University of São Paulo - USP
Institute of Physics | Department of General Physics

Master's Project

Quantification of the Impact of Polarization Leakage on BINGO Cosmological Observables

Aluno: Gabriel Silva Costa
Advisor: Élcio Abdalla

03/2024

1 Abstract

The use of the 21 cm line of neutral hydrogen (HI) emerges as a powerful and complementary source of information for describing the Universe on large scales, as well as aiding in constraining explanations for the accelerated expansion of the Universe. However, the observation of such a weak signal is done indirectly through the integrated signal collected, which requires precise instrumental control over various contaminating and calibration aspects. Among these contaminations, an important source is through polarization leakage—an instrumental effect that inserts spurious signals into the total intensity parameter (Stokes I), making it difficult to separate the cosmological signal from astrophysical foreground emissions. This limitation represents a critical obstacle for experiments that use the 21 cm signal, such as the Brazilian-led radio telescope BINGO (BAO from Integrated Neutral Gas Observations), designed to detect baryon acoustic oscillations (BAO) through neutral hydrogen emission. This project seeks to address, estimate, and characterize polarization leakage in BINGO observations, considering its optical design. To achieve this goal, computational simulations will be performed using the GRASP software, which models the propagation of electromagnetic radiation in complex optical systems. The analysis will include the calculation of polarization parameters (Stokes parameters) and the determination of the Mueller matrix associated with the BINGO optical system, which quantifies how different polarization states transform as they pass through it. The expected results include a detailed understanding of the origin and magnitude of the polarization leakage, as well as the development of strategies for its control or mitigation. These advances will contribute to the robustness of the cosmological observations performed by the BINGO radio telescope, strengthening its ability to explore the large-scale Universe.

2 Introduction

The integrated neutral hydrogen (HI) signal through the emission of the characteristic 21 cm line emerges as a crucial resource for large-scale cosmological studies, with more than two decades of research highlighting its value (Furlanetto; Oh; Briggs, 2006). Recent advances in radio telescope technology, particularly the construction of BINGO, emphasize its use in the detection of the characteristic 21 cm HI line (Abdalla et al., 2021). This line allows for the measurement of the matter distribution in the Universe, providing *insights* into its evolution, dynamics, and composition, complementing traditional techniques such as galaxy distribution (Prat, J. et al., 2018), cosmic microwave background (CMB) (Bass, 2010), gravitational lensing (Meneghetti et al., 2020), and supernovae (Riess et al., 1998).

The BINGO radio telescope, under construction in the Brazilian hinterland, will focus on the detection of 21 cm signals from the post-reionization epoch ($z < 7$). In this cosmological period, the remnant neutral hydrogen is predominantly present in dark matter halos (Villaescusa-Navarro, Francisco et al., 2018). This reinforces the utility of neutral hydrogen (HI) as a tracer of dark matter on large scales, allowing for the investigation of cosmological dark energy and dark matter.

However, the weak 21 cm signal relative to other sky emissions makes its detection challenging. This requires exceptionally stable observations and rigorous control over contaminants and systematic effects (Chapman et al., 2012). Sources of *foregrounds* of galactic and extragalactic origin, for example, are four to five orders of magnitude stronger than the post-reionization 21 cm signals. These *foregrounds* are spectrally smooth, in contrast to the Gaussian-like spectral behavior of the 21 cm signal, allowing for their effective modeling and subtraction from the observed data (Abdalla et al., 2021).

Another significant source of contamination comes from instrumental effects, particularly calibration errors, which impede the acquisition of precise data (Kumar; Dutta; Roy, 2020). Although noise sources, such as white noise and $1/f$ noise, are well-known and analyzed (Vernotte, F; Lantz, E, 2015; Paladino, E. et al., 2014), polarization leakage suffers from the imprecise modeling of polarized foreground emissions in the 21 cm frequency regime (Carucci; Irfan; Bobin, 2020).

Currently, 21 cm work for single-dish telescopes—such as BINGO, SKA, MeerKAT, and FAST—that deal with polarization contamination either use the CRIME code (Alonso; Ferreira; Santos, 2014) based on Faraday rotation, or assume a linear model (Gao, Li-Yang et al., 2023). Furthermore, these studies also concentrate on analyses of the U and Q polarizations, assuming that V is negligible. However, this assumption is not true for the BINGO telescope (Abdalla, F. et al., 2022), where $V \gg Q, U$ is observed.

This leakage refers to the unintentional detection of polarized signal components in unpolarized modes due to optical imperfections, misalignments, or non-linearities in detection systems (Ansah-Narh et al., 2018). The problem of polarization leakage is particularly critical in Intensity Mapping techniques, where the precise subtraction of foregrounds is essential due to the relative weakness of the 21 cm signal (Battye et al., 2013). Minimizing polarization leakage is crucial to prevent galactic and extragalactic foregrounds from obscuring the faint cosmological signal. Furthermore, the non-smooth, spectral nature of the cosmological signal adds an extra difficulty, preventing removal by conventional methods used for the galactic and extragalactic sources (Emma Chapman et al., 2016).

Thus, the main objective of this study is to estimate and characterize polarization leakage by calculating the **Stokes parameters** and the **Mueller matrix** (Nunhokee et al., 2017) in the context of BINGO's 21 cm observations. The results will provide *insights* important for the understanding and mitigation of polarization leakage effects in Intensity Mapping, increasing the precision and reliability of the data from BINGO and similar telescopes.

3 Objectives

The procurement of the Mueller matrix (Objective 1) is the first fundamental step for the instrumental characterization of the BINGO radio telescope. This matrix, which describes the electromagnetic interaction of the target radiation with the optical system, is generated using the GRASP software, which already incorporates the telescope's optical design (Abdalla, F. et al., 2022). However, to assess the real impact of this matrix on scientific observations, it is essential to develop a functional model. To do this, we will decompose the beams generated by GRASP into Zernike polynomials (Objective 2). This approach is more suitable because it accounts for the asymmetries of the beams from the horns positioned further away from the focal plane center. These Zernike models will act as an "instrumental response beam" which, unlike an ideal Gaussian beam, will carry the signature of polarization leakage.

Subsequently, the generated beams decomposed into Zernike polynomials will be integrated into the BINGO pipeline, through the adaptation of the HIDE&SEEK software[1]. This software will perform the convolution of the beams with portions of the sky during the scanning process. The sky will be simulated by combining 21 cm cosmological emissions with galactic and extragalactic emissions in the telescope's operational band (980-1260 MHz) and in specific channels (Objective 3).

The final step of the analysis will consist of comparing the angular power spectrum (C_ℓ) of these contaminated maps with the spectrum of ideal maps. This comparison will allow us to robustly quantify the magnitude of the systematic error introduced by polarization leakage across the angular scales relevant to BINGO. Additionally, we will compare the results obtained with those that would be achieved if we used models generated by the CRIME software[2] and by the linear approximation commonly assumed by the scientific community (Objective 4).

4 Work Plan and Schedule of Activities

1. Bibliographic survey and literature review (2 months):

    a. Comprehensive review of literature related to the observation of the 21cm neutral hydrogen line and the challenges associated with polarization leakage in radio telescopes;

    b. Identification of previous studies on modeling and analysis methods for polarization leakage in similar telescopes;

2. Familiarization with the Ticra-GRASP software (3 months):

    a. Study of the GRASP package and its functionalities for modeling radiation propagation and polarization in astronomical optical systems;

    b. Performance of preliminary simulations to understand the influence of telescope parameters on polarization leakage;

3. Data processing and analysis (10 months):

    a. Production of BINGO radio telescope beam simulations, using a Gaussian beam for a single frequency and six different polarizations (linear_x, linear_y, linear_45°, linear_−45°, circular_rh, circular_lh);

    b. Derivation of the Mueller matrix from the manipulation of Stokes parameters obtained with the simulated beams;

    c. Modeling of the Mueller matrix elements using Zernike polynomials for generating sky intensity maps;

    d. Production of sky intensity maps using the Zernike models.

4. Analysis and interpretation of the generated maps (3 months):

[1] https://cosmology.ethz.ch/research/software-lab/hide—seek.html

[2] http://intensitymapping.physics.ox.uk/CRIME.html

a. Comparison of the generated maps and analysis of their spectra;

b. Comparison with the results from models generated by the CRIME software and by the linear approximation;

c. Identification of patterns and correlations that indicate the presence and magnitude of polarization leakage.

5. Discussion of results (2 months):

   a. Discussion of the results in light of existing literature and comparison with previous studies on polarization leakage in radio telescopes;

6. Drafting of the final report and dissemination of results (3 months):

   a. Elaboration of the final research project report, documenting all work stages, results obtained, and conclusions reached;

   b. Presentation of the results at scientific conferences and workshops for sharing with the academic community.

5 Methodology

5.1 Data Set

Assuming the telescope configuration, the GRASP software[3] produces radiation field data and their distributions collected at each focal plane location, saving them in text files. The BINGO telescope configuration is already implemented in the software, as described in (Abdalla, F. et al., 2022), as an off-axis crossed-Dragone optical design, in which the center of the focal plane is not aligned with the main axis of the primary mirror. Such a configuration was designed and optimized to exhibit low levels of spillover and sidelobes, as well as low cross-polarization, as presented in the previously mentioned work.

GRASP propagates a monochromatic, linearly polarized beam through the optical system, providing the response as a two-dimensional complex electric field in $u - v$ coordinates. This data is transformed from Cartesian coordinates to celestial coordinates (RA, DEC), and a linear interpolation is performed. GRASP propagates a monochromatic, Gaussian, and linearly polarized beam through the optical system and provides the result as a two-dimensional complex electric field in $u - v$ coordinates on the surface of a sphere. However, for this analysis, the results are presented in celestial coordinates (RA, DEC), chosen at a specific time during the survey (defined as the moment when RA $= 0$ is located directly to the south), considering that BINGO operates as a transit telescope.

To achieve this, a linear interpolation is applied based on the sequence of transformations described below. Initially, the transformation from (u, v) coordinates to standard Cartesian coordinates (x, y, z) on the unit sphere is performed:

[3] https://www.ticra.com/software/grasp/

$$x = u \tag{1}$$
$$y = v \tag{2}$$
$$z = \sqrt{1 - u^2 - v^2} \tag{3}$$

The $u - v$ coordinates represent a plane tangent to the sphere at the center of the beam, which is at $x = y = 0$ and $z = 1$ in this coordinate system. To convert to celestial coordinates, a new Cartesian system is defined, where the z-axis points towards the celestial North Pole. The optical system is rotated to position the center of the focal plane at a declination of $-15°$[4]. To center the pointing at a declination of $-15°$, a rotation of $105°$ (i.e., $90° + 15°$) is performed from the celestial pole, the point where GRASP was configured to generate the beam:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{bmatrix} cos(105°) & 0 & sin(105°) \\ 0 & 1 & 0 \\ -sin(105°) & 0 & cos(105°) \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{4}$$

The celestial coordinates are then defined directly.

$$\mathrm{DEC} = \arcsin(z'), \tag{5}$$
$$\mathrm{RA} = \arctan 2(y', x') = 2 \arctan \left(\frac{y'}{\sqrt{x'^2 + y'^2} + x'} \right). \tag{6}$$

The intensity I (in watts) is calculated as the sum of the squares of the two orthogonal components of the electric field.

$$I = |E_{co}|^2 + |E_{cx}|^2. \tag{7}$$

Here, E_{co} is the electric field measured in the x direction, and E_{cx} is the electric field measured in the y direction. For the analysis of a single horn, the number of points N_p used for the field outputs in the GRASP software was set to 512×512 within a $u - v$ range of -0.05 to 0.05, over a frequency range of 980 MHz to 1280 MHz, values that can be easily varied and adapted according to the demands of the analyses.

5.2 Stokes Parameters

The polarization state of electromagnetic radiation can be described by a set of values called Stokes parameters (Trippe, 2014). The four Stokes parameters, I, Q, U, and V, of the incident beam follow the definition:

$$I = |E_{co}|^2 + |E_{cx}|^2 = |E_a|^2 + |E_b|^2 = |E_{rhc}|^2 + |E_{lhc}|^2 \tag{8}$$
$$Q = |E_{co}|^2 - |E_{cx}|^2 \tag{9}$$
$$U = |E_a|^2 - |E_b|^2 \tag{10}$$
$$V = |E_{rhc}|^2 - |E_{lhc}|^2 \tag{11}$$

[4] As presented in (Abdalla, F. et al., 2022), an additional rotation of the optical system, on the order of 7 degrees, is necessary to compensate for the radio telescope site's location, i.e., to compensate for the telescope's latitude.

The subscripts correspond to three different bases in the Jones vector space. With (E_{co}, E_{cx}) representing the standard Cartesian base defined above, E_a and E_b are defined by rotating the Cartesian base used for (E_{co}, E_{cx}) by 45°, while E_{rhc} and E_{lhc} correspond to the right- and left-hand circular polarization bases, defined as:

$$E_{a/b} = \frac{1}{\sqrt{2}}(\pm E_{co} + E_{cx}) \tag{12}$$

$$E_{lhc/rhc} = \frac{1}{\sqrt{2}}(E_{co} \pm iE_{cx}) \tag{13}$$

The four Stokes parameters represent different aspects of polarization:

- I (Total intensity): Represents the total intensity of the light, regardless of its polarization. It is the sum of the intensities of all polarization components.
- Q (Linear polarization on one axis): Measures the difference in intensity between light polarized along the horizontal and vertical axes.
- U (Linear polarization on another axis): Measures the difference in intensity between light polarized at $+45°$ and $-45°$.
- V (Circular polarization): Measures the difference between right- and left-hand circularly polarized light.

The Stokes parameters can be combined into a vector called the *Stokes vector*, written as $\mathbf{S} = [I, Q, U, V]$.

5.3 Mueller Matrix

The Mueller matrix is a 4×4 matrix that describes how the state of polarization of light changes upon interacting with a material or optical system. Each component of the Mueller matrix represents a specific effect on the Stokes parameters. This transformation is represented by the equation

$$\mathbf{s}' = \mathbf{M}\mathbf{s}. \tag{14}$$

The Mueller matrix can be schematically written as

$$\mathbf{M} = \begin{bmatrix} I \to I' & Q \to I' & U \to I' & V \to I' \\ I \to Q' & Q \to Q' & U \to Q' & V \to Q' \\ I \to U' & Q \to U' & U \to U' & V \to U' \\ I \to V' & Q \to V' & U \to V' & V \to V' \end{bmatrix} \tag{15}$$

The diagonal elements describe how the system preserves or modifies specific polarization states, while the off-diagonal elements represent cross-conversions between different types of polarization.

5.4 Instrumental Beam Modeling with Zernike Polynomials

The Mueller matrix $\mathbf{M}$, obtained from the GRASP simulations, is a discrete representation of the telescope's response on a grid of points on the sky. To create a continuous and computationally efficient beam model, each of the 16 elements of the Mueller matrix, $M_{ij}(\text{RA, DEC})$, will be decomposed into a basis of Zernike polynomials.

Zernike polynomials form a set of orthogonal polynomials over the unit circle, making them ideal for describing aberrations and complex shapes in optical systems. The process involves fitting a linear combination of Zernike polynomials to each M_{ij} surface:

$$M_{ij}(\rho, \phi) \approx \sum_{n,m} c_{nm}^{ij} Z_n^m(\rho, \phi) \tag{16}$$

where c_{nm}^{ij} are the Zernike coefficients that describe the contribution of each mode Z_n^m to the matrix element M_{ij}. This decomposition will provide us with an analytical and compact model of the instrumental response, which encapsulates all polarization effects, including leakage. The analysis will be carried out using scientific computing libraries in Python, such as `Zernike`.

5.5 Intensity Map Simulation with HIDE & SEEK

To evaluate the impact of polarization leakage on scientific observations, end-to-end simulations will be performed with the HIDE (HI Data Emulator) package. HIDE is part of the HIDE & SEEK software suite, developed at ETH Zurich, a suite that is designed to *forward model the entire radio survey system chain*, from the sky to the final data. The adaptation of HIDE for BINGO was developed by project members and will be used in this work to simulate the telescope's temporal observations, according to its scanning model and observational characteristics. However, the current version of the adaptation assumes beams with axial symmetries, which, as shown in (Abdalla, F. et al., 2022), does not correspond to the most realistic scenario. This is because horns further away from the focal plane center exhibit higher levels of aberrations. Thus, this project plans to carry out the evaluation of two scenarios:

Using HIDE, two main scenarios will be simulated:

1. Ideal Scenario (Control): Sky maps will be generated by convolving the astrophysical components (21 cm signal, synchrotron, free-free, and dust foregrounds) with a Gaussian beam, i.e., with radial symmetry. This scenario represents an observation where aberration effects do not introduce additional systematic effects into the generated maps.

2. Realistic Scenario (Contaminated): The simulation will be repeated, but instead of a simple convolution, we will apply our full beam model. For each sky pixel, the Stokes vector of the astrophysical emission, $\mathbf{s}_{\text{sky}} = [I, Q, U, V]_{\text{sky}}$, will be multiplied by our Mueller matrix modeled with Zernikes, $\mathbf{M}(\text{RA, DEC})$ (15), to obtain the observed Stokes vector:

$$\mathbf{s}_{\text{obs}} = \mathbf{M}(\text{RA, DEC}) \cdot \mathbf{s}_{\text{sky}} \tag{17}$$

The contaminated intensity map will be the first element of the resulting vector, I'_{obs}. This map will contain the actual sky intensity summed with the polarization leakage terms (e.g., $M_{12}Q_{\text{sky}} + M_{13}U_{\text{sky}}$).

The output of this process will be two sets of sky intensity maps: one ideal and one contaminated by polarization leakage.

5.6 Power Spectrum Analysis and Contamination Quantification

The final quantification of the impact of polarization leakage will be performed in the spherical harmonics space, through the analysis of the angular power spectrum (C_ℓ). The C_ℓ is the equivalent of the correlation function in harmonic space; that is, it is a two-point function that quantifies correlations on spherical shells. In the context of 21 cm signal fluctuations in tomographic analyses, it represents the correlations of intensity fluctuations at different angular scales (ℓ), between different redshift bins.

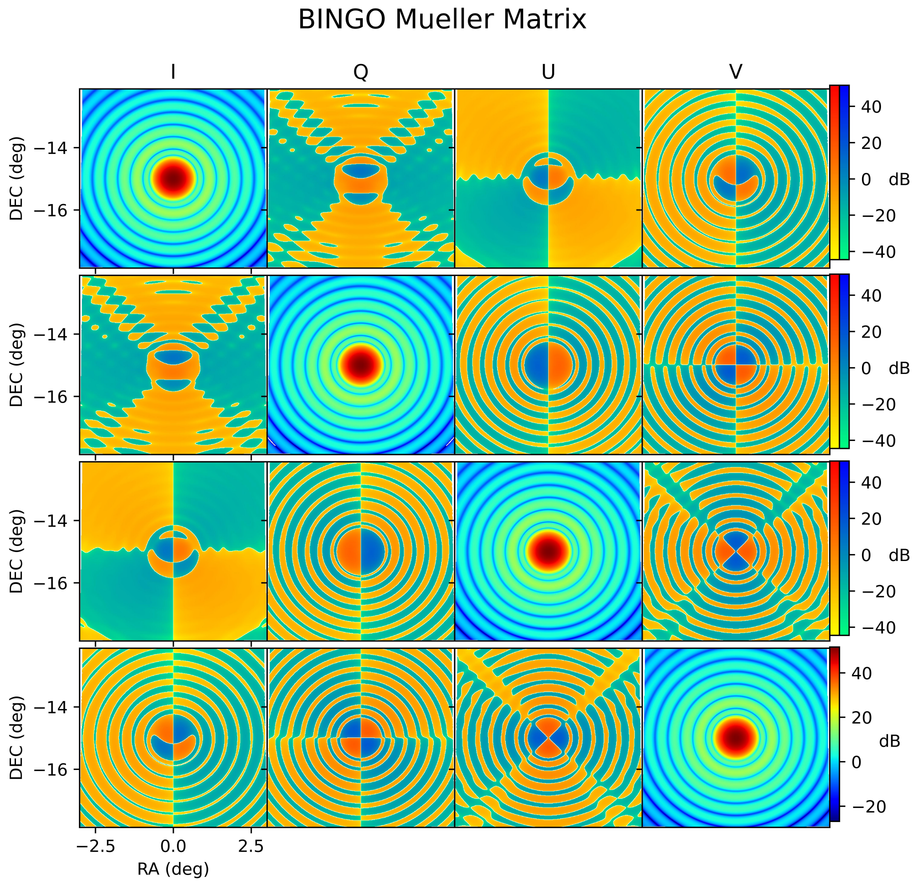

Figure 1: Full Mueller matrix (4×4) calculated for a single horn at 1100 MHz, using a simplified optical model. The axes represent the angular deviation from the center of the field of view, in units of dB ($X_{\mathrm{dB}} = 10 \log_{10}(\pm X)$). The two color bars indicate the positive (red) and negative (blue) values for the off-diagonal elements.

The analysis procedure will be:

1. Spectrum Calculation: Using packages such as `NaMaster`[5] and `HEALPy`[6], we will calculate the power spectra for the ideal (C_ℓ^{ideal}) and contaminated ($C_\ell^{\text{contaminated}}$) maps.

2. Residue Quantification: The difference between the two spectra will be quantified to evaluate the magnitude of the contamination as a function of angular scale. The absolute difference ($D(\ell) = C_\ell^{\text{contaminated}} - C_\ell^{\text{ideal}}$) and the relative difference ($R(\ell) = D(\ell)/C_\ell^{\text{ideal}}$) will be calculated.

3. Comparison with Uncertainties: The magnitude of the contamination, $D(\ell)$, will be compared with the expected statistical uncertainties for BINGO, which include cosmic variance and instrumental noise. This step is crucial for determining whether polarization leakage is a negligible or dominant systematic error.

4. Morphological Analysis: The shape of the contamination spectrum, $D(\ell)$, will be analyzed to verify the existence of oscillatory structures that might mimic or distort the cosmological Baryon Acoustic Oscillations (BAO) signal, BINGO's main scientific objective.

This complete analysis will allow us not only to characterize the instrument but also to determine the requirements for the mitigation of this systematic effect, ensuring the robustness of the experiment's cosmological results.

5.7 Preliminary Results and Methodology Validation

To ensure the viability of the project in the face of the high computational cost of full simulations, an incremental development strategy has been adopted. Firstly, a complete analysis pipeline is being developed and will be validated using a set of simulated data under simplified, yet representative, conditions. These simplifications include:

- The simulation of the response for a single frequency (1100 MHz), in the center of the BINGO band;
- The analysis of a single horn (signal receiver) out of the 28 existing ones;
- The use of a preliminary and simplified optical model of the telescope.

This approach allowed for the efficient development and debugging of the pipeline, which executes the following sequence:

1. Extraction of the complex electric field data from the GRASP simulations.

2. Processing of the data in `Python` to calculate the Stokes parameters and generate the full Mueller matrix for the optical system.

3. Decomposition of each element of the Mueller matrix into Zernike polynomials, generating an analytical beam model.

[5] https://namaster.readthedocs.io/en/latest/
[6] https://healpy.readthedocs.io/en/latest/

4. Use of the beam models as input for the HIDE software to simulate sky observations.

The successful development of this prototype pipeline confirms the technical feasibility of the project. As proof of this validation, we present the results obtained for this simplified scenario below.

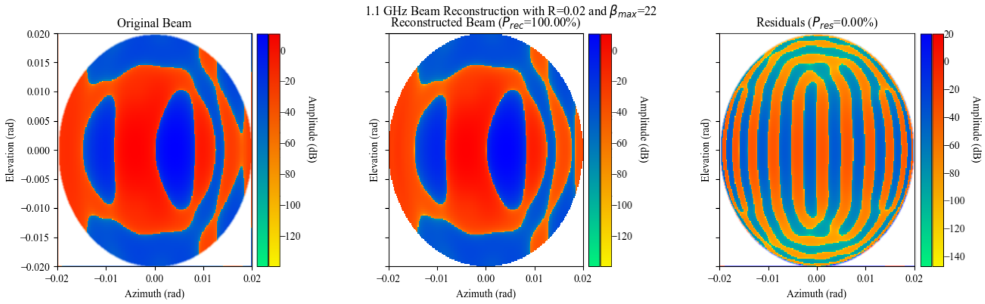

Figure 2: Validation of the reconstruction with Zernike polynomials for the M_{12} element (row 1, column 2) of the Mueller matrix in the simplified scenario. From left to right: the map of the original element simulated in GRASP; its reconstruction from the Zernike model; and the residue map (Original - Reconstruction). The two color bars indicate the positive (red) and negative (blue) values. Quantitative analysis of the normalized power indicates a reconstruction fidelity (P_{rec}) of 100.00%, with a residual power (P_{res}) of 0.00%, which validates the high precision of the modeling methodology.

Figure 1 presents the Mueller matrix calculated under these conditions. Even in this simplified case, the off-diagonal elements, especially in the terms that convert polarization into intensity (first row), demonstrate a low presence of polarization leakage, which this project aims to fully quantify.

A crucial step in the methodology is the ability to accurately model each element of the Mueller matrix. Figure 2 demonstrates the high fidelity of the Zernike reconstruction approach. The low-amplitude residue map confirms that the decomposition is an excellent representation of the instrumental response, validating the method for its application to the full data set.

With the robustness of the beam characterization pipeline now confirmed, the central objective of this project is to apply this validated methodology to perform an end-to-end impact analysis.

A crucial part of the methodology validation will consist of comparing the results obtained through our detailed physical model with simplified approximations found in the literature. The full approach, via the Mueller Matrix, describes the observed intensity as:

$$I_{\text{observed}} = M_{11} I_{\text{sky}} + M_{12} Q_{\text{sky}} + M_{13} U_{\text{sky}} + M_{14} V_{\text{sky}}$$

In contrast, simpler models, such as the linear approximation (Gao, Li-Yang et al., 2023), describe the leakage as follows:

$$I_{\text{observed}} \approx I_{\text{sky}} + \epsilon_Q Q_{\text{sky}} + \epsilon_U U_{\text{sky}}$$

In these equations, the terms I_{sky}, Q_{sky}, and U_{sky} represent sky maps that include astrophysical foregrounds, such as synchrotron radiation, and can be generated using simulation packages such as PySM 3[7]. In the simplified linear approach, the coefficients ϵ_Q and

[7] https://pysm3.readthedocs.io/en/latest/

ϵ_U are often treated as a single number, representing an average system error. By applying this model, the contamination manifests as a linear combination of the Q and U polarization maps overlaid on the total intensity I map, resulting in sky maps contaminated by polarized synchrotron emission, such as in figures 3. Such a model, however, ignores the contribution of circular polarization (Stokes V), an assumption that is not valid for BINGO, where the circular polarization induced by the optics is dominant compared to the linear polarization ($V \gg Q, U$) (Abdalla, F. et al., 2022). The comparison between the results of our model and this approximation will allow us to quantify the precision gained with our more fundamental approach.

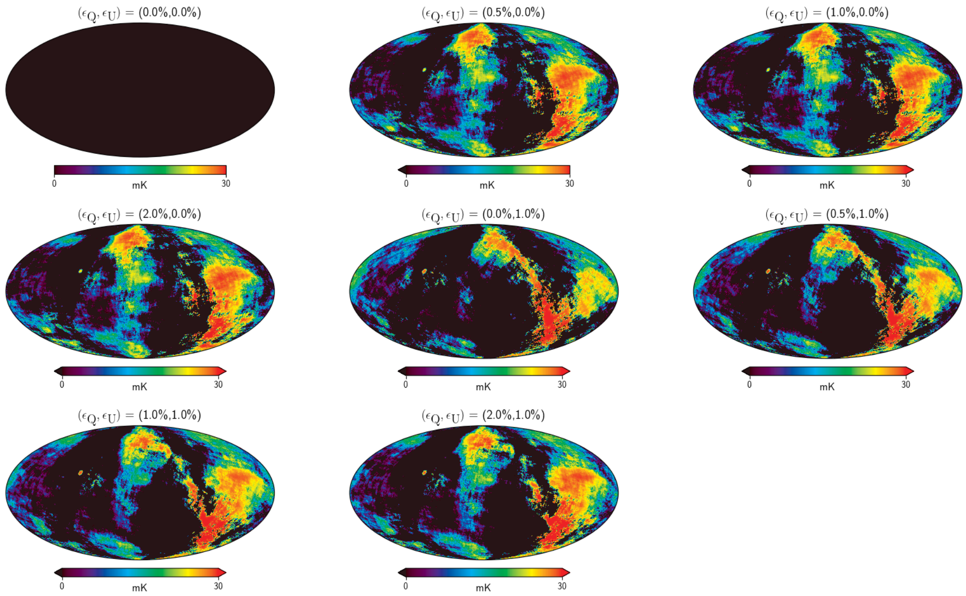

Figure 3: Visual demonstration of polarization leakage contamination according to the simplified linear model ($I_{\text{observed}} \approx I_{\text{sky}} + \epsilon_Q Q_{\text{sky}} + \epsilon_U U_{\text{sky}}$). The reference panel (top left, with $\epsilon_Q = 0\%$ and $\epsilon_U = 0\%$) shows the synchrotron foreground intensity map, simulated with the PySM 3 package. The other panels illustrate how the additive contamination increases with the introduction of different percentages of leakage (ϵ_Q, ϵ_U), generating artifacts that overlap the original signal. These maps exemplify the effect of a simplified model that will be compared with the results of our complete methodology.

Future work, detailed in the schedule, will follow the main steps below:

1. **Generation of Realistic Models:** Application of the pipeline to generate the Zernike models for the complete data set, covering the entire frequency band (980 to 1260 MHz) and using BINGO's final optical configuration.

2. **Intensity Map Simulation:** Use of these realistic beam models in HIDE to generate sky maps that include polarization leakage contamination.

3. **Spectral Analysis and Contamination Quantification:** Calculation of the angular power spectra (C_ℓ) of the contaminated maps and their rigorous comparison with the ideal case, allowing for the quantification of the magnitude of the systematic error.

4. **Validation and Comparison with Simplified Models:** Direct comparison of the power spectra of our model with those generated by the linear approximation, in order to evaluate the magnitude of the error incurred by such simplifications and validate the necessity of our detailed approach.

The successful conclusion of these steps will provide an unprecedented understanding of BINGO's instrumental effects, which is crucial for the success of its scientific objectives.

6 Method of Results Analysis

The analysis of the results will focus on three central objectives: (1) to characterize BINGO's instrumental response to polarization leakage; (2) to quantify the impact of this systematic effect on cosmological observables; and (3) to develop and evaluate strategies for its mitigation. The approach will combine the analysis of the Mueller matrix, modeled with Zernike polynomials, and the statistical analysis of the simulated intensity maps.

The steps of the analysis are detailed below:

- **Characterization of the Instrumental Response:** The first analysis will focus on the Zernike coefficients obtained for each element of the Mueller matrix. We will investigate the spatial morphology of the main leakage terms (e.g., M_{IQ} and M_{IU}), identifying the contribution of specific modes (such as dipoles, quadrupoles) and correlating them with the telescope's optical design. The goal is to answer: what are the primary aberrations that cause the leakage and in which regions of the field of view are they dominant?
- **Quantification of the Impact on the Power Spectrum:** This is the central step of the impact analysis. Using the angular power spectra calculated in the methodology, we will perform a rigorous comparative analysis:
  - We will calculate the power spectrum of the contamination, $D(\ell) = C_\ell^{\text{contaminated}} - C_\ell^{\text{ideal}}$, to quantify the spurious power added at each angular scale.
  - We will compare the magnitude of $D(\ell)$ with the expected uncertainty level for BINGO ($\Delta C_\ell^{\text{total}}$), determining the scales at which polarization leakage becomes a significant effect.
  - We will analyze the morphology of $D(\ell)$ to identify whether the contamination introduces oscillatory features that might mimic or distort the Baryon Acoustic Oscillations (BAO) signal.
  - Additionally, we will compare our results with those obtained by alternative methods, such as Faraday rotation-based modeling via the CRIME software (Alonso; Ferreira; Santos, 2014) and the linear approximation, commonly employed by the scientific community. This comparison will allow us to assess the robustness and advantages of our Mueller matrix-based approach relative to more simplified or specific models.

Upon completion of this analysis, we will have a complete quantitative understanding of the effects of polarization leakage in BINGO, a fundamental requirement for ensuring the reliability of the experiment's future cosmological measurements.

References

Abdalla, E. et al. The bingo project: I. baryon acoustic oscillations from integrated neutral gas observations. *Astronomy & Astrophysics*, EDP Sciences, v. 664, p. A14, ago. 2021. ISSN 1432-0746. Disponível em: <http://dx.doi.org/10.1051/0004-6361/202140883>. 1, 2

Abdalla, F. et al. The bingo project: iii. optical design and optimization of the focal plane. *Astronomy & Astrophysics*, EDP Sciences, v. 664, p. A16, ago. 2022. ISSN 1432-0746. Disponível em: <http://dx.doi.org/10.1051/0004-6361/202141382>. 2, 4, 5, 7, 11

Alonso, D.; Ferreira, P. G.; Santos, M. G. Fast simulations for intensity mapping experiments. *Monthly Notices of the Royal Astronomical Society*, Oxford University Press (OUP), v. 444, n. 4, p. 3183–3197, set. 2014. ISSN 0035-8711. Disponível em: <http://dx.doi.org/10.1093/mnras/stu1666>. 2, 12

Ansah-Narh, T. et al. Simulations of systematic direction-dependent instrumental effects in intensity mapping experiments. *mnras*, v. 481, n. 2, p. 2694–2710, dez. 2018. 2

Bass, M. *Handbook of Optics: Geometrical and Physical Optics, Polarized Light, Components and Intruments*. [S.l.]: McGraw-Hill, 2010. 1

Battye, R. A. et al. Hi intensity mapping: a single dish approach. *Monthly Notices of the Royal Astronomical Society*, Oxford University Press (OUP), v. 434, n. 2, p. 1239–1256, jul. 2013. ISSN 1365-2966. Disponível em: <http://dx.doi.org/10.1093/mnras/stt1082>. 2

Carucci, I.; Irfan, M. O.; Bobin, J. Recovery of 21-cm intensity maps with sparse component separation. *Monthly Notices of the Royal Astronomical Society*, Oxford University Press (OUP), v. 499, n. 1, p. 304–319, set. 2020. ISSN 1365-2966. Disponível em: <http://dx.doi.org/10.1093/mnras/staa2854>. 2

Chapman, E. et al. Foreground removal using fastica : a showcase of lofar-eor. *Monthly Notices of the Royal Astronomical Society*, v. 423, n. 3, p. 2518–2532, 06 2012. ISSN 0035-8711. Disponível em: <https://doi.org/10.1111/j.1365-2966.2012.21065.x>. 1

Emma Chapman et al. *The Effect of Foreground Mitigation Strategy on EoR Window Recovery*. 2016. Disponível em: <https://arxiv.org/abs/1408.4695>. 2

Furlanetto, S.; Oh, S. P.; Briggs, F. H. Cosmology at low frequencies: The 21 cm transition and the high-redshift universe. *Physics Reports*, v. 433, n. 4-6, p. 181–301, 2006. 1

Gao, Li-Yang et al. Eliminating polarization leakage effect for neutral hydrogen intensity mapping with deep learning. *Monthly Notices of the Royal Astronomical Society*, Oxford University Press (OUP), v. 525, n. 4, p. 5278–5290, set. 2023. ISSN 1365-2966. Disponível em: <http://dx.doi.org/10.1093/mnras/stad2646>. 2, 10

Kumar, J.; Dutta, P.; Roy, N. Calibration requirements for epoch of reionization 21-cm signal observations – i. effect of time-correlated gains. *Monthly Notices of the Royal Astronomical Society*, v. 495, n. 4, p. 3683–3694, 05 2020. ISSN 0035-8711. Disponível em: <https://doi.org/10.1093/mnras/staa1371>. 2

Meneghetti, M. et al. An excess of small-scale gravitational lenses observed in galaxy clusters. *Science*, American Association for the Advancement of Science (AAAS), v. 369, n. 6509, p. 1347–1351, set. 2020. ISSN 1095-9203. Disponível em: <http://dx.doi.org/10.1126/science.aax5164>. 1

Nunhokee, C. D. et al. Constraining polarized foregrounds for eor experiments. ii. polarization leakage simulations in the avoidance scheme. *The Astrophysical Journal*, American Astronomical Society, v. 848, n. 1, p. 47, out. 2017. ISSN 1538-4357. Disponível em: <http://dx.doi.org/10.3847/1538-4357/aa8b73>. 2

Paladino, E. et al. 1/f noise: Implications for solid-state quantum information. *Rev. Mod. Phys.*, American Physical Society, v. 86, p. 361–418, Apr 2014. Disponível em: <https://link.aps.org/doi/10.1103/RevModPhys.86.361>. 2

Prat, J. et al. Dark energy survey year 1 results: Galaxy-galaxy lensing. *Phys. Rev. D*, American Physical Society, v. 98, p. 042005, Aug 2018. Disponível em: <https://link.aps.org/doi/10.1103/PhysRevD.98.042005>. 1

Riess, A. et al. Observational evidence from supernovae for an accelerating universe and a cosmological constant. *The Astronomical Journal*, American Astronomical Society, v. 116, n. 3, p. 1009–1038, set. 1998. ISSN 0004-6256. Disponível em: <http://dx.doi.org/10.1086/300499>. 1

Trippe, S. Polarization and polarimetry: a review. *arXiv preprint arXiv:1401.1911*, 2014. 5

Vernotte, F; Lantz, E. Metrology and 1/fnoise: linear regressions and confidence intervals in flicker noise context. *Metrologia*, IOP Publishing, v. 52, n. 2, p. 222–237, fev. 2015. ISSN 1681-7575. Disponível em: <http://dx.doi.org/10.1088/0026-1394/52/2/222>. 2

Villaescusa-Navarro, Francisco et al. Ingredients for 21 cm intensity mapping. *The Astrophysical Journal*, American Astronomical Society, v. 866, n. 2, p. 135, out. 2018. ISSN 1538-4357. Disponível em: <http://dx.doi.org/10.3847/1538-4357/aadba0>. 1