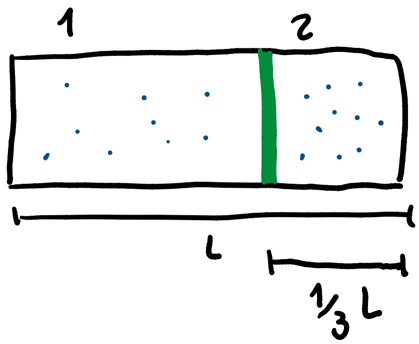
0.3_1

1) a)



$T_1 = 400K \quad T_2 = ?$

$V_1 = \frac{2}{3} V_T \quad V_2 = \frac{1}{3} V_T$

Para que el sistema esté estable $P_1 = P_2$

$$P_1 V_1 = nRT_1 \Rightarrow P_1 = \frac{nRT_1}{V_1}$$

$$P_2 V_2 = nRT_2 \Rightarrow P_2 = \frac{nRT_2}{V_2}$$

$$\frac{n\cancel{R}T_1}{V_1} = \frac{n\cancel{R}T_2}{V_2}$$

$$T_2 = \frac{T_1 V_2}{V_1} = \frac{400K \cdot \frac{1}{\cancel{3}} \cancel{V_T}}{\frac{2}{\cancel{3}} \cancel{V_T}} = 200K$$

b) De la primera ley de la termodinámica

$$\Delta U = Q - W$$

Como no se realiza trabajo

$$\Delta U = Q$$

$$\Delta U = n c_v \Delta T$$

$$Q = n c_v \Delta T \qquad (1)$$

De la ley de transferencia de Fourier

$$\dot{Q} = -kA \frac{dT}{dx}$$

$$\frac{Q}{dt} = -\frac{kA}{l} \, dT \qquad (2)$$

Reemplazando (1) en (2)

$$\frac{n c_v \Delta T}{dt} = -\frac{kA}{l} \, dT$$

Para el cambio de T_1

$$n c_v \frac{dT_1}{dt} = -\frac{kA}{l} \int_{T_2}^{T_1} dT$$

$$n c_v \frac{dT_1}{dt} = -\frac{kA}{l} (T_1 - T_2) \qquad (3)$$

Para el cambio de T_2

$$ncu\frac{dT_2}{dt} = -\frac{\kappa A}{\ell}\int_{T_1}^{T_2} dT$$

$$ncu\frac{dT_2}{dt} = -\frac{\kappa A}{\ell}(T_2 - T_1)$$

$$ncu\frac{dT_2}{dt} = \frac{\kappa A}{\ell}(T_1 - T_2) \quad (4)$$

Reemplazando $C = \frac{\kappa A}{ncu\ell}$ en (3) y (4)

$$\left.\frac{dT_1}{dt}\right|_{t=0} = -C(T_1^0 - T_2^0)$$

$$\left.\frac{dT_2}{dt}\right|_{t=0} = C(T_1^0 - T_2^0)$$

c) Del sistema de ecuaciones tenemos

$$\begin{cases} T_1' = -C(T_1 - T_2) \\ T_2' = C(T_1 - T_2) \end{cases}$$

Resolviendo por operadores

$$D[T_1] + CT_1 - CT_2 = 0$$
$$D[T_2] + CT_2 - CT_1 = 0$$

$$(D + C)[T_1] + (-C)[T_2] = 0$$
$$(-C)[T_1] + (D+C)[T_1] = 0$$

$$\Delta = \begin{vmatrix} D+C & -C \\ -C & D+C \end{vmatrix} = D^2 + 2CD$$

$$(D^2 + 2CD)[T_1] = 0$$
$$(D^2 + 2CD)[T_2] = 0$$

$$T_1'' + 2CT_1' = 0 \qquad \Rightarrow \qquad r_1^2 + 2Cr_1 = 0$$
$$r_1 = 0,\ -2C$$

$$T_2'' + 2CT_2' = 0 \qquad \Rightarrow \qquad r_2^2 + 2Cr_2 = 0$$
$$r_2 = 0,\ -2C$$

$$T_1 = A_1 e^{-2ct} + A_2 \quad \rightarrow \quad T_1' = -2ct A_1 e^{-2ct}$$

$$T_2 = B_1 e^{-2ct} + B_2 \quad \rightarrow \quad T_2' = -2ct B_1 e^{-2ct}$$

Del sistema original, se sabe que

$$T_1' = -T_2'$$

$$-\cancel{2}\cancel{c}\cancel{t}A_1\cancel{e^{-2ct}} = \cancel{2}\cancel{c}\cancel{t}B_1\cancel{e^{-2ct}}$$

$$A_1 = -B_1$$

Reemplazando en el sistema origi

$$-\cancel{2ct}\cancel{A_1e^{-2ct}} = -\cancel{c}(\cancel{A_1e^{-2ct}} + A_2 - \cancel{B_1e^{-2ct}} - B_2)$$

$$0 = A_2 - B_2$$

$$A_2 = B_2$$

De las condiciones iniciales,

$$T_1(0) = 400 \qquad T_2(0) = 200$$

$$A_1 + A_2 = 400 \qquad B_1 + B_2 = 200$$

$$-B_1 + B_2 = 400 \qquad B_2 = 200 - B_1$$

$$-2B_1 = 200$$

$$B_1 = -100$$

$$B_2 = 300, \quad A_2 = 300$$

$$A_1 = 100$$

Entonces,

$$T_1(t) = 300 + 100e^{-2ct}$$

$$T_2(t) = 300 - 100e^{-2ct}$$

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