Exercise 1

Out of 300 students in an Iowa high-school, 95 play cricket only, 120 play football only, 80 play volleyball only and 5 play no games. If one student is chosen at random, find the probability that

(a) (4 points) the student plays volleyball.

$$P(\text{ student plays volleyball }) = \frac{80}{300} = \frac{4}{15}$$

(b) (4 points) the student plays either cricket or volleyball.

$$P(\text{ student plays either cricket or volleyball }) = \frac{95}{300} + \frac{80}{300} = \frac{175}{300} = \frac{7}{12}$$

(c) (4 points) the student plays neither football nor volleyball.

$$P(\text{ student plays neither football nor volleyball }) = \frac{95}{300} + \frac{5}{300} = \frac{100}{300} = \frac{1}{3}$$

Exercise 2

The seniors from an Ankeny high-school are required to participate in exactly one after-school sport. Data were gathered from a sample of 120 students regarding their choice of sport. The following data were recorded

Gender	Football	Tennis	Basketball	Total
Male	17	8	10	35
Female	31	17	37	85
Total	48	25	47	120

(a) (6 points) For this group of students, do the data suggest that gender and sports are independent of each of other? Justify your answer.

We need to check the following:

$$P(\text{ Female and Tennis }) = P(\text{ Female })P(\text{ Tennis })$$
$$\frac{17}{120} \neq \left(\frac{85}{120}\right)\left(\frac{25}{120}\right)$$

Thus, gender and sports are not independent.

(b) (6 points) Two students are chosen at random from 120 students. Find the probability that:

(i) both play tennis

$$\left(\frac{25}{120}\right)\left(\frac{24}{119}\right)$$

(ii) neither play football

$$\left(\frac{72}{120}\right)\left(\frac{71}{119}\right)$$

(c) (6 points) One student is chosen at random. What is the probability that the student plays basketball given that the student is female?

$$\begin{aligned} P(\text{ student plays basketball} \mid \text{student is male }) &= \frac{P(\text{ student plays basketball and student is male })}{P(\text{ student is male })} \\ &= \frac{10/120}{35/120} \\ &= \frac{2}{7} \end{aligned}$$

Exercise 3

(5 points) Events A and B are independent. Suppose $P(B) = 0.6$ and $P(A \cap B) = 0.12$. Find $P(A)$.

$$P(A \cap B) = P(A)P(B) \quad \Rightarrow \quad P(A) = \frac{P(A \cap B)}{P(B)} = \frac{0.12}{0.6} = 0.2$$

Exercise 4

The table below shows the number of left and right handed tennis players in a sample of 50 males and females

Gender	Left handed	Right handed	Total
Male	3	29	32
Female	2	16	18
Total	5	45	50

If a tennis player was selected at random from the group, find the probability that the player is

(a) (4 points) female and right handed.

$$P(\text{ female and right handed }) = \frac{16}{50} = \frac{8}{25}$$

(b) (4 points) left handed.

$$P(\text{ left handed }) = \frac{5}{50} = \frac{1}{10}$$

(c) (4 points) right handed given that the player selected is male.

$$\begin{aligned} P(\text{ right handed} \mid \text{male }) &= \frac{P(\text{ right handed and male })}{P(\text{ male })} \\ &= \frac{29/50}{32/50} \\ &= \frac{29}{32} \end{aligned}$$

Exercise 5

(6 points) A soccer team wins 60% of its games when it scores the first goal, and 10% of its games when the opposing team scores first. If the team scores the first goal about 30% of the time, what is the probability of wining a game?

First, we need to define the events. Let

$$\begin{aligned} W &: \text{the soccer team wins the game} \\ F &: \text{the soccer team scores first} \end{aligned}$$

We are given that $P(W|F) = 60\%$, $P(W|F^c) = 10\%$ and $P(F) = 30\%$. We want to find $P(W)$. Then, by the law of total probability, we have

$$\begin{aligned} P(W) &= P(W|F)P(F) + P(W|F^c)P(F^c) \\ &= (60\%)(30\%) + (10\%)(70\%) \\ &= 0.25 \end{aligned}$$

Exercise 6

Cristiano Ronaldo is one of the most popular athletes in the worlds. From 2009 to 2018, he player for Real Madrid. Let X denote the number of goals that Cristiano scored per game in Real Madrid.

x	0	1	2	3	4	5
$p(X = x)$	0.20	0.45	0.20	0.11	0.03	0.01

(a) (3 points) Find $P(X \leq 2)$

$$\begin{aligned} P(X \leq 2) &= P(X = 0) + P(X = 1) + P(X = 2) \\ &= 0.2 + 0.45 + 0.2 \\ &= 0.85 \end{aligned}$$

(b) (3 points) Find $P(X > 1)$

$$\begin{aligned} P(X > 1) &= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) \\ &= 0.2 + 0.11 + 0.03 + 0.01 \\ &= 0.35 \end{aligned}$$

(c) (5 points) Find $E(X)$

$$\begin{aligned} E(X) &= 0(0.2) + 1(0.45) + 2(0.2) + 3(0.11) + 4(0.03) + 5(0.01) \\ &= 1.35 \end{aligned}$$