Homework 3.
Due: Thursday, January 31, 2019 before 9:30am EDT.

Suplementary material: watch lectures of prof. Vigoda (the link is available on Canvas) DC4: FFT1 and DC5: FFT2.

Suggested reading: Chapter 2 of the book.

FFT Homework

Problem 1 (Problem 2.8 DPV)

(a) What is the FFT of $(1, 0, 0, 0)$? What is the appropiate value of ω in this case? Of which sequence is $(1, 0, 0, 0)$ the FFT?

(b) Repeat with $(1, 0, 1, -1)$.

Problem 2 (FFT practice)

Do polynomial multiplication by FFT for the pair of polynomials $1 + x + 2x^2$ and $2 + 3x$. **Show your work**.

Problem 3 (Integer multiplication using FFT)

(a) Given an $n-$bit integer number $a = a_0 a_1 a_2 \ldots a_{n-1}$ define a polynomial $A(x)$ satisfying $A(2) = a$.

(b) Given two $n-$bit integers a and b, give an algorithm to multiply them in $O(n \log(n))$ time. Use the FFT algorithm from class as a black-box (i.e. don't rewrite the code, just say run FFT on ...).Explain your algorithm in words and analyse its running time.

Problem 4 (sum of two sets)

Consider two sets A and B, each having n integers in the range from 0 to $10n$. We wish to compute the Cartesian sum of A and B, defined by:

$$C = \{x + y : x \in A,\ y \in B\}.$$

Note that integers in C are in the range from 0 to $20n$. We want to find the set of elements in C and also the number of times each element of C is realized as a sum of elements in A and B. Show that the problem can be solved in $O(n \log(n))$ time by reducing it to the polynomial multiplication algorithm. You just use the polynomial multiplication algorithm without modifying it. You need to explain what input you put into the polynomial multiplication algorithm and what you do with the output to get the solution to the Cartesian Sum problem.

Example: $A = [1, 2, 3]$, $B = [2, 3]$ and the solution to the Cartesian Sum problem is: $C = [3, 4, 5, 6]$
3 appears and is obtainable in 1 way,
4 appears and is obtainable in 2 ways,
5 appears and is obtainable in 2 ways,
6 appears and is obtainable in 1 way.