Fourier ejercicios métodos computacionales

Saturday, 5 February 2022 7:58 PM

1.1. Demostrar que $\int_{t_1}^{t_2} f(t)\,dt = \frac{a_0(t_2-t_1)}{2} + \sum_{n=1}^{\infty} \frac{1}{nW_0}\left[-b_n\left(\cos(nW_0t_2)-\cos(nW_0t_1)\right) + a_n\left(\operatorname{sen}(nW_0t_2)-\operatorname{sen}(nW_0t_1)\right)\right]$

Sea $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos(nW_0t) + b_n\sin(nW_0t)\right)$

1 Multipliquemos por $1 = \frac{dt}{dt}$

$$f(t)\,dt = \frac{a_0}{2}\,dt + \sum_{n=1}^{\infty} a_n\cos(nW_0t)\,dt + b_n\sin(nW_0t)\,dt$$

2 Integremos de t_1 a t_2

$$\int_{t_1}^{t_2} f(t)\,dt = \int_{t_1}^{t_2}\frac{a_0}{2}\,dt + \sum_{n=1}^{\infty}\int_{t_1}^{t_2} a_n\cos(nW_0t)\,dt + \int_{t_1}^{t_2} b_n\sin(nW_0t)\,dt$$
$$\int_{t_1}^{t_2} f(t)\,dt = \frac{a_0}{2}\int_{t_1}^{t_2} dt + \sum_{n=1}^{\infty} a_n\int_{t_1}^{t_2}\cos(nW_0t)\,dt + b_n\int_{t_1}^{t_2}\sin(nW_0t)\,dt$$

Sea $u = nW_0t$, $du = nW_0\,dt$ y $dt\ \frac{du}{nW_0}$

$$\int_{t_1}^{t_2} f(t)\,dt = \frac{a_0}{2}\int_{t_1}^{t_2} dt + \sum_{n=1}^{\infty}\frac{a_n}{nW_0}\int\cos(u)\,du + \frac{b_n}{nW_0}\int\sin(u)\,du$$
$$\int_{t_1}^{t_2} f(t)\,dt = \frac{a_0}{2}[t_2-t_1] + \sum_{n=1}^{\infty}\frac{a_n}{nW_0}[\sin(u)] + \frac{b_n}{nW_0}[-\cos(u)]$$
$$\int_{t_1}^{t_2} f(t)\,dt = \frac{a_0}{2}[t_2-t_1] + \sum_{n=1}^{\infty}\frac{a_n}{nW_0}[\sin(nW_0t)]_{t_1}^{t_2} + \frac{b_n}{nW_0}[-\cos(nW_0t)]_{t_1}^{t_2}$$
$$\int_{t_1}^{t_2} f(t)\,dt = \frac{a_0(t_2-t_1)}{2} + \sum_{n=1}^{\infty}\frac{1}{nW_0}\left[-b_n\left(\cos(nW_0t_2)-\cos(nW_0t_1)\right) + a_n\left(\operatorname{sen}(nW_0t_2)-\operatorname{sen}(nW_0t_1)\right)\right] \therefore$$

1.2 Encontrar la serie de fourier de $f(t) = t$ para $(-\pi,\pi)$ y $f(t+2\pi) = f(t)$

$$f(t) = 2\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\sin(nt)}{n}$$

Sea $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nW_0t) + \sum_{n=1}^{\infty} b_n\sin(nW_0t)$

$$\frac{a_0}{2} = \frac{1}{T}\int_0^T f(t)\,dt, \text{ donde } T \text{ es } 2\pi$$
$$\frac{a_0}{\cancel{2}} = \frac{1}{\cancel{2}\pi}\int_{-\pi}^{\pi} t\,dt$$
$$a_0 = \int_{-\pi}^{\pi} t\,dt = \left[\frac{t^2}{2}\right]_{-\pi}^{\pi} = \frac{\pi^2}{2} - \frac{\pi^2}{2} = 0$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(nW_0t)\,dt$$
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t\cos(nW_0t)\,dt \Rightarrow u = t \quad dv = \cos(nW_0t)\,dt$$
$$du = dt \quad v = \frac{\sin(nW_0t)}{nW_0}$$
$$a_n = \frac{1}{\pi}\left[t\,\frac{\sin(nW_0t)}{nW_0} - \int_{-\pi}^{\pi}\frac{\sin(nW_0t)}{nW_0}\,dt\right]$$
$$a_n = \frac{1}{\pi}\left[t\,\frac{\sin(nW_0t)}{nW_0} + \left[\frac{\cos(nW_0t)}{n^2W_0^2}\right]_{-\pi}^{\pi}\right]$$
$$a_n = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(nW_0t)\,dt$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} t\sin(nW_0t)\,dt$$
$$b_n = \frac{1}{\pi}\left[-t\,\frac{\cos(nW_0t)}{nW_0} - \left[\frac{\sin(nW_0t)}{n^2W_0^2}\right]_{-\pi}^{\pi}\right]$$
$$b_n = \frac{2(-1)^{nW_0-1}}{nW_0}\sin(nW_0t), \text{ Con } n = nW_0 \text{ ya que } W=1$$
$$b_n = \frac{2(-1)^{n-1}}{n}\sin(nt)$$

$$f(t) = 2\sum_{n}^{\infty}\frac{(-1)^{n-1}}{n}\sin(nt) \therefore$$

1.3 Integrar la serie de Fourier de $f(t) = t^2$

Sea $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(nW_0t) + \sum_{n=1}^{\infty} b_n\sin(nW_0t)$

$$\frac{a_0}{2} = \frac{1}{T}\int_0^T f(t)\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2\,dt$$
$$a_0 = \frac{1}{\pi}\left[\frac{t^3}{3}\right]_{-\pi}^{\pi} \Rightarrow \frac{1}{\pi}\left[\frac{\pi^3}{3} + \frac{\pi^3}{3}\right] = \frac{1}{\pi}\left[\frac{2\pi^3}{3}\right] = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(nW_0t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t^2\cos(nt)\,dt, \quad n = nW_0 \text{ Con } W_0 = 1$$
$$a_n = \frac{1}{\pi}\left[\frac{(n^2t^2-2)}{n^3}\cancel{(\sin(nt))}^{0} + \frac{2nt\cos(nt)}{n^3}\right]_{-\pi}^{\pi}$$
$$a_n = \frac{2}{\pi n^3}\left[2\pi n\cos(\pi n)\right]$$
$$a_n = \frac{4}{n^2}\left[\cos(\pi n)\right] = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(nW_0t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t^2\sin(nt)\,dt, \quad n = nW_0 \text{ Con } W_0 = 1$$
$$b_n = \frac{1}{\pi}\left[\cancel{\frac{2nt\sin(nt)}{n^3}}^{0} + \frac{(2-n^2t^2)\cos(nt)}{n^3}\right]_{-\pi}^{\pi}$$
$$b_n = \frac{1}{\pi}\left[\frac{(2-n^2t^2)\cos(nt)}{n^3}\right]_{-\pi}^{\pi}$$
$$b_n = 0$$

$$\int_0^t f(t)\,dt = \frac{a_0}{2}\int_0^t dt + \sum_{n=1}^{\infty} a_n\int_0^t\cos(nt)\,dt$$
$$\int_0^t f(t)\,dt = \frac{\pi^2}{3}\int_0^t dt + \sum_{n=1}^{\infty}\frac{4}{n^2}(-1)^n\int_0^t\cos(nt)\,dt$$
$$\int_0^t f(t)\,dt = \frac{\pi^2}{3}t + \sum_{n=1}^{\infty}\frac{4(-1)^n}{n^3}\sin(nt)$$
$$\frac{t^3}{3} - \frac{\pi^2}{3}t = 4\sum_{n=1}^{\infty}\frac{4(-1)^n}{n^3}\sin(nt)$$
$$\frac{t}{12}(t^2-\pi^2) = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\sin(nt)$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{t}{12}(t^2-\pi^2)\right)^2 dt = \sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n^3}\sin(nt)\right)^2$$