EFFECTS OF UNKNOWN FORCES AND CONTROL MECHANISMS TO SYSTEM DYNAMICS

A STUDY ON A THEORETICAL EPIDEMIC OUTBREAK

BASED ON DIRK HELBING AND CHRISTIAN KÜHNERT. 2003. ASSESSING INTERACTION NETWORKS WITH APPLICATIONS TO CATASTROPHE DYNAMICS AND DISASTER MANAGEMENT. PHYSICA A: STATISTICAL MECHANICS AND ITS APPLICATIONS 328, 3 (2003), 584–606. HTTPS://DOI.ORG/10.1016/S0378-4371(03)00519-3

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EnE 305



AGENDA

- Why do we need to study interactions and their cascading effects?
- Catastrophe Dynamics of an Epidemic Outbreak
- Temporal Evolution of the Catastrophe
- Development of the Catastrophe with Unknown Forces
- Devising Control Strategies for Mitigation of Cascading Effects

• More frequent disasters in the past years are caused by triggered or cascading hazards.



Source: https://singingrooster.org/deforestation-hurricanes-haiti/

Source: https://commons.wikimedia.org/wiki/File:Flooded_areas_after_Hurricane_Jeanne.jpg

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Source: https://www.climatecentral.org/news/european-heat-wave-chances-rise-19225



Source: https://nerc.ukri.org/planetearth/stories/1849



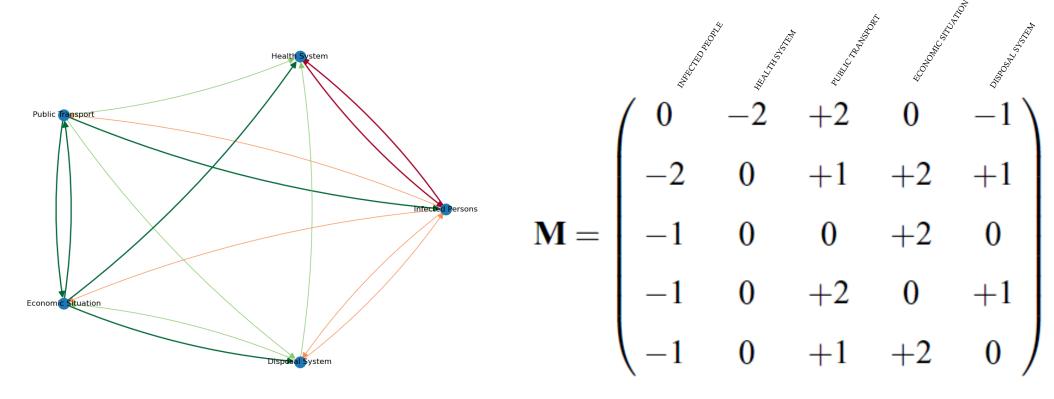
<u>Source</u>: https://blog.ucsusa.org/rachel-cleetus/new-noaa-report-shows-2017-was-the-costliest-year-on-record-for-us-disasters/

- More frequent disasters in the past years are caused by triggered or cascading hazards.
- Rapid urbanization increases the risk of the population to cascading disasters.
- There is a need to shift the focus from addressing risks of multiple single hazards to the overall risk of multi-hazard situation.

CATASTROPHE DYNAMICS

- Loosely related to "catastrophe theory", more of system thinking and dynamics
- Catastrophe defined as accumulation of direct and indirect interactions between events or factors investigated
- Uses an interaction network as basis
 - Can be used as a decision support tool when managing multi-factors situation by assessing the gap between the ideal dynamics of the network versus the current dynamics

CATASTROPHE DYNAMICS: INTERACTION NETWORK AND MATRIX



CATASTROPHE DYNAMICS: SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS

$$\frac{d}{d\tau}\vec{X}(\tau) = \begin{pmatrix} 0 & -2 & +2 & 0 & -1 \\ -2 & 0 & +1 & +2 & +1 \\ -1 & 0 & 0 & +2 & 0 \\ -1 & 0 & +2 & 0 & +1 \\ -1 & 0 & +1 & +2 & 0 \end{pmatrix} \cdot \vec{X}(\tau)$$

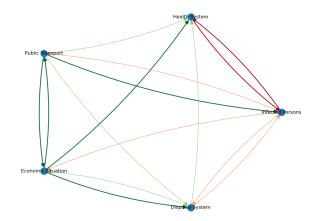
CATASTROPHE DYNAMICS: SYSTEM OF LINEAR DIFFERENTIAL EQUATIONS

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This only looks after direct interactions. How about the indirect interactions?

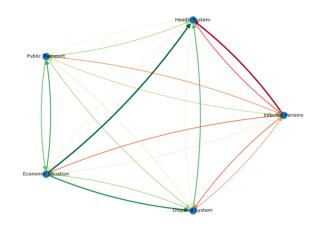
CATASTROPHE DYNAMICS: **ASSESSMENT MATRIX**

$$\mathbf{M} = \begin{pmatrix} 0 & -2 & +2 & 0 & -1 \\ -2 & 0 & +1 & +2 & +1 \\ -1 & 0 & 0 & +2 & 0 \\ -1 & 0 & +2 & 0 & +1 \\ -1 & 0 & +1 & +2 & 0 \end{pmatrix}$$



$$\mathbf{M} = \begin{pmatrix} 0 & -2 & +2 & 0 & -1 \\ -2 & 0 & +1 & +2 & +1 \\ -1 & 0 & 0 & +2 & 0 \\ -1 & 0 & +2 & 0 & +1 \\ -1 & 0 & +1 & +2 & 0 \end{pmatrix}$$

$$\mathbf{A} = (A_{ij}) = \begin{pmatrix} 0.9 & -2.2 & 1.3 & -0.8 & -1.6 \\ -3.4 & 1.1 & 1.5 & 3.5 & 2.3 \\ -1.7 & 0.6 & 0.5 & 2.5 & 0.8 \\ -2.0 & 0.6 & 2.1 & 1.5 & 1.6 \\ -2.0 & 0.6 & 1.5 & 2.9 & 0.9 \end{pmatrix}$$



CATASTROPHE DYNAMICS: SOLUTION TO THE SYSTEM OF ODES

$$\frac{d}{d\tau}\vec{X}(\tau) = \mathbf{M} \cdot \vec{X}(\tau)$$

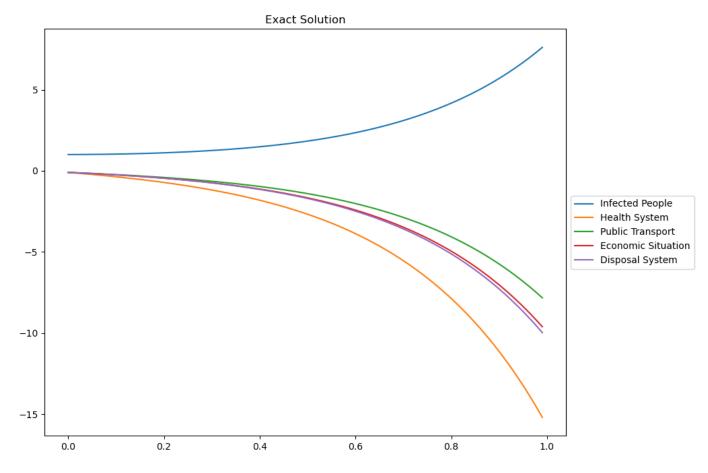
Exact Solution (where $\vec{X}(0) = \vec{X}$):

$$\vec{X}(\tau) = e^{\tau M} \cdot \vec{X}(0)$$

$$\vec{X}(\tau) = (\tau A_{\tau} + I) \cdot \vec{X}(0)$$

TEMPORAL EVOLUTION OF THE CATASTROPHE

- $\vec{X}(0) = [1, -0.1, -0.1, -0.1, -0.1]^T$
- Iterated $\vec{X}(\tau)$ through $\tau \in [0,1]$
- Interactions grow exponentially.
 - Infectives grow.
 - Other sectors collapse.
- Problematic!



DEVELOPMENT OF THE CATASTROPHE WITH UNKNOWN FORCES

$$d\vec{X}(\tau) = \mathbf{M} \cdot \vec{X}(\tau) d\tau + \mathbf{C} \cdot \vec{\xi}(\tau) d\tau$$

Why random processes?

- A lot of unaccounted uncertainties in the environment
- Unknown forces usually modelled as random processes

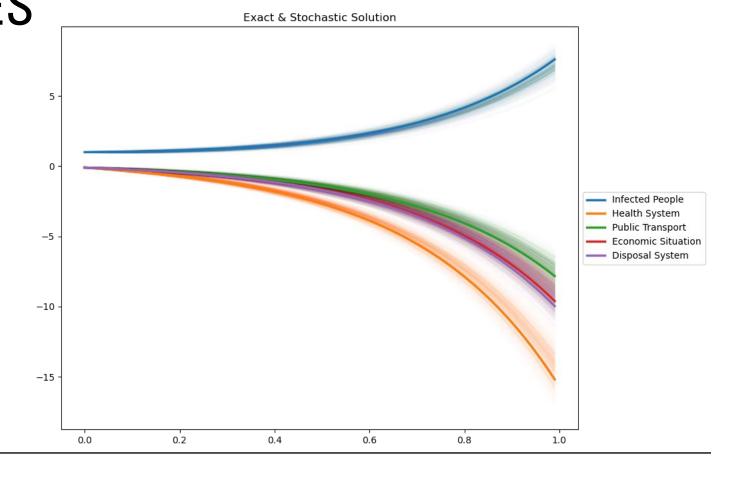
DEVELOPMENT OF THE CATASTROPHE WITH UNKNOWN FORCES

Exact & Stochastic Solution

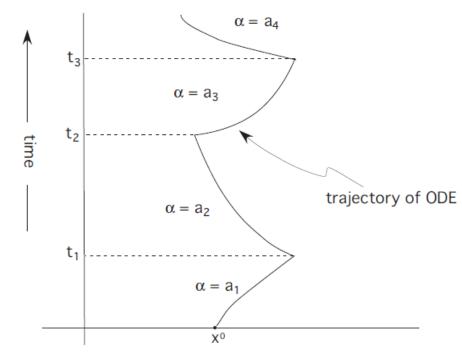
Follows the same exponential trend

• Possible trajectories fall within a range from the deterministic solution

• Still problematic!

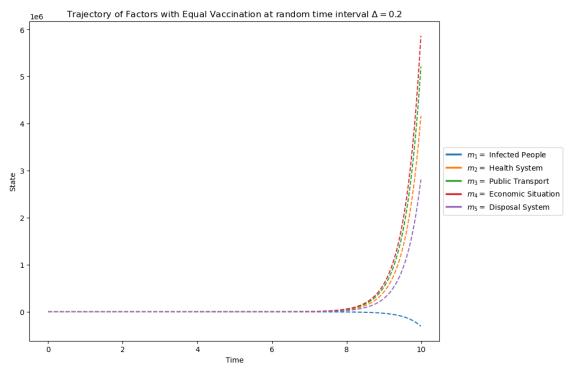


- Damping effect in the form of disaster responses
- Controls result to staggered trajectories.
- For this experiment:
 - Vaccination for medical workers and disposal workers (+1 effect)



Source: Lawrence Evans. 2010. An Introduction to Mathematical Optimal Control Theory Version 0.2.

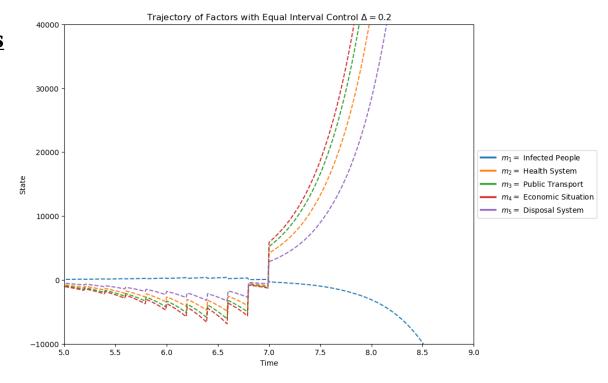
- Constant Vaccination on Constant Time Intervals
- Doubling Vaccination on Constant Time Intervals
- Constant Vaccination on Random Time Intervals
- Doubling Vaccination on Random Time Intervals



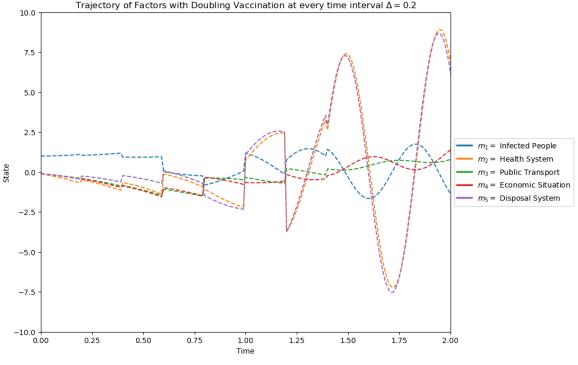
Goal: end of pandemic and recovery of other sectors

Constant Vaccination on Constant Time Intervals

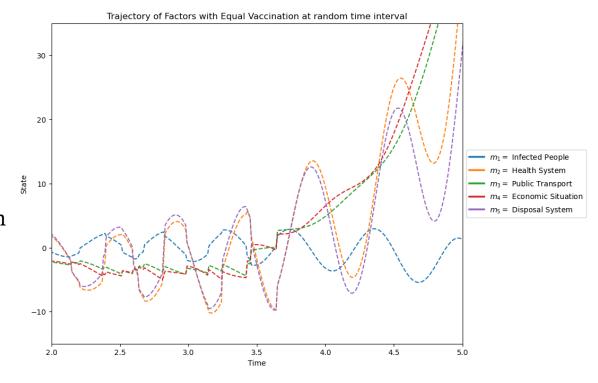
- Catastrophe Lifespan: t = 7.0
- Assuming time scale is years, equal vaccination for every 2.4 months ends the pandemic in 7 years.
- Doubling Vaccination on Constant Time Intervals
- Constant Vaccination on Random Time Intervals
- Doubling Vaccination on Random Time Intervals



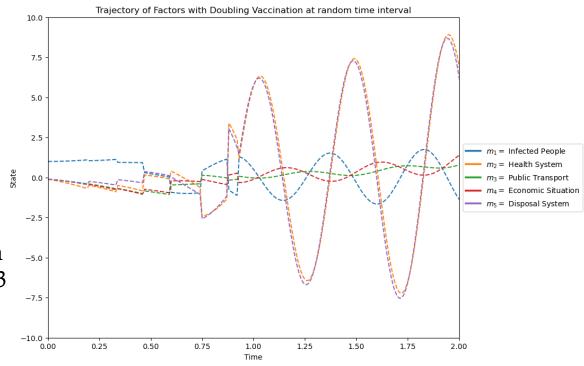
- Constant Vaccination on Constant Time Intervals
- **Doubling Vaccination on Constant Time Intervals**
 - Catastrophe Lifespan: t = 1.4
 - Assuming time scale is years, doubling vaccination for every 2.4 months ends the pandemic in 1.4 years.
 - Oscillation occurs! (Must be stability issues)
- Constant Vaccination on Random Time Intervals
- Doubling Vaccination on Random Time Intervals



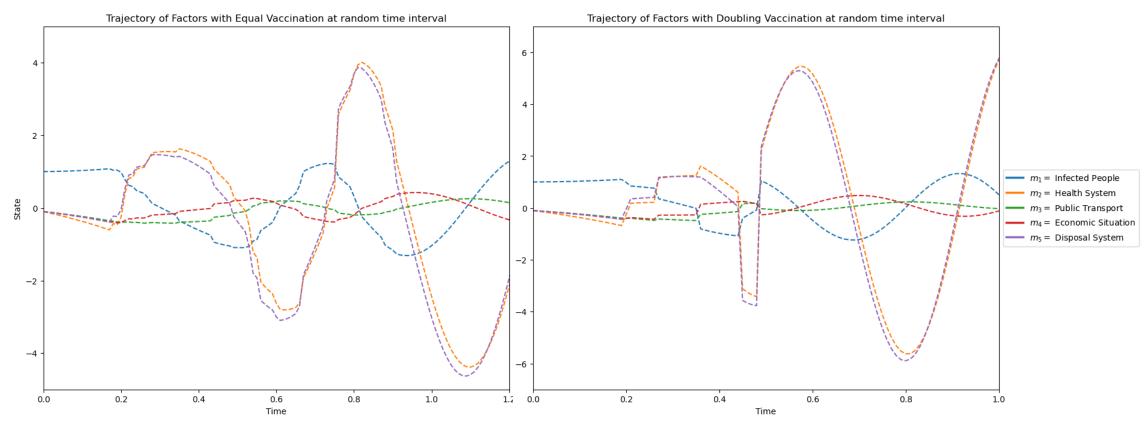
- Constant Vaccination on Constant Time Intervals
- Doubling Vaccination on Constant Time Intervals
- Constant Vaccination on Random Time Intervals
 - Random Interval: t = 0.2 + dt; $dt \in [0.1,2]$
 - Catastrophe Lifespan: t = 3.65
 - Assuming time scale is years, constant vaccination on random time intervals ends the pandemic in 3.65 years.
 - Oscillation occurs! (Must be stability issues)
- Doubling Vaccination on Random Time Intervals



- Constant Vaccination on Constant Time Intervals
- Doubling Vaccination on Constant Time Intervals
- Constant Vaccination on Random Time Intervals
- **Doubling Vaccination on Random Time Intervals**
 - Random Interval: t = 0.2 + dt; $dt \in [0.1,2]$
 - Catastrophe Lifespan: t = 0.93
 - Assuming time scale is years, doubling vaccination on random time interval ends the pandemic in 0.93 years.
 - Oscillation occurs! (Must be stability issues)



OTHER SIMULATIONS $(T = 0.2 + DT; DT \in [-0.1, +0.1])$



Adjusting random interval can show interesting effects: Lifespan = 0.91 (equal) and 0.49 (doubling)

CONCLUSION

- Effects cascaded from interacting events/factors can grow in uncontrollably, with or without considering unaccounted forces.
- Adjusting control parameters (intensity and frequency) can change the trajectory of the overall system.
- Stability needs to be investigated to avoid inputting parameters that cause oscillation.
- Theoretical framework must be verified with real life scenario and experts' advice.

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