

# **A Survey of Radiosity and Ray-tracing Methods in Global Illumination**

**Submitted by Ge Jin**

**12<sup>th</sup> Dec 2000**

**To**

**Dr. James Hahn**

**Final Project of CS368**

**Advanced Topics in Computer Graphics**

## Contents

Abstract.....	3
Introduction .....	3
Definition and Solutions.....	6
Radiosity Method .....	8
Hemi-cube method .....	8
Ray tracing Form factor .....	9
Hybrid Form Factors .....	10
Combined Radiosity and Ray Tracing method .....	11
Two-path method .....	11
General Two-pass method.....	13
Ray Tracing method .....	14
Path-tracing method .....	14
Radiance Method (collection of methods) .....	15
Bi-directional ray tracing .....	18
Metropolis light tracing.....	20
Photon map method.....	21
Conclusions .....	23
References .....	24

# Abstract

This paper surveys Radiosity and Ray-tracing methods used in global illumination. Global illumination is based on light transport mechanism in real world. The light transport mechanism can be expressed in terms of BRDF of each element, and the most common and practical way is dividing the BRDF into specular and diffuse component. Mathematically Global illumination is a problem of solving numerical equations concerning with the convergence, converging speed and if it converges to right answer. The initial value is the given light sources and their characteristics. The energy of the light source is propagated into the geometrical space. The Radiosity and Ray-tracing methods are used to calculate the energy propagation in each iteration step. The BRDF of geometrical element is important to the efficiency of each method. If the BRDF is ideal diffuse, Radiosity method will converge and converge to right answer. However, Ray-tracing algorithm works more efficiently in calculating specular reflection, refraction and caustic surfaces. Those Radiosity and Ray-tracing algorithms can be measured in two aspects, the accuracy and efficiency in BRDF simulation and the rendering speed.

# Introduction

The Global illumination techniques are inspired by Radiosity method<sup>1</sup>. It's a robust method to ideal diffuse inter-reflection within a closed environment. The environment is subdivided into patches or elements, over which the intensity of light is constant. A hemi-

cube method is exploited to calculate form factor—measurement of any other patch's contribution. Kajia<sup>2</sup> established a concrete equation for the global illumination.

The early works in Radiosity method is to find out more efficient and accurate way to calculate form-factors. Instead of hemi-cube method, Wallace<sup>3</sup> introduced a ray-tracing method to calculate form factor, while eliminating the aliasing and inadequate sampling problems in hemi-cube method. Meanwhile Baum et al.<sup>4</sup> introduced an analytically determined form factor calculation method when the traditional hemi-cube method can not treat well.

Global illumination problem can be seen through the BRDF measurement. If we consider the light transport mechanism in two patches, there would be four types of transfer mechanisms: diffuse to diffuse, diffuse to specular, specular to specular, specular to diffuse. The former radiosity methods only dealt with diffuse to diffuse reflection. A two-pass solution combining Enhanced Ray-tracing and Enhanced Radiosity method<sup>5</sup> is used to solve the specular and diffuse reflection in global illumination. Path tracing method suggested by kajia is a non-uniform importance sampling in infinite ray-space. And the variance reduction techniques are used to control sampling rates. Later, Sillion<sup>6</sup> extended the two-pass method, which overcome the limitation of planar specular surface and included any number of specular reflections and refraction in its path.

An alternative trend in radiosity is progressive refinement method<sup>7</sup>. Instead of calculating final radiance at each patch, update radiosity of every patch in each step. Chen<sup>8</sup> extended progressive refinement method to an interactive system, which allows incremental rendering with changing geometry and lighting condition. Recently, an interactive updating of global illumination using line-space hierarchy introduced by Drettakis<sup>9</sup>.

Global illumination method, in another viewpoint, is a kind of finite element method. Some traditional techniques used by Finite element method are also useful in global illumination. The error associated with each patch should be at the same level. Baum et al.<sup>10</sup> Suggested an automatic meshing technique to generate accurate radiosity solution. Pat Harahan<sup>11</sup> presented a rapid hierarchical radiosity algorithms for illuminating scenes with large patches.

This paper will concentrate on light transport and calculation models using ray-tracing method to generate global illumination. Gregory Ward<sup>12</sup> introduced an efficient ray tracing method to calculate interreflection between diffuse and specular component. It's based on important sampling of light source and adaptive calculation of diffuse interreflection using Monte Carlo ray-tracing. It's also used caching mechanism for reuse pre rendered radiance in the successive frames. Monte Carlo method is inefficient in most cases and Eric Veach<sup>13</sup> proposed a metropolis sampling method incorporated with Monte Carlo Ray Tracing. Metropolis method generates a sequence of light path and randomly mutates a single current path, and each mutation is accepted or reject by a probability function. To render a participating media Eric Lafortune and Yves D. Willems<sup>14</sup> suggested a technique integrating light shooting and gathering for participating media. Henrik Wann Jensen and Per H. Christensen<sup>15</sup> introduced a photon map algorithm to simulate light transport in scenes. It's also used bi-directional Monte Carlo ray tracing with photon maps to increase the efficiency and reduced the noise.

In the following chapters, we will describe fundamental issues in global illumination and compare various techniques in ray tracing global illumination methods. We will mainly choose the hem-cube method, two-pass method, extended two pass method for the

fundamental issues, and compare Kaija's path tracing method, Radiance method, Metropolis light tracing, bi-directional light tracing and photon map method.

## Definition and Solutions

Rendering equation suggested by Kaija set a concrete framework to solve the global illumination problem.

$$I(x, x') = g(x, x') \left[ \mathcal{E}(x, x') + \int_s \rho(x, x', x'') I(x', x'') dx'' \right]$$

The transport intensity light from one surface point to another is simply the sum of emitted light and the total light intensity which is scattered toward viewing point from all other surface points.

To solve this equation we should take into account the reflectance model of each surface patch. So the analysis of BRDF should be accomplished. BRDF is the abbreviation of bi-directional reflectance distribution function.

$$f_r(w', x, w) = \frac{w(w', x, w)}{\cos \theta}$$

The main characteristics of BRDF is positive, symmetric and energy conserving. To solve the problem practically, most works have divided the surface reflectance to Diffuse and specular component. Later, people also pay attention to the light transporting among non-uniform media.

Radiosity and Ray tracing algorithms exploited in global illumination can be measured with their solution to the surface BRDF. We can classify the major solutions into a table illustrating the capability of each method dealing with BRDF and its complexity.

Category	BRDF and its complexity	Solutions
Radiosity	Diffuse only	Hemi-cube Radiosity Progressive Radiosity Analytical form-factor method
Radiosity and ray tracing	Diffuse and specular (planar surface only) and single pass	Two-pass method
	Diffuse and specular (non-planar surface) multi pass	General Two-pass method
Ray tracing (for solving the diffuse component, various techniques are suggested)	Forward statistic ray tracing enables caustic	Kajia's Path tracing
	Diffuse and specular (non-planar surface) deterministic forward ray tracing	Radiance method
	Forward and backward combination	Bi-directional path tracing
	Path mutation in ray tracing	Metropolis light transfer method
	Bi-directional Photon tracing	Photon map method

However, these solutions have common interest in some points. One major thing is important sampling. To solve a problem using iteration or numerical integration, the variables with large value should be considered first. It is a kind of importance sampling. The other thing is error level adjustment. For the radiosity method, it presented as hierarchical or meshing techniques. In ray tracing, it is represented as ray path level and ray sampling rates.

We will discuss these solutions in some detail.

# Radiosity Method

## Hemi-cube method

Hemi-cube method, in terms of rendering equation, is simplification of light transfer model as directionally invariant diffuse transport. So the equation can be written as

$$B_i dA = E_i dA_i + \rho_i \sum_j B_j dA_j F_{dA_j dA_i}$$

For the symmetric of light transfer model the relationship of patches can be expressed as

$$F_{ij} A_i = F_{ji} A_j$$

$$F_{ij} = F_{ji} \frac{A_j}{A_i}$$

So for n patches, it is an n simultaneous equation with n unknowns.

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \dots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \dots & -\rho_2 F_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \dots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

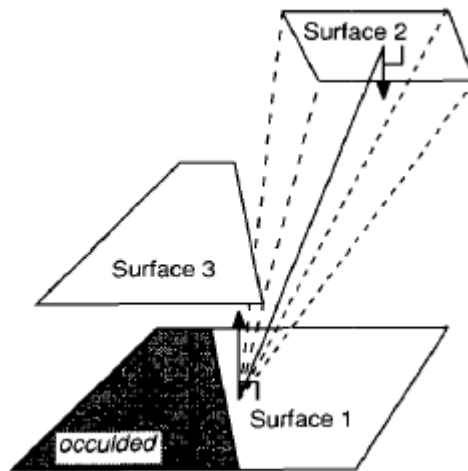
The form-factor between two patches an expression of radiance exchange between them.

In hemi-cube method form factor is calculated by projecting other patches to the surface of a centered patch.

The hemi-cube method has some limitations in Anti-aliasing. The patches are projected into the uniform pixels of the hemi-cube. So the distance between the center of cube and the projected patches are very important to calculating form factor. If the patches are close to the heme-cube, the aliasing caused by the variance of the distance is quite obvious. For the large area light source, insufficient subdivision also causes aliasing.



To render a geometric object, we should know the radiance of the vertex to do some interpolation among the polygons. But the vertex radiance should be extrapolated from the radiance of nearby patches. It may lead to visual discontinuity in curved surface. The most important disadvantage in radiosity method is intervening problem. The projected patches are projected into one point in the other patch.



### Ray tracing Form factor

In order to solve the problems in hemi-cube method, Wallace suggested a ray tracing method to calculate form factor. It's a kind of light source to vertex calculation.

The aliasing due to the inadequate sampling of the surface patch is overcome by adaptively increasing the ray samples. For large light source, it can also be subdivided into smaller patch and the form factor is the accumulation of the smaller patch form factor. This will eliminate the intervene problem in hem-cube method.

There is a problem, how many ray samplings should be cast to accurate calculation of form factor? The residual error could be used to measure the accuracy of the form factor.

$$|F_n - F_{n+1}| < \epsilon$$

The  $F_n$  is the form factor using  $n$  samples and  $F_{n+1}$  is using  $n+1$  samples. If the consecutive form factors are close enough, there is no need to shoot more rays.

$$F_{A_s dA_v} = dA_v \frac{1}{n} \sum_{i=1}^n H_i \frac{\cos \phi_{vi} \cos \phi_{si}}{\pi r^2 + A_s / n}$$

$$B_v = \rho_v B_s A_s \frac{1}{n} \sum_{i=1}^n H_i \frac{\cos \phi_{vi} \cos \phi_{si}}{\pi r^2 + A_s / n}$$

The equation for this method is described as below. Subdivision of source patch will eliminate the intervening problem.

The ray tracing method has resolved most of the problems embedded in Radiosity method. However, the ray tracing method to calculating form factor is a quite expensive method.

### Hybrid Form Factors

Hybrid method to Calculate form factor is to maintain the efficiency in hemi-cube method and calculate it analytically.

$$F_{ij} = \frac{1}{A_j} \int_{A_i} \int_{A_j} \frac{\cos \phi_i \cos \phi_j}{\pi r^2} dA_j dA_i$$

Instead of projecting surface patch to the hemi-cube and count the number of pixels. It calculates each element form factor and takes the weighted average to get the form factor. The analytic form factor calculation only done when the distance falls in some certain criteria. The intervening problems solved by subdivide the projecting patch until it is fully visible or invisible.

# Combined Radiosity and Ray Tracing method

## Two-path method

Neither radiosity nor ray tracing method can deal with global illumination completely.

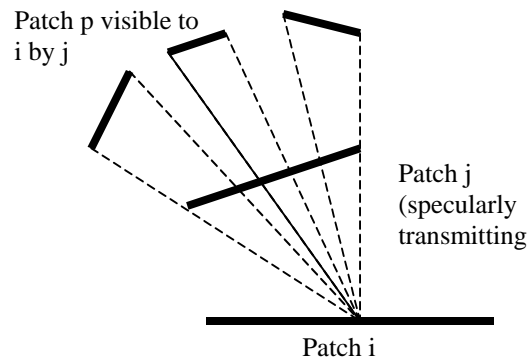
For the radiosity, it can gracefully solve the diffuse interreflection. But if you want to solve specular reflection using radiosity method, because the specular effect will be very little part of surface patch, and you have to subdivide the patch into very small part, in worst case even to the pixel level. It will be degraded into pure Monte Carlo ray tracing.

For the ray tracing method, it handles specular reflections quite well, however, to calculate diffuse term, a lot of ray samples are to be shot to calculate the diffuse reflection factor. Wallace suggested a method combining radiosity and ray tracing method. They used enhanced radiosity method to handle view independent diffuse to diffuse and specular to diffuse reflection. These parts are preprocessing part, and the specular component is calculated only to the necessary extent for correct calculation of diffuse component. The specular to specular and diffuse to specular reflection, they used the ray tracing method.

In radiosity part, they extend the form factor to include the translucency.

$$I_i = I_{e,i} + \rho_{d,i} \sum_{j=1}^n \{I_j F_{ij} + \tau_{s,j} \sum_{p=1}^n I_p T_{f,ijp}\} + \tau_{d,i} \sum_{j=1}^n \{I_j T_{ij} + \tau_{s,j} \sum_{p=1}^n I_p T_{b,ijp}\}$$

In implementing translucency, the used backward hemisphere to calculate form factor for transmitting energy.



For the specular to diffuse reflection, they suggested a virtual light source method. But the reflection is restricted to planar mirror. At the reflection position generate a virtual patch and calculate the form factor.

To deal with specular to specular and specular to diffuse reflection, ray-tracing method is used. To coincident with form factor method, they constructed a square pyramid for the specific out going direction and truncate the BRDF into square pyramid and sampling at this square pyramid. A low-resolution Z-buffer algorithm used to catch the surface element projected into this specular pyramid. And used the intensity of the pixels to calculate specular component. For the depth of recursion, reduce the resolution of patch and do the successive bounce. To reduce the aliasing caused by uniform sampling in specular pyramid. They adopted some stochastic method. Randomly rotate the specular pyramid and get the anti-aliased result.

However the Two-pass method only support planar surface reflection and did not included refraction. The virtual patch generation has some limitation in recursive reflection.

## General Two-pass method

General two-pass method suggested an extended form factor calculation method. The form factor included the portion of energy after any number of specular reflection and refraction.

$$I(x, x') = g(x, x') \left\{ \epsilon(x') + \rho_d(x') \int_s I(x', x'') dx'' \right\} + g(x, x') \int_s \rho_s(x, x', x'') I(x', x'') dx''$$

Arbitrary paths and complexity are included in these extended form factors and ray-tracing method generates these extended form factors. The extended form factor compared with two-pass method included any number of specular reflections, and that gives the generality.

After calculating extended form factor, they used standard radiosity method to calculate diffuse to diffuse reflection. And in second path, they used standard ray tracing method.

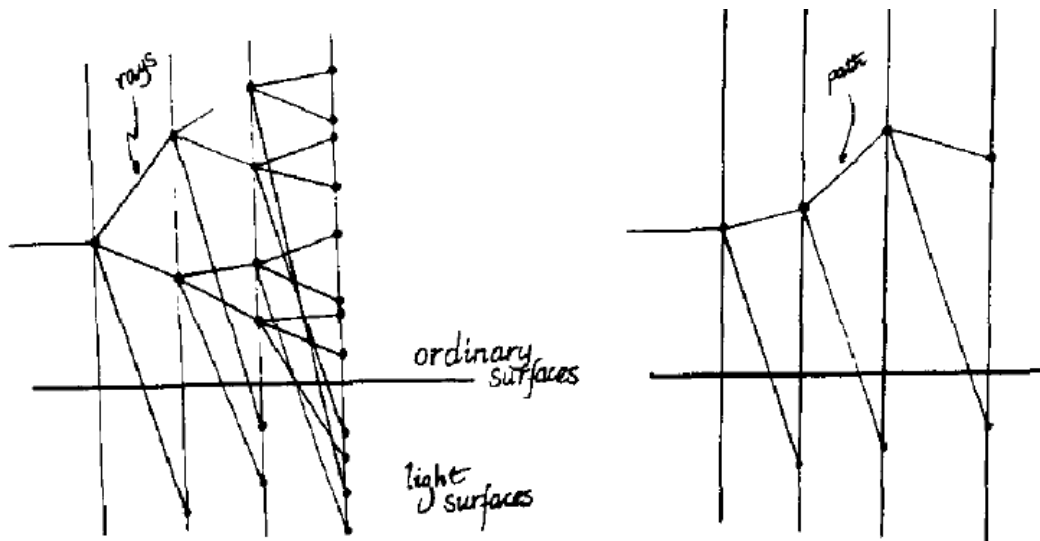
They also extended the hemi-cube sampling to an alternative way. By project the hemi-cube to a planar rectangular region and adaptively subdivide the planar region. Use this they calculate elementary form factors and extended form factors. Elementary form factor is like the traditional radiosity form factor. And the extended part is calculated by subdividing the region until the ray tracing tree matches. And propagate the elementary form factor already calculated to the each patch in the ray tree.

To efficiently simulate view dependent mirror-like perfect specular reflection, they used mirror much bigger than actual mirror, and calculate the successive specular reflection using pre-computed mirror image.

# Ray Tracing method

## Path-tracing method

Kajia's ray tracing algorithm is a forward ray tracing, which is tracing ray from pixel to light source. There will be infinite number of ray path generated, instead of calculating all the paths, path-tracing method estimated the integral using intellectually selecting some paths and weighted average them to approximate the integral.



In order to accurately estimate the pixel value using some of the ray path, the paths is selected to minimize the variance of the pixel value. Kajia's path tracing is a non-uniform importance sampling of the ray path. The first path of the ray is the most important radiance term of final integral. The method of selecting or distribute ray is "Hierarchical integration". It keeps track of ray types closely match the desired distribution by varying the probability of each type to match the distribution. And randomly scale the contribution of each type of ray to generate the weighted sample distribution.

However, when the importance sampling criteria does not meet, the path tracing can not provide enough accurate estimation of the integral, and will be degraded into the pure Monte Carlo ray tracing.

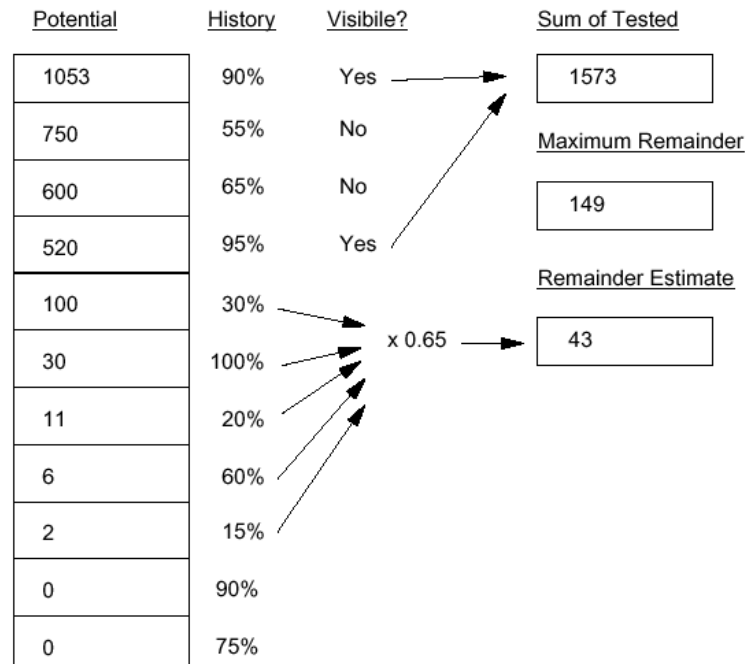
### **Radiance Method (collection of methods)**

Radiance method was expressed as collection of methods. It is because that Radiance uses direct shadow test and adaptive and statistical light source sampling. It's also uses virtual light source generation like original two-pass method and added secondary light source. They also used ochre node o caching the indirect illumination to reuse. In using Monte Carlo ray tracing to get the diffuse to diffuse radiance, the recursive evaluation only occurs at sampling point. Participating media is approximated at certain level.

Direct shadow test:

Forward ray tracing method is used to calculate direct illumination of pixels in scene. It's equal to the first path in path-tracing method. Assume there is a lot of light sources in the space, for every pixel, calculate the entire light source will cost a lot of time. So, statistical method is used to reduce the time spent on minor light sources. If the remaining total intensity of the light sources are less than 10% of already calculated radiance. Use a statistic value multiplied by the total remaining intensity and add it to pixel value. Furthermore, they even reduced time to look into whole remaining light sources using certainty value. If the certainty value is 1, using above statistic method and if it is 0 only look at next largest light source. The light sources are sorted and have their visibility test.

Light source subdivision:



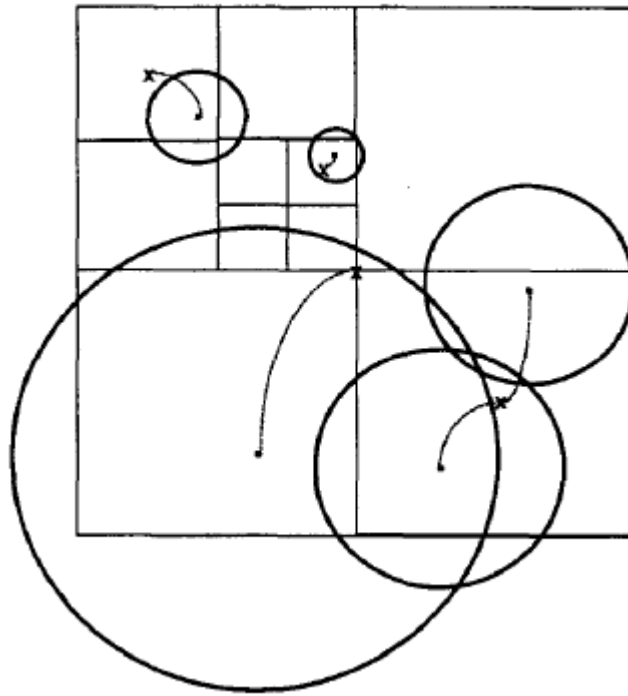
Like radiosity and ray tracing method, the light sources are subdivided enough to eliminate the aliasing or insufficient illumination.

Radiance methods also provide “second light source”. It’s an area of diffuse surface patch with high intensity. Two-pass method provide “virtual light source”, which is highly specular reflection patch. Non of the ray tracing method specially consider high intensity diffuse area. Using forward ray tracing, it will takes long time to get the intensity value of these areas.

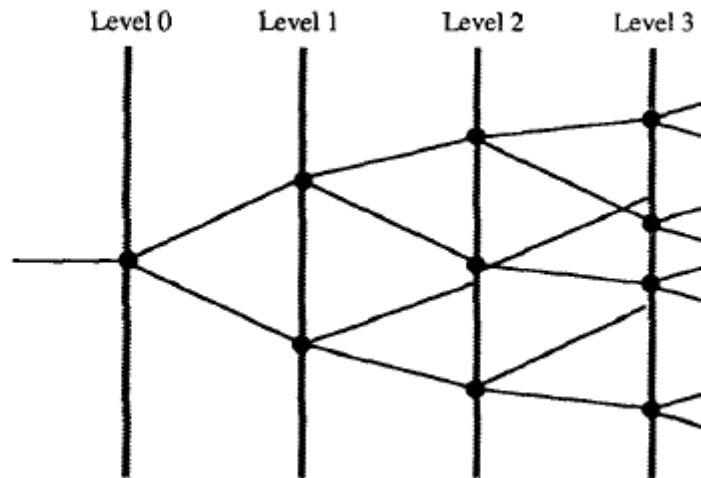
After calculating direct illumination, radiance calculates indirect specular and diffuse illumination. In non-perfectly specular surface sampling in certain weight in certain direction. There is only one ray in primary BRDF’s specular direction



Hierarchical octree is used to caching the indirect radiance value. And each node contains the calculated radiance value. The sampling point subdivides the octree nodes by the variance. The points need to calculate indirect radiance is based on the variance of gradient with consideration of rotational and translation variance. And with the point of geometrical discontinuity and rotational variance, also need to calculate new indirect diffuse intensity. Other points values near the calculated points are interpolated either using linear or cubic interpolation.



However, the recursive calculation only occurs at the sampling points. So it will reduce much of minor rays to be calculated. For the sampling point is limited to some bound the recursive calculation also bound in some limit.



The main contribution of radiance method is caching and interpolation scheme in calculating indirect diffuse component. And the direct and adaptive statistical calculation of shadow rays (direct component). The sampling rays are reduced by calculating variance of the gradient.

The restrictions of radiance method are unable to calculate non-planar specular surface and specular rays are simulated using only one ray and filtering to eliminate the aliasing.

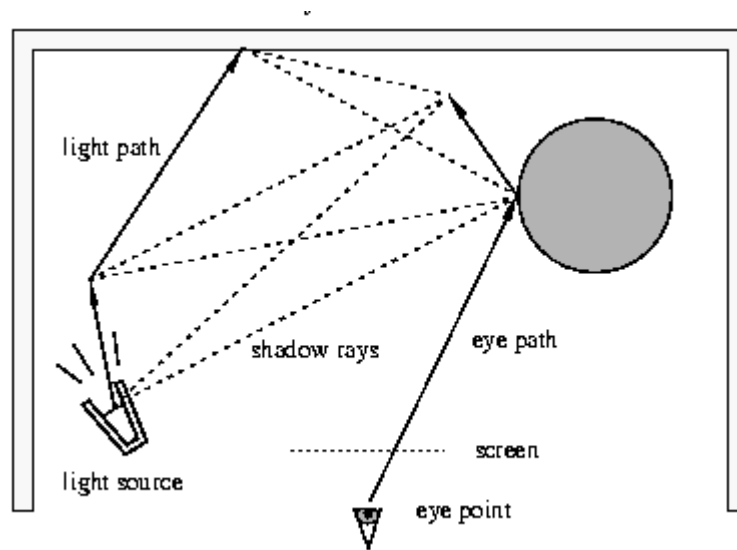
The adaptive statistical shadow ray test will degrade to the pure Monte Carlo method, if the light sources have the similar intensity.

### **Bi-directional ray tracing**

The bi-directional path tracing method integrates previous light shooting and light gathering techniques in global illumination. It combines the idea of path tracing and light tracing by creating random walks. Starting not only from the eye point but also from light sources. Eye path start from the eye point, through the pixel that being computed, as in classical path tracing. Light paths start from the sample point of light source in sample

direction. The rays can be scattered or absorbed in the medium as well as being reflected or absorbed by the surface.

This algorithm tries to optimize the sampling process at light source by using adaptive Probability density functions. Thus the potential function is actually used to guide the sampling process.



Bi-directional light tracing algorithm is a stochastic rendering algorithm based on particle tracing using adaptive PDFs. And thus, all kinds of light transport are handled in a uniform way, no mesh is required, all light paths are generated correctly, and particles are shot in directions with high potential capability.

However this algorithm is view-dependent, and the image quality is strongly dependent on the number of particles being shot. And high frequency details such as reflection, refraction and caustic are difficult to render accurately.

Bi-directional path tracing consider two parts: the paths are carrying relatively large amount of lights. And they have large weights in measurement of final image. From light source and eye position generate random bounce light paths. And these two independent

paths are concatenated to generate whole light path. For efficiency, generate maximum length eye path and maximum length light path and joining each prefix with suffix of light path using probability estimation value. The visibility test can be optimized using Russian roulette to randomly suppress small potential contribution without adding bias.

The weighting of each sample can be estimated using power heuristics. With importance sampling, the weight could be 1, and samplings can be used power heuristics to estimate. There is some work to be done in considering the sampling method. And high frequency details such as reflection, refraction and caustic are difficult to render accurately.

### **Metropolis light tracing**

Metropolis light tracing algorithm generates a sequence of light transport paths by randomly mutating a single current path. For example, adding a new vertex to the path. Each mutation is accepted or rejected with a carefully chosen probability, to ensure that paths are sampled according to the contribution they make to the ideal image.

The path space is explored locally, and the average cost per sample is small. Once the important path is found, the nearby paths are explored as well, and the mutation set is easily extended.

However some deterministic methods are also used in direct lighting. And for the importance sampling, the average acceptance value is high for importance sampling direction.

Firstly, generate initial path, and calculate the luminance of the path. Importance value is estimation using calculated luminance. Add normally estimated (tone mapping or normal rendering) multiplied by luminance of the path and the importance of the path.

Then mutate path to another path. Also calculate luminance and the corresponding importance. If the importance value is 0, reject this path and generate a new path. If path is accepted, calculate new importance value and add it to pixel value. And start from mutated path and start new mutation.

Metropolis light transform is very effective in strange lighting condition. Where light sources are extremely expensive to reach using standard Monte Carlo method. And the mutation condition is simple and generic. Metropolis light transform is good at estimating view-independent part, and view dependent part using some standard method.

However, it is a kind of statistical method and the parameters are needed to be studied for the efficiency.

### **Photon map method**

Photon maps makes it possible to efficiently compute global illumination including caustics, diffuse color bleeding, and participating media. Photon maps can be used in scenes containing many complex objects of general type. The method is capable of handling advanced material descriptions based on a mixture of specular, diffuse, and non-diffuse components.

Photon map is extension of bi-directional path tracing method. However, they used some techniques to searching and storing light path efficiently. Unlike other methods, it used and estimates object scene with their variance in photon map value, and adaptively generate more photons or just interpolate them.

It is an unbiased method tracing with light source, and the photon numbers are normalized with all light sources, so that even the intensity and light source numbers are increased, the photon numbers are remain statistically equal.

The bi-directional path tracing and metropolis methods are probabilistic method. But the photon map is deterministic in some way. It separates the specular, diffuse and caustic part and shoot photons in different manner.

Photons are stored in k-d tree, and it is very efficient in searching. The k-d trees are balanced for searching efficiency. Some modifications are added to k-d tree to efficient tracing in neighbor photons. For the radiance value is interpolate using photons. And if the neighbor photons have great difference, add new node to k-d tree. Photons are stored in node, if it is diffuse or caustic then store photon value, if specular reflect the photons until it is absorbed. Global photon map is generated once for certain scene.

In visualization part, using distributed ray-tracing algorithm in final rendering. The reason is directly visualize global photon map is very time-consuming. In final rendering, in-direct illumination using importance sampling and distributed tracing to access the photon maps. Interpolated photon values need to be filtered to get the smooth image.

Monte Carlo ray tracing, bi-directional light tracing and metropolis can simulate global illumination with low memory consume. The cost of photon map is the storage space for global photon maps. But it is very efficient in rendering. The statistical part only lies in distributed ray tracing part. The photon map algorithm is much faster than those probability algorithms and less noticeable to the high frequency noises. And it can render specular, caustic and participating media. Another advantage is that the photon map is free-source and anyone can add it to his ray-tracing program.

## Conclusions

This paper surveys on the global illumination using radiosity and ray tracing. From the original radiosity method to photon map and other statistical path tracing method.

Earlier works in global illumination is focus on global diffuse phenomena. Which is considered as ambient term in local illumination model. Some works has been done to revise the radiosity method, like form factor calculation and progressive method.

Later, some combination of ray tracing and radiosity method to simulate specular reflection and refraction. Further, in order to render caustic and participating media, more sophisticated ray tracing algorithms are suggested. Recently, There is some works done in the area of interactive and instance global illumination.

These days, there is not much paper presented in global illumination in SIGGRAPH. For the reason that many methods have done to accurately rendering global illumination scenes. However, it is not likely to get a real-time rendering or semi-real time rendering of global illumination. The image-based rendering is a trend which says, it is very expensive to render global illumination, why not use photo images it simulate the global scene.

The future works will be in the area of image-based rendering to simulate more accurately rendering of the image, and the interactive techniques in global illumination and other kind of statistical method using ray-tracing method. With the hardware development, there will be some algorithms using ray tracing benefit from hardware related works.

# References

- 
- <sup>1</sup> M.F.Cohen, D.P.GreenBurg “The Hemi-cube: A Radiosity solution for complex it is hard to achieve environments”, Proceedings of SIGGRAPH85 in Computer Graphics Volume 19 No3 pp 31-40 1985
- <sup>2</sup> Kajia, James T., “The Rendering Equation” Proceedings of SIGGRAPH86 In Computer Graphics, Volume 20 No 4 Aug 1986 pp.143-150
- <sup>3</sup> Wallace, John R.,Michael Cohen and Donald P. GreenBurg “ A Ray Tracing Algorithm For Progressive Radiosity” Proceedings of SIGGRAPH87 In Computer Graphics, Volume 21 No 4 Jul 1987 pp.311-320
- <sup>4</sup> Daniel R. Baum, Holly E. Rushmeier, and James M. Winget “Improving Radiosity Solutions Through the use of Analytically Determined Form-Factors” Proceedings of SIGGRAPH89 In Computer Graphics, Volume 23 No 3 Jul 1989 pp.325-334
- <sup>5</sup>Wallace John R, Michael Cohen, Donald P GreenBurg “A Two Pass Solution To The Rendering Equation: A Synthetic Of Ray Tracing and Radiosity method” Proceedings of SIGGRAPH87 In Computer Graphics, Volume 21 No 4 Jul 1987 pp.311-320
- <sup>6</sup> Francois Sillion, Claude Puech “ A General Two-Pass method Integrating Specular and Diffuse Reflection” Proceedings of SIGGRAPH89 In Computer Graphics, Volume 23 No 3 Jul 1989 pp.335-344
- <sup>7</sup> Michael F. Cohen, Shenchang Eric Chen, John F. Wallace, Donald P. GreenBurg “A progressive Refinement Approach to Fast Radiosity Image Generation” Proceedings of SIGGRAPH88 In Computer Graphics, Volume 22 No 4 Jul 1988 pp.75-84
- <sup>8</sup> Shenchang Eric Chen “Incremental Radiosity: An Extension of Progressive Radiosity to an Interactive Image Synthesis System” Proceedings of SIGGRAPH90 In Computer Graphics, Volume 24 No 4 Jul 1990 pp.135-144
- <sup>9</sup> George Dretakis and Francois X. Sillion “Interactive Update of Global Illumination using A Line-space Hierarchy” Proceedings of SIGGRAPH97 pp.57-64
- <sup>10</sup> Daniel R.Baum, Stephen Mann, Kevin P. Smith, and James M. Winget “Making Radiosity Usable: Automatic Preprocessing and Meshing Techniques for the Generation of Accurate Radiosity Solutions” Proceedings of SIGGRAPH91 In Computer Graphics, Volume 25 No 4 Jul 1991 pp.51-60
- <sup>11</sup> Pat Hanrahan, David Salzman,Larry Aupperle “A Rapid Hierarchical Radiosity Algorithm” Proceedings of SIGGRAPH91 In Computer Graphics, Volume 25 No 4 Jul 1991 pp.197-206
- <sup>12</sup> Gregory J.Ward, Francis M.Rubinstein, Robert D Clear “ A Ray Racing Solution for Diffuse Interreflection” Proceedings of SIGGRAPH88 In Computer Graphics, Volume 22 No 4 Jul 1988 pp.85-92
- <sup>13</sup> Eric Veach, Leonidas J. Guibas “Metropolis Light Transport” Proceedings of SIGGRAPH97 In Computer Graphics, Jul 1997 pp.65-76
- <sup>14</sup> Eric P.Lafortne and Yves D.Willems “Rendering Participating Media with Bidirectional Path Tracing” 7<sup>th</sup> Eurographics workshop on Rendering
- <sup>15</sup> Henrik wann Jenson,Per H. Christensen “Efficient Simulation of Light Transport in scences with participating Media using photon maps” Proceedings of SIGGRAPH97 In Computer Graphics, Jul 1998 pp.65-76