# Homework 2

Simulation and Performance Evaluation – University of Trento

**DEADLINE: 17:29 on April 29, 2025**

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You can solve the following assignments using any programming language. Try to do the homework by yourselves, without help from AI tools. You can use utility functions made available by the programming language of your choice, including functions to extract random numbers.

You will see a facility on Moodle to upload your homework. Please upload your code, and separately upload a **short** report where you describe your findings (no more than 2-3 pages). Upload **two versions** of the report: one with your names, and a second, fully anonymized one.

## Exercise 1

Watch the brief video labeled "Theorem on Poisson arrivals" on Moodle. Then use simulation to prove the following result.
Given that a Poisson process of rate $\lambda$ yields $N$ arrivals in some interval $[0, T]$, we can draw these arrivals in two ways:

1. By drawing $N$ arrival times uniformly at random in the interval $[0, T]$;
2. By drawing a set of $N$ exponential inter-arrival times of average value $1/\lambda$ in the interval $[0, T]$.

For example, show that the inter-arrival times of method 1 are also exponentially distributed. Then show that arrivals drawn with method 2 lead to arrival times that are uniformly distributed in $[0, T]$.
(*Hint 1*: start by choosing $\lambda$, $T$ and $N$ such that $\lambda T \approx N$ to make things easier. Then see if you observe anything different when you choose other values.)
(*Hint 2*: for method 2, are there extra checks you should implement?)

## Exercise 2

Consider the following "weird" probability density function:

$$f(x) = \frac{1}{A} x^2 \sin^2(\pi x), \quad -3 \le x \le 3\,,$$

where $A = 8.8480182$ is a normalization factor such that $\int_{-3}^{3} f(x)\,\mathrm{d}x \approx 1$, so $f(x)$ can be interpreted as a PDF.

1. Employ rejection sampling to draw a large number of samples from the above PDF. (Question: do you really need to know the value of $A$?)
2. Plot the resulting empirical PDF (e.g., through a histogram) and compare it against the theoretical PDF. Make sure you draw a sufficiently large number of variates, so that the histogram convincingly fits the theoretical PDF.
3. Draw 20 000 variates from the above distribution, and consider the first 200 variates. Apply the formulas seen in class to compute a 95% confidence interval for the mean, median and 0.9-quantile of the dataset you drew. Then compute these confidence intervals again using the bootstrap procedure.
4. Subdivide the 20 000 variates in 100 disjoint sets of 200 variates each. For each sub-dataset, compute the mean and the confidence interval for the mean using any method you prefer. How many confidence intervals contain the true mean?