<u>**TendonForces Forcing Module as an add-on to the PyElastica simulation software**</u>

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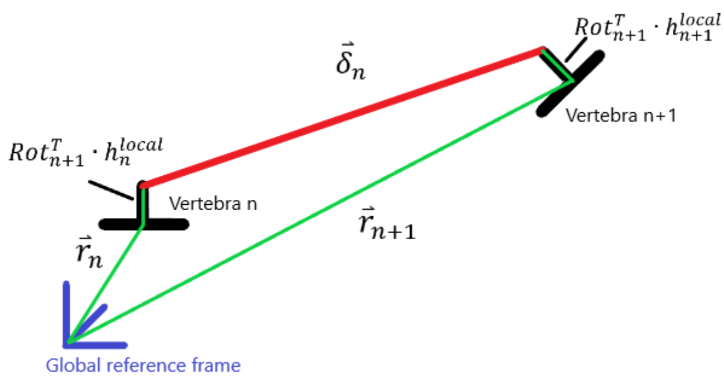
Mathematical procedure

The nature of the interaction between vertebrae, the tendon and the rod are nontrivial and are not so easily expressed mathematically. This is because these three entities interact in a nonlinear fashion: as tension increases, so does the deflection of the rod and thus the resultant forces in each of the vertebrae because of the tendon tension, which in turn affects the deflection of the rod. In a sense, it is a more complex version of the P delta problem in classical structural mechanics.
However, given the validated ability of PyElastica to numerically solve the governing equations of the rod and output the state of many variables in the system, the interaction of vertebra-tendon-rod can be implemented in a simpler manner that is better suited to match the numerical computations that the software is already doing.
The following procedure details the steps that the TendonForces module takes in order to apply tendon actuated forces and moments to the rod being simulated in the PyElastica simulator:

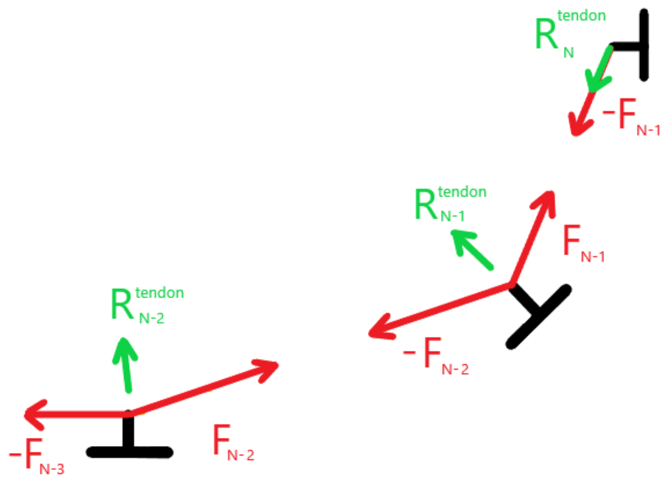
1. Using the position_collection data obtained from the simulator, the nodes which contain a vertebra are located in the 3D workspace, as well as their rotation matrices constructed using directors_collection. Using this data, along with the height of the vertebra, a vector is constructed which describes the relative change in position in space between the top of one vertebra to the next, as seen in the following image:



   Thus giving the following equation:
$$\vec{\delta}_n = \left(\vec{r}_{n+1} + Rot_{n+1}^T \cdot \vec{h}_{n+1}^{local}\right) - \left(\vec{r}_n + Rot_{n+1}^T \cdot \vec{h}_n^{local}\right)$$
   Where $\vec{r}_n$ is the position vector of the n vertebra's node in the global reference frame (taken from position_collection), $\vec{h}_n^{local}$ is the vector which describes the local orientation and the height of the vertebra, and Rot_n is the rotation matrix associated with the n vertebra (the transposing of this matrix is done to be able to rotate the $\vec{h}_n^{local}$ vector from the local reference frame to the global reference frame, so it can then be added to $\vec{r}_n$).

2. Now, the unit-normed vector of $\vec{\delta}_n$ is calculated and then scaled by the tension T applied to the tendon, as such:
$$\vec{F}_n = T \cdot \frac{\vec{\delta}_n}{\left\|\vec{\delta}_n\right\|}$$
   Where $\vec{F}_n$ is the force vector that acts on the n vertebra, pointing in the same direction as $\vec{\delta}_n$.

3. Given that for every vertebra there are two force vectors that appear because of the tendon tension, as seen in the following image:



Then the resultant is found by a vector sum:

$$\vec{R}_n^{tendon} = \vec{F}_n - \vec{F}_{n-1}$$

Where $\vec{R}_n^{tendon}$ is the resultant force caused by the tendon tension in each vertebra n. It is noted that this vector sum includes a negative force, with subindex $n-1$. This is because the vector that describes this force is no other than $-\vec{\delta}_{n-1}$.

4. Because the weight of the vertebrae must also be taken into account, the final resultant force for each vertebra node is the following:

$$\vec{R}_n = \vec{F}_n - \vec{F}_{n-1} + W_n$$

Where W_n denotes the weight vector of each vertebra n, and $\vec{R}_n$ is the final resultant force being applied to the vertebra node.

5. The effect of the torques generated along the rod because of the tendon-vertebra-rod interaction must be considered because there are forces, namely the tendon generated forces which act at points that are away from the center line of the rod. This generates torques along the rod because there are now forces and lever arms on which they act. To apply these torques, first it must be understood that PyElastica handles forces in the global reference frame and torques in the local reference frame. What follows is the rotation of the tendon forces obtained previously from the global reference frame to the local reference frame, as such:

$$\vec{R}_n^{local} = Rot_n \cdot \vec{R}_n^{tendon}$$

Note that it is specifically $\vec{R}_n^{tendon}$ and not $\vec{R}_n$ being used, and this is because the weight included in $\vec{R}_n$ does not generate torque as it is acting upon the center line of the rod.

6. Next, the locally rotated resultant tendon force vector is cross-multiplied with the local vertebra height vector to obtain the resulting torque that will be applied to the vertebra n:

$$\vec{M}_n = h_n^{local} \times \vec{R}_n$$

7. Once calculated, $\vec{R}_n$ and $\vec{M}_n$ are applied to the system at their corresponding vertebra nodes and at the current time step.

Physical experimentation / validation

Overview

The physical experimentation and validation of TendonForces was done with a set of three experiments. The details of these experiments and their results can be found in the work (undergraduate thesis by Gabriel Tuzlaci): "*Desarrollo de Entorno en ROS2 para el Simulado y Control de un Robot Continuo Accionado por Tendones*". The purpose of each experiment was the following:

- The building of the physical experiment environment, which consisted of an elastic silicone rod fixed in one end and fitted with 3D printed PLA discs vertebrae and a monofilament nylon wire as tendons. Measuring of the rod's tip's position was done using a leveled laser and grid paper placed behind the rod.

1. Experiment No. 1: calibration of the simulator's elasticity modulus by allowing the rod to deflect under its own weight and measuring the vertical position of the rod's tip. These measurements were taken quickly after the rod reached equilibrium at its deflection, and shortly after the rod was supported so that it would not have any deflection (this is to reduce the effect of creep exhibited by these polymers). The measurements taken (5) were identical and these were used to run simulations in the PyElastica simulator under the exact same conditions, changing only the young's modulus until the simulated tip deflection reached the measured tip deflection.
2. Experiment No. 2: calibration of the simulator's shear modulus by allowing the rod to deflect under its own weight as well as an actuated tendon on the rod's horizontal plane. This was done because the rod exhibited a very slight twist when the horizontal tendon was activated, which then allowed for the calibration of the shear modulus to match the tip's deflection. The same procedure as Experiment No. 1 was followed, except for an additional condition which was the activation of a tendon on the horizontal plane. Many simulations were carried out until the simulation's vertical tip deflection matched the experiment's measured vertical tip deflection.
3. Experiment No. 3: the validation of the TendonForces external forcing module was examined by carrying out a series of ten experiments and comparing the results between the simulations and the measurements. The experiments consisted on allowing for the deflection of the rod under its own weight, as well as with the activation of a tendon on the vertical plane and on the horizontal plane. The tensions applied to the tendons were not identical and they were ever increasing until the tenth experiment. The 3D position of the rod's tip was measured and compared with the 3D position of the simulated rod's tip. The results showed an average error in the range of 2%, low enough to be considered valid.

Calibration method

Before carrying out the experimental procedures, it is important to understand how the calibration of the simulated model would work. The process functions as follows:

1. **Defining the system function**: The chosen function in this case is the PyElastica simulation. PyElastica takes input values and computes simulations; for this process, the final deflection value of the simulated rod's tip will be used as the output.

2. **Structuring a separate function**: A separate function is created to execute multiple simulations until the desired values are reached. This will be achieved using SciPy's `fsolve` function (Virtanen et al., 2020), which focuses on finding roots of nonlinear functions.

3. **Input for `fsolve`**: The `fsolve` function will receive an initial value for the elasticity modulus (for Experiment No. 1) or the shear modulus (for Experiment No. 2). After running a simulation via PyElastica, the final deflection value of the tip will be taken, and the desired tip deflection value will be subtracted from it — in this case, the value measured in experiments, corrected to account for its offset.
4. **Iteration with `fsolve`**: Step 3 is repeated automatically due to the internal workings of `fsolve`, adjusting the respective input parameter until a root of the provided function is found (in this case, until the desired deflection is achieved).
5. **Recording the parameter**: The obtained parameter is printed and recorded to be used in subsequent experiments.

Materials

The following materials are needed to carry out the three experiments:

- **2 silicone cylinders**
    - 27.0 cm in length
    - 1.1 cm in diameter
- **3D printing material**: PLA
- **Monofilament nylon thread**
- **Grid paper** (measured in centimeters)
- **2 plastic bottles**
- **1.6L of water** in a container
- **Syringe** with a capacity of 50–60 mL
- **Wood screws/nails**
- **Wooden blocks and/or boards**
- **2 bearings** with an inner diameter of 8 mm and an outer diameter of 22 mm
    - Code: 608 ZZ
- **2 camera tripods**
- **Precision scale**
- **Quick-adhesion glue**
- **Low-power laser pointer**
- **Level**
- **Square ruler**

Experiment No. 1:

The procedure for Experiment No. 1 is detailed below:

1. The volume of the silicone cylinder was calculated using its diameter and length.
2. The density of the silicone cylinder was calculated using the measured mass and computed volume.
3. Using a wooden board and machine screws, 2 cm of a silicone cylinder was fixed in place, leaving 25 cm of the cylinder free. The cylinder was initially supported to prevent any deflection.
4. Six discs were designed to be used as vertebrae. These discs were designed considering the rod's diameter and followed the design shown in Figure 3-7.

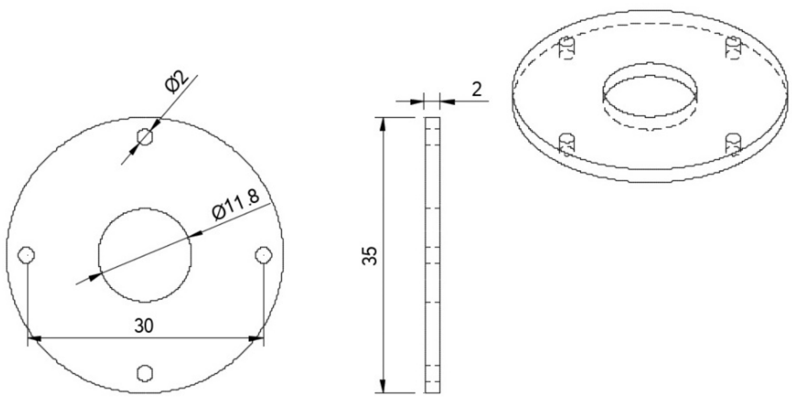


Figure 3-7. Basic design for the vertebrae discs used in the experiments.

5. With 25 cm of free cylinder, 0.5 cm was marked from both the fixed and free ends, resulting in a usable length of 24 cm for the cylinder.
6. The usable length of 24.0 cm was divided into five segments, yielding 4.8 cm each. Starting from one end of the usable length, six discs were placed along the cylinder, spaced 4.8 cm apart. Adhesive was applied at the contact points between the cylinder and each disc to secure them.
7. Monofilament nylon thread was threaded through the holes (only the horizontal holes pointing outward and the vertical ones pointing upward) in the discs along the silicone cylinder. A knot was tied at the hole of the last disc, located at the free end of the cylinder, leaving some slack in the thread at the fixed end.
8. The two bearings were carefully positioned in the system's fixation, ensuring that the nylon thread rested on the bearing with an angle close to or equal to zero relative to the horizontal axis. This setup was applied to both nylon threads passed through the discs, as shown in Figure 3-8.

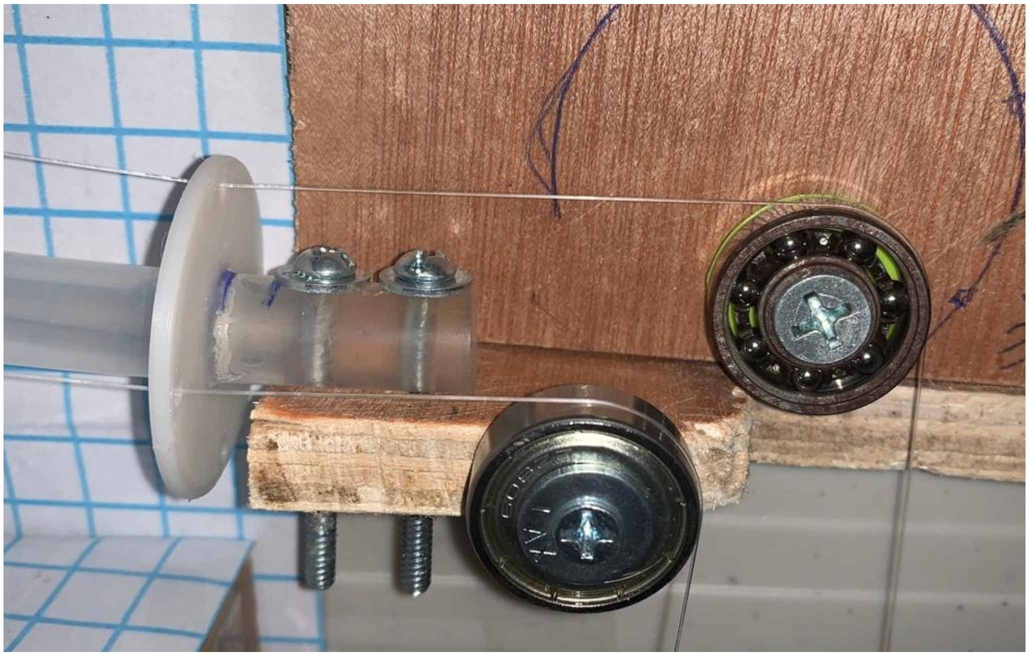


Figure 3-8. The fixation setup for the rod using machine screws, as well as the proper alignment of the bearings to ensure an angle close to zero with the horizontal for the monofilament nylon thread.

9. A grid paper sheet was placed on a flat, vertical surface.
10. The system, including the silicone cylinder, was placed as close as possible to the grid paper without making contact, ensuring that the entire cylinder fit within the grid.
11. A tripod-mounted camera was adjusted to capture clear images of the entire cylinder and the grid paper.
12. A tripod-mounted laser was adjusted for easy repositioning when the silicone cylinder was released and flexed. The laser was verified to be level and aligned with the grid paper.
13. The support holding the silicone cylinder was removed.
14. Once the deflection of the rod stabilized, the laser tripod was adjusted to point the laser directly at the tip of the silicone cylinder. An image of the system was captured, ensuring the entire setup was included.
15. The silicone cylinder was supported again to prevent deflection, and a 5-minute waiting period was observed.
16. Step 13 was repeated a total of five times, with identical results observed each time.
17. The vertical deflection (Z-direction) of the rod's tip was recorded and used to calibrate the elasticity modulus in the simulation.
18. The silicone cylinder was supported again to prevent deflection.

As mentioned in the procedure, the measurements were repeated five times, yielding identical results, which increases their reliability. In Figure 3-9, the image shows the system in deflection under its own weight, with the laser pointing at the tip of the silicone cylinder and the measurement recorded on the grid paper.

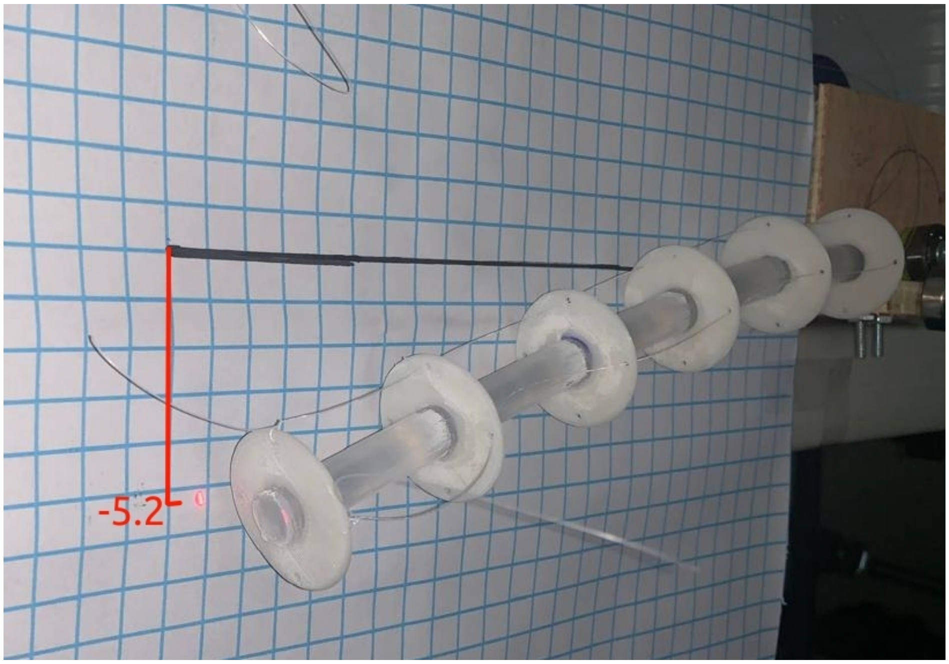


Figura 3-9. Measurement taken using the laser for Experiment No. 1. It is noted that the tendons are not tensioned and that the laser is centered on the tip of the cylinder, with a measurement of -5.2 centimeters.

Following the aforementioned **Calibration** procedure, the following result was obtained:

Experiment No. 1		
Z measured (cm)	Z corrected (cm)	Result
		Young's Modulus (Pa)
-5.2	-5.4	1.6598637E+07

Table 3-1. Results obtained for the calibration of the simulation carried out in the experiment No. 1.

The result of the simulation can be observed int he **Figure 3-10**:

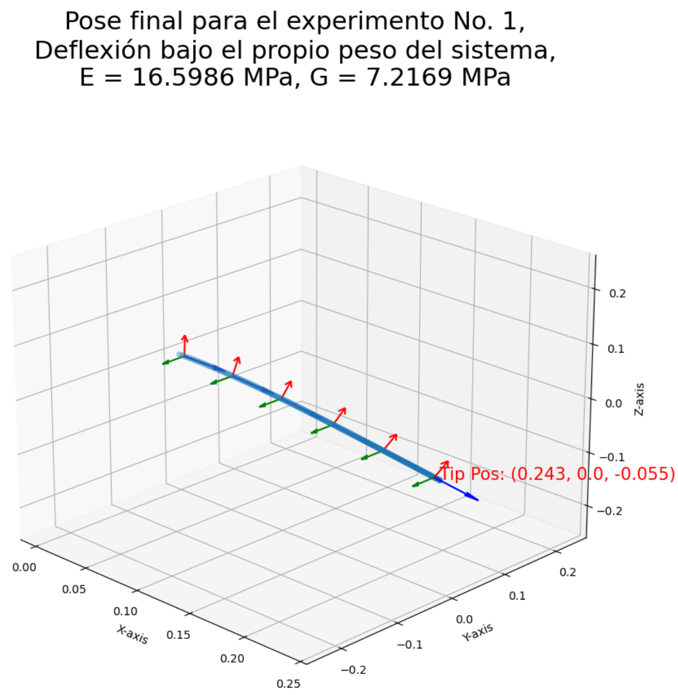


Figura 3-10. Final pose of the simulated robot after the rod deflected under the weight of itself and the attached vertebrae.

Experiment No. 2

The procedure for Experiment No. 2 is detailed below:

1. The empty plastic bottles were weighed using a precision scale (in this case, they weighed 55.37 g).
2. The tripod with the camera was set up to capture a clear view of the system.
3. The tripod with the laser was adjusted, ensuring it was level and aligned with the grid paper.
4. A knot was tied on the loose nylon monofilament thread corresponding to the tendon in the horizontal plane of the disc at the fixed end, attaching it to one of the plastic bottles.
5. A total of 150 mL of water was measured using the syringe and poured into the plastic bottle attached to the horizontal tendon.
6. The nylon monofilament thread was placed over its respective bearing, and the weight of the plastic bottle with water was gradually released until the tension in the thread fully supported the bottle's weight.
7. The tripod with the laser was adjusted so that the laser pointed directly at the tip of the flexed cylinder.
8. A photo of the flexed system was taken with the laser pointing at the tip.
9. The weight of the plastic bottle with water was supported again, and the silicone cylinder was returned to its unflexed state.
10. The laser was activated again, and a photo was taken of the system, capturing the exact point marked by the laser on the grid paper.
11. The deflection in the Z direction (vertical) at the tip of the silicone cylinder was measured.
    Steps 8–11 were repeated five times, with a 5-minute waiting period between trials. Identical results were observed in all trials. This result was then used to calibrate the shear modulus of the simulation in PyElastica.

As mentioned in the procedure, the measurements were repeated five times, yielding identical results, which increases their reliability. In Figure 3-11, the image shows the system deflected under its own weight and the action of the horizontal tendon, with the laser pointing at the tip of the silicone cylinder. Subsequently, the cylinder is supported, and the measurement is displayed on the grid paper in Figure 3-12.

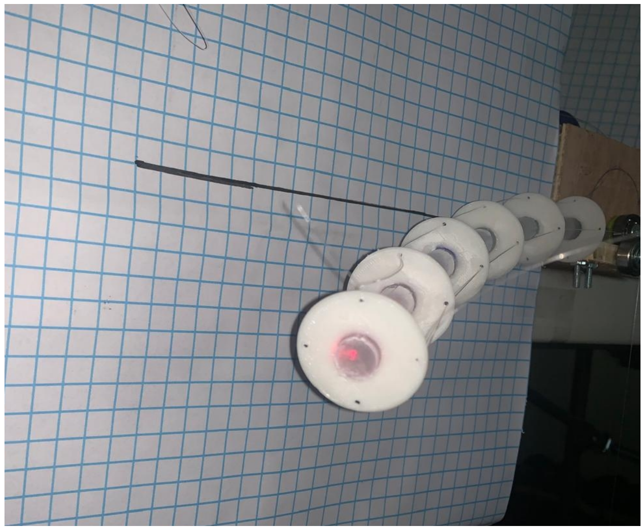


Figura 3-11. The laser is correctly leveled and pointing at the center of the deflected rod's tip.

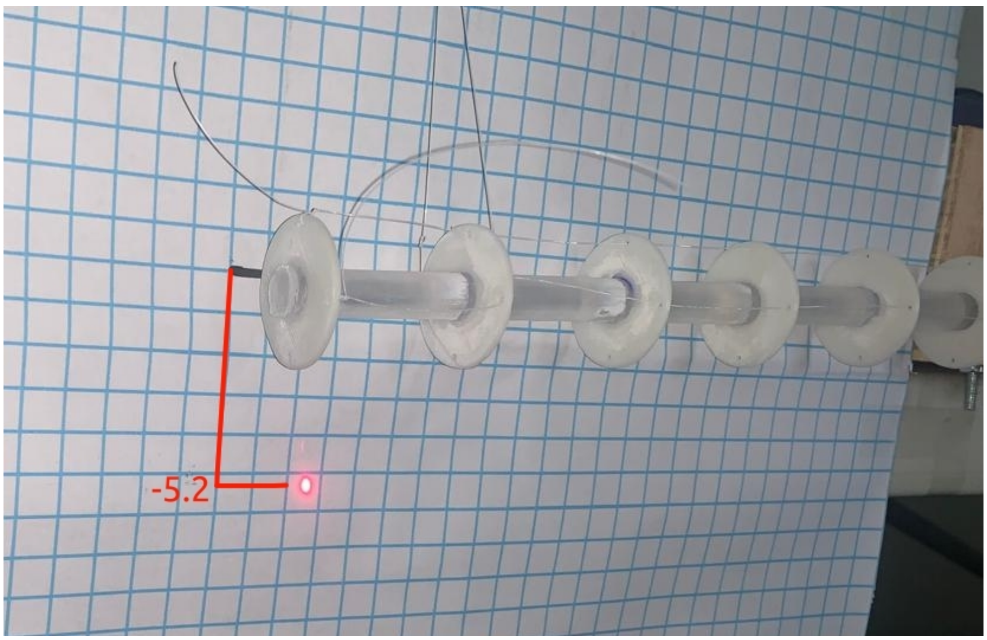


Figura 3-12. Measurement of the rod's deflection after the rod is once again supported and the laser is kept at the same spot.

Following the aforementioned **Calibration** procedure, the following result was obtained:

Experiment No. 2		
Z measured (cm)	Z corrected (cm)	Result
		Shear Modulus (Pa)
-5.2	-5.4	7.2168800E+06

Tabla 3-2. Results obtained for the calibration of the simulation in the experiment No. 2.

The result of the simulation can be observed in the following **Figure 3-13**:

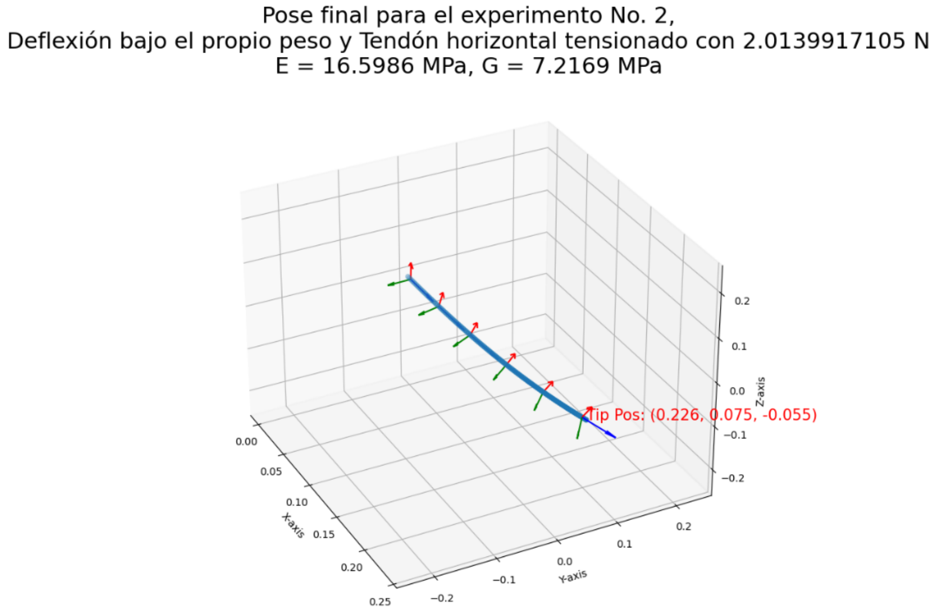


Figura 3-13. Simulation of the rod under the loads of its own weight, the weight of the vertebrae, and a tension of 2.0139917105 N in the tendon located in the row pointing in the +Y direction.

Experiment No. 3

The procedure for this experiment is described below:

1. Using the same setup as in Experiment No. 2, the first step was to empty the plastic bottle containing water.
2. Two plastic bottles were tied to the tendons at the clamped end that had loose nylon thread. These bottles were temporarily supported.
3. A sheet was placed on a horizontal and flat surface, perpendicular to the vertical grid paper, ensuring it was positioned at a height of approximately -3.0 cm relative to the zero axis marked on the vertical grid paper.
4. The tripod with the camera and the tripod with the laser were adjusted and prepared.
5. Fifty milliliters of water were added to the plastic bottle tied to the tendon in the vertical plane.
6. Twenty-five milliliters of water were added to the plastic bottle tied to the tendon in the horizontal plane.
7. The plastic bottles were gently released to avoid applying tension too abruptly to the tendon.
8. The tripod with the laser was adjusted so that the laser pointed directly at the tip of the flexed silicone cylinder.
9. A photo of the system was taken with the laser activated, after which the laser was turned off.
10. Using a square tool, it was placed perpendicular to the horizontal surface with the grid paper and aligned with the center of the flexed silicone cylinder to measure the displacement of the tip along the Y-axis (horizontal direction), which was recorded.
11. The two plastic bottles with water and the silicone cylinder were then supported again.
12. The laser was activated, and a photo was taken to capture the exact position where the laser marked the tip on the grid paper. This measurement was used for the X-axis (axial direction) and the Z-axis (vertical direction), both of which were recorded.
13. Steps 5–12 were repeated a total of 10 times, gradually adding up to 500 mL of water in the bottle attached to the tendon in the vertical plane and 250 mL in the bottle attached to the tendon in the horizontal plane.
14. The results obtained in Step 13 were compared with the simulations performed in PyElastica using the same parameters and conditions as in the corresponding experiments. These results were analyzed, and conclusions were drawn regarding the validation of the tendon-driven actuation model.

The results obtained for this experiment are detailed below, starting with the measurements conducted during the physical experiment, along with their respective corrections to account for offsets:

	Mass (g)		(cm)	+0.3 (cm)	(cm)	-2.0 (cm)	(cm)	-0.2 (cm)
Trial	Vertical Tendon	Horizontal Tendon	X measured	X Corrected	Y Measured	Y Corrected	Z Measured	Z Corrected
1	105.37	80.37	24.2	24.5	5.2	3.45	-1.3	-1.5
2	155.37	105.37	24	24.3	6.3	4.3	0.8	0.6
3	205.37	130.37	23.6	23.9	7	5	2.7	2.5
4	255.37	155.37	22.9	23.2	8	6	4.7	4.5
5	305.37	180.37	21.6	21.9	9.8	7.8	6.7	6.5
6	355.37	205.37	19.9	20.2	10.5	8.5	8.9	8.7

7	405.37	230.37	18.5	18.8	12	10	10.1	9.9
8	455.37	255.37	16.2	16.5	12	10	11.8	11.6
9	505.37	280.37	13.9	14.2	12.7	10.7	12.7	12.5
10	555.37	305.37	11.1	11.4	13.6	11.6	13.8	13.6

Table 3-3. Results from the measurements, along with their corrections for the 10 trials carried out in experiment No. 3.

It is noted that the measurements were corrected by reversing the offset associated with them. Subsequently, the simulation results are detailed and compared with the results obtained from the physical experiment, which can be observed in the difference columns of **Table 3-4**.

	(cm)					
Trial	X simu-lated	Y simu-lated	Z simu-lated	Difference X	Difference Y	Difference Z
1	24.6	3.1	-1.7	0.1	-0.35	-0.2
2	24.4	4.2	0.2	0.1	-0.1	-0.4
3	23.9	5.2	2.2	0	0.2	-0.3
4	23.1	6.3	4.2	-0.1	0.3	-0.3
5	22	7.3	6.1	0.1	-0.5	-0.4
6	20.5	8.2	8	0.3	-0.3	-0.7
7	18.7	9	9.8	-0.1	-1	-0.1
8	16.5	9.7	11.3	0	-0.3	-0.3
9	14.1	10.2	12.6	-0.1	-0.5	0.1
10	11.4	10.4	13.5	0	-1.2	-0.1

Table 3-4. Table showcasing the differences between the simulated and measured XYZ positions for the experiment No. 3.

To better understand the results, a third table was calculated showing the percentage error in position obtained for each of the trials, which can be seen below:

	Difference / Robot Length * 100%			
Trial	Error X	Error Y	Error Z	Average error
1	0.40%	-1.40%	-0.80%	-0.60%
2	0.40%	-0.40%	-1.60%	-0.53%
3	0.00%	0.80%	-1.20%	-0.13%
4	-0.40%	1.20%	-1.20%	-0.13%
5	0.40%	-2.00%	-1.60%	-1.07%
6	1.20%	-1.20%	-2.80%	-0.93%
7	-0.40%	-4.00%	-0.40%	-1.60%
8	0.00%	-1.20%	-1.20%	-0.80%
9	-0.40%	-2.00%	0.40%	-0.67%
10	0.00%	-4.80%	-0.40%	-1.73%

Tabla 3-5. Results for the average relative error between the simulated and the measured XYZ positions for the experiment No. 3.

It is noted that the error calculation considers the total length of the robot's rod. As detailed at the top of **Table 3-5**, the error is calculated by dividing the measured difference by the total length of the robot, which is 25 cm.

To visualize the results, the following graph illustrates the error values calculated in **Table 3-5**:

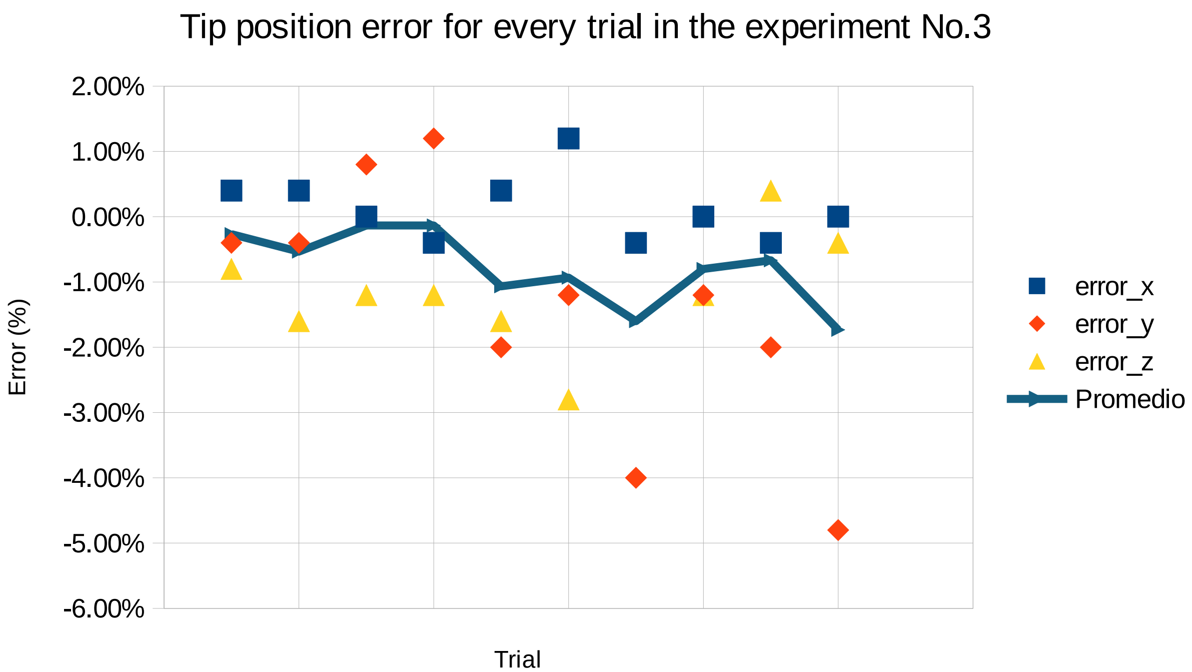
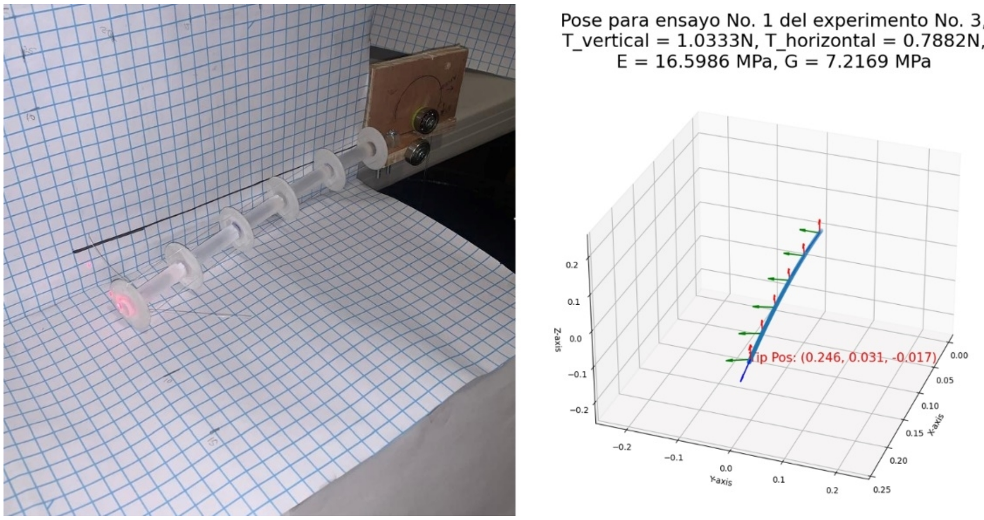


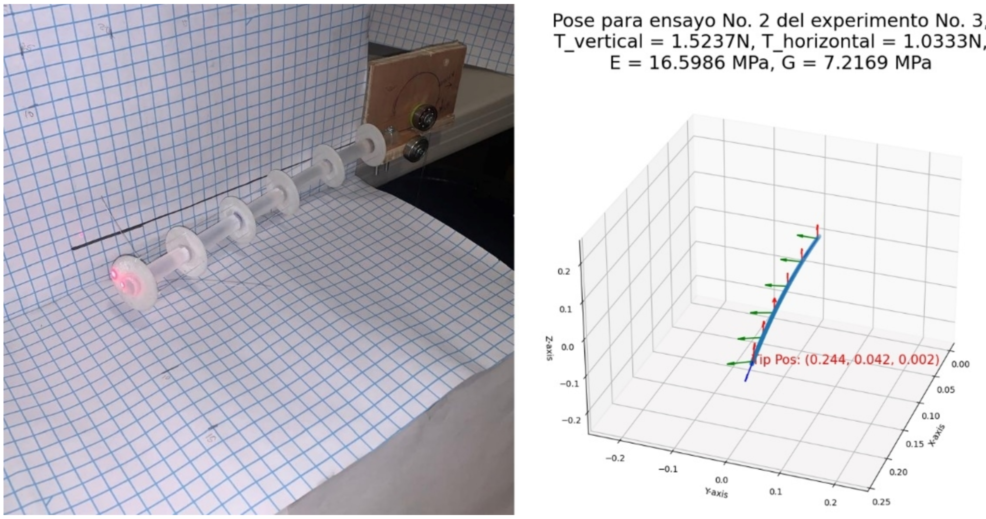
Figura 3-14. Error graph for Experiment No. 3, showing that the maximum error occurred in the Y-axis during the last trial. It is also noted that the average error for each trial remains below a magnitude of 2%.

Based on the results obtained, it is concluded that the margin of error remains within an acceptable average of below ±2%. This indicates that the force and moment model for tendon-driven actuation is validated, as demonstrated by the results obtained in the PyElastica simulations.
The images of the measurements taken, along with the corresponding simulations, can be observed in the following **Annex** section.

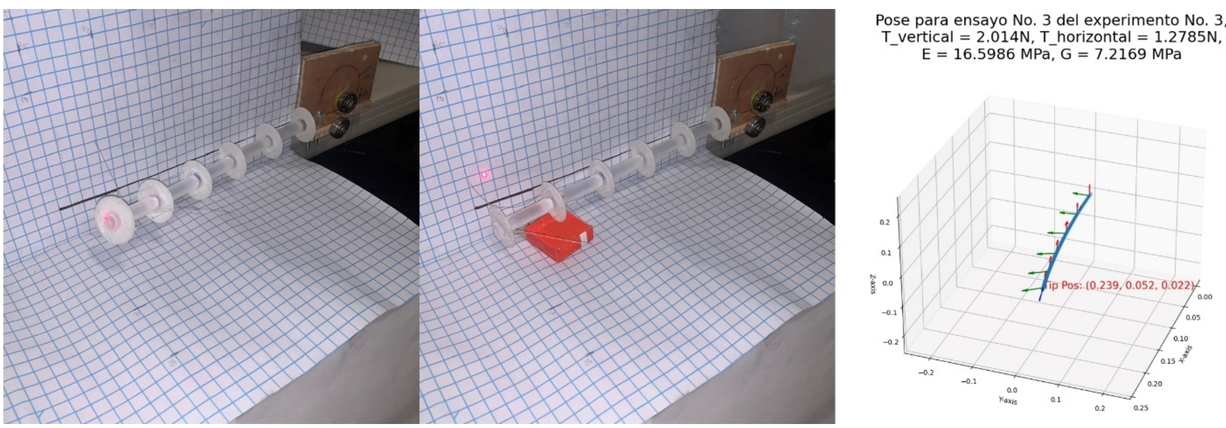
Annex



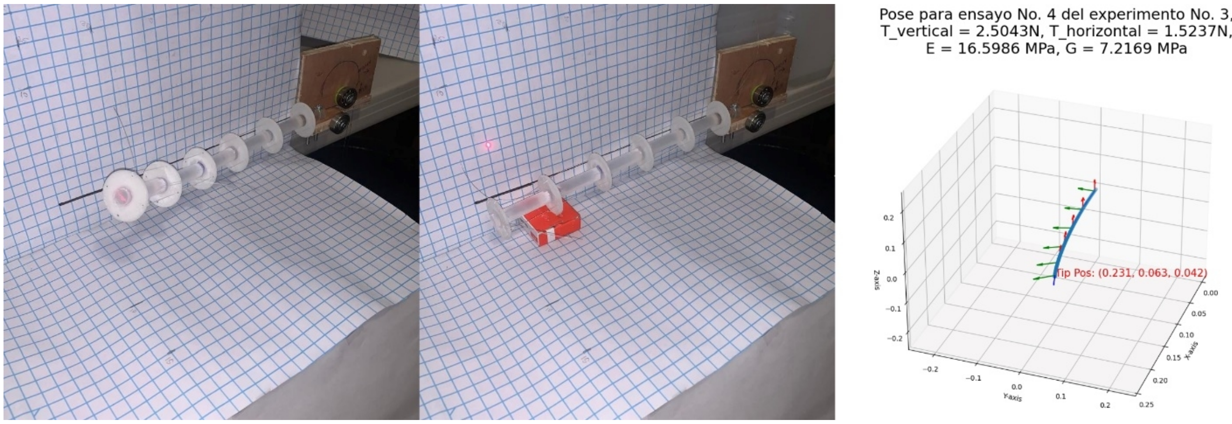
Annex-1. Measurement and simulation for trial 1 of the experiment No. 3.



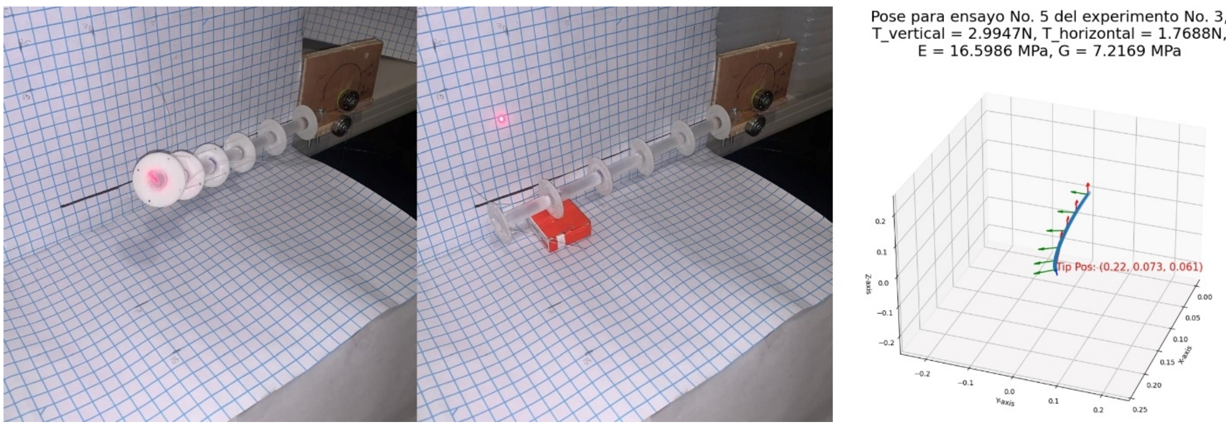
Annex-2. Measurement and simulation for trial 2 of the experiment No. 3..



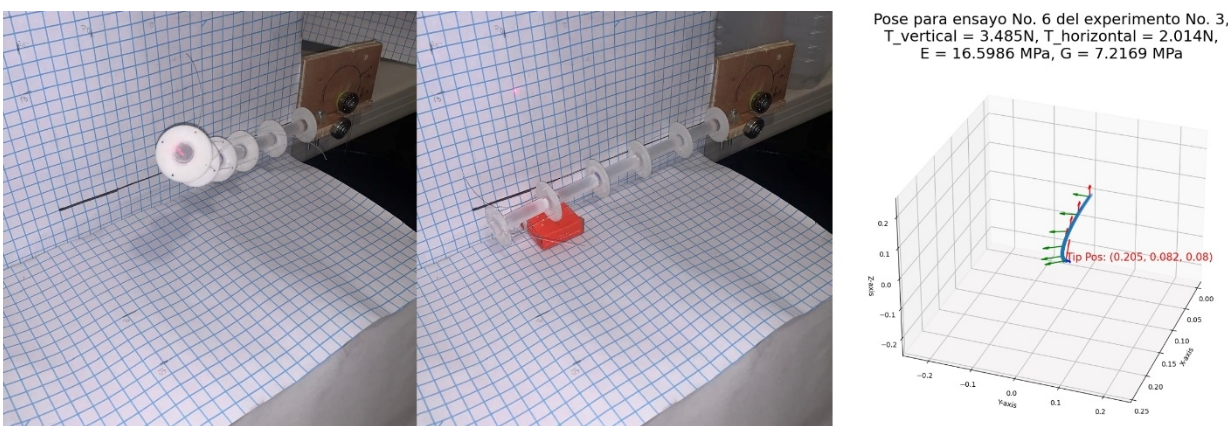
Annex-3. Measurement and simulation for trial 3 of the experiment No. 3.



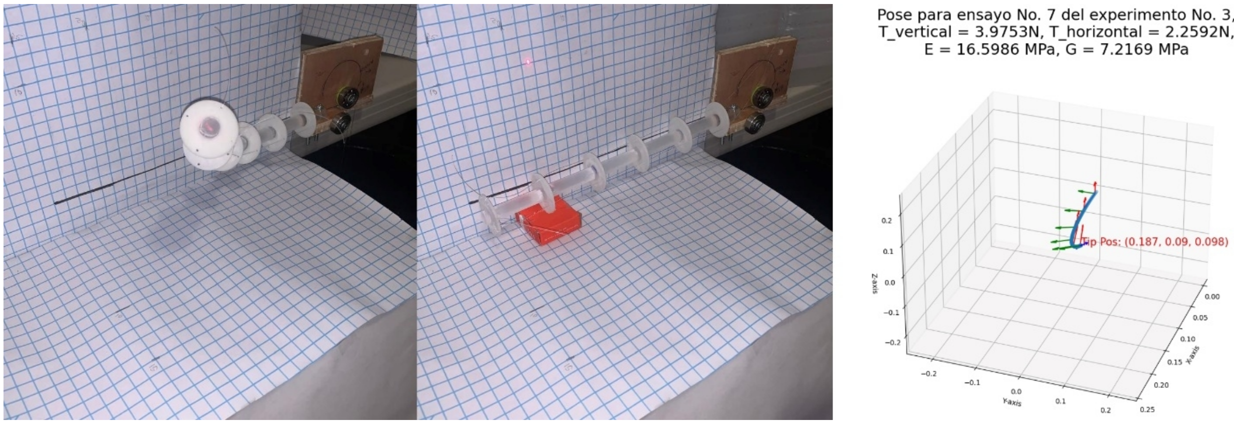
Annex-4. Measurement and simulation for trial 4 of the experiment No. 3.



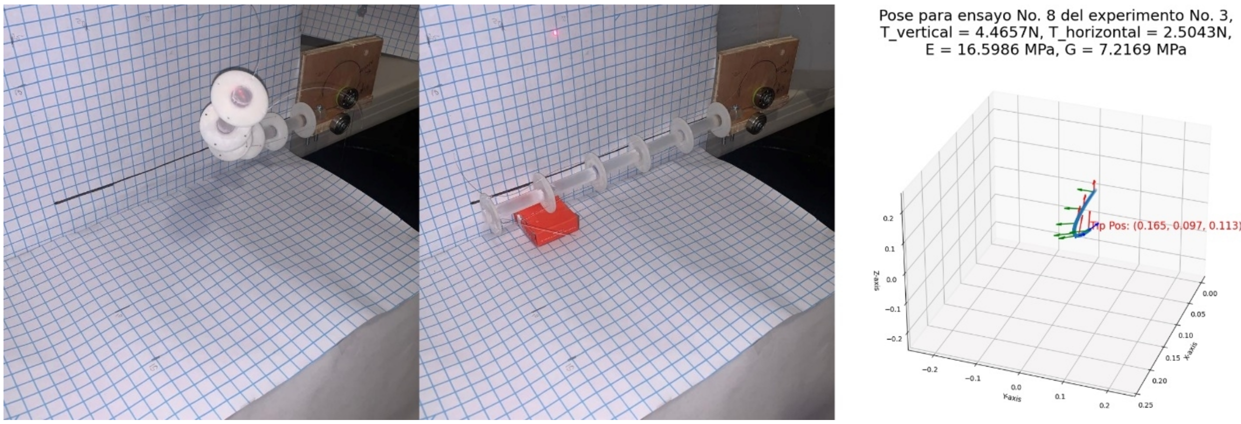
Annex-5. Measurement and simulation for trial 5 of the experiment No. 3.



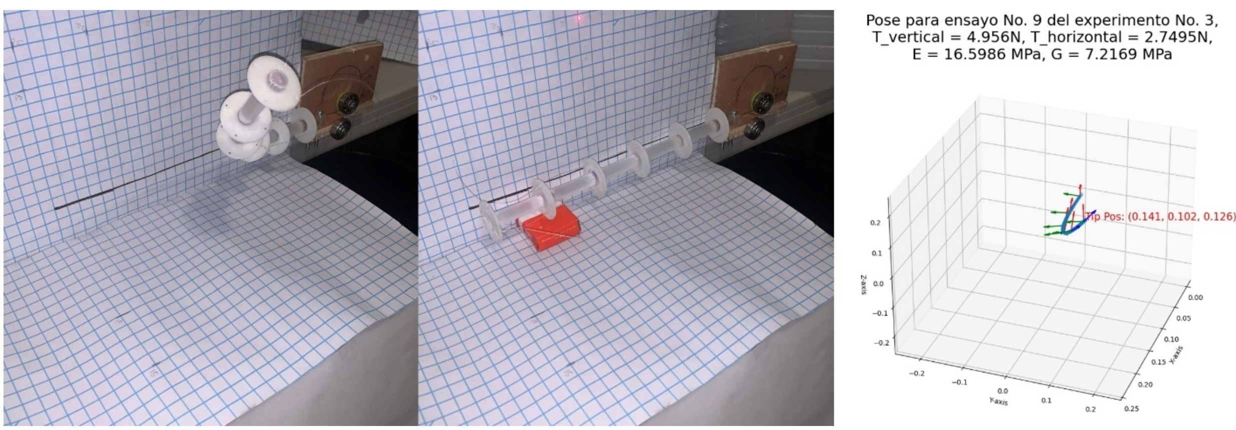
Annex-6. Measurement and simulation for trial 6 of the experiment No. 3.



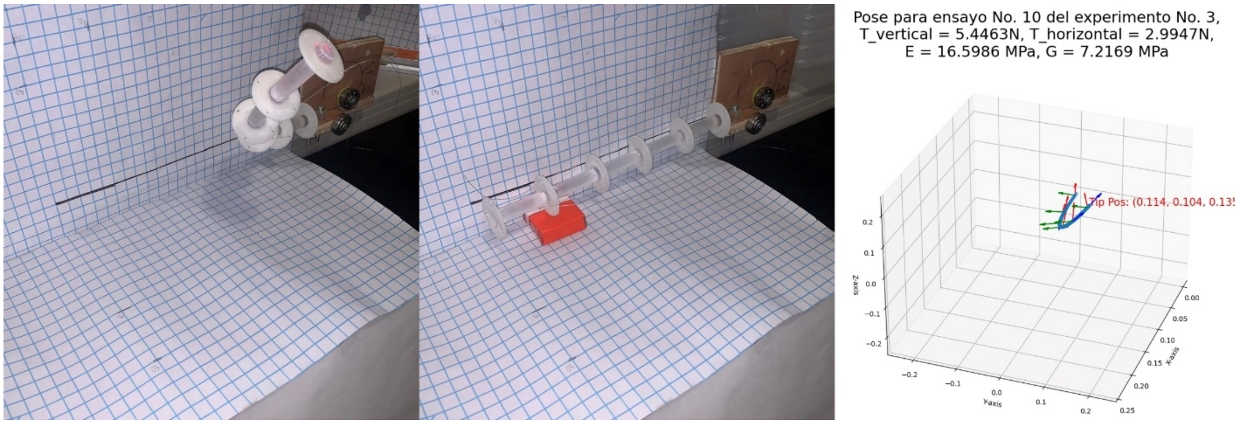
Annex-7. Measurement and simulation for trial 7 of the experiment No. 3.



Annex-8. Measurement and simulation for trial 8 of the experiment No. 3.



Annex-9. Measurement and simulation for trial 9 of the experiment No. 3.



Annex-10. Measurement and simulation for trial 10 of the experiment No. 3.