Nominal C Unification

Seminário de Computação - UnB

Gabriel Silva, Advisor: Mauricio Ayala

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Department of Mathematics - University of Brasília

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Background

Nominal Terms, Permutations and Substitutions

Atoms and Variables

Consider a set of variables $\mathbb{X} = \{X, Y, Z, \dots\}$ and a set of atoms $\mathbb{A} = \{a, b, c, \dots\}$.

Permutations

A permutation π represents an exchange of a finite amount of atoms in $\mathbb{A}$ and is usually represented by a list of swappings:

$$\pi = (a_1, b_1) :: ... :: (a_n, b_n) :: nil$$

Nominal Terms

Definition (Nominal Terms)

Nominal terms are inductively generated according to the grammar:

$$s, t \quad ::= \quad \langle\rangle \;\mid\; a \;\mid\; \pi \cdot X \;\mid\; [a]t \;\mid\; \langle s, t\rangle \;\mid\; f\ t$$

The symbols denote respectively: unit, atom term, moderated variable, abstraction and function application.

Permutation Action on an Atom

Definition (Permutation Action on an Atom)

The action of a permutation on an atom is defined by:

- $nil \cdot a := a$
- $(b\ c) :: \pi \cdot a := \pi \cdot a$
- $(b\ c) :: \pi \cdot b := \pi \cdot c$

Permutation Action on a Term

Definition (Permutation Action on a Term)
The action of a permutation π on a term t is generated by induction on the structure of the term t.

$$
\begin{aligned}
\pi \cdot \langle\rangle &= \langle\rangle & \pi \cdot (\pi' \cdot X) &= (\pi \oplus \pi') \cdot X \\
\pi \cdot \langle s, t\rangle &= \langle \pi \cdot s, \pi \cdot t\rangle & \pi \cdot ([a]t) &= [\pi \cdot a](\pi \cdot t) \\
\pi \cdot (f\ \ t) &= f(\pi \cdot t) & &
\end{aligned}
$$

Substitution

Definition (Substitution)

A substitution σ is a mapping from variables to terms, such that $\{X \mid X \neq X\sigma\}$ is finite.

Substitution Action on Terms

Definition (Substitution Action on a Term)
The action of a substitution σ on a term t is denoted by σt and defined recursively by:

$$\begin{aligned} \langle\rangle\sigma &= \langle\rangle & (\pi \cdot X)\sigma &= \pi \cdot X\sigma \\ \langle s, t\rangle\sigma &= \langle \sigma s, \sigma t\rangle & ([a]t)\sigma &= [a](t\sigma) \\ (f\ \ t)\sigma &= f(t\sigma) & & \end{aligned}$$

OBS: Substitutions and permutations commute.

Background

Freshness and α-Equality

Intuition Behind the Concepts

Two important predicates are the freshness predicate $\#$ and the α-equality predicate $\approx_\alpha$:

- $a\#t$ intuitively means that if a occurs in t then it must do so under an abstractor $[a]$.
- $s \approx_\alpha t$ intuitively means that s and t are α-equivalent.

Derivation Rules for Freshness

$$\frac{}{a\#b}\,(\#atom) \qquad \frac{\pi^{-1}(a)\#X}{a\#\pi \cdot X}\,(\#X)$$

$$\frac{}{a\#b}\,(\#ab)$$