Functional Nominal C-Unification

Seminário de Computação - UnB

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Background

Nominal Terms, Permutations and Substitutions

Atoms and Variables

Consider a set of variables $\mathbb{X} = \{X, Y, Z, \dots\}$ and a set of atoms $\mathbb{A} = \{a, b, c, \dots\}$.

Permutations

An atom permutation π represents an exchange of a finite amount of atoms in $\mathbb{A}$ and is represented by a list of swappings:

$$\pi = (a_1, b_1) :: ... :: (a_n, b_n) :: nil$$

Nominal Terms

Definition (Nominal Terms)

Nominal terms are inductively generated according to the grammar:

$$s, t \quad ::= \quad \langle\rangle \;\mid\; a \;\mid\; \pi \cdot X \;\mid\; [a]t \;\mid\; \langle s, t\rangle \;\mid\; f\ t$$

The symbols denote respectively: unit, atom term, suspended variable, abstraction, pair and function application.

Examples of Permutation Actions

Permutations acts on atoms and terms, here are some examples:

- $t = a$, $\pi = (a\ b)$, $\pi \cdot t = b$.
- $t = f(a, c)$, $\pi = (a\ b)$ and $\pi \cdot t = f(b, c)$.
- $t = [a]a$, $\pi = (a\ b) :: (b\ c)$, $\pi \cdot t = [b]b$.

Substitution

Definition (Substitution)
A substitution σ is a mapping from variables to terms, such that $\{X \mid X \neq X\sigma\}$ is finite.

Examples of Substitutions Acting on Terms

Substitutions also act on terms:

- $\sigma = \{X \to a\}$, $t = f(X, X)$, $t\sigma = f(a, a)$.
- $\sigma = \{X \to f(a, b)\}$, $t = f(X, c)$, $t\sigma = f(f(a, b), c)$.

Background

Freshness and α-Equality

Intuition Behind the Concepts

Two important predicates are the freshness predicate $\#$ and the α-equality predicate $\approx_\alpha$:

- $a\#t$ intuitively means that if a occurs in t then it must do so under an abstractor $[a]$.
- $s \approx_\alpha t$ intuitively means that s and t are α-equivalent.

Contexts

A context is a set of constraints of the form $a\#X$. Contexts are denoted by the letters Δ, ∇ or Γ.

Derivation Rules for Freshness

$$\frac{}{\Delta \vdash a\#\langle\rangle}\,(\#\langle\rangle) \qquad \frac{}{\Delta \vdash a\#b}\,(\#atom)$$

$$\frac{(\pi^{-1}(a)\#X) \in \Delta}{\Delta \vdash a\#\pi \cdot X}\,(\#X) \qquad \frac{}{\Delta \vdash a\#[a]t}\,(\#[a]a)$$

$$\frac{\Delta \vdash a\#t}{\Delta \vdash a\#[b]t}\,(\#[a]b) \qquad \frac{\Delta \vdash a\#s \quad \Delta \vdash a\#t}{\Delta \vdash a\#\langle s, t\rangle}\,(\#pair)$$

$$\frac{\Delta \vdash a\#t}{\Delta \vdash a\#f\ \ t}\,(\#app)$$

Derivation Rules for α-Equivalence

$$\frac{}{\Delta \vdash \langle\rangle \approx_\alpha \langle\rangle}\,(\approx_\alpha \langle\rangle) \qquad \frac{}{\Delta \vdash a \approx_\alpha a}\,(\approx_\alpha \mathit{atom})$$

$$\frac{\Delta \vdash s \approx_\alpha t}{\Delta \vdash fs \approx_\alpha ft}\,(\approx_\alpha \mathit{app}) \qquad \frac{\Delta \vdash s \approx_\alpha t}{\Delta \vdash [a]s \approx_\alpha [a]t}\,(\approx_\alpha [a]a)$$

$$\frac{\Delta \vdash s \approx_\alpha (ab)\cdot t,\ a\#t}{\Delta \vdash [a]s \approx_\alpha [b]t}\,(\approx_\alpha [a]b) \qquad \frac{ds(\pi,\pi')\#X \subseteq \Delta}{\Delta \vdash \pi\cdot X \approx_\alpha \pi'\cdot X}\,(\approx_\alpha \mathit{var})$$

$$\frac{\Delta \vdash s_0 \approx_\alpha t_0,\ \ \Delta \vdash s_1 \approx_\alpha t_0}{\Delta \vdash \langle s_0, s_1\rangle \approx_\alpha \langle t_0, t_1\rangle}\,(\approx_\alpha \mathit{pair})$$

Additional Rule for α-Equivalence with Commutative Symbols

We need to add a rule to take into account commutative function symbols. Therefore, if a function symbol is commutative, the following rule can be applied:

$$\frac{\Delta \vdash s_0 \approx_\alpha t_1, \;\; \Delta \vdash s_1 \approx_\alpha t_0}{\Delta \vdash f(\langle s_0, s_1\rangle) \approx_\alpha f(\langle t_0, t_1\rangle)} \, (\approx_\alpha C - pair)$$

Examples of Derivation Rules as a Calculus

The derivation rules for freshness and α-equivalence can be seen as a calculus: