Gabe Coyote
Vineel Kesavarapu
Mark Peralta

4330 Assignment 6

hw6-Coyote, peralta, kesavarapu.pdf

1. $\gcd(1644, 1264)$ $1644 - 1264 = \underline{380}$

$\gcd(1264, \underline{380})$ $1264 - (380 \times 3) = \underline{124}$

$\gcd(380, \underline{124})$ $380 - (124 \times 3) = \underline{8}$

$\gcd(124, \underline{8})$ $124 - (8 \times 15) = \underline{4}$

$\gcd(8, \underline{4})$ $8 / 4 = 2$ no remainder $\underline{0}$

$\gcd(4, \underline{0})$ Total of 5 divisions

2. Show for every pair m, n of positive integers with $m > n$,

$$(m \bmod n) < m/2,$$

where $m \bmod n$ is the remainder of m/n as given by D.A.

Proof:

(1) If $m \geq 2n$ by the D.A $\exists!$ q, r s.t

$m = nq + r$ and $0 \leq r < n$.

But $r < n < {}^{n}/_{2}$

$(m \bmod n) < n < {}^{m}/_{2}$

(2) Seperate $n < m < 2n$

$m = nq + r$, $0 \leq r < n$

$0 < n - r < m - r < 2n - r < 2n$

$$\frac{0}{n} < \frac{m-r}{n} < 2$$

$0 < q < 2$ $\underline{q = 1}$ positive integer

$m = n + r$ $\exists$ $r < n$.

$(m \bmod n) = r$ $r = m - n$

$r = m - n < m - {}^{m}/_{2} \rightarrow r = m - n < {}^{m}/_{2}$

$\therefore$ $(m \bmod n) < {}^{m}/_{2}$ $\blacksquare$

3. $b_{k+2} = a_{k+1} \bmod b_{k+1}$

$< a_{k+1}/2$ (by probum 2)

$= b_k/2$

$0 < b_k \bmod a_k \bmod b_k < b_k/2 < b_k$

$0 < b_k \bmod b_{k+1} < b_k/2 < b_k$

$0 < a_{k+1} \bmod b_{k+1} < b_k/2 < b_k$

$0 < b_{k+2} < b_k/2 < b_k$

size of b_{k+2} compared with b_k.

$$b_{2x+2} < \frac{b_k}{2^x}, \quad \text{for } x \geq 1$$

If $b_1, b_2, b_3, \ldots, b_{1+2\log_2 b_2}$ are non zero then

$$b_{2+2\log_2 b_2} < \frac{b_2}{2^{\log_2 b_2}} = 1.$$

And, for some number $[b_2 + 2\log_2 b_2] < 1$, then it must be zero.

Let $b_{k+2} < \frac{b_k}{2}$, so lets state for some value b_2

$$b_4 < \frac{b_2}{2},$$

$$b_6 < \frac{b_4}{2^1} < \frac{b_2}{2^2},$$

$$b_8 < \frac{b_2}{2^3},$$

$$b_{10} < \frac{b_2}{2^4},$$

So $b_{2x+2} < \frac{b_2}{2^x}$, for $x \geq 1$.

Upperbound on maximum size k is $2 + 2\log_2 b$.