Libraries

In [1]:
```python
import numpy as np
import pandas as pd
```

Read data

In [2]:
```python
real_estate_data = pd.read_csv("../../data/Real_estate.csv")
print (real_estate_data.shape)
real_estate_data.head()
```

```
(414, 8)
```

Out[2]:

	No	X1 transaction date	X2 house age	X3 distance to the nearest MRT station	X4 number of convenience stores	X5 latitude	X6 longitude	Y house price of unit area
0	1	2012.917	32.0	84.87882	10	24.98298	121.54024	37.9
1	2	2012.917	19.5	306.59470	9	24.98034	121.53951	42.2
2	3	2013.583	13.3	561.98450	5	24.98746	121.54391	47.3
3	4	2013.500	13.3	561.98450	5	24.98746	121.54391	54.8
4	5	2012.833	5.0	390.56840	5	24.97937	121.54245	43.1

Spltting data into train and test sets

In [3]:
```python
X = real_estate_data.iloc[:, 1:7]
y = real_estate_data.iloc[:, -1]

# Normalize the features to the [0, 1] range using min max
x_max = X.max(axis=0)
x_min = X.min(axis=0)
X_normalized = (X - x_min) / (x_max - x_min)
```

2. a)

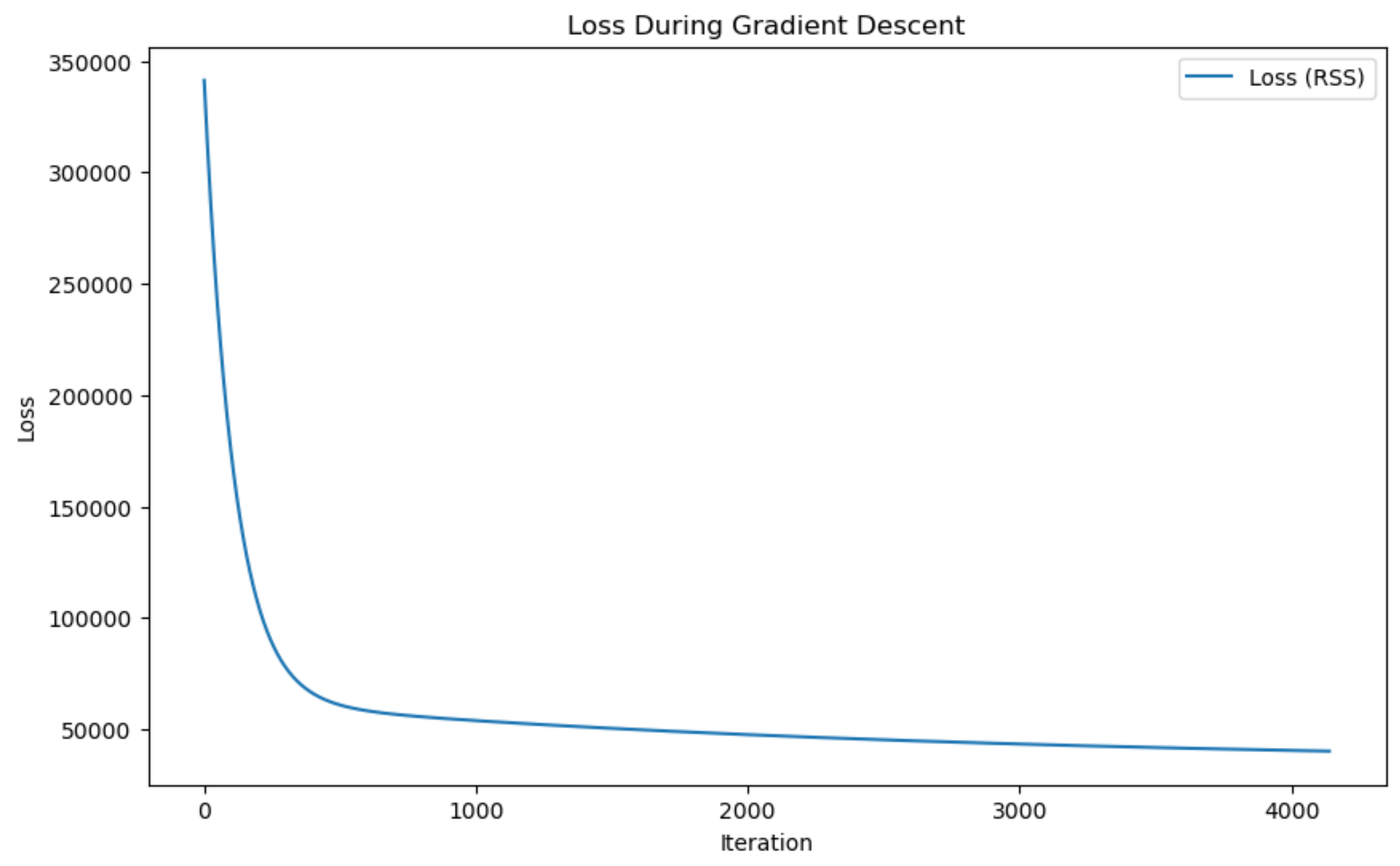
In [4]:
```python
w = np.array([1, 1, 1, 1, 1, 1])
b = 10
learning_rate = .001
steps = 4140
loss_values = []

for step in range(steps):

        y_pred = np.dot(X_normalized, w) + b
        loss = np.sum((y - y_pred) ** 2)
        loss_values.append(loss)

        gradient_w = -2 * (X_normalized.T.dot(y - y_pred) / len(y))
        gradient_b = -2 * (np.sum(y - y_pred) / len(y))

        w = w - (learning_rate * gradient_w)
        b = b - (learning_rate * gradient_b)

# Calculate predictions for the entire dataset with final w and b
final_y_pred = X_normalized.dot(w) + b

# Calculate R-squared using final predictions
SSR = ((y - final_y_pred) ** 2).sum()
SST = ((y - y.mean()) ** 2).sum()
r2 = 1 - (SSR / SST)


print("Optimized weights: \n", w, "\n")
print("Optimized bias: ", b, "\n")
print("R^2: ", r2)
```

```
Optimized weights: 
 X1 transaction date                        4.896591
X2 house age                              -2.170361
X3 distance to the nearest MRT station    -5.584158
X4 number of convenience stores           13.027288
X5 latitude                               10.220503
X6 longitude                              11.918416
dtype: float64 

Optimized bias:  19.688189362333627 

R^2:  0.47503616118575687
```

b)

In [5]:
```python
import matplotlib.pyplot as plt
```

In [6]:
```python
# Plotting the loss over iterations
plt.figure(figsize=(10, 6))
plt.plot(loss_values, label='Loss (RSS)')
plt.xlabel('Iteration')
plt.ylabel('Loss')
plt.title('Loss During Gradient Descent')
plt.legend()
plt.show()
```



Yes, with a sufficiently large number of steps, gradient descent is expected to converge to a solution that is not the same but it is close to the optimal, as evidenced by the loss graph which shows a rapid initial decrease in cost and subsequent stabilization, indicating that the algorithm is approaching an optimal set of parameters.

c)

In [7]:
```python
w = np.array([1, 1, 1, 1, 1, 1])
b = 10
learning_rate = 0.001
steps = 4140
n_points = len(y)
loss_values = []

for step in range(steps):
    i = step % n_points
    x_i = X_normalized.iloc[i]
    y_i = y.iloc[i]
    y_pred_i = np.dot(w, x_i) + b

    loss = (y_i - y_pred_i) ** 2
    loss_values.append(loss)

    gradient_w_i = -2 * x_i * (y_i - y_pred_i)
    gradient_b_i = -2 * (y_i - y_pred_i)

    w = w - learning_rate * gradient_w_i
    b = b - learning_rate * gradient_b_i

# Calculate predictions for the entire dataset with final w and b
final_y_pred = X_normalized.dot(w) + b

# Calculate R-squared using final predictions
SSR = ((y - final_y_pred) ** 2).sum()
SST = ((y - y.mean()) ** 2).sum()
r2 = 1 - (SSR / SST)

print("Optimized weights: \n", w, "\n")
print("Optimized Bias: ", b, "\n")
print("R^2: ", r2)
```

```
Optimized weights: 
 X1 transaction date                        4.864761
X2 house age                              -2.178003
X3 distance to the nearest MRT station    -5.602656
X4 number of convenience stores           13.009351
X5 latitude                               10.227864
X6 longitude                              11.884743
dtype: float64 

Optimized Bias:  19.620826963629238 

R^2:  0.4751042583786431
```

For 2(a), $R^2 \approx 0.4750 < 0.4751$. Hence, SGD was slightly better than GD.

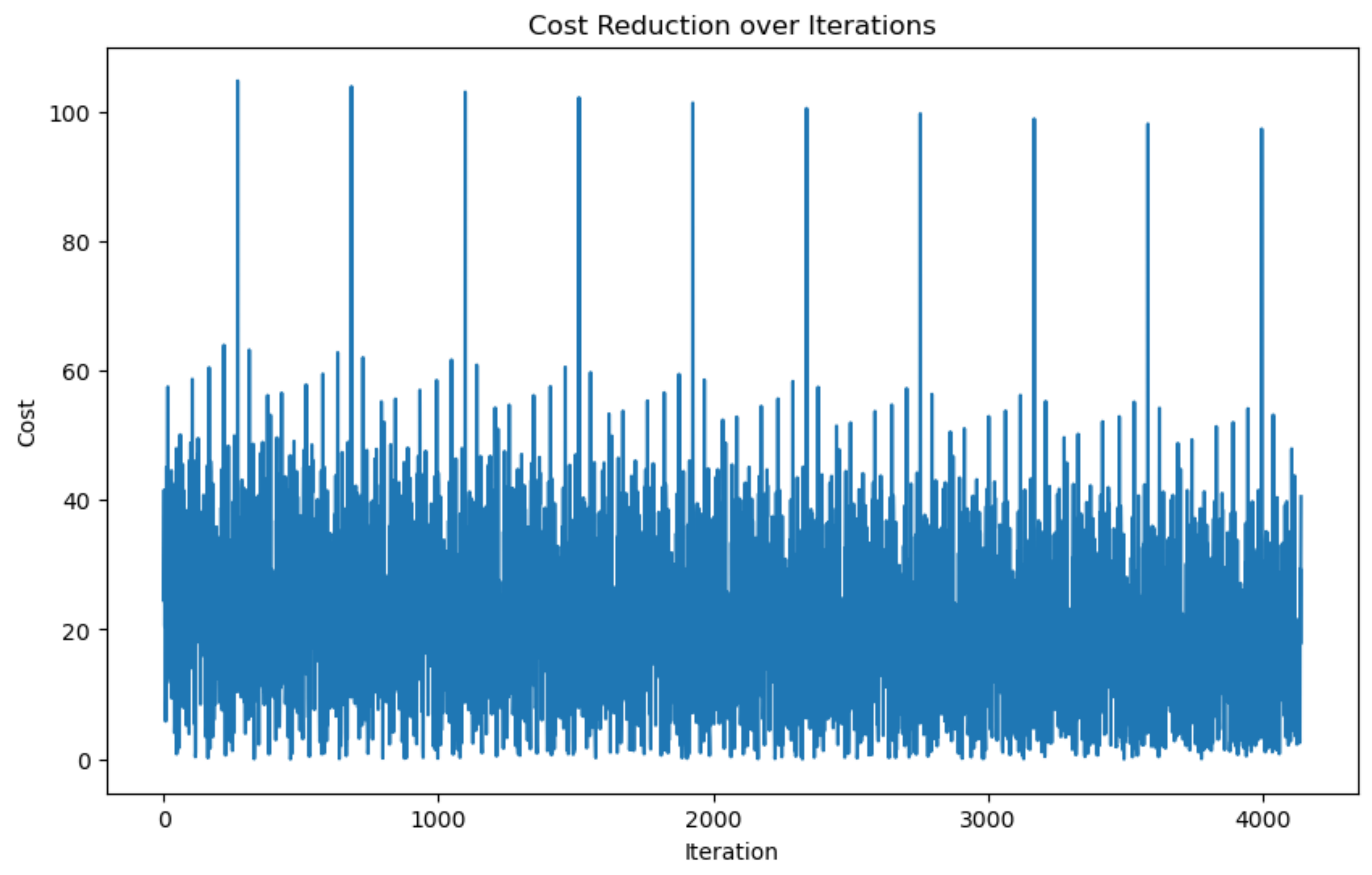
The slight improvement of SGD over GD in terms of R^2 can be attributed to SGD's ability to better navigate the cost function landscape through its frequent, varied updates, potentially finding a slightly more optimal set of parameters for the given dataset.

d)

In [8]:
```python
w = np.array([1, 1, 1, 1, 1, 1])
b = 10
learning_rate = 0.001
steps = 4140
n_points = len(y)

costs = []

for step in range(steps):
    i = step % n_points
    x_i = np.array(X_normalized.iloc[i]).reshape(1, -1)
    y_i = y[i]

    y_pred_i = np.dot(x_i, w) + b
    error_i = y_i - y_pred_i

    cost_i = np.abs(error_i).sum()
    costs.append(cost_i)


    subgrad_w = np.sign(y_pred_i - y_i) * x_i
    subgrad_b = np.sign(y_pred_i - y_i)


    w = w - learning_rate * subgrad_w.flatten()
    b = b - learning_rate * subgrad_b

# After optimization, calculate final predictions
final_y_pred_l1 = np.dot(X_normalized, w) + b

# Compute R-squared for the optimized model
SSR_l1 = ((y - final_y_pred_l1) ** 2).sum()
SST_l1 = ((y - y.mean()) ** 2).sum()
r2_l1 = 1 - (SSR_l1 / SST_l1)

print("Updated weights:", w)
print("Updated bias:", b)
print("R^2 score:", r2_l1)
```

```
Updated weights: [3.00745852 2.48712785 1.47364676 2.6724     2.78491881 3.54698879]
Updated bias: [13.778]
R^2 score: -1.424736969733107
```

In [9]:
```python
# Plotting the cost over iterations
plt.figure(figsize=(10, 6))
plt.plot(costs)
plt.xlabel('Iteration')
plt.ylabel('Cost')
plt.title('Cost Reduction over Iterations')
plt.show()
```



In [ ]: