P$ | 6 &E |
| {2, 3} | 8. | $\sim\sim Q$ | 4, 7 RAA |
| {2, 3} | 9. | Q | 8 DNE |
| {3} | 10. | $P \rightarrow Q$ | 2, 9 CP |
| {11} | 11. | Q | Assumption for vE |
| {2, 11} | 12. | $P \ \& \ Q$ | 2, 11 &I |
| {2, 11} | 13. | Q | 12 &E |
| {11} | 14. | $P \rightarrow Q$ | 2, 13 CP |
| {1} | 15. | $P \rightarrow Q$ | 1, 3, 10, 11, 14 vE |

3. $P \vee Q : \sim P \rightarrow Q$

| | | | |
|-----------|-----|---------------------------|---------------------|
| {1} | 1. | $P \vee Q$ | Premise |
| {2} | 2. | $\sim P$ | Assumption for CP |
| {3} | 3. | P | Assumption for vE |
| {4} | 4. | $\sim Q$ | Assumption for RAA |
| {2, 3} | 5. | $P \& \sim P$ | 2, 3 &I |
| {2, 3, 4} | 6. | $(P \& \sim P) \& \sim Q$ | 4, 5 &I |
| {2, 3, 4} | 7. | $P \& \sim P$ | 6 &E |
| {2, 3} | 8. | $\sim \sim Q$ | 4, 7 RAA |
| {2, 3} | 9. | Q | 8 DNE |
| {3} | 10. | $\sim P \rightarrow Q$ | 2, 9 CP |
| {11} | 11. | Q | Assumption for vE |
| {2, 11} | 12. | $\sim P \& Q$ | 2, 11 &I |
| {2, 11} | 13. | Q | 12 &E |
| {11} | 14. | $\sim P \rightarrow Q$ | 2, 13 CP |
| {1} | 15. | $\sim P \rightarrow Q$ | 1, 3, 10, 11, 14 vE |

4. $\sim P \rightarrow Q : P \vee Q$

| | | | |
|--------|-----|--------------------------------|--------------------|
| {1} | 1. | $\sim P \rightarrow Q$ | Premise |
| {2} | 2. | $\sim(P \vee Q)$ | Assumption for RAA |
| {3} | 3. | P | Assumption for RAA |
| {3} | 4. | $P \vee Q$ | 3 vI |
| {2, 3} | 5. | $(P \vee Q) \& \sim(P \vee Q)$ | 2, 4 &I |
| {2} | 6. | $\sim P$ | 3, 5 RAA |
| {1, 2} | 7. | Q | 1, 6 MP |
| {1, 2} | 8. | $P \vee Q$ | 7 vI |
| {1, 2} | 9. | $(P \vee Q) \& \sim(P \vee Q)$ | 2, 8 &I |
| {1} | 10. | $\sim \sim(P \vee Q)$ | 2, 9 RAA |
| {1} | 11. | $P \vee Q$ | 10 DNE |

3 The proof is the following.

| | | | |
|------------|-----|--|---------------------------|
| {1} | 1. | $\sim P \leftrightarrow Q$ | Premise |
| {2} | 2. | $P \leftrightarrow Q$ | Assumption for RAA |
| {3} | 3. | Q | Assumption for RAA |
| {1} | 4. | $(\sim P \rightarrow Q) \& (Q \rightarrow \sim P)$ | 1 \leftrightarrow E |
| {2} | 5. | $(P \rightarrow Q) \& (Q \rightarrow P)$ | 2 \leftrightarrow E |
| {2} | 6. | $Q \rightarrow P$ | 5 &E |
| {1} | 7. | $Q \rightarrow \sim P$ | 4 &E |
| {2, 3} | 8. | P | 3, 6 MP |
| {1, 3} | 9. | $\sim P$ | 3, 7 MP |
| {1, 2, 3} | 10. | $P \& \sim P$ | 8, 9 &I |
| {1, 2} | 11. | $\sim Q$ | 3, 10 RAA |
| {12} | 12. | P | Assumption for RAA |
| {2} | 13. | $P \rightarrow Q$ | 5 &E |
| {2, 12} | 14. | Q | 12, 13 MP |
| {1, 2, 12} | 15. | $Q \& \sim Q$ | 11, 14 &I |
| {1, 2} | 16. | $\sim P$ | 12, 15 RAA |
| {1} | 17. | $\sim P \rightarrow Q$ | 4 &E |
| {1, 2} | 18. | Q | 16, 17 MP |
| {1, 2} | 19. | $Q \& \sim Q$ | 11, 18 &I |
| {1} | 20. | $\sim(P \leftrightarrow Q)$ | 2, 19 RAA |

4 The proofs are the followings.

(i) $P \rightarrow Q, \sim Q \vdash \sim P$

| | | | |
|-----------|----|-------------------|--------------------|
| {1} | 1. | $P \rightarrow Q$ | Premise |
| {2} | 2. | $\sim Q$ | Premise |
| {3} | 3. | P | Assumption for RAA |
| {1, 3} | 4. | Q | 1, 3 MP |
| {1, 2, 3} | 5. | $Q \& \sim Q$ | 2, 4 &I |
| {1, 2} | 6. | $\sim P$ | 3, 5 RAA |

- (ii) For this to work, instead of assuming P for RAA, P should be assumed for CP, and then conclude, that $P \rightarrow (Q \& \sim Q)$. Then the following needs to be proven.

$P \rightarrow (Q \& \sim Q) \vdash \sim P$

| | | | |
|-----------|-----|---|-------------------|
| {1} | 1. | $P \rightarrow (Q \& \sim Q)$ | Premise |
| {2} | 2. | P | Assumption for CP |
| {1, 2} | 3. | $Q \& \sim Q$ | 1, 2 MP |
| {1, 2} | 4. | Q | 3 &E |
| {1, 2} | 5. | $\sim Q$ | 3 &E |
| {6} | 6. | $P \rightarrow (Q \& \sim Q)$ | Assumption for CP |
| {1, 2, 6} | 7. | $(P \rightarrow (Q \& \sim Q)) \& Q$ | 4, 6 &I |
| {1, 2, 6} | 8. | Q | 7 &E |
| {1, 2} | 9. | $(P \rightarrow (Q \& \sim Q)) \rightarrow Q$ | 6, 8 CP |
| {1, 2} | 10. | $\sim(P \rightarrow (Q \& \sim Q))$ | 5, 9 MT |
| {1} | 11. | $P \rightarrow \sim(P \rightarrow (Q \& \sim Q))$ | 2, 10 CP |
| {1} | 12. | $\sim\sim(P \rightarrow (Q \& \sim Q))$ | 1 DNI |
| {1} | 13. | $\sim P$ | 11, 12 MT |

5 $P, \sim P \vdash Q$

| | | | |
|-----------|----|---------------------------|--------------------|
| {1} | 1. | P | Premise |
| {2} | 2. | $\sim P$ | Premise |
| {3} | 3. | $\sim Q$ | Assumption for RAA |
| {1, 2} | 4. | $P \& \sim P$ | 1, 2 &I |
| {1, 2, 3} | 5. | $(P \& \sim P) \& \sim Q$ | 3, 4 &I |
| {1, 2, 3} | 6. | $P \& \sim P$ | 5 &E |
| {1, 2} | 7. | $\sim\sim Q$ | 3, 6 RAA |
| {1, 2} | 8. | Q | 7 DNE |

Chapter Four: Formal Logic and Formal Semantics #1

EXERCISE 4.1

1 The complete truth-tables are the followings.

1. $P \rightarrow (P \ \& \ P)$

| P | P | \rightarrow | (| P | $\&$ | P |) |
|---|---|---------------|---|---|------|---|---|
| F | F | T | | F | F | F | |
| T | T | T | | T | T | T | |

2. $P \ \& \ \sim P$

| P | P | $\&$ | \sim | P |
|---|---|----------|--------|---|
| F | F | F | T | F |
| T | T | F | F | T |

3. $P \vee \sim P$

| P | P | \vee | \sim | P |
|---|---|----------|--------|---|
| F | F | T | T | F |
| T | T | T | F | T |

4. $P \rightarrow (Q \rightarrow P)$

| P | Q | P | \rightarrow | (| Q | \rightarrow | P |) |
|---|---|---|---------------|---|---|---------------|---|---|
| F | F | F | T | | F | T | F | |
| F | T | F | T | | T | F | F | |
| T | F | T | T | | F | T | T | |
| T | T | T | T | | T | T | T | |

5. $(P \ \& \ Q) \leftrightarrow (Q \ \& \ P)$

| P | Q | (| P | $\&$ | Q |) | \leftrightarrow | (| Q | $\&$ | P |) |
|---|---|---|---|------|---|---|-------------------|---|---|------|---|---|
| F | F | | F | F | F | | T | | F | F | F | |
| F | T | | F | F | T | | T | | T | F | F | |
| T | F | | T | F | F | | T | | F | F | T | |
| T | T | | T | T | T | | T | | T | T | T | |

6. $(P \vee Q) \leftrightarrow \sim Q$

| P | Q | (| P | \vee | Q |) | \leftrightarrow | \sim | Q |
|---|---|---|---|--------|---|---|-------------------|--------|---|
| F | F | | F | F | F | | F | T | F |
| F | T | | F | T | T | | F | F | T |
| T | F | | T | T | F | | T | T | F |
| T | T | | T | T | T | | F | F | T |

7. $(P \rightarrow Q) \rightarrow (\sim Q \rightarrow \sim P)$

| P | Q | (P → Q) | → | (~ Q → ~ P) |
|---|---|-----------|----------|---------------|
| F | F | F | T | T |
| F | T | F | T | F |
| T | F | T | T | F |
| T | T | T | T | T |

8. $\sim(P \vee Q) \leftrightarrow (\sim P \& \sim Q)$

| P | Q | ~ (P ∨ Q) | ↔ | (~ P & ~ Q) |
|---|---|-------------|----------|---------------|
| F | F | T | T | T |
| F | T | F | F | F |
| T | F | F | F | F |
| T | T | F | T | F |

9. $\sim P \& (Q \vee R)$

| P | Q | R | ~ P & (Q ∨ R) |
|---|---|---|-----------------|
| F | F | F | F |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | F |
| T | F | T | F |
| T | T | F | T |
| T | T | T | T |

10. $\sim(P \& (Q \vee R))$

| P | Q | R | ~ (P & (Q ∨ R)) |
|---|---|---|---------------------|
| F | F | F | T |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | T |
| T | F | T | F |
| T | T | F | F |
| T | T | T | F |

11. $\sim\sim(P \& \sim P)$

| P | ~ (P & ~ P) |
|---|---------------|
| F | F |
| T | T |

12. $((P \rightarrow Q) \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow R)$

| P | Q | R | ((P → Q) → (Q → R)) → (P → R) |
|---|---|---|---------------------------------------|
| F | F | F | T |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | F |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

13. $\sim((P \rightarrow Q) \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow R)$

| P | Q | R | $\sim((P \rightarrow Q) \rightarrow (Q \rightarrow R)) \rightarrow (P \rightarrow R)$ |
|---|---|---|---|
| F | F | F | T |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | T |
| T | F | T | T |
| T | T | F | F |
| T | T | T | T |

14. $(P \rightarrow ((Q \rightarrow R) \vee \sim R)) \vee \sim Q$

| P | Q | R | $(P \rightarrow ((Q \rightarrow R) \vee \sim R)) \vee \sim Q$ |
|---|---|---|---|
| F | F | F | T |
| F | F | T | T |
| F | T | F | T |
| F | T | T | T |
| T | F | F | T |
| T | F | T | T |
| T | T | F | T |
| T | T | T | T |

15. $((P \& Q) \rightarrow (R \vee \sim S)) \rightarrow T$

| T | S | R | Q | P | (| (| P | & | Q |) | → | (| R | ∨ | ~ | S |) |) | → | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----------|---|
| F | F | F | F | F | | | F | F | F | | T | | F | T | T | F | | | F | F |
| F | F | F | F | T | | | T | F | F | | T | | F | T | T | F | | | F | F |
| F | F | F | T | F | | | F | F | T | | T | | F | T | T | F | | | F | F |
| F | F | F | T | T | | | T | T | T | | T | | F | T | T | F | | | F | F |
| F | F | T | F | F | | | F | F | F | | T | | T | T | T | F | | | F | F |
| F | F | T | F | T | | | T | F | F | | T | | T | T | T | F | | | F | F |
| F | F | T | T | F | | | F | F | T | | T | | T | T | T | F | | | F | F |
| F | F | T | T | T | | | T | T | T | | T | | T | T | T | F | | | F | F |
| F | T | F | F | F | | | F | F | F | | T | | F | F | F | T | | | F | F |
| F | T | F | F | T | | | T | F | F | | T | | F | F | F | T | | | F | F |
| F | T | F | T | F | | | F | F | T | | T | | F | F | F | T | | | F | F |
| F | T | F | T | T | | | T | T | T | | F | | F | F | F | T | | | T | F |
| F | T | T | F | F | | | F | F | F | | T | | T | T | F | T | | | F | F |
| F | T | T | F | T | | | T | F | F | | T | | T | T | F | T | | | F | F |
| F | T | T | T | F | | | F | F | T | | T | | T | T | F | T | | | F | F |
| F | T | T | T | T | | | T | T | T | | T | | T | T | F | T | | | F | F |
| T | F | F | F | F | | | F | F | F | | T | | F | T | T | F | | | T | T |
| T | F | F | F | T | | | T | F | F | | T | | F | T | T | F | | | T | T |
| T | F | F | T | F | | | F | F | T | | T | | F | T | T | F | | | T | T |
| T | F | F | T | T | | | T | T | T | | T | | F | T | T | F | | | T | T |
| T | F | T | F | F | | | F | F | F | | T | | T | T | T | F | | | T | T |
| T | F | T | F | T | | | T | F | F | | T | | T | T | T | F | | | T | T |
| T | F | T | T | F | | | F | F | T | | T | | T | T | T | F | | | T | T |
| T | F | T | T | T | | | T | T | T | | T | | T | T | T | F | | | T | T |
| T | T | F | F | F | | | F | F | F | | T | | F | F | F | T | | | T | T |
| T | T | F | F | T | | | T | F | F | | T | | F | F | F | T | | | T | T |
| T | T | F | T | F | | | F | F | T | | T | | F | F | F | T | | | T | T |
| T | T | F | T | T | | | T | T | T | | F | | F | F | F | T | | | T | T |
| T | T | T | F | F | | | F | F | F | | T | | T | T | F | T | | | T | T |
| T | T | T | F | T | | | T | F | F | | T | | T | T | F | T | | | T | T |
| T | T | T | T | F | | | F | F | T | | T | | T | T | F | T | | | T | T |
| T | T | T | T | T | | | T | T | T | | T | | T | T | F | T | | | T | T |

EXERCISE 4.2

1 The formulas are the following kinds:

- 1 tautologous,
- 2 inconsistent,
- 3 tautologous,
- 4 tautologous,
- 5 tautologous,
- 6 contingent,
- 7 tautologous,
- 8 contingent,
- 9 contingent,
- 10 contingent,
- 11 inconsistent,
- 12 contingent,
- 13 contingent,
- 14 tautologous,
- 15 contingent.

2 (i) The negation of any tautologous formula is an inconsistent formula, (ii) and the negation of any contingent formula is a contingent formula.

3 The answer depends on what we mean by "have to test". There are simple ways to reduce the number of tests. We need to find if there is a case where the formula is false. We only need to test in case T is false, and the antecedent is true. The antecedent is true when either

- 1 $(P \ \& \ Q)$ is false or
- 2 $(P \ \& \ Q)$ is true and $(R \vee \sim S)$ is true.

The first case can be false when P is false or Q is false. In effect, we only tested one case: T is false and P is false.

EXERCISE 4.3

1 The complete truth-tables are the followings. For each of these formulas, an IPLI is constructed by substituting "0=0" for true atomic formulas, and "0=1" for false

atomic formulas in the case all of the premises are true, and the conclusion is false.

1. $P \vee Q : P$

Semantically invalid.

| P | Q | P | \vee | Q | P |
|---|---|---|----------|---|----------|
| F | F | F | F | F | F |
| F | T | F | T | T | F |
| T | F | T | T | F | T |
| T | T | T | T | T | T |

2. $P \vee \sim Q, \sim Q : \sim P$

Semantically invalid.

| P | Q | P | \vee | \sim | Q | \sim | Q | \sim | P |
|---|---|---|----------|--------|---|----------|---|----------|---|
| F | F | F | T | T | F | T | F | T | F |
| F | T | F | F | F | T | F | T | T | F |
| T | F | T | T | T | F | T | F | F | T |
| T | T | T | T | F | T | F | T | F | T |

3. $Q : \sim(\sim P \& \sim Q)$

Semantically valid.

| P | Q | Q | \sim | (| \sim | P | $\&$ | \sim | Q |) |
|---|---|----------|----------|---|--------|---|------|--------|---|---|
| F | F | F | F | | T | F | T | T | F | |
| F | T | T | T | | T | F | F | F | T | |
| T | F | F | T | | F | T | F | T | F | |
| T | T | T | T | | F | T | F | F | T | |

4. $\sim(P \vee \sim Q) : \sim P \& Q$

Semantically valid.

| P | Q | \sim | (| P | \vee | \sim | Q |) | \sim | P | $\&$ | Q |
|---|---|----------|---|---|--------|--------|---|---|--------|---|----------|---|
| F | F | F | | F | T | T | F | | T | F | F | F |
| F | T | T | | F | F | F | T | | T | F | T | T |
| T | F | F | | T | T | T | F | | F | T | F | F |
| T | T | F | | T | T | F | T | | F | T | F | T |

5. $P \rightarrow Q, Q \rightarrow R : R \rightarrow P$
Semantically invalid.

| P | Q | R | P | \rightarrow | Q | Q | \rightarrow | R | R | \rightarrow | P |
|---|---|---|---|---------------|---|---|---------------|---|---|---------------|---|
| F | F | F | F | T | F | F | T | F | F | T | F |
| F | F | T | F | T | F | F | T | T | T | F | F |
| F | T | F | F | T | T | T | F | F | F | T | F |
| F | T | T | F | T | T | T | T | T | T | F | F |
| T | F | F | T | F | F | F | T | F | F | T | T |
| T | F | T | T | F | F | F | T | T | T | T | T |
| T | T | F | T | T | T | T | F | F | F | T | T |
| T | T | T | T | T | T | T | T | T | T | T | T |

6. $P \rightarrow (Q \rightarrow R) : Q \rightarrow (P \rightarrow R)$
Semantically valid.

| P | Q | R | P | \rightarrow | (| Q | \rightarrow | R |) | Q | \rightarrow | (| P | \rightarrow | R |) |
|---|---|---|---|---------------|---|---|---------------|---|---|---|---------------|---|---|---------------|---|---|
| F | F | F | F | T | | F | T | F | | F | T | | F | T | F | |
| F | F | T | F | T | | F | T | T | | F | T | | F | T | T | |
| F | T | F | F | T | | T | F | F | | T | T | | F | T | F | |
| F | T | T | F | T | | T | T | T | | T | T | | F | T | T | |
| T | F | F | T | T | | F | T | F | | F | T | | T | F | F | |
| T | F | T | T | T | | F | T | T | | F | T | | T | T | T | |
| T | T | F | T | F | | T | F | F | | T | F | | T | F | F | |
| T | T | T | T | T | | T | T | T | | T | T | | T | T | T | |

7. $P \& \sim Q : \sim(P \rightarrow Q)$
Semantically valid.

| P | Q | P | $\&$ | \sim | Q | \sim | (| P | \rightarrow | Q |) |
|---|---|---|----------|--------|---|----------|---|---|---------------|---|---|
| F | F | F | F | T | F | F | | F | T | F | |
| F | T | F | F | F | T | F | | F | T | T | |
| T | F | T | T | T | F | T | | T | F | F | |
| T | T | T | F | F | T | F | | T | T | T | |

8. $Q \rightarrow P, P \vee Q : P \vee R$
Semantically valid.

| P | Q | R | Q | \rightarrow | P | P | \vee | Q | P | \vee | R |
|---|---|---|---|---------------|---|---|----------|---|---|----------|---|
| F | F | F | F | T | F | F | F | F | F | F | F |
| F | F | T | F | T | F | F | F | F | F | T | F |
| F | T | F | T | F | F | F | T | T | F | F | F |
| F | T | T | T | F | F | F | T | T | F | T | T |
| T | F | F | F | T | T | T | T | F | T | T | F |
| T | F | T | F | T | T | T | T | F | T | T | F |
| T | T | F | T | T | T | T | T | T | T | T | F |
| T | T | T | T | T | T | T | T | T | T | T | T |

9. : $((\sim P \rightarrow Q) \rightarrow \sim P) \rightarrow \sim P$
Semantically valid.

| P | Q | $((\sim P \rightarrow Q) \rightarrow \sim P) \rightarrow \sim P$ | | | | | | | | | | |
|---|---|--|--|--|--|--|--|--|--|--|--|--|
| F | F | | | | | | | | | | | |
| F | T | | | | | | | | | | | |
| T | F | | | | | | | | | | | |
| T | T | | | | | | | | | | | |

10. : $\sim(P \vee \sim Q) \rightarrow (\sim P \& Q)$
Semantically valid.

| P | Q | $\sim(P \vee \sim Q) \rightarrow (\sim P \& Q)$ | | | | | | | | | | |
|---|---|---|--|--|--|--|--|--|--|--|--|--|
| F | F | | | | | | | | | | | |
| F | T | | | | | | | | | | | |
| T | F | | | | | | | | | | | |
| T | T | | | | | | | | | | | |

11. $\sim R \rightarrow Q : (P \vee Q) \rightarrow (\sim R \rightarrow P)$
Semantically invalid.

| P | Q | R | $(P \vee Q) \rightarrow (\sim R \rightarrow P)$ | | | | | | | | | | |
|---|---|---|---|--|--|--|--|--|--|--|--|--|--|
| F | F | F | | | | | | | | | | | |
| F | F | T | | | | | | | | | | | |
| F | T | F | | | | | | | | | | | |
| F | T | T | | | | | | | | | | | |
| T | F | F | | | | | | | | | | | |
| T | F | T | | | | | | | | | | | |
| T | T | F | | | | | | | | | | | |
| T | T | T | | | | | | | | | | | |

12. $\sim P \rightarrow (Q \vee R), \sim P \rightarrow \sim R : Q$
Semantically invalid.

| P | Q | R | $\sim P \rightarrow (Q \vee R), \sim P \rightarrow \sim R : Q$ | | | | | | | | | | |
|---|---|---|--|--|--|--|--|--|--|--|--|--|--|
| F | F | F | | | | | | | | | | | |
| F | F | T | | | | | | | | | | | |
| F | T | F | | | | | | | | | | | |
| F | T | T | | | | | | | | | | | |
| T | F | F | | | | | | | | | | | |
| T | F | T | | | | | | | | | | | |
| T | T | F | | | | | | | | | | | |
| T | T | T | | | | | | | | | | | |

13. $P \& (Q \vee (Q \rightarrow R)) : (P \& Q) \vee ((P \& \sim Q) \vee (P \& R))$
Semantically valid.

| P | Q | R | P | & | (| Q | ∨ | (| Q | → | R |) |) | (| P | & | Q |) | ∨ | (| (| P | & | ∼ | Q |) | ∨ | (| P | & | R |) |) |
|---|---|---|---|----------|---|---|---|---|---|---|---|---|---|---|---|---|---|----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | F | F | F | F | | F | T | | F | T | F | | | | F | F | F | F | | | | F | F | T | F | | F | | F | F | F | | |
| F | F | T | F | F | | F | T | | F | T | F | | | | F | F | F | F | | | | F | F | T | F | | F | | F | F | F | | |
| F | T | F | F | F | | T | T | | T | F | F | | | | F | F | T | F | | | | F | F | F | T | | F | | F | F | F | | |
| F | T | T | F | F | | T | T | | T | T | T | | | | F | F | T | F | | | | F | F | F | T | | F | | F | F | F | | |
| T | F | F | T | T | | F | T | | F | T | F | | | | T | F | F | T | | | | T | T | T | F | | T | | T | F | F | | |
| T | F | T | T | T | | F | T | | F | T | T | | | | T | F | F | T | | | | T | T | T | F | | T | | T | T | T | | |
| T | T | F | T | T | | T | T | | T | F | F | | | | T | T | T | T | | | | T | F | F | T | | F | | T | F | F | | |
| T | T | T | T | T | | T | T | | T | T | T | | | | T | T | T | T | | | | T | F | F | T | | T | | T | T | T | | |

14. $:(P \vee Q) \leftrightarrow \sim(\sim(Q \& \sim P) \& \sim(P \rightarrow Q))$
Semantically invalid.

| P | Q | (| P | ∨ | Q |) | ↔ | ∼ | (| ∼ | (| Q | & | ∼ | P |) | & | ∼ | (| P | → | Q |) |) |
|---|---|---|---|---|---|---|----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | F | | F | F | F | | F | T | | T | | F | F | T | F | | F | F | | F | T | F | | |
| F | T | | F | T | T | | F | T | | F | | T | T | T | F | | F | F | | F | T | T | | |
| T | F | | T | T | F | | F | F | | T | | F | F | F | T | | T | T | | T | F | F | | |
| T | T | | T | T | T | | T | T | | T | | T | F | F | T | | F | F | | T | T | T | | |

15. $(P \vee Q) \leftrightarrow (\sim R \vee S) : R \rightarrow (P \leftrightarrow \sim(R \& \sim S))$
Semantically invalid.

| P | Q | R | S | P | ∨ | Q | ↔ | ∼ | R | ∨ | S | R | → | P | ↔ | ∼ | R | & | ∼ | S |
|---|---|---|---|---|---|---|----------|---|---|---|---|---|----------|---|---|---|---|---|---|---|
| F | F | F | F | F | F | F | F | T | F | T | F | F | T | F | F | T | F | F | T | F |
| F | F | F | T | F | F | F | F | T | F | T | T | F | T | F | F | T | F | F | T | T |
| F | F | T | F | F | F | F | T | F | T | F | T | T | T | F | F | T | F | T | T | F |
| F | F | T | T | F | T | T | T | T | F | T | F | T | T | F | F | T | F | F | T | F |
| F | T | F | F | F | T | T | T | T | F | T | T | T | T | F | F | T | F | F | T | T |
| F | T | T | F | F | T | T | F | F | T | F | F | T | T | F | F | T | F | T | T | F |
| F | T | T | T | F | T | T | T | F | T | T | T | T | F | F | F | T | T | F | T | T |
| T | F | F | F | T | F | F | F | T | F | T | F | F | T | T | T | T | F | F | T | F |
| T | F | F | T | T | F | F | T | F | T | F | T | T | F | T | F | T | F | T | T | F |
| T | F | T | F | T | F | F | F | F | T | T | T | T | T | T | T | T | T | F | T | T |
| T | F | T | T | T | F | F | T | T | F | T | T | T | T | T | T | T | T | F | T | T |
| T | T | F | F | T | T | T | T | T | F | T | T | T | F | T | T | T | F | F | T | F |
| T | T | F | T | T | T | T | T | T | F | T | T | T | T | T | T | T | F | F | T | T |
| T | T | T | F | T | T | T | F | F | T | F | F | T | T | T | F | F | T | T | T | F |
| T | T | T | T | T | T | T | T | F | T | T | T | T | T | T | T | T | T | F | T | T |

EXERCISE 4.4

1 The truth-tables are the followings.

1. $\sim(P \vee Q) : \sim P \ \& \ Q$

| P | Q | $\sim (P \vee \sim Q)$ | $\& \sim (\sim P \& Q)$ |
|---|---|------------------------|-------------------------|
| F | F | F | F |
| F | T | T | F |
| T | F | F | F |
| T | T | F | F |

2. $P \rightarrow Q, Q \rightarrow R, P : R$

| P | Q | R | $((P \rightarrow Q) \& (Q \rightarrow R)) \& P$ | $\& \sim R$ |
|---|---|---|---|-------------|
| F | F | F | F | F |
| F | F | T | F | F |
| F | T | F | F | F |
| F | T | T | F | F |
| T | F | F | T | F |
| T | F | T | T | F |
| T | T | F | T | F |
| T | T | T | T | F |

3. $\sim Q \rightarrow (\sim P \rightarrow Q), \sim Q : (\sim P \rightarrow Q)$

| P | Q | $((\sim Q \rightarrow (\sim P \rightarrow Q)) \& \sim Q) \& \sim (\sim P \rightarrow Q)$ |
|---|---|--|
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | F |

4. $P \rightarrow (Q \rightarrow R), P, \sim R : \sim Q$

| P | Q | R | $((P \rightarrow (Q \rightarrow R)) \& P) \& \sim R$ | $\& \sim \sim Q$ |
|---|---|---|--|------------------|
| F | F | F | F | F |
| F | F | T | F | F |
| F | T | F | F | F |
| F | T | T | F | F |
| T | F | F | T | F |
| T | F | T | T | F |
| T | T | F | F | F |
| T | T | T | T | F |

5. : $(P \rightarrow Q) \vee (Q \rightarrow R)$

| P | Q | R | \sim | (| (| P | \rightarrow | Q |) | \vee | (| Q | \rightarrow | R |) |) |
|---|---|---|----------|---|---|---|---------------|---|---|--------|---|---|---------------|---|---|---|
| F | F | F | F | | | F | T | F | | T | | F | T | F | | |
| F | F | T | F | | | F | T | F | | T | | F | T | T | | |
| F | T | F | F | | | F | T | T | | T | | T | F | F | | |
| F | T | T | F | | | F | T | T | | T | | T | T | T | | |
| P | F | F | F | | | T | F | F | | T | | F | T | F | | |
| P | F | T | F | | | T | F | F | | T | | F | T | T | | |
| P | T | F | F | | | T | T | T | | T | | T | F | F | | |
| P | T | T | F | | | T | T | T | | T | | T | T | T | | |

2 The truth-tables are the followings.

1. $P \rightarrow Q, Q \rightarrow R : P \rightarrow R$

| P | Q | R | (| (| P | → | Q |) | & | (| Q | → | R |) |) | → | (| P | → | R |) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----------|---|---|---|---|---|---|
| F | F | F | | | F | T | F | | T | | F | T | F | | T | | F | T | F | | |
| F | F | T | | | F | T | F | | T | | F | T | T | | T | | F | T | T | | |
| F | Q | F | | | F | T | T | | F | | T | F | F | | T | | F | T | F | | |
| F | Q | T | | | F | T | T | | T | | T | T | T | | T | | F | T | T | | |
| P | F | F | | | T | F | F | | F | | F | T | F | | T | | T | F | F | | |
| P | F | T | | | T | F | F | | F | | F | T | T | | T | | T | T | T | | |
| P | Q | F | | | T | T | T | | F | | T | F | F | | T | | T | F | F | | |
| P | Q | T | | | T | T | T | | T | | T | T | T | | T | | T | T | T | | |

2. $P \rightarrow Q, Q \rightarrow R : P \rightarrow R$

| P | Q | R | (| P | → | (| Q | → | R |) |) | → | (| Q | → | (| P | → | R |) |) |
|---|---|---|---|---|---|---|---|---|---|---|----------|---|---|---|---|---|---|---|---|---|---|
| F | F | F | | F | T | | F | T | F | | T | | F | T | | F | T | F | | | |
| F | F | T | | F | T | | F | T | T | | T | | F | T | | F | T | T | | | |
| F | T | F | | F | T | | T | F | F | | T | | T | T | | F | T | F | | | |
| F | T | T | | F | T | | T | T | T | | T | | T | T | | F | T | T | | | |
| T | F | F | | T | T | | F | T | F | | T | | F | T | | T | F | F | | | |
| T | F | T | | T | T | | F | T | T | | T | | F | T | | T | T | T | | | |
| T | T | F | | T | F | | T | F | F | | T | | T | F | | T | F | F | | | |
| T | T | T | | T | T | | T | T | T | | T | | T | T | | T | T | T | | | |

3. $P \rightarrow (Q \rightarrow R)$, $P, \sim R : \sim Q$

| P | Q | R | (| (| (| P | \rightarrow | (| Q | \rightarrow | R |) |) | & | P |) | & | \sim | R |) | \rightarrow | \sim | Q |
|---|---|---|---|---|---|---|---------------|---|---|---------------|---|---|---|---|---|---|---|--------|---|---|---------------|--------|---|
| F | F | F | | | | F | T | | F | T | F | | | F | F | | F | T | F | | T | T | F |
| F | F | T | | | | F | T | | F | T | T | | | F | F | | F | F | T | | T | T | F |
| F | T | F | | | | F | T | | T | F | F | | | F | F | | F | T | F | | T | F | T |
| F | T | T | | | | F | T | | T | T | T | | | F | F | | F | F | T | | T | F | T |
| T | F | F | | | | T | T | | F | T | F | | | T | T | | T | T | F | | T | T | F |
| T | F | T | | | | T | T | | F | T | T | | | T | T | | F | F | T | | T | T | F |
| T | T | F | | | | T | F | | T | F | F | | | F | T | | F | T | F | | T | F | T |
| T | T | T | | | | T | T | | T | T | T | | | T | T | | F | F | T | | T | F | T |

4. $P, (Q \& R) : (P \& Q) \vee (P \& R)$

| P | Q | R | (| P | & | (| Q | & | R |) |) | \rightarrow | (| (| P | & | Q |) | \vee | (| P | & | R |) |) |
|---|---|---|---|---|---|---|---|---|---|---|---|---------------|---|---|---|---|---|---|--------|---|---|---|---|---|---|
| F | F | F | | F | F | | F | F | F | | | T | | | F | F | F | | F | | F | F | F | | |
| F | F | T | | F | F | | F | F | T | | | T | | | F | F | F | | F | | F | F | T | | |
| F | T | F | | F | F | | T | F | F | | | T | | | F | F | T | | F | | F | F | F | | |
| F | T | T | | F | F | | T | T | T | | | T | | | F | F | T | | F | | F | F | T | | |
| T | F | F | | T | F | | F | F | F | | | T | | | T | F | F | | \vee | | T | F | F | | |
| T | F | T | | T | F | | F | F | T | | | T | | | T | F | F | | T | | T | T | T | | |
| T | T | F | | T | F | | T | F | F | | | T | | | T | T | T | | T | | T | F | F | | |
| T | T | T | | T | T | | T | T | T | | | T | | | T | T | T | | T | | T | T | T | | |

5. $P \leftrightarrow Q, Q \leftrightarrow R : P \leftrightarrow R$

| P | Q | R | (| (| P | \leftrightarrow | Q |) | & | (| Q | \leftrightarrow | R |) |) | \rightarrow | (| P | \leftrightarrow | R |) |
|---|---|---|---|---|---|-------------------|---|---|---|---|---|-------------------|---|---|---|---------------|---|---|-------------------|---|---|
| F | F | F | | | F | T | F | | T | | F | T | F | | | T | | F | T | F | |
| F | F | T | | | F | T | F | | F | | F | F | T | | | T | | F | F | T | |
| F | T | F | | | F | F | T | | F | | T | F | F | | | T | | F | T | F | |
| F | T | T | | | F | F | T | | F | | T | T | T | | | T | | F | F | T | |
| T | F | F | | | T | F | F | | F | | F | T | F | | | T | | T | F | F | |
| T | F | T | | | T | F | F | | F | | F | F | T | | | T | | T | T | T | |
| T | T | F | | | T | T | T | | F | | T | F | F | | | T | | T | F | F | |
| T | T | T | | | T | T | T | | T | | T | T | T | | | T | | T | T | T | |

3 The truth-tables are the followings.

1. $\sim(P \vee \sim Q) : \sim P \& Q$

| P | Q | (| \sim | (| P | \vee | \sim | Q |) |) | \rightarrow | (| \sim | P | & | Q |) |
|---|---|---|--------|---|---|--------|--------|---|---|---|---------------|---|--------|---|---|---|---|
| F | F | | F | | F | T | T | F | | | T | | T | F | F | F | |
| F | T | | T | | F | F | F | T | | | T | | T | F | T | T | |
| T | F | | F | | T | T | T | F | | | T | | F | T | F | F | |
| T | T | | F | | T | T | F | T | | | T | | F | T | F | T | |

2. $P \rightarrow Q, Q \rightarrow R, P : R$

| P | Q | R | (P \rightarrow Q) | \rightarrow | ((Q \rightarrow R) | \rightarrow | (P \rightarrow R)) |
|---|---|---|-----------------------|---------------|-------------------------|---------------|-------------------------|
| F | F | F | F | T | F | T | F |
| F | F | T | F | T | F | T | T |
| F | T | F | F | T | T | F | F |
| F | T | T | F | T | T | T | T |
| T | F | F | T | F | F | F | F |
| T | F | T | T | F | F | T | T |
| T | T | F | T | T | T | F | F |
| T | T | T | T | T | T | T | T |

3. $\sim Q \leftrightarrow (\sim P \rightarrow Q), \sim Q : (\sim P \rightarrow Q)$

| P | Q | (\sim Q \rightarrow (\sim P \rightarrow Q)) | \rightarrow | (\sim Q \rightarrow (\sim P \rightarrow Q)) |
|---|---|---|---------------|---|
| F | F | T | F | F |
| F | T | F | T | T |
| T | F | T | F | F |
| T | T | F | T | T |

4. $P \rightarrow (Q \rightarrow R), P, \sim R : \sim Q$

| P | Q | (\sim Q \rightarrow (\sim P \rightarrow Q)) | \rightarrow | (\sim Q \rightarrow (\sim P \rightarrow Q)) |
|---|---|---|---------------|---|
| F | F | T | F | F |
| F | T | F | T | T |
| T | F | T | F | F |
| T | T | F | T | T |

5. $:(P \rightarrow Q) \vee (Q \rightarrow R)$

| P | Q | R | (P \rightarrow Q) | \vee | (Q \rightarrow R) |
|---|---|---|-----------------------|--------|-----------------------|
| F | F | F | F | T | F |
| F | F | T | F | T | T |
| F | T | F | F | T | F |
| F | T | T | F | T | T |
| P | F | F | T | F | F |
| P | F | T | T | F | T |
| P | T | F | T | T | F |
| P | T | T | T | T | T |

4 The interpretations are the followings.

1. $P, \sim(P \& Q) : \sim Q$

Valid sequent.

| P | \sim | (P & Q) | \sim Q |
|---|--------|-----------|----------|
| 1 | 2 | 5 4 6 | 3 |
| T | T | T F F | F |

2. $P \rightarrow (Q \rightarrow R) : Q \rightarrow (P \rightarrow R)$

Valid sequent.

| $P \rightarrow (Q \rightarrow R)$ | | | | $Q \rightarrow (P \rightarrow R)$ | | | |
|-----------------------------------|---|---|---|-----------------------------------|---|---|-----|
| 7 | 1 | 9 | 8 | 3 | 2 | 5 | 4 6 |
| T | T | T | T | T | F | T | F F |

3. $Q \rightarrow R : (\sim Q \rightarrow \sim P) \rightarrow (P \rightarrow R)$

Valid sequent.

| $Q \rightarrow R$ | | | $(\sim Q \rightarrow \sim P) \rightarrow (P \rightarrow R)$ | | | | | | | | | | |
|-------------------|---|---|---|----|---|---|----|---|---|---|---|---|-----|
| 8 | 1 | 7 | (| 10 | 9 | 3 | 11 | P |) | 2 | (| 5 | 4 6 |
| F | T | F | (| T | F | T | T | |) | F | (| T | F F |

4. $\sim(P \rightarrow Q), Q \vee (R \& S) : R \& S$

Valid sequent.

| $\sim (P \rightarrow Q)$ | | | | $Q \vee (R \& S)$ | | | | | $R \& S$ | |
|--------------------------|---|---|---|-------------------|---|---|---|----|----------|----|
| 1 | 5 | 4 | 6 | 7 | 2 | 9 | 8 | 10 | 3 | 11 |
| T | T | F | F | F | T | T | T | T | F | T |

5. $(P \& \sim Q) \vee (Q \& \sim P) : P \leftrightarrow Q$

Invalid sequent.

| $(P \& \sim Q) \vee (Q \& \sim P)$ | | | | | | | | | | $P \leftrightarrow Q$ | | |
|------------------------------------|---|---|---|---|----|---|----|----|--|-----------------------|---|---|
| 5 | 8 | 7 | 6 | 1 | 10 | 9 | 11 | 12 | | 3 | 2 | 4 |
| F | F | F | T | T | T | T | T | F | | F | F | T |