



UNIVERSIDADE FEDERAL
DO RIO DE JANEIRO

SISTEMAS LINEARES 1

Trabalho Final

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Curso: Engenharia Eletrônica

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DRE: 118044310

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1 Questão 1

1.1 Circuito 1

Letra A

O circuito 1 consiste em um RC ligado em série, uma boa aplicação comercial para este circuito é um filtro passa-baixa. O filtro passa-baixa permite que apenas frequências abaixo da frequência de corte sejam levadas ao amplificador. No caso em questão o filtro será utilizado para deixar passar apenas frequências abaixo de $\omega = 100 \text{ rad/s}$.

Letra B

$$R = 10\,000\,\Omega$$

$$C = 10^{-6}\,F$$

Letra C

De acordo com a lei de Kirchhoff das tensões a soma das tensões na malha devem ser iguais a zero. Deste modo, substituindo as tensões de cada componente temos:

$$V(t) = R \cdot \frac{\partial q(t)}{\partial t} + q(t) \cdot \frac{1}{C}$$

Letra D

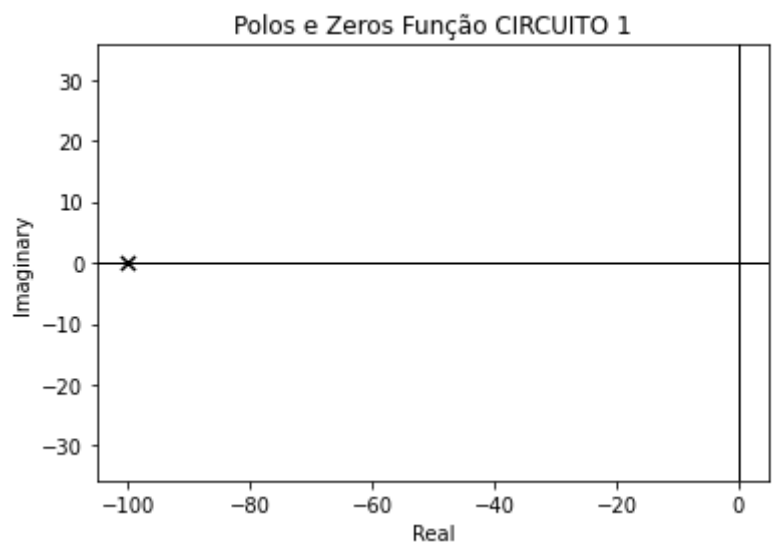
Para encontrar a função de transferência primeiro aplicamos a transformada de Laplace em todos os termos e então dividimos a função de saída pela entrada:

$$Q(s)V(s)^{-1} = RC^{-1} \cdot \frac{1}{s+RC^{-1}}$$

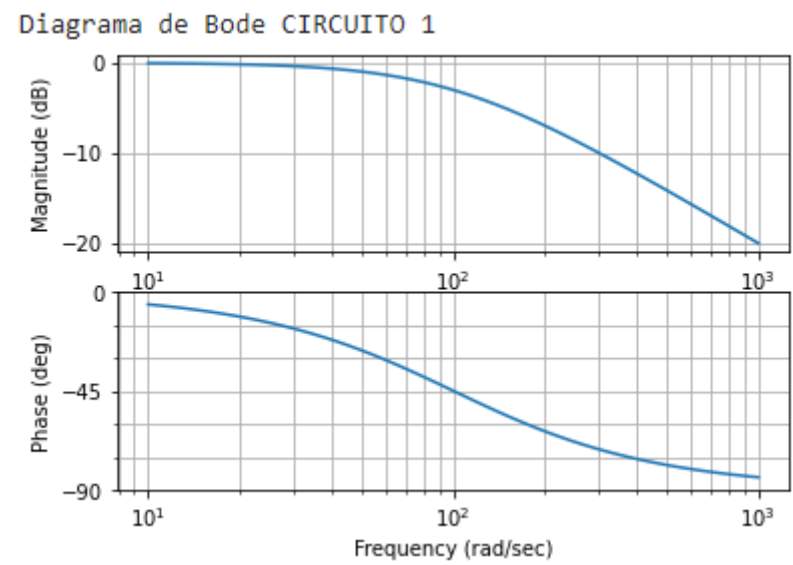
Substituindo os valores temos:

$$Q(s)V(s)^{-1} = 100 \cdot \frac{1}{s+100}$$

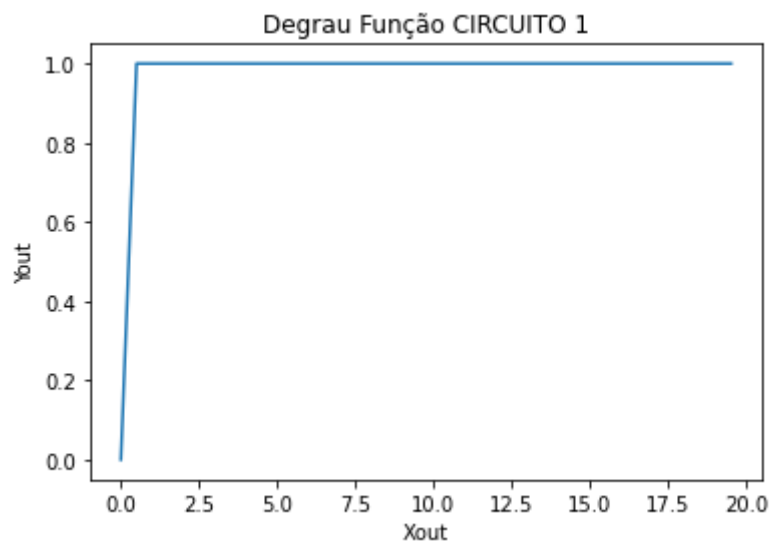
Letra E



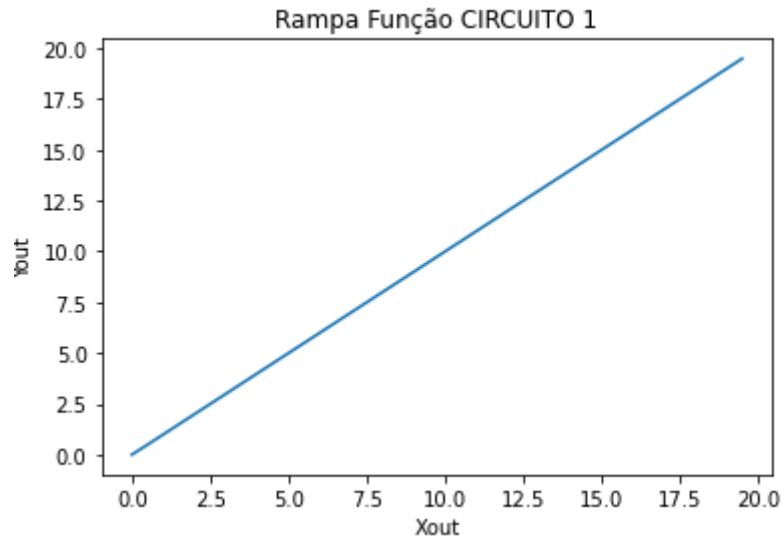
Letra F



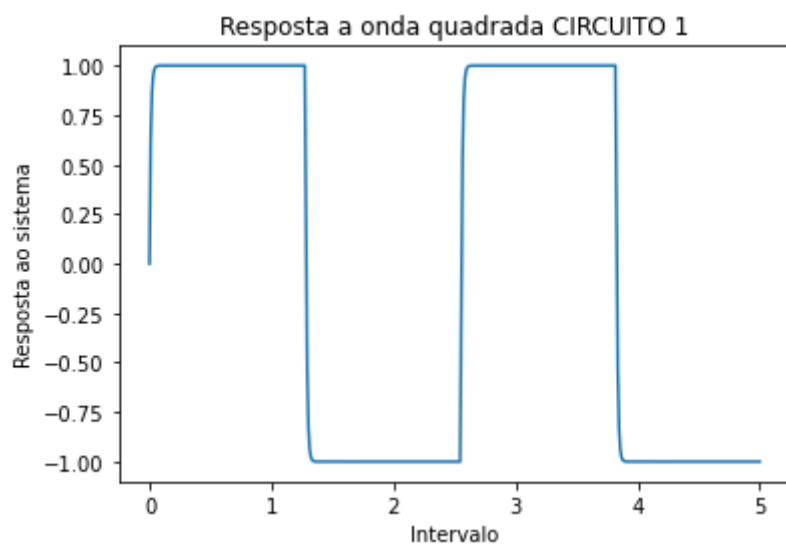
Letra G



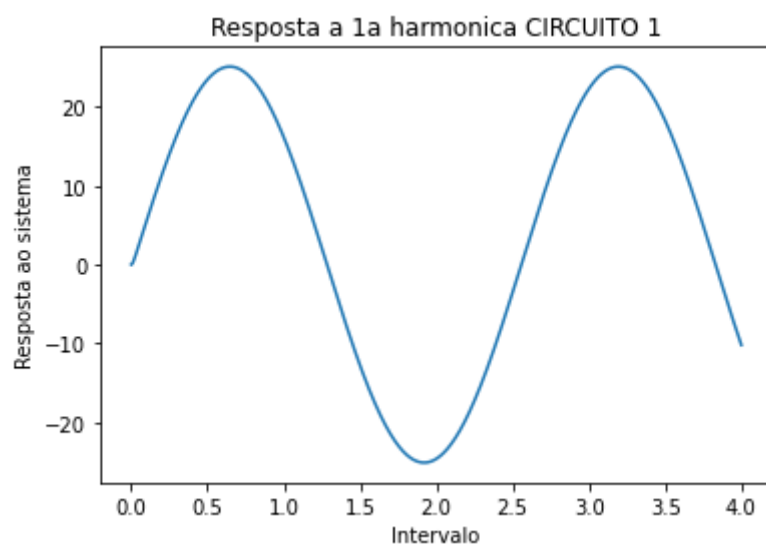
Letra H

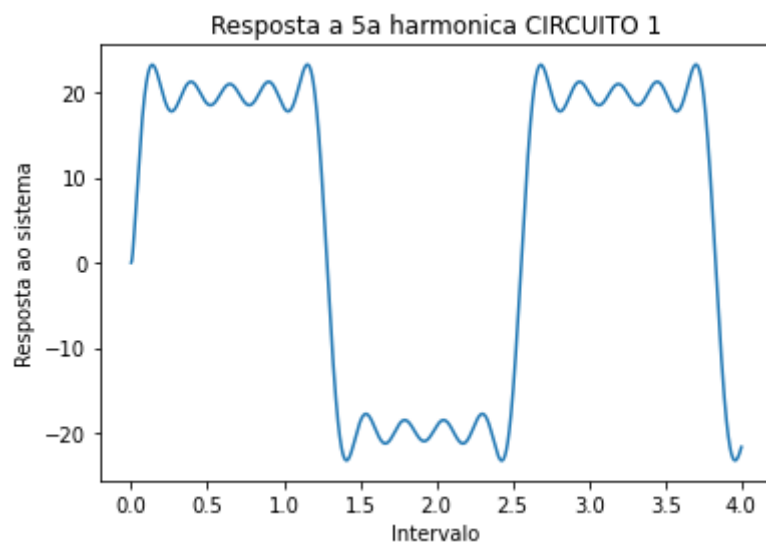
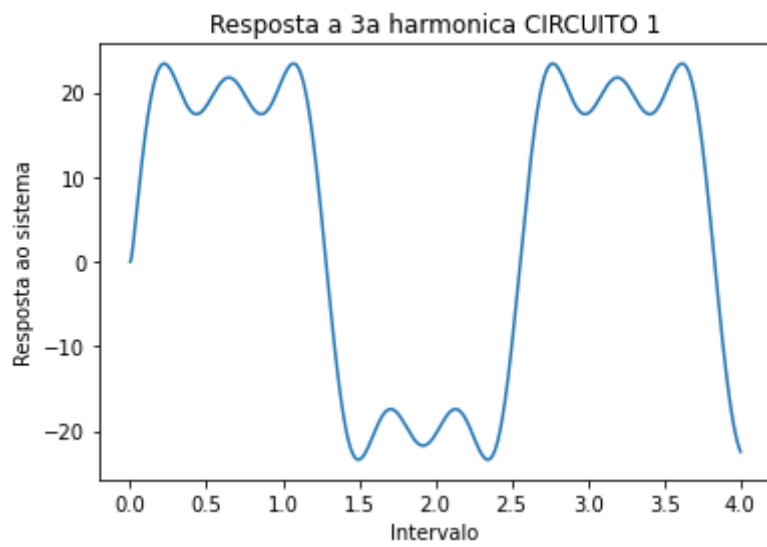


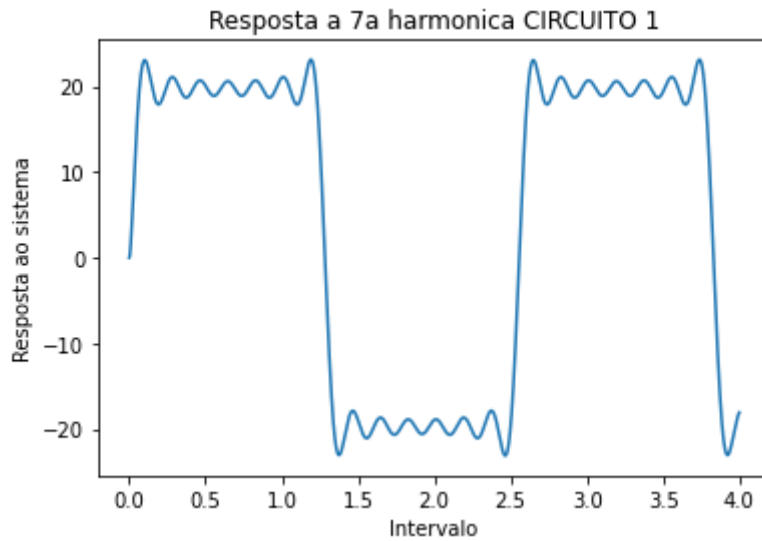
Letra I



Letra J,K,L e M







1.2 Circuito 2

Letra A

O circuito 2 consiste em um RL ligado em série, uma boa aplicação comercial para este circuito é a de filtro passa-alta. Analogamente ao filtro passa-baixa, o filtro passa-alta permite que apenas frequências acima da frequência de corte sejam passadas para a saída do sistema. No presente trabalho a frequência de corte foi de $\omega = 10^{-7} \text{ rad/s}$.

Letra B

$$R = 10\,000\Omega$$

$$L = 10^{-3}H$$

Letra C

Utilizando a lei de Kirchhoff das tensões e destrinchando as tensões de cada componente obtemos:

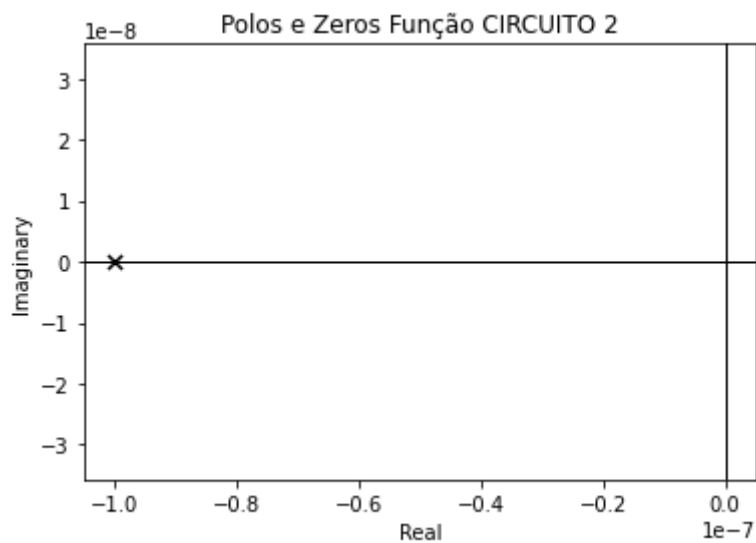
$$V(t) = Ri(t) + L \cdot \frac{\partial i(t)}{\partial t}$$

Letra D

$$I(s) \cdot V(s)^{-1} = \frac{1}{R + Ls}$$

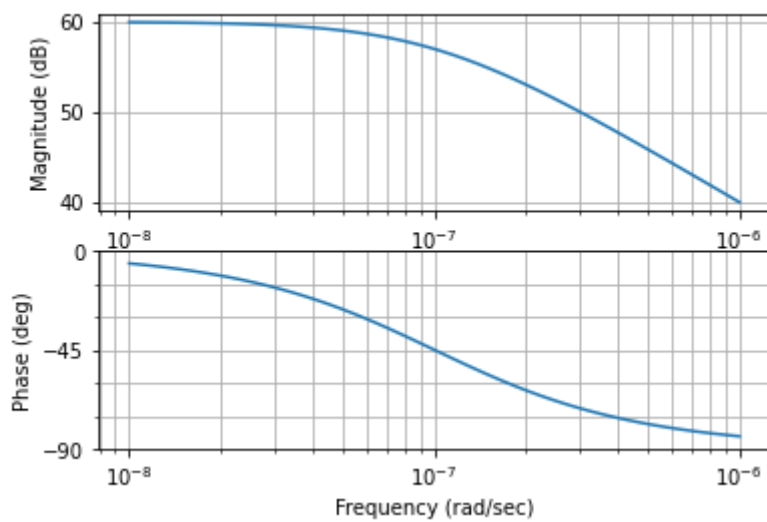
$$\text{substituindo pelos valores dos componentes: } I(s) \cdot V(s)^{-1} = \frac{1}{10^4 + 10^{-3}s}$$

Letra E

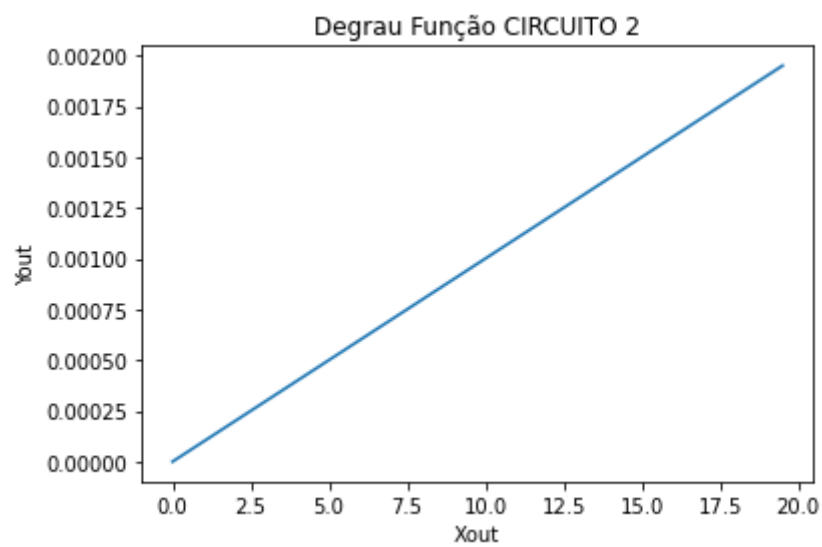


Letra F

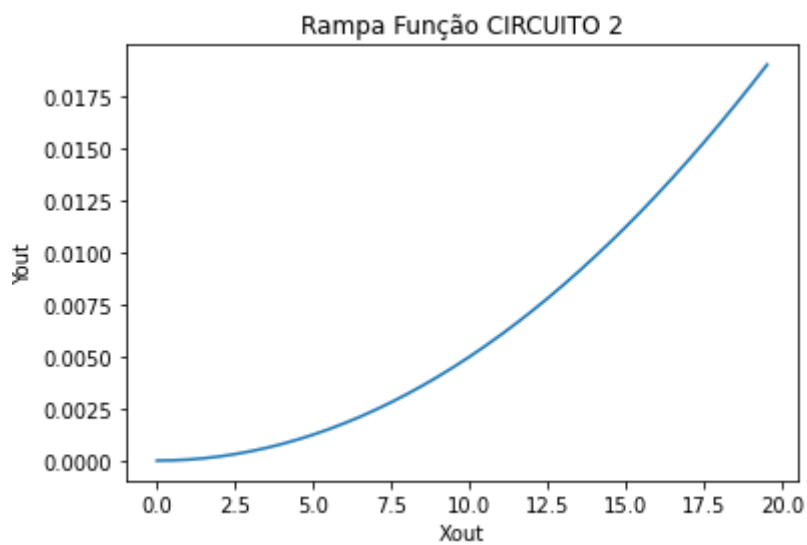
Diagrama de Bode CIRCUITO 2



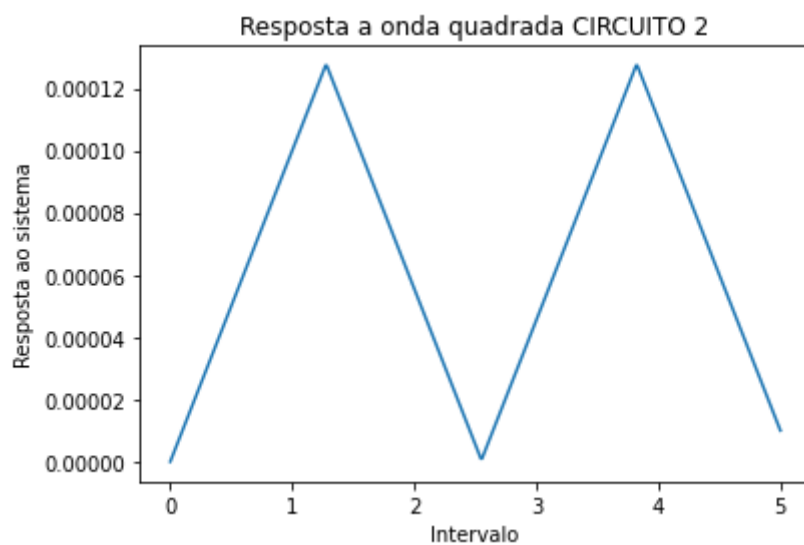
Letra G



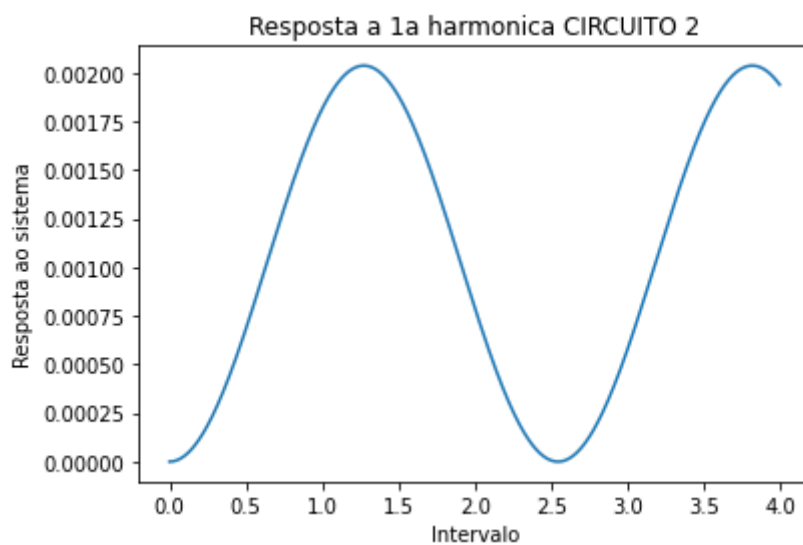
Letra H

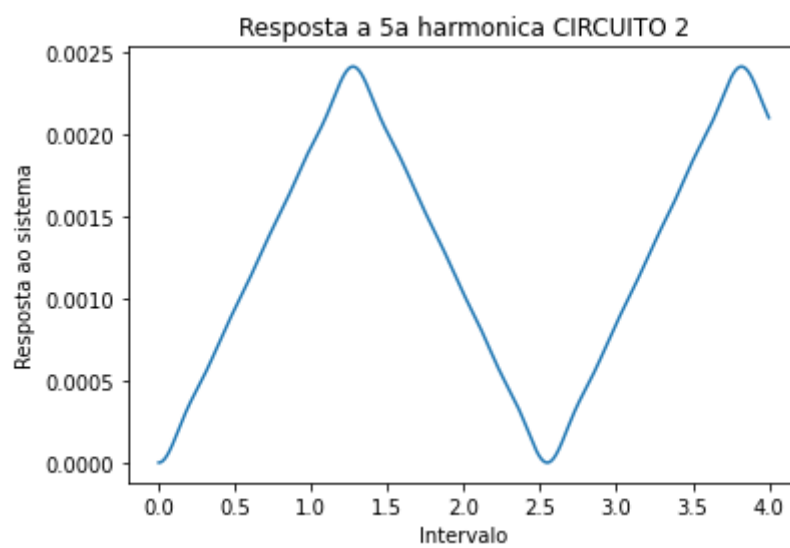
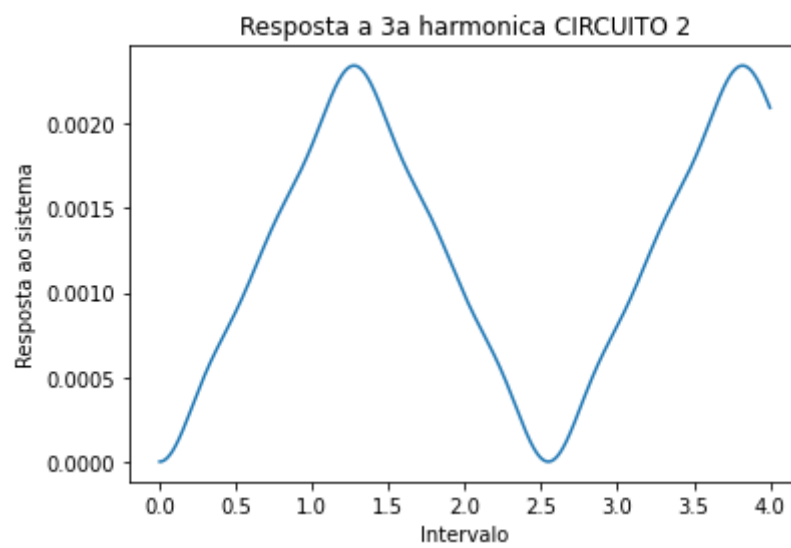


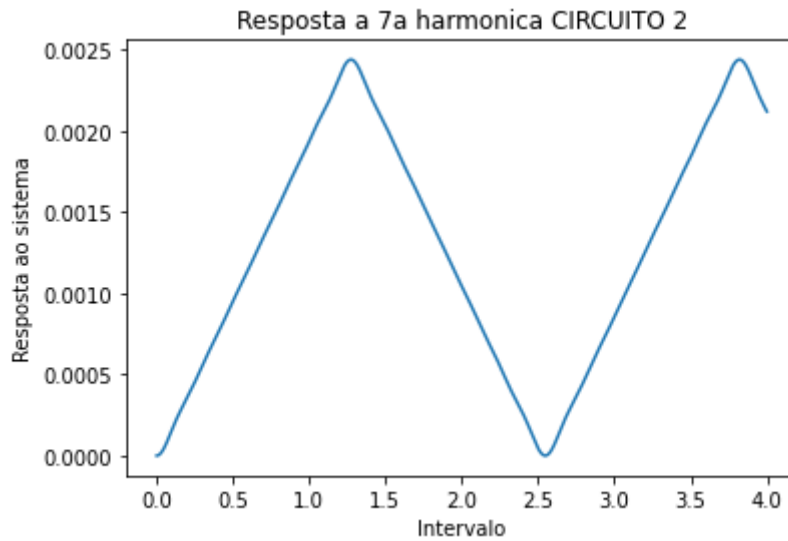
Letra I



Letra J,K,L e M







1.3 Circuito 3

Letra A

O terceiro circuito consiste em um RC paralelo. Esse circuito pode exercer papel de filtro de corrente. No caso, a corrente que queremos filtrar é aquela que tem até $\omega = 10^6 \text{ rad/s}$.

Letra B

$$R = 10^4 \Omega$$

$$C = 10^{-6} F$$

Letra C

Utilizando a lei de kirchoff das correntes e substituindo pelas correntes de seus respectivos componentes temos:

$$i(t) = V(t) \cdot \frac{1}{R} + C \cdot \frac{\partial V(t)}{\partial t}$$

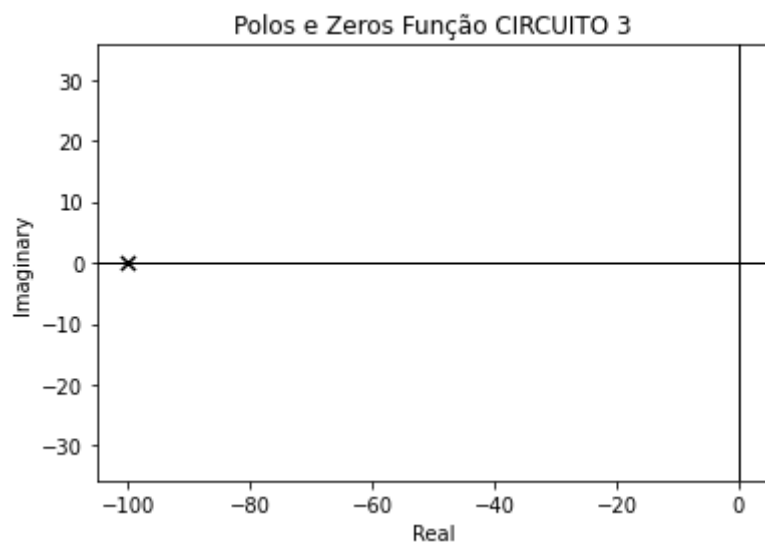
Letra D

$$V(s) \cdot I(s)^{-1} = C^{-1} \cdot \frac{1}{(RC)^{-1} + s}$$

substituindo pelos componentes do circuito temos:

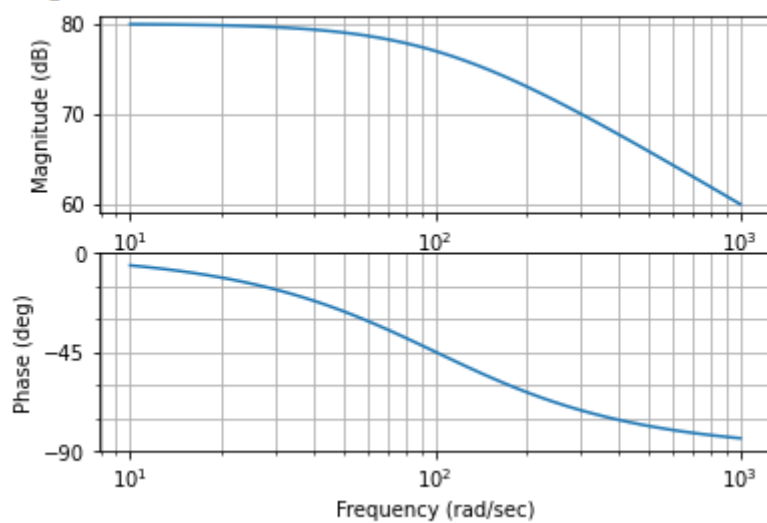
$$V(s) \cdot I(s)^{-1} = 10^6 \cdot \frac{1}{10^2 + s}$$

Letra E

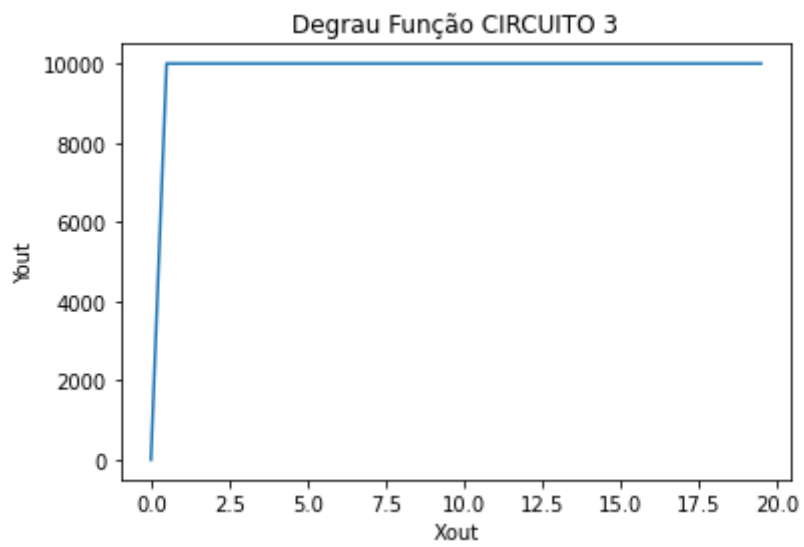


Letra F

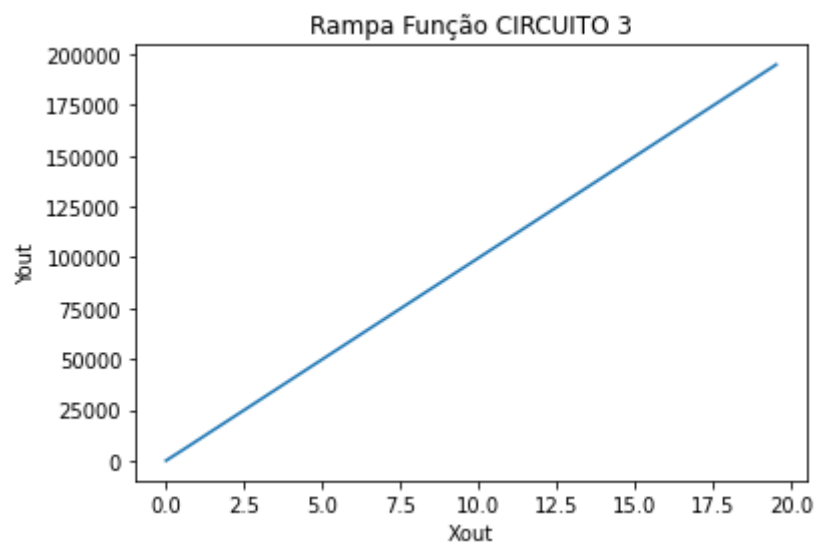
Diagrama de Bode CIRCUITO 3



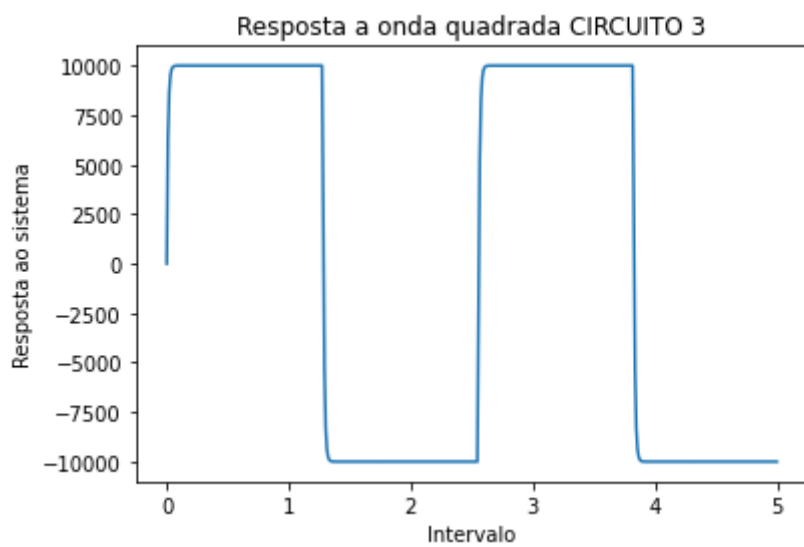
Letra G



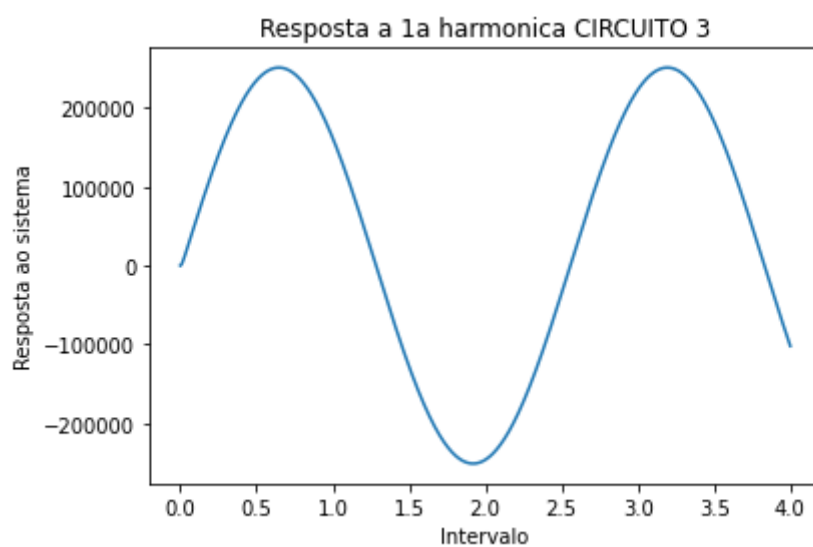
Letra H

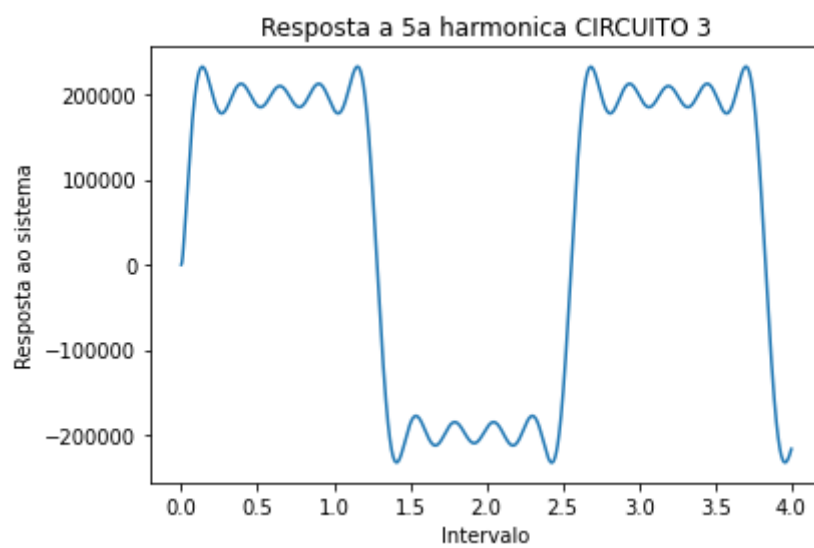
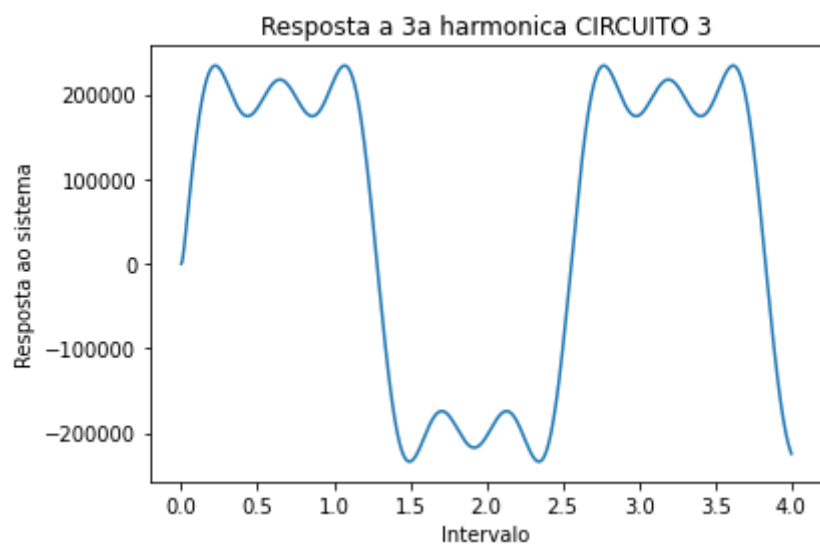


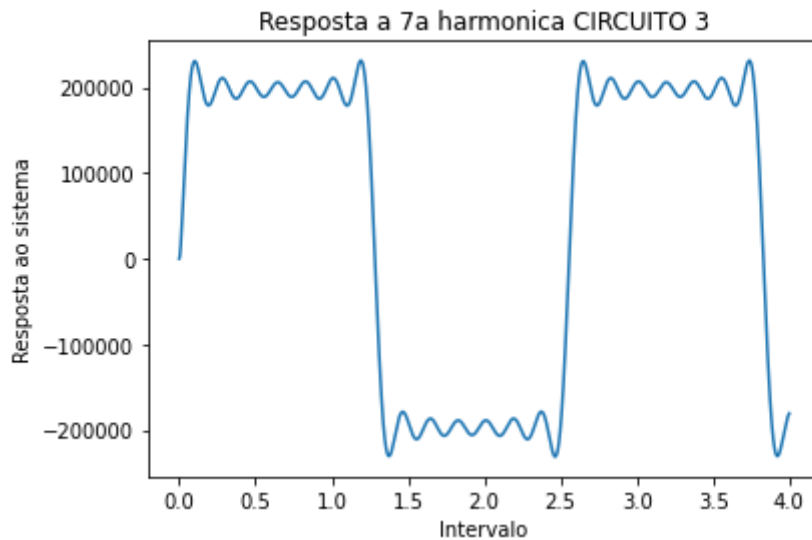
Letra I



Letra J,K,L e M







1.4 Circuito 4

Letra A

O circuito 4 consiste em um circuito RL paralelo. Assim como o paralelo de RC uma boa aplicação para o circuito RL é a filtragem de corrente. Nesse projeto queremos permitir passagem de corrente de até $\omega = 0,7 \text{ rad/s}$.

Letra B

$$R = 10\Omega$$

$$L = 10^{-3}H$$

Letra C

Analogamente ao RC paralelo ao aplicarmos a lei de Kirchhoff das correntes temos:

$$1 \cdot \frac{\partial i(t)}{\partial t} = R^{-1} \cdot \frac{\partial v(t)}{\partial t} + L^{-1} \cdot v(t)$$

Letra D

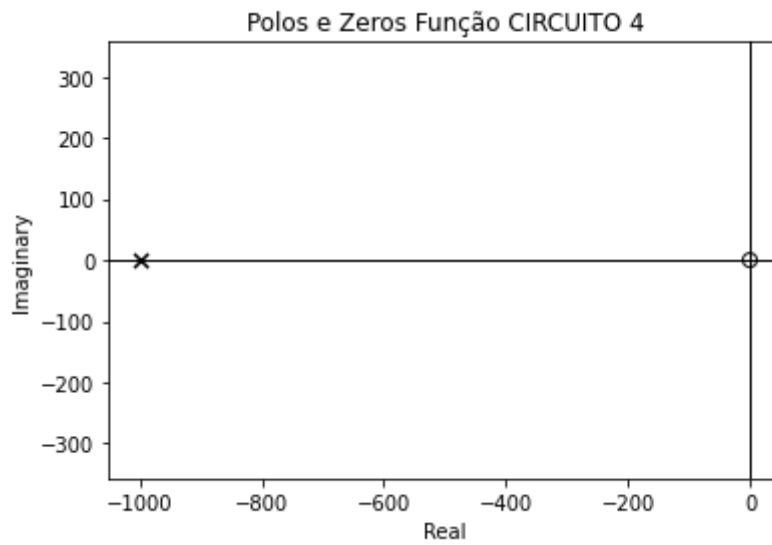
Aplicando a transformada de Laplace nos elementos e dividindo a saída pela entrada temos a função de transferência:

$$V(s) \cdot I(s)^{-1} = Rs \cdot \frac{1}{s + RL^{-1}}$$

Substituindo os elementos:

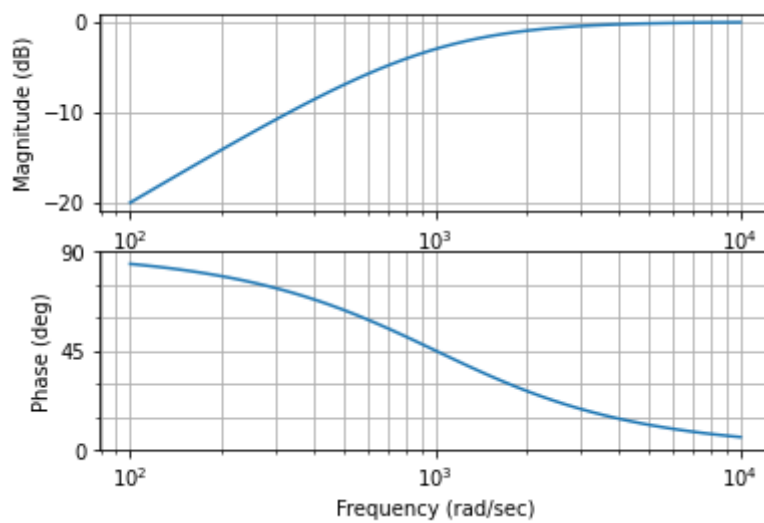
$$V(s) \cdot I(s)^{-1} = 10s \cdot \frac{1}{s+10^3}$$

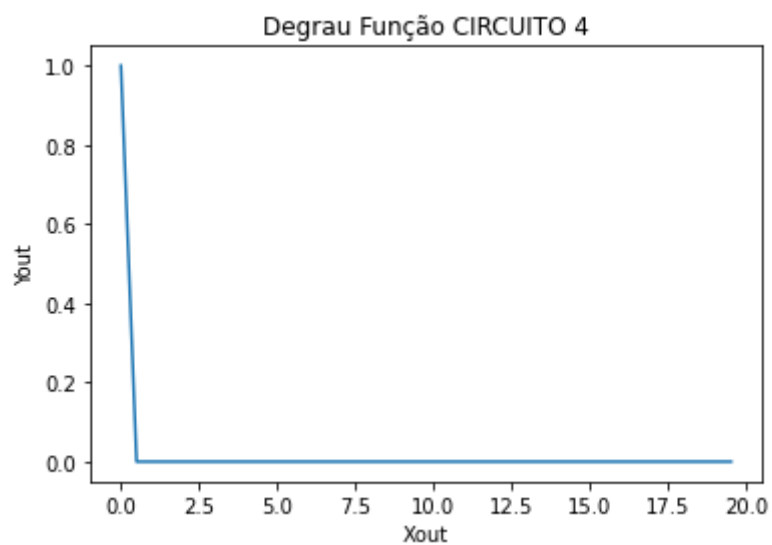
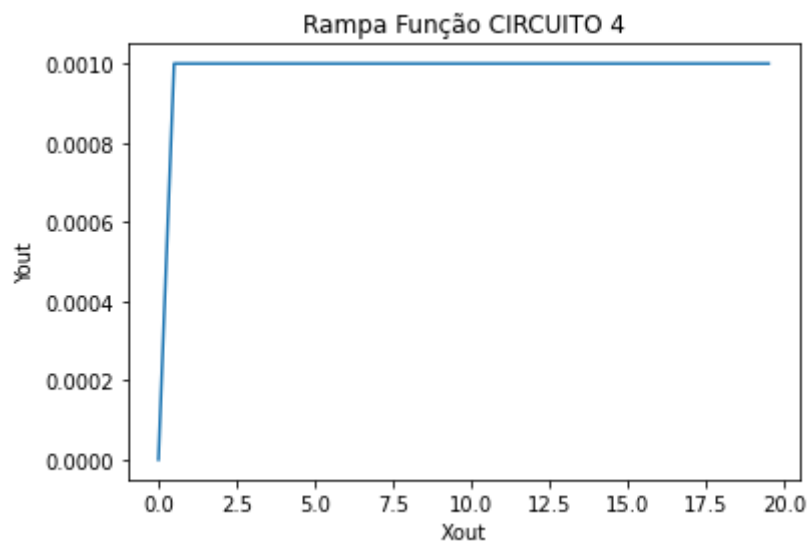
Letra E

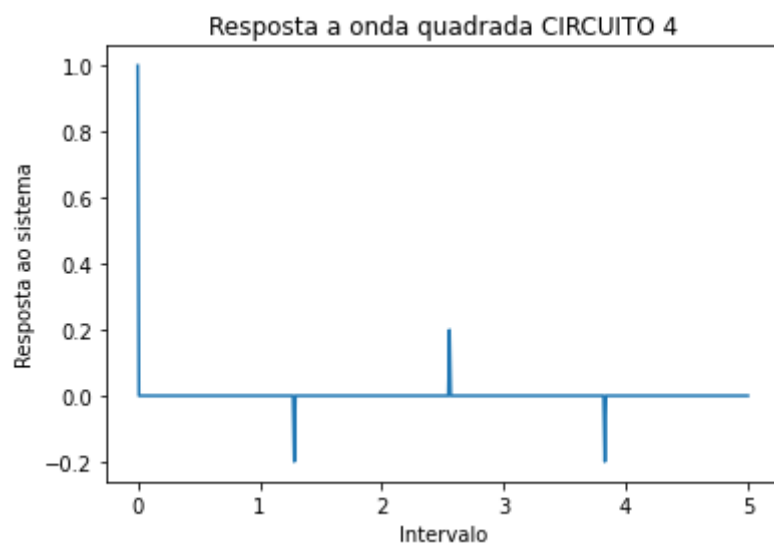


Letra F

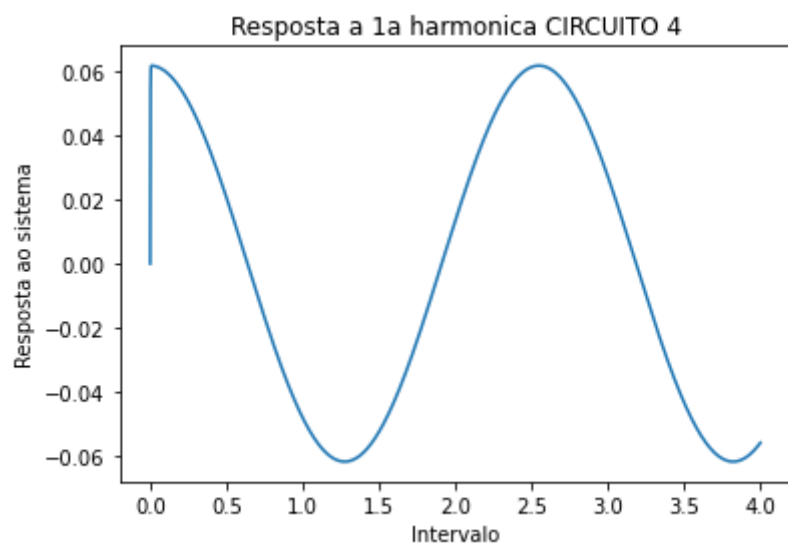
Diagrama de Bode CIRCUITO 4

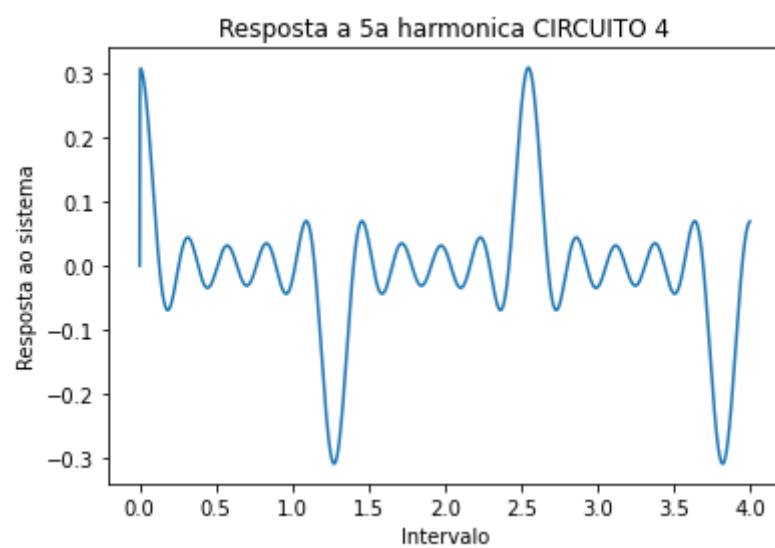
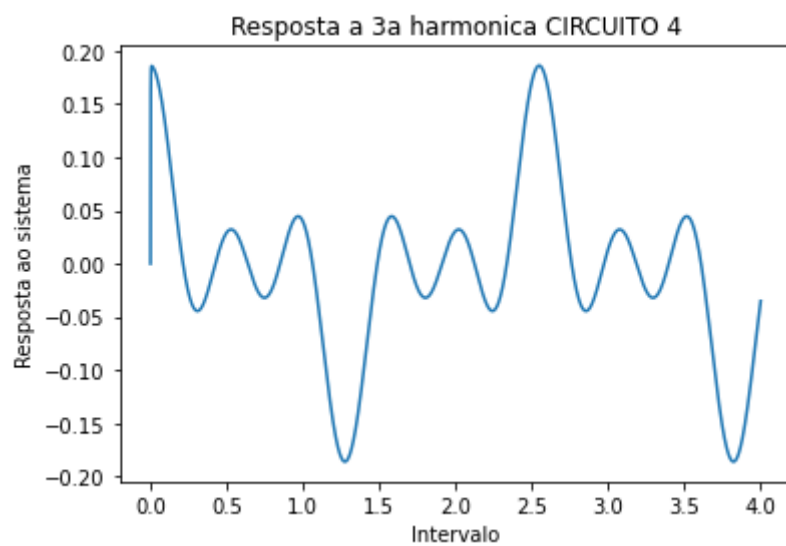


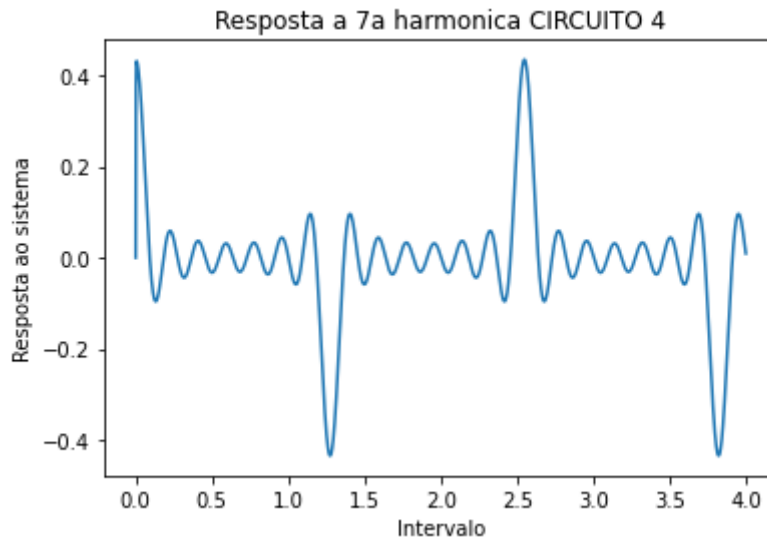
Letra G**Letra H****Letra I**



Letra J,K,L e M







1.5 Circuito 5

Letra A

O circuito 5 consiste em um circuito RCL paralelo. Uma boa aplicação para este circuito é a de filtro passa-faixa. Esse filtro permite a passagem de frequência de valores presentes em uma faixa determinada.

Letra B

$$L = 1,5 \cdot 10^{-3} H$$

$$R = 1 \Omega$$

$$C = 1 F$$

Letra C

EDO do circuito aplicando lei de Kirchhoff das correntes:

$$1 \cdot \frac{\partial i(t)}{\partial t} = R^{-1} \cdot \frac{\partial v(t)}{\partial t} + C \cdot \frac{\partial^2 v(t)}{\partial t^2} + L^{-1} \cdot v(t)$$

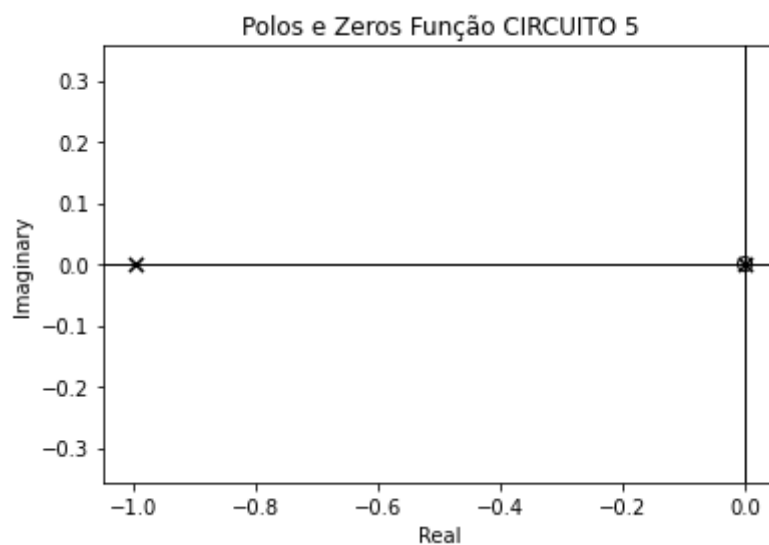
Letra D

A função de transferência:

$$V(s) \cdot I(s)^{-1} = C^{-1} \cdot \frac{s}{s^2 + s \cdot (CR)^{-1} + (CL)^{-1}}$$

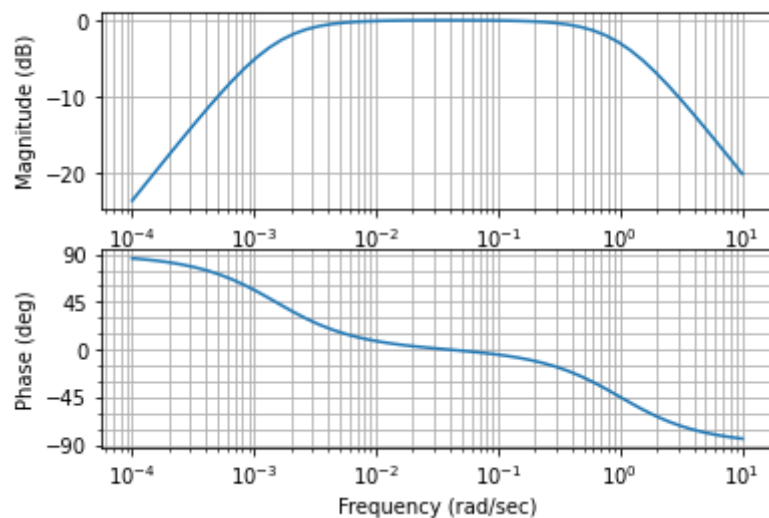
$$V(s) \cdot I(s)^{-1} = 1 \cdot \frac{s}{s^2 + s + 1,5^{-3}}$$

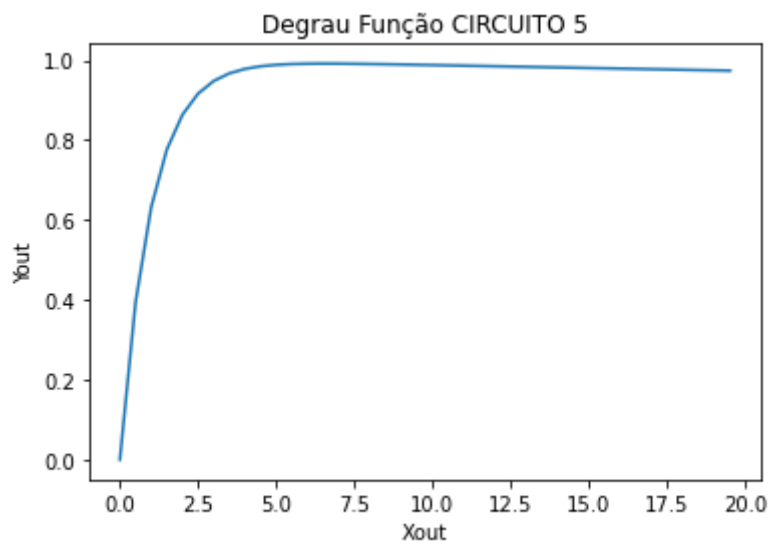
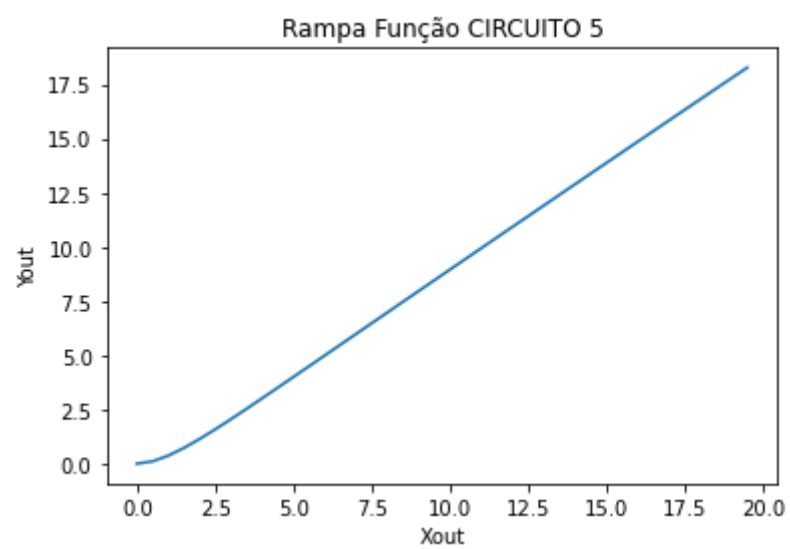
Letra E

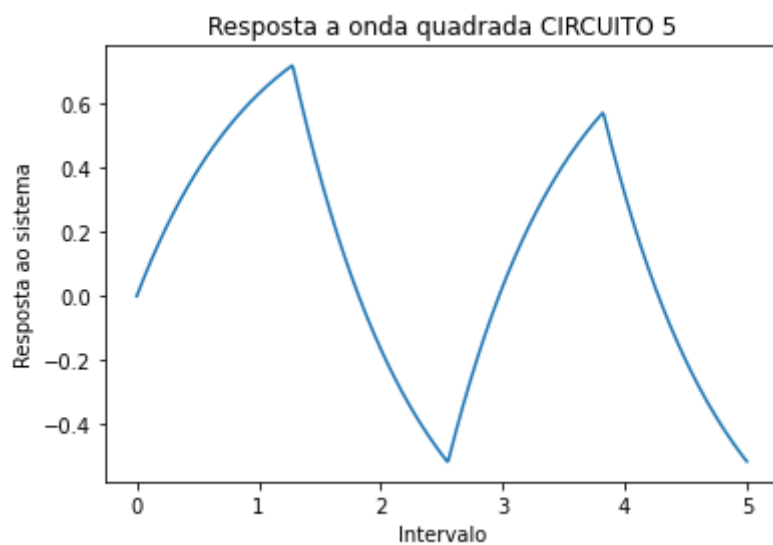


Letra F

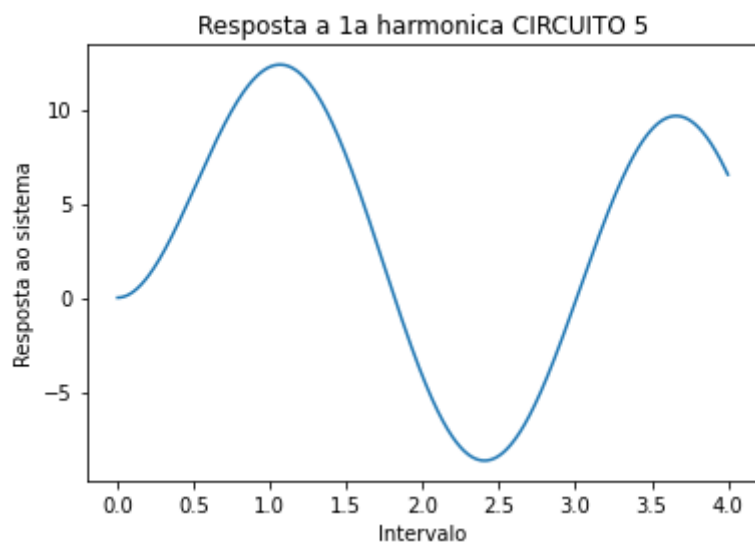
Diagrama de Bode CIRCUITO 5

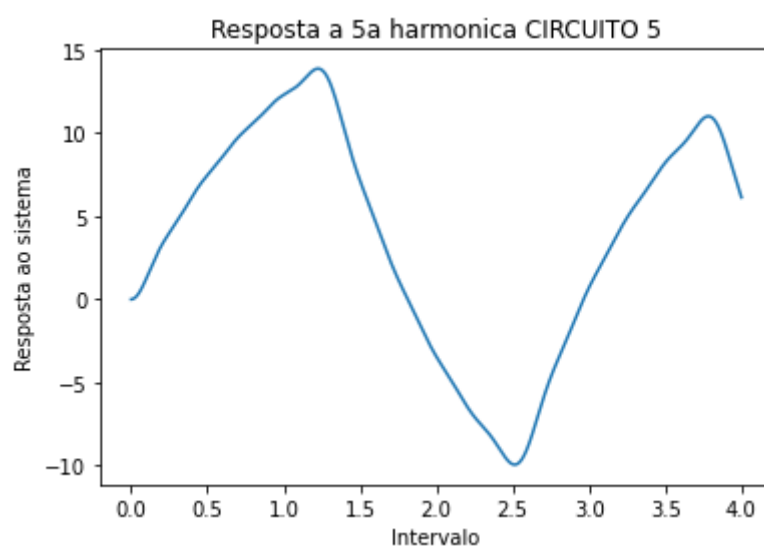
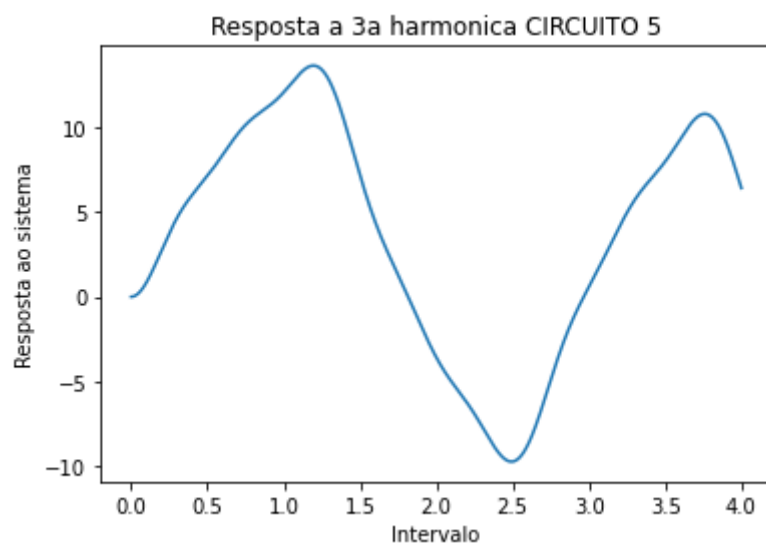


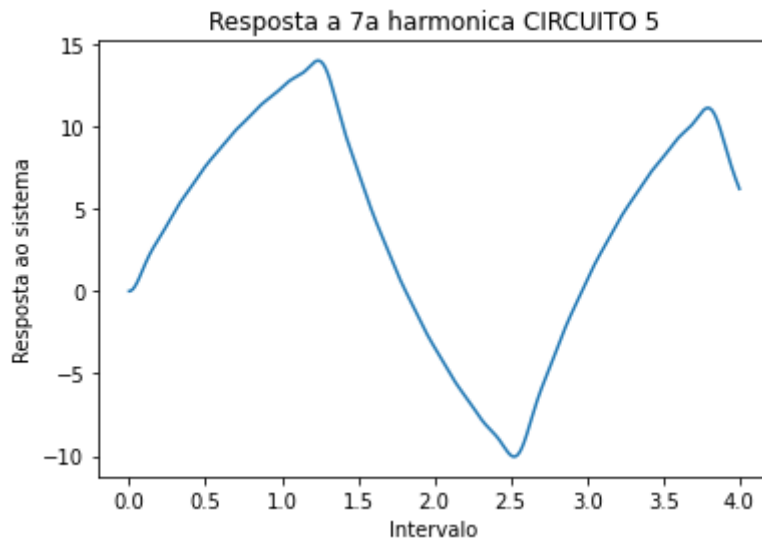
Letra G**Letra H****Letra I**



Letra J,K,L e M







1.6 Circuito 6

Letra A

O circuito 6 consiste em um RCL em série. Assim como no circuito anterior uma boa aplicação comercial para este circuito é a do filtro passa-faixa.

Letra B

$$L = 1,5 \cdot 10^{-3} H$$

$$R = 1 \Omega$$

$$C = 1 F$$

Letra C

Aplicando a lei de kirchoff das tensões temos a EDO:

$$1 \cdot \frac{\partial v(t)}{\partial t} = L \cdot \frac{\partial^2 i(t)}{\partial t^2} + R \cdot \frac{\partial i(t)}{\partial t} + C^{-1} \cdot i(t)$$

Letra D

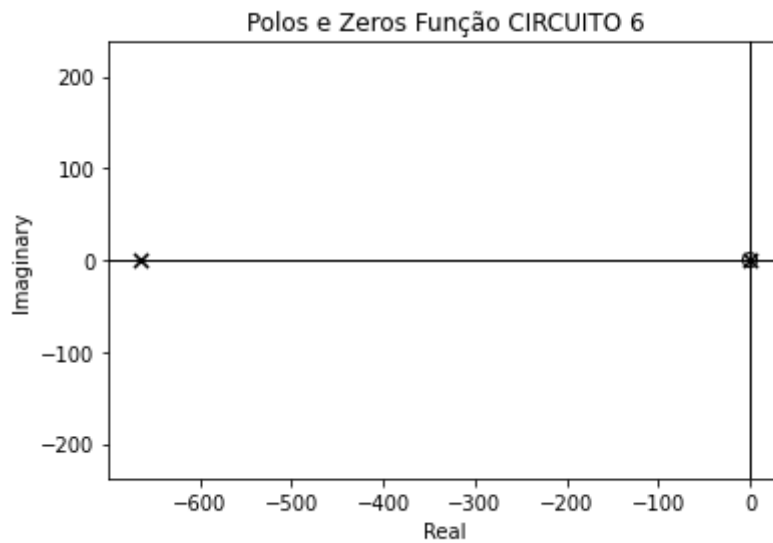
Aplicando a transformada de Laplace temos:

$$I(s) \cdot V(s)^{-1} = L^{-1} \cdot \frac{s}{s^2 + R \cdot L^{-1} + C \cdot L^{-1}}$$

substituindo os valores:

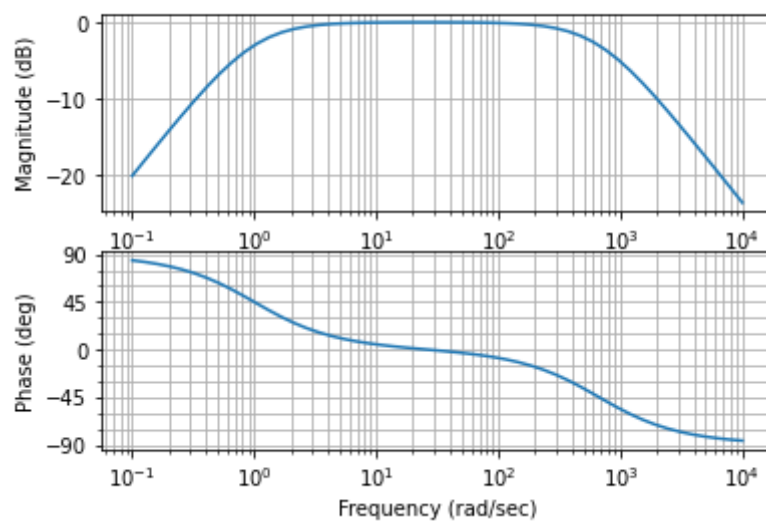
$$I(s) \cdot V(s)^{-1} = 1,5^{-3} \cdot \frac{s}{s^2 + 1,5 \cdot 10^3 s + 1,5 \cdot 10^{-3}}$$

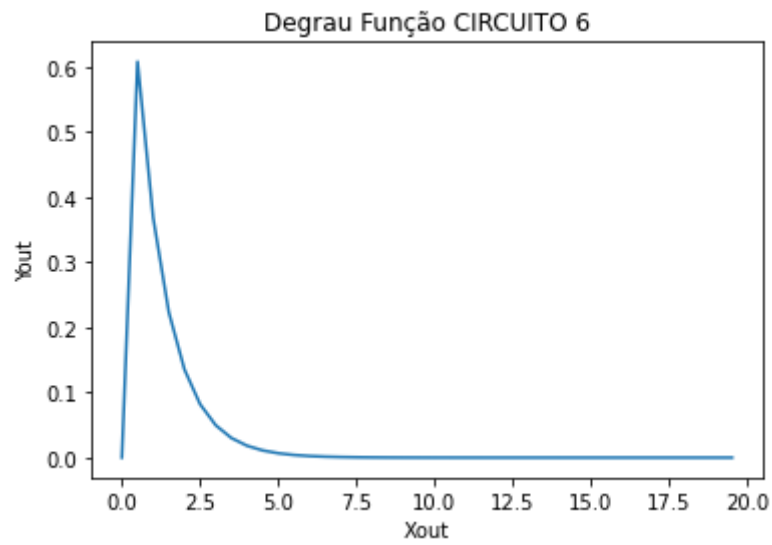
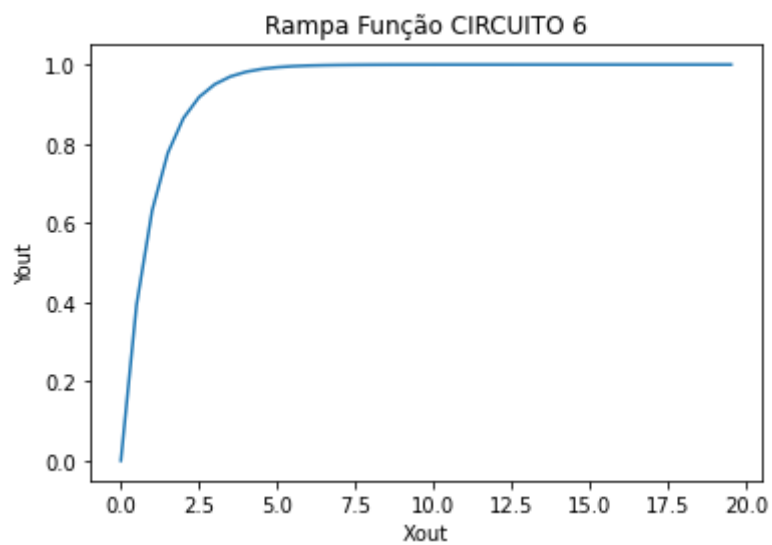
Letra E

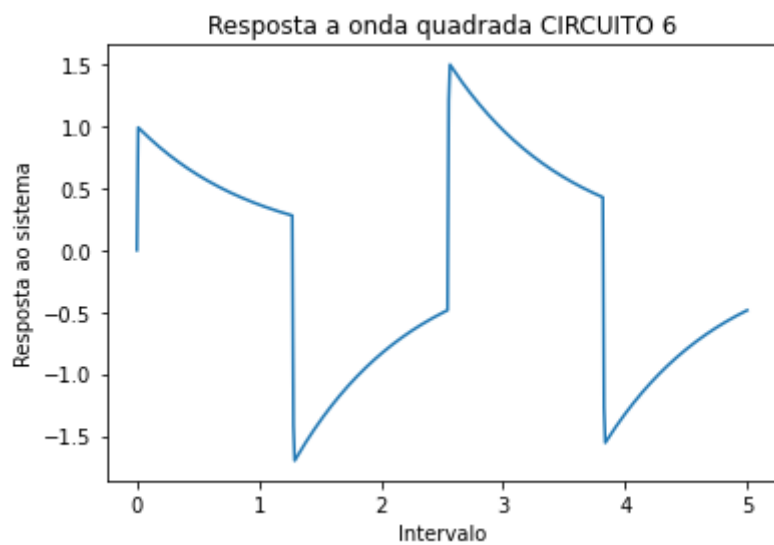


Letra F

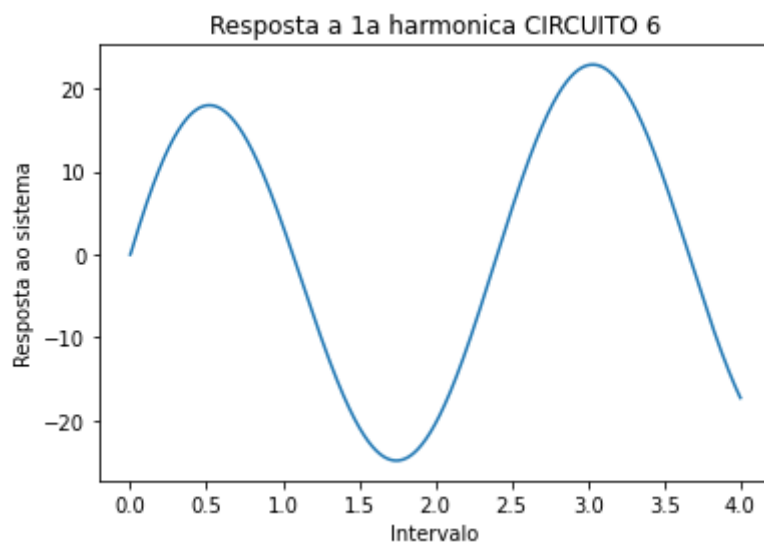
Diagrama de Bode CIRCUITO 6

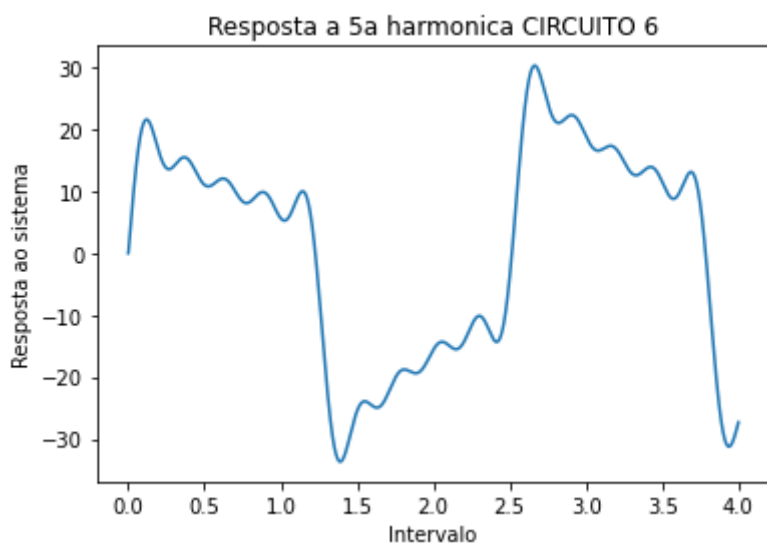
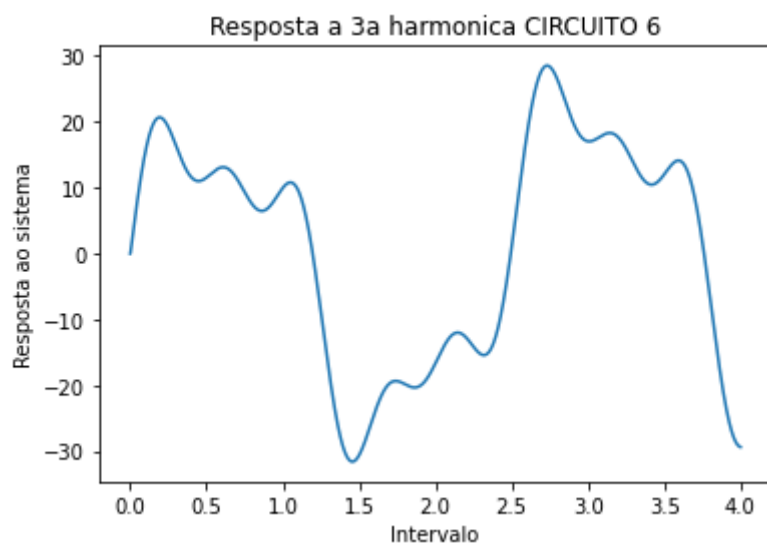


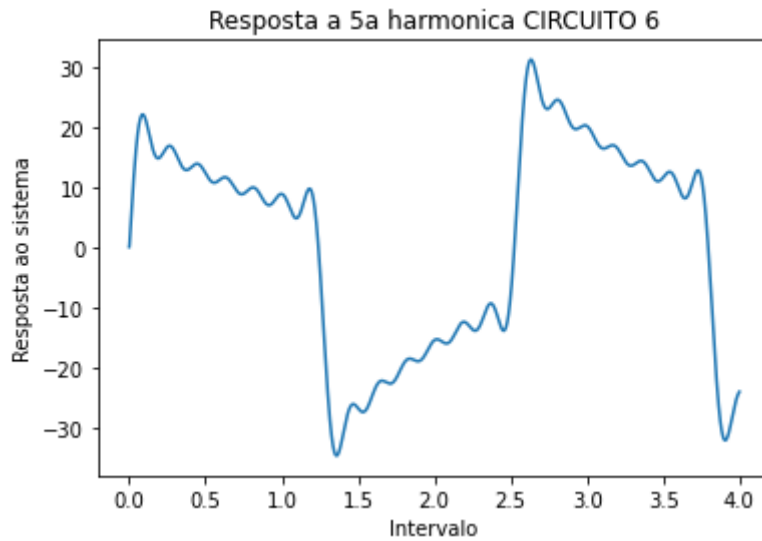
Letra G**Letra H****Letra I**



Letra J,K,L e M







2 Questão 2

Tendo por DRE = 118044310, os valores dos coeficientes do Diagrama de Blocos foram $A = 8$; $B = 12$; $C = 4$; e $D = 5$;

2.1 Letra A

A análise do Diagrama de Blocos nos retorna:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Ao utilizar a transformada de Laplace nas equações encontramos por

$$y(t) \text{ como } Y(s) = CX(s) + DU(s)$$

$$\dot{x}(t) \text{ como } sX(s) = AX(s) + BU(s).$$

$$\text{Temos que } sX(s) - AX(s) = BU(s) \text{ nos retorna } X(s) = U(s) \cdot \frac{B}{s-A}$$

$$\text{Com isso temos que } Y(s) = CX(s) + DU(s) \rightarrow Y(s) = U(s) \cdot \frac{CB}{s-A} + DU(s)$$

$$\text{A função de transferência será } Y(s) \cdot \frac{1}{U(s)} = \frac{CB}{s-A} + D$$

Substituindo os valores dos coeficientes temos:

$$Y(s) \cdot \frac{1}{U(s)} = (5s + 8) \cdot \frac{1}{s-8}$$

2.2 Letra B e C

Os diagramas de Polos e zeros e de Bode da função de transferência.

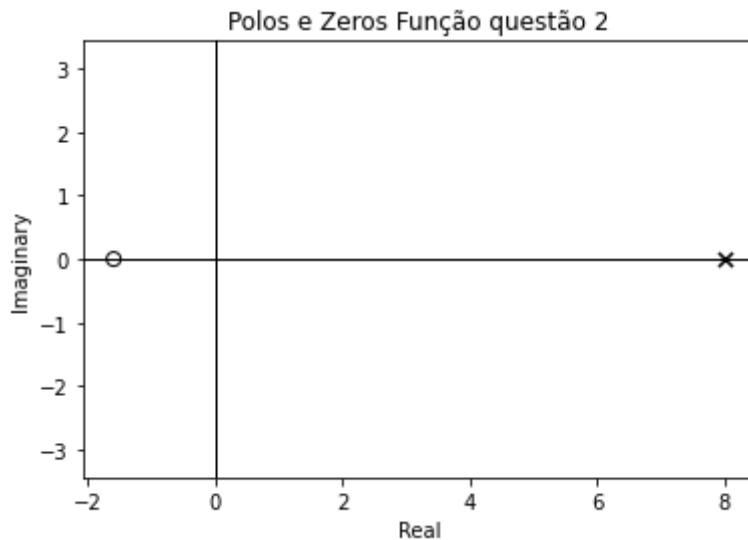
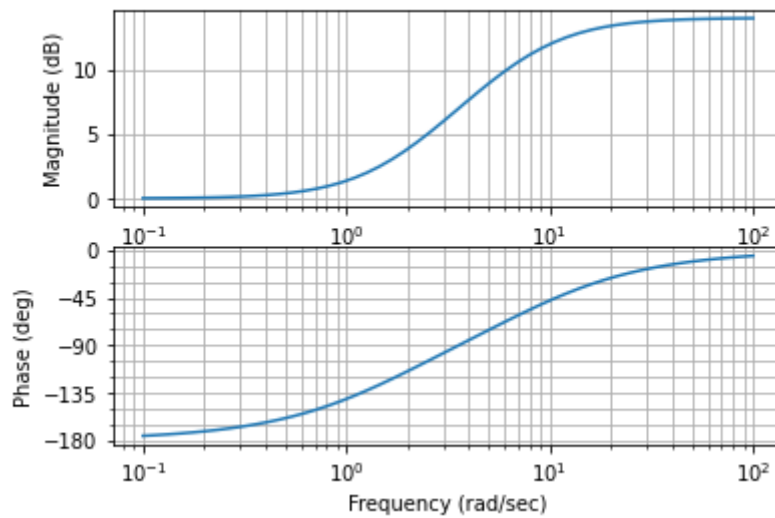


Diagrama de Bode Função questão 2



2.3 Letra D e E

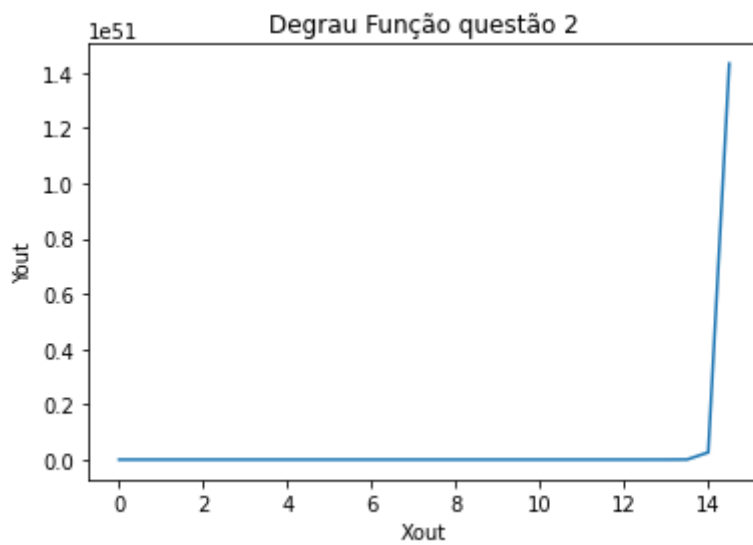
Substituindo os valores do diagrama de blocos nas equações encontradas no item acima temos:

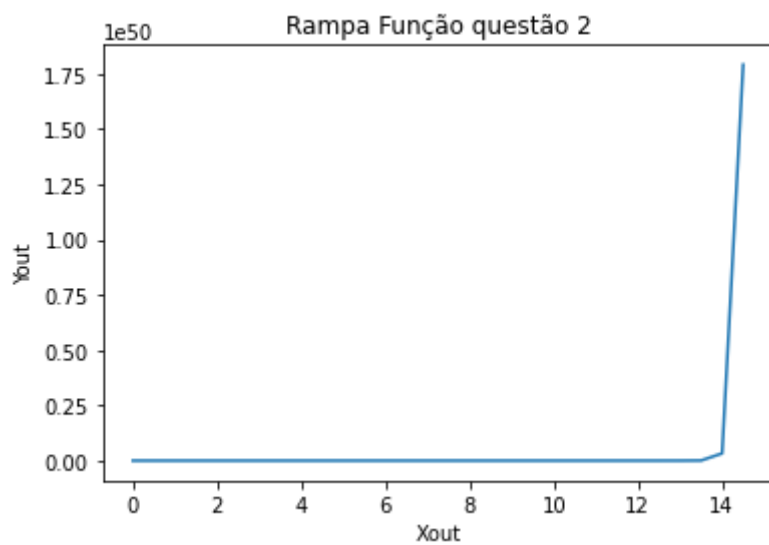
$$x'(t) = 8x(t) + 12u(t)$$

$$y(t) = 4x(t) + 5u(t)$$

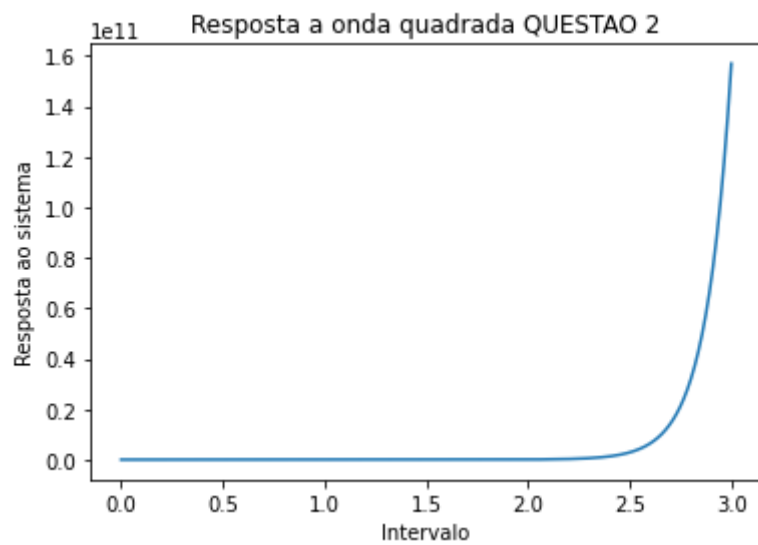
2.4 Letra F e G

As respostas ao Degrau unitário e a Rampa unitária são:

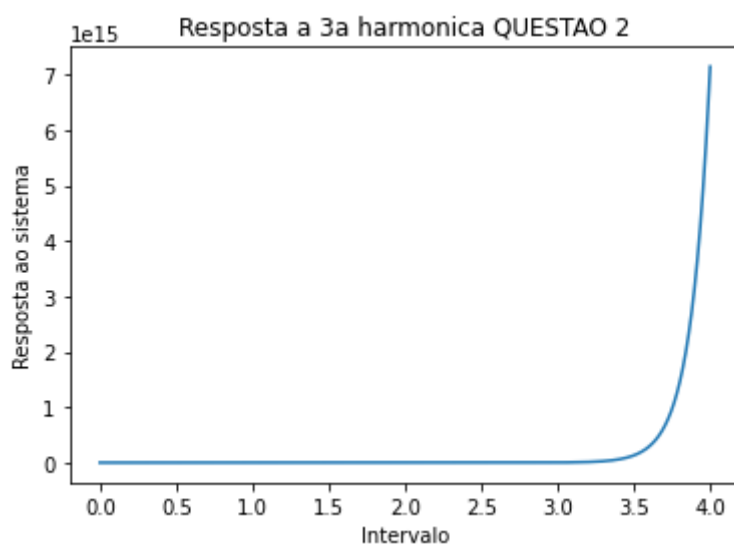
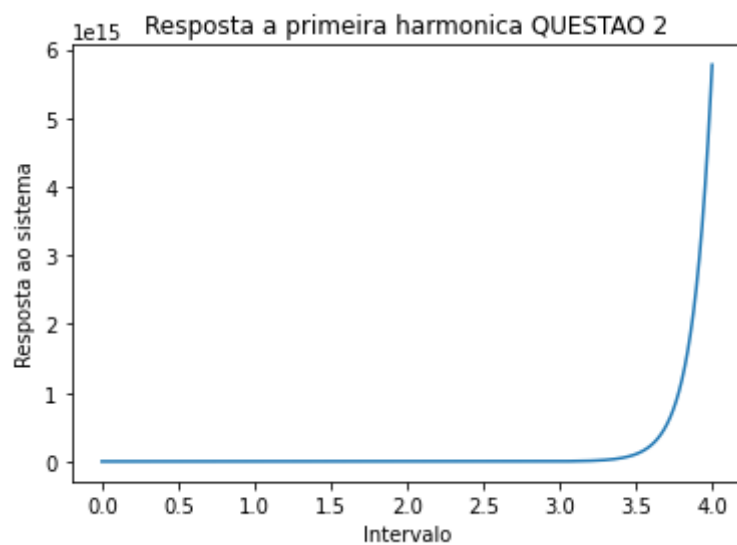


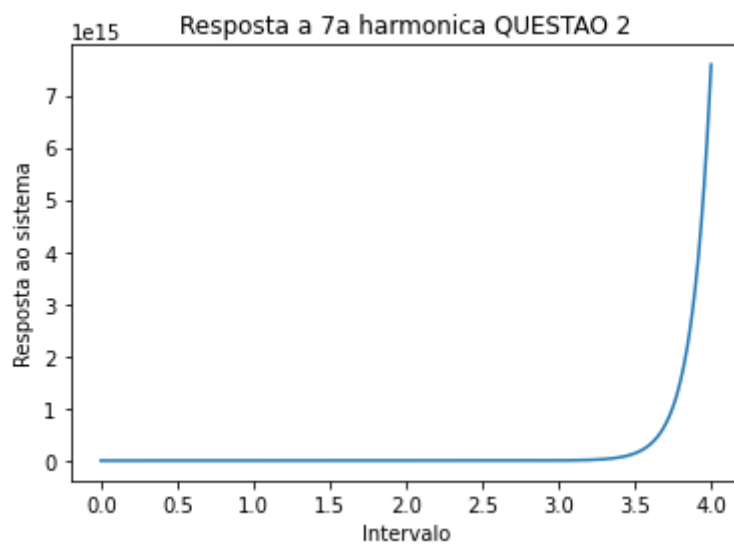
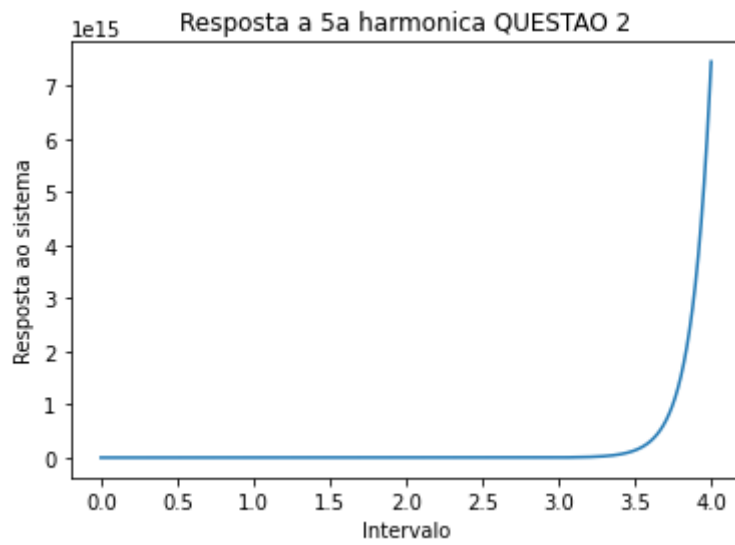


2.5 letra H



2.6 Letra I, J, K e L





3 Questão 3

Para encontrarmos a EDO de uma função de transferência sabemos que:

$$H(s) = Y(s) \cdot \frac{1}{X(s)}$$

3.1 Função 1

Para a função de transferência $H(s) = (1+\alpha s) \cdot \frac{1}{s^2+2s+2}$

$$(1+\alpha s) \cdot \frac{1}{s^2+2s+2} = Y(s) \cdot \frac{1}{X(s)}$$

multiplicando os termos de maneira cruzada temos:

$$(s^2 + 2s + 2) \cdot Y(s) = (1 + \alpha s)X(s)$$

Aplicando a transformada inversa de Laplace encontramos:

$$y''(t)+2y'(t)+2y(t)=\alpha x'(t) + x(t)$$

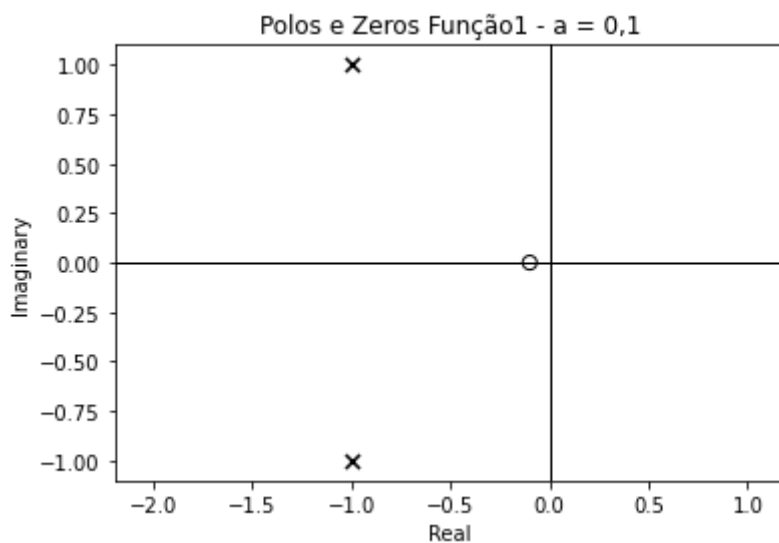
3.1.1 Para $\alpha = 0.1$

Letra A: EDO

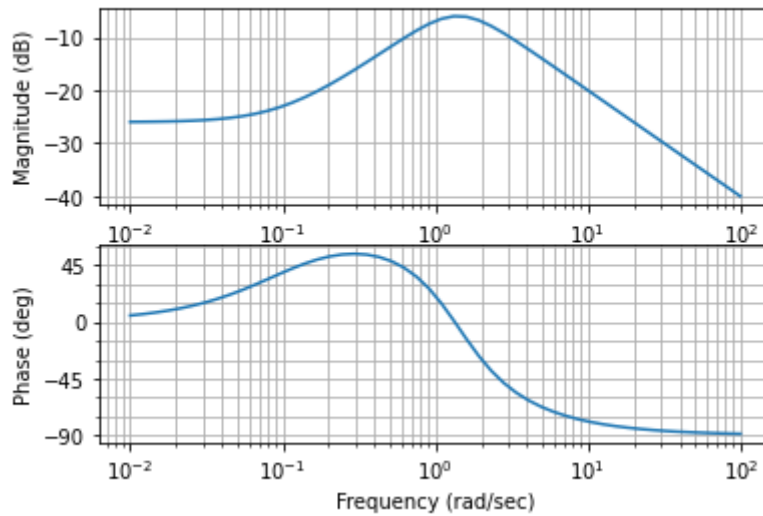
A função de transferência é $H(s) = (1+0.1s) \cdot \frac{1}{s^2+2s+2}$

$$y''(t)+2y'(t)+2y(t) = 0.1x'(t)+x(t)$$

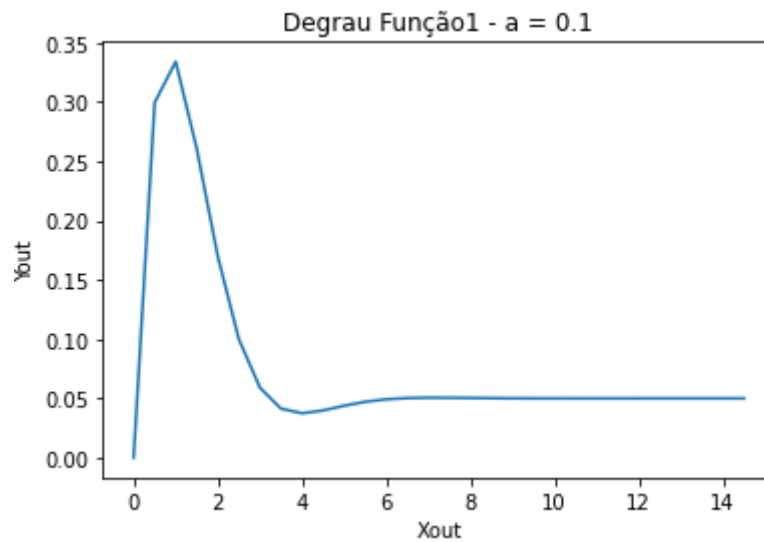
Letra B: Diagrama de Polos e zeros



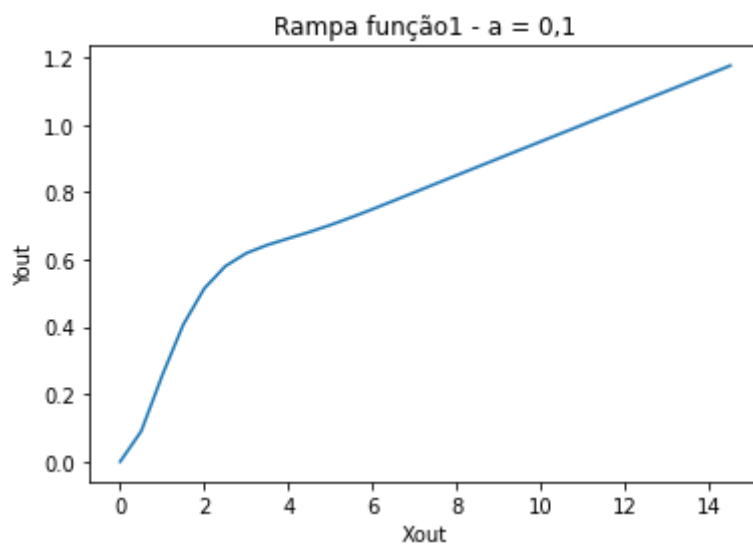
Letra C: Diagrama de Bode

Diagrama de Bode Função1, $a = 0,1$ 

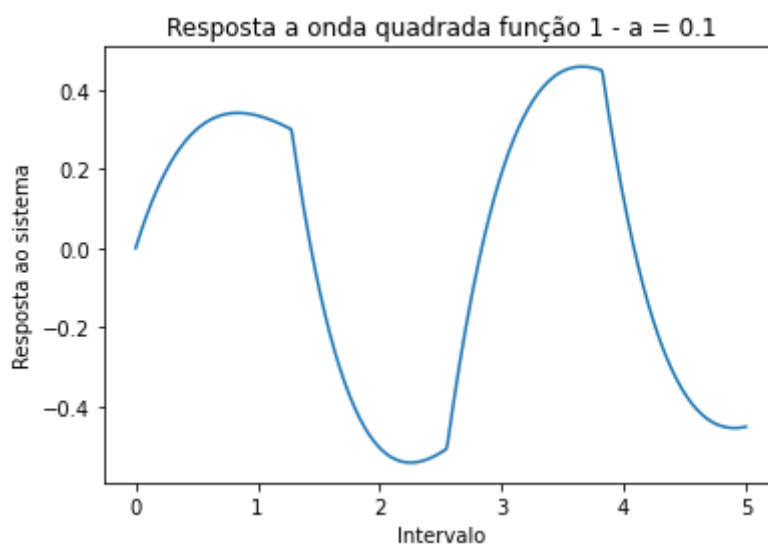
Letra D: Resposta ao degrau unitário



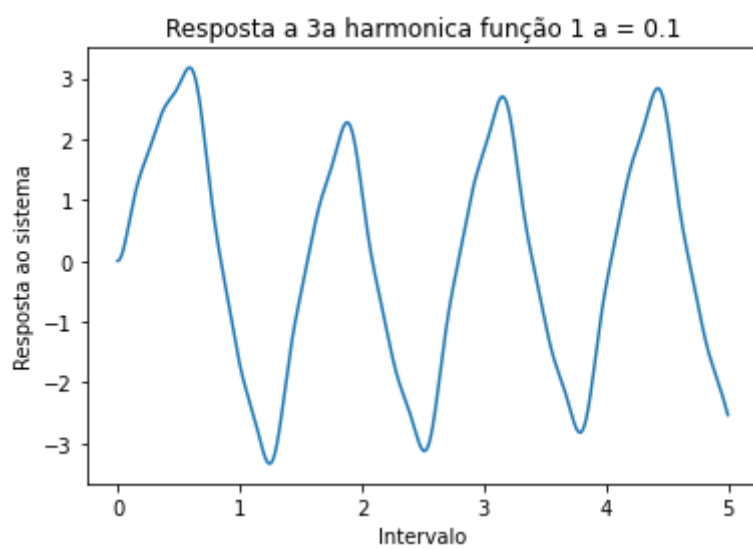
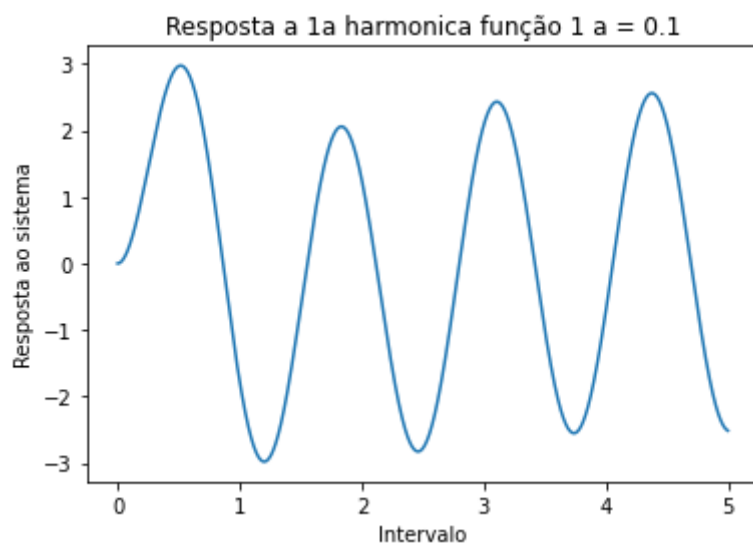
Letra E: Resposta a rampa unitária

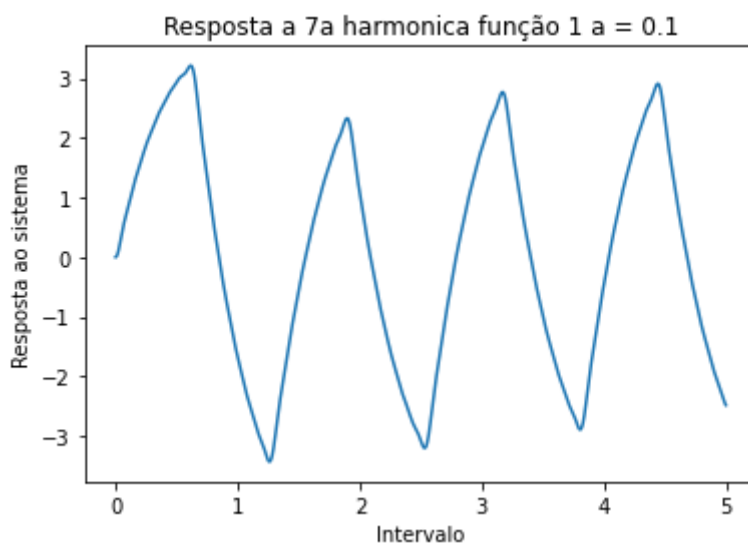
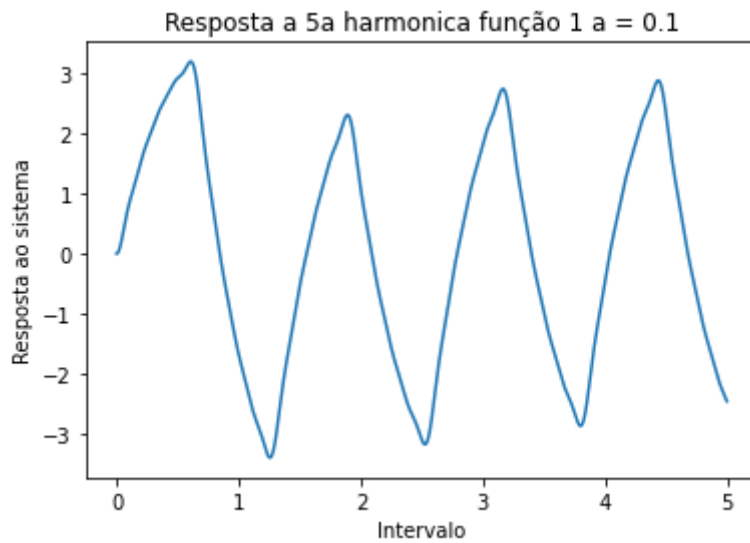


Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmonicas



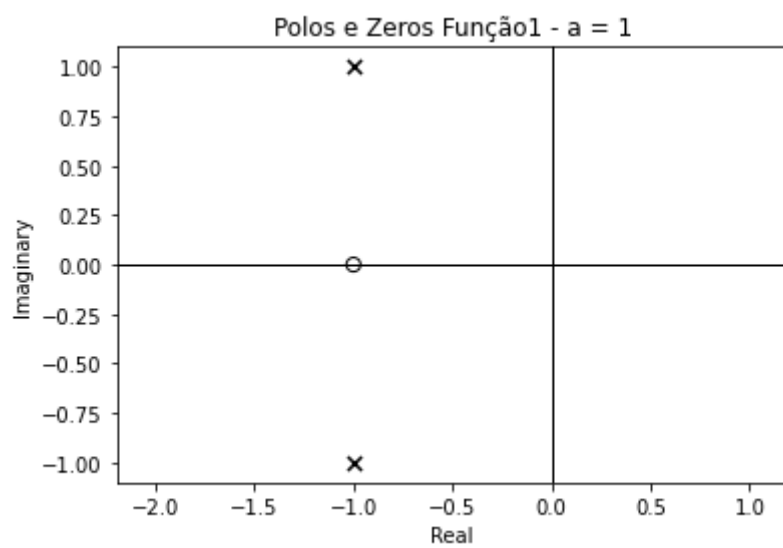


3.1.2 Para $\alpha = 1$

Letra A: EDO A função de transferência é $H(s) = (1+1s) \cdot \frac{1}{s^2+2s+2}$

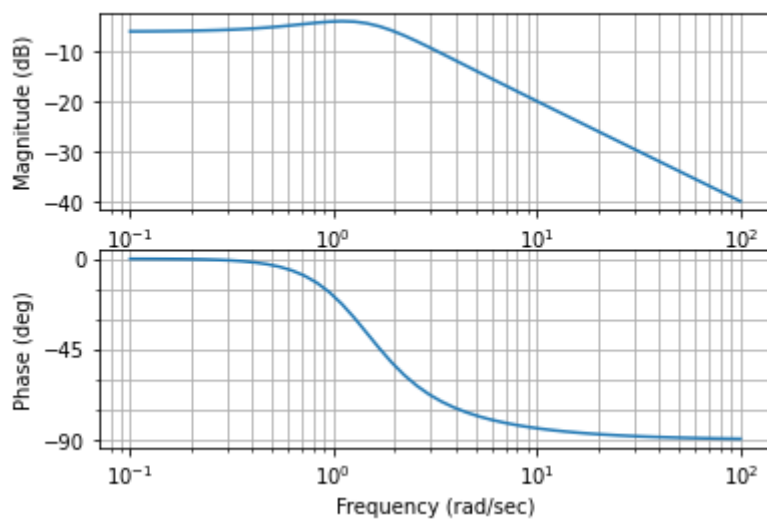
$$y''(t) + 2y'(t) + 2y(t) = x'(t) + x(t)$$

Letra B: Diagrama de Polos e zeros

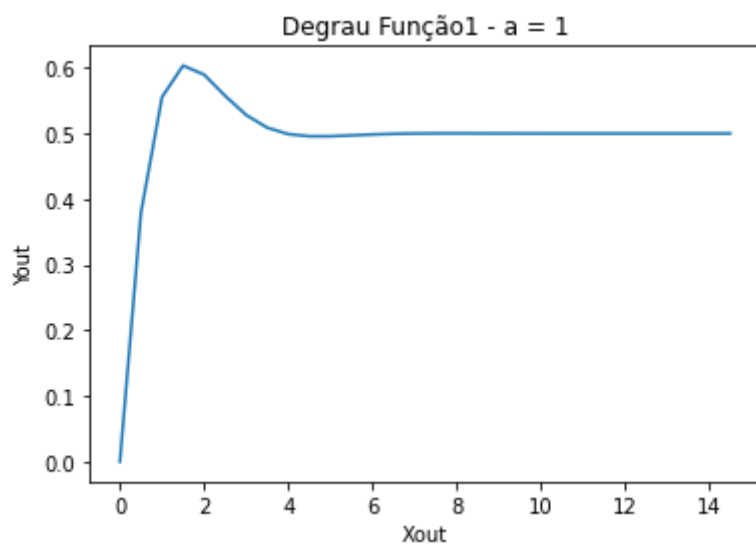


Letra C: Diagrama de Bode

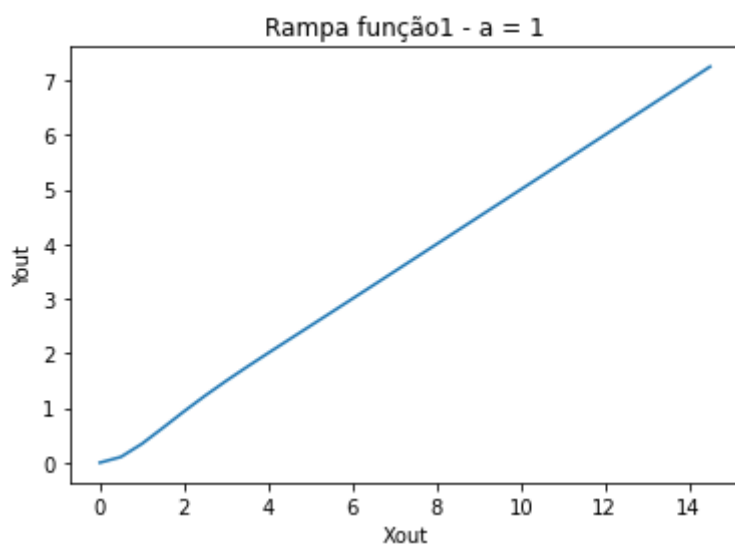
Diagrama de Bode Função1, a = 1



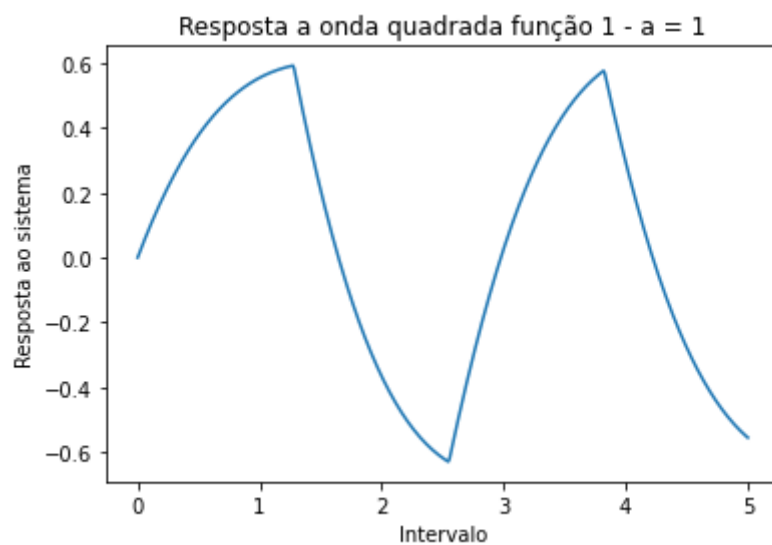
Letra D: Resposta ao degrau unitário



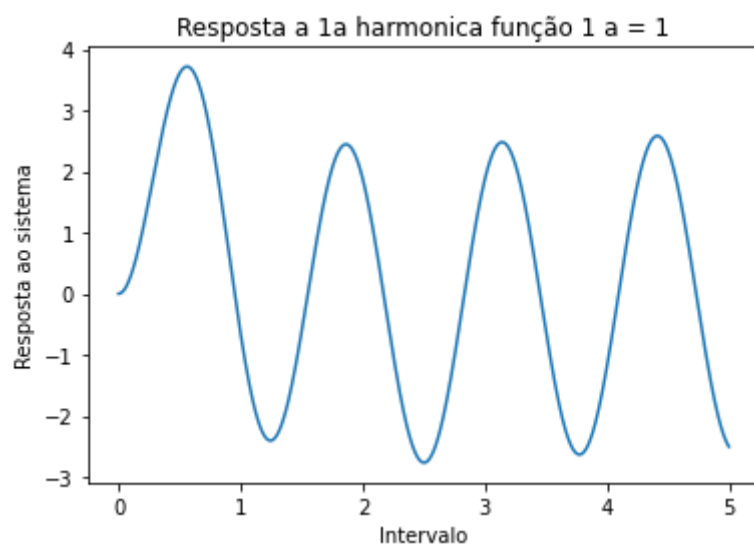
Letra E: Resposta a rampa unitária

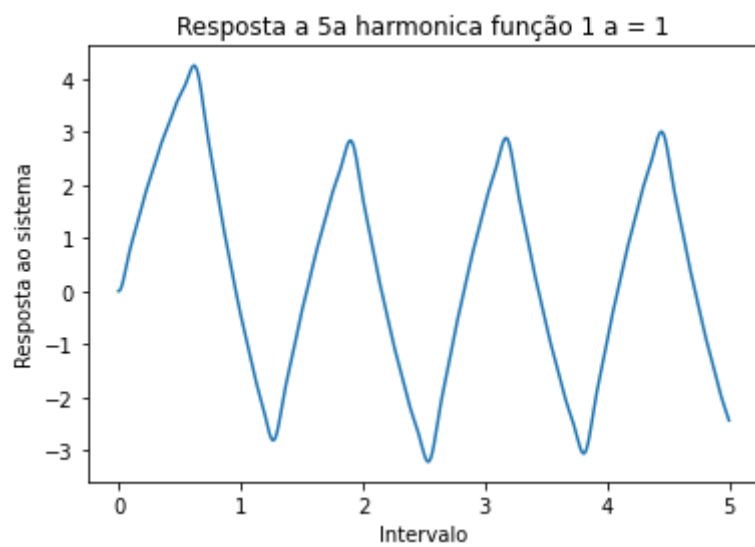
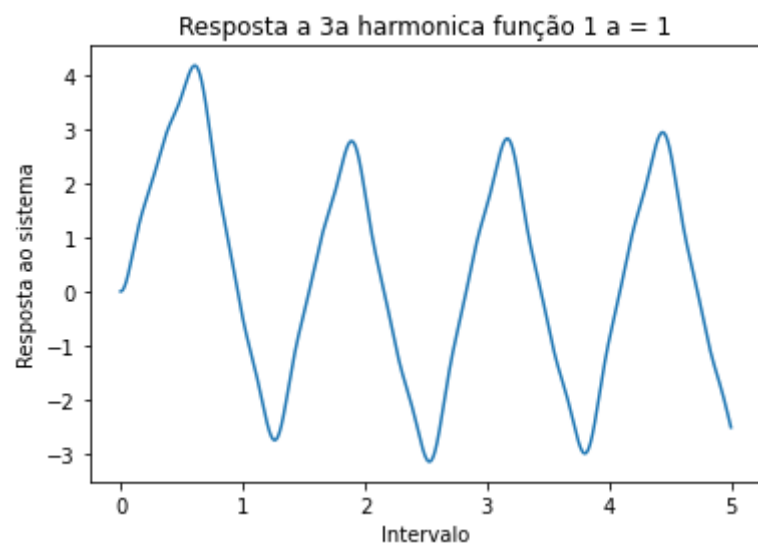


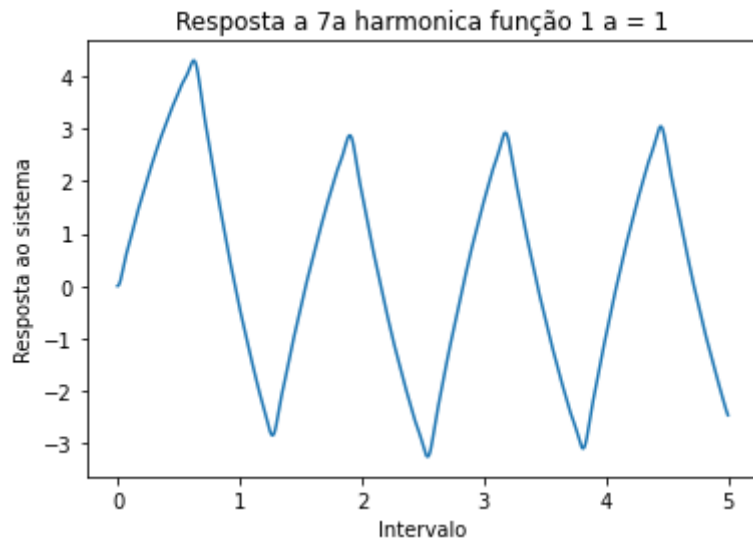
Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmônicas





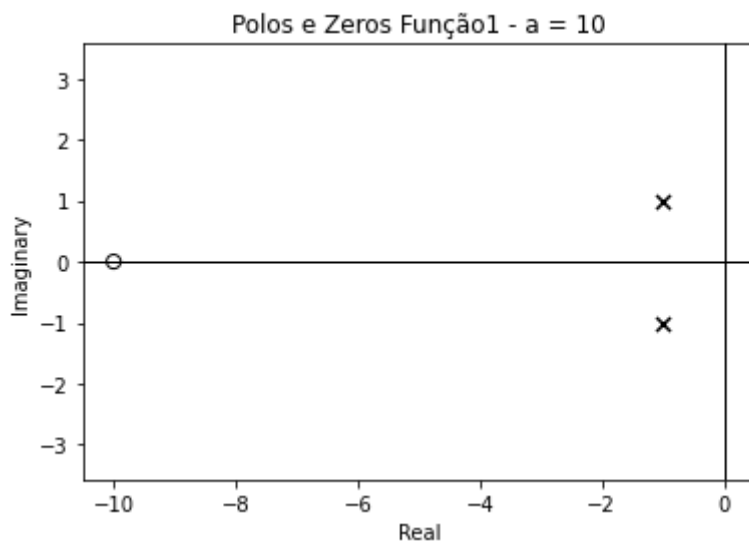


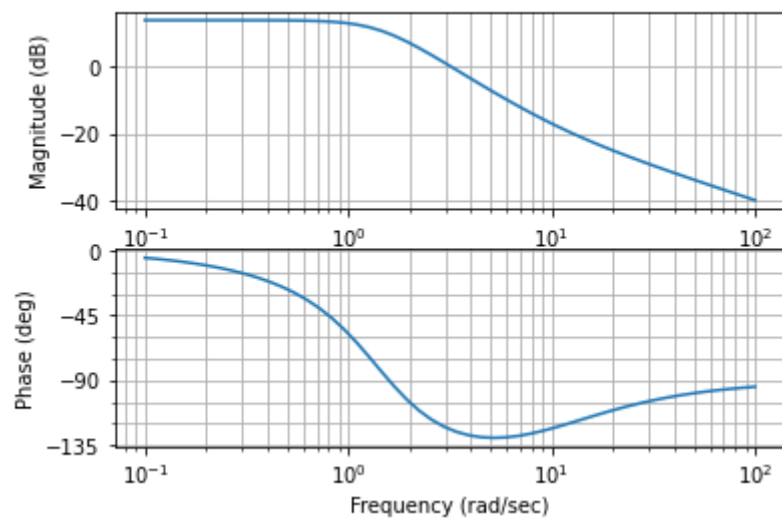
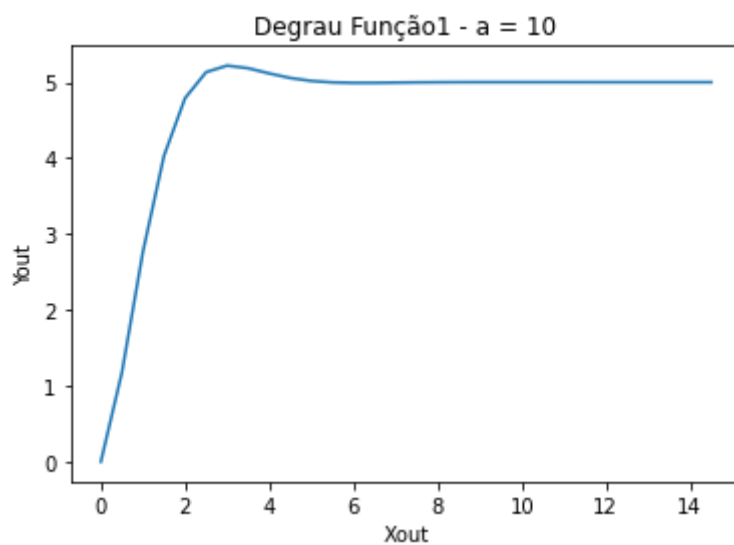
3.1.3 Para $\alpha = 10$

Letra A: EDO A função de transferência é $H(s) = (1+10s) \cdot \frac{1}{s^2+2s+2}$

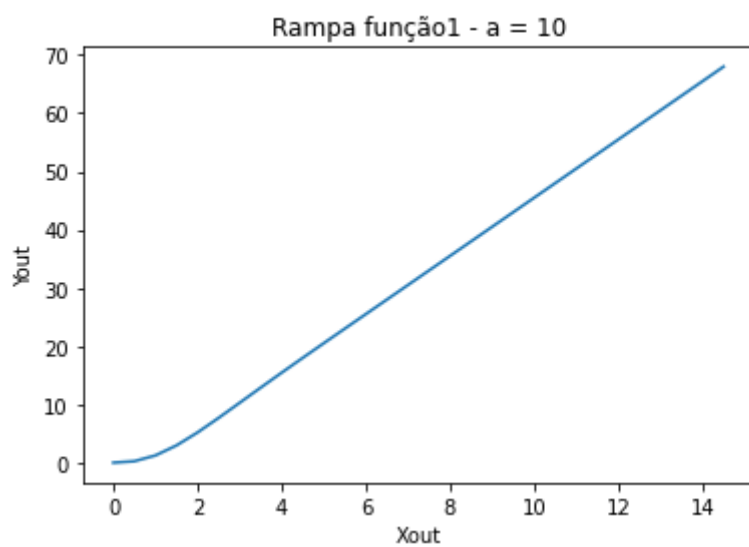
$$y''(t) + 2y'(t) + 2y(t) = 10x'(t) + x(t)$$

Letra B: Diagrama de Polos e zeros



Letra C: Diagrama de Bode**Diagrama de Bode Função1, $a = 10$** **Letra D: Resposta ao degrau unitário**

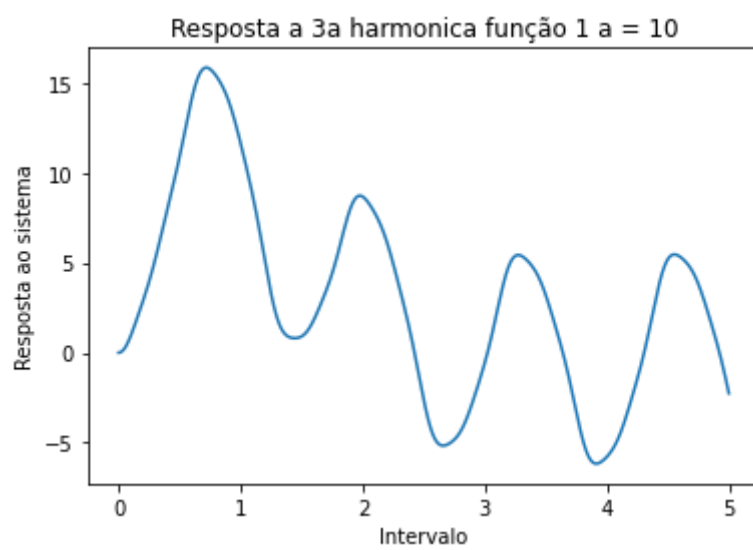
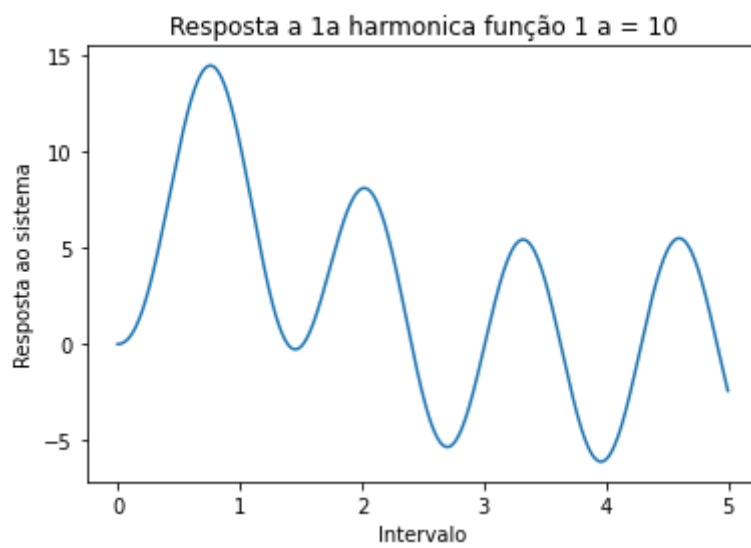
Letra E: Resposta a rampa unitária

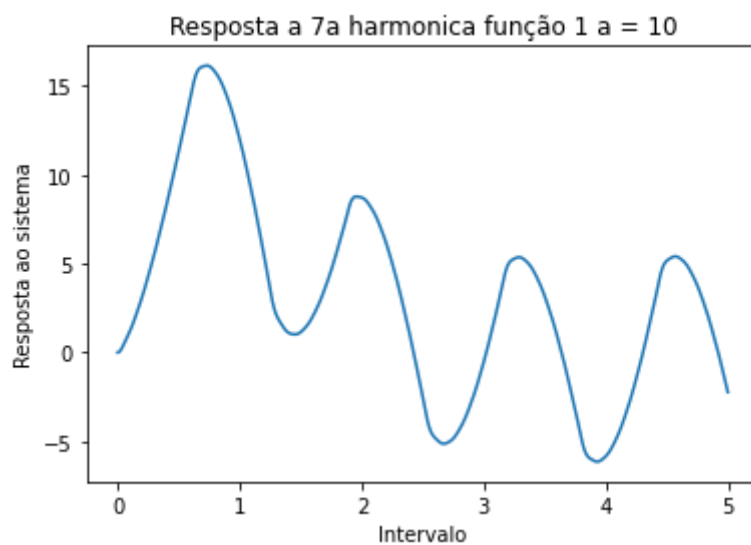
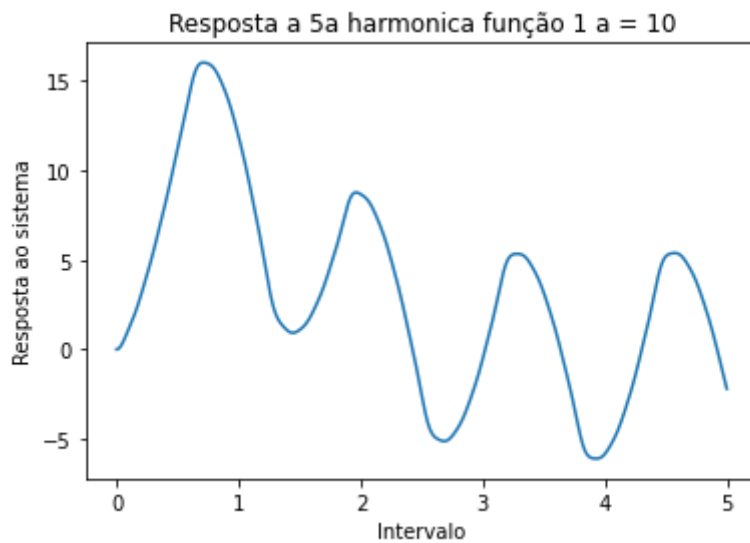


Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmonicas



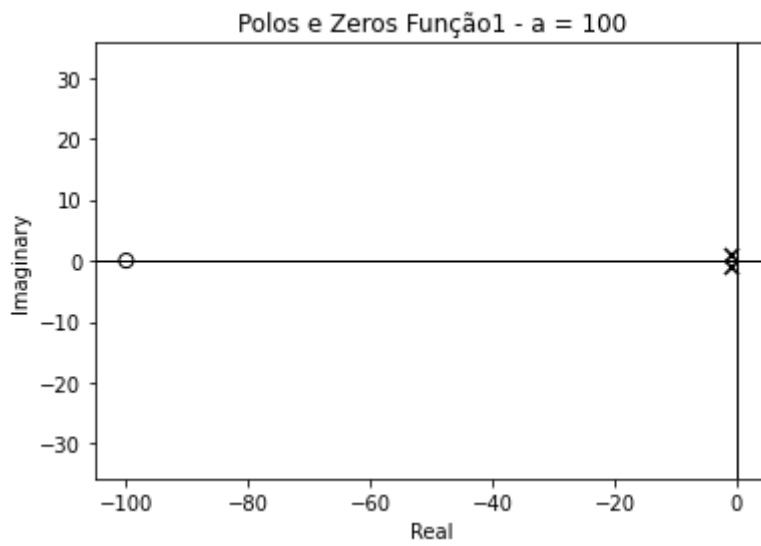
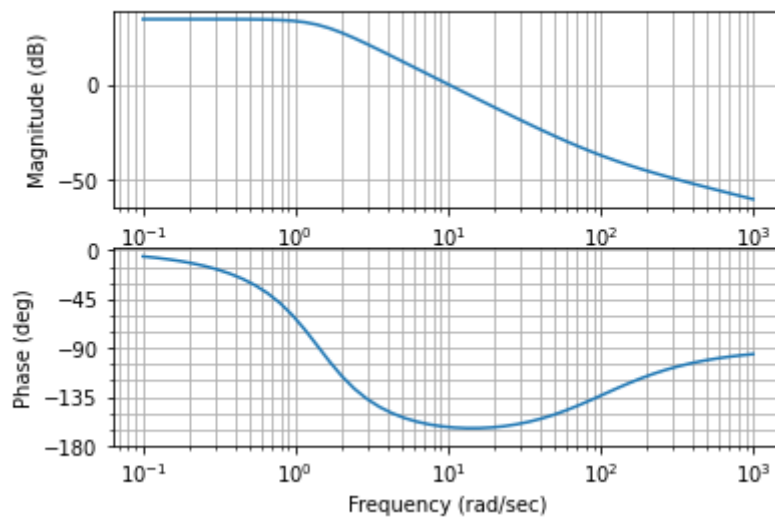


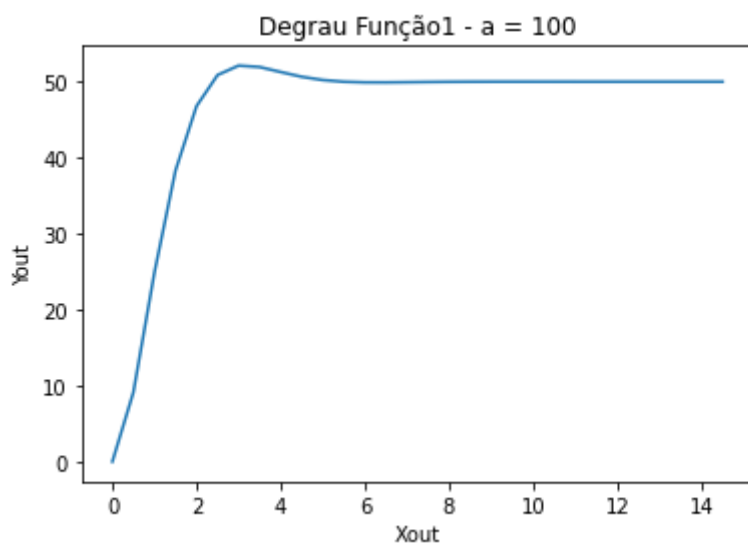
3.1.4 Para $\alpha = 100$

Letra A: EDO A função de transferência é $H(s) = (1+100s) \cdot \frac{1}{s^2+2s+2}$

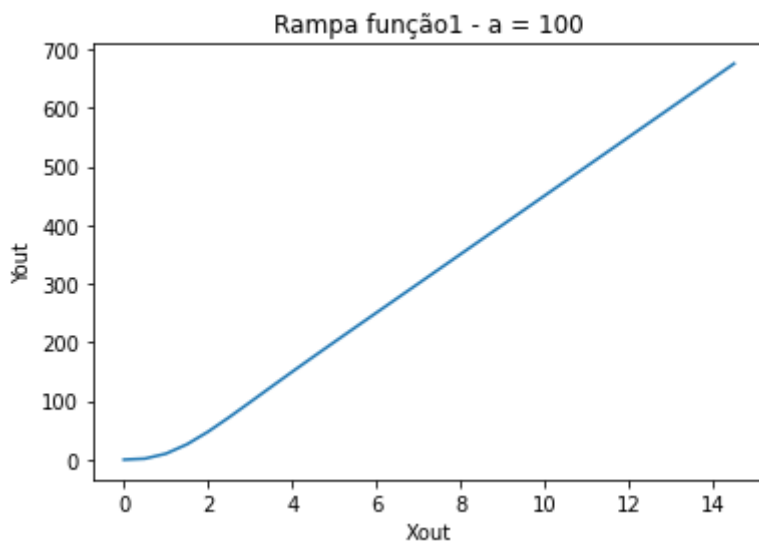
$$y''(t) + 2y'(t) + 2y(t) = 100x'(t) + x(t)$$

Letra B: Diagrama de Polos e zeros

**Letra C:** Diagrama de BodeDiagrama de Bode Função1, $a = 100$ **Letra D:** Resposta ao degrau unitário



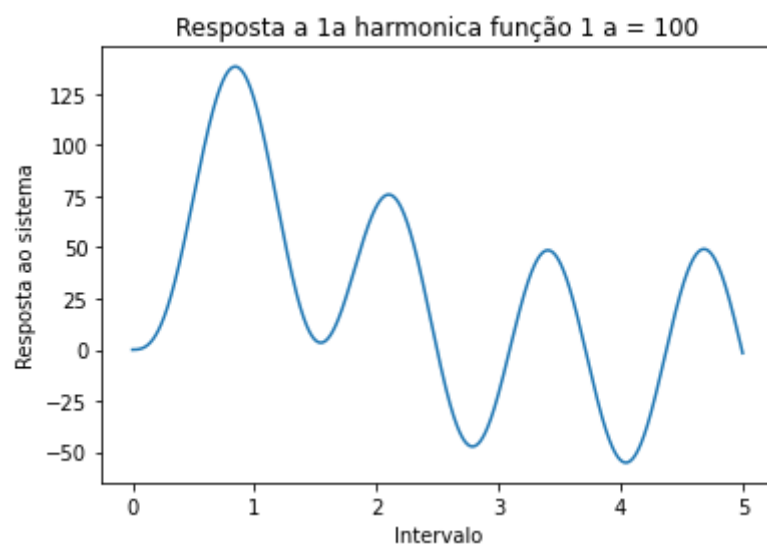
Letra E: Resposta a rampa unitária

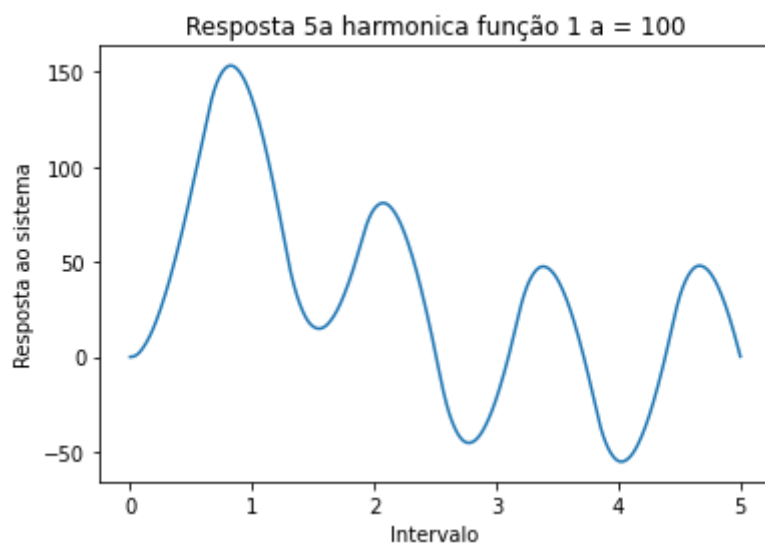


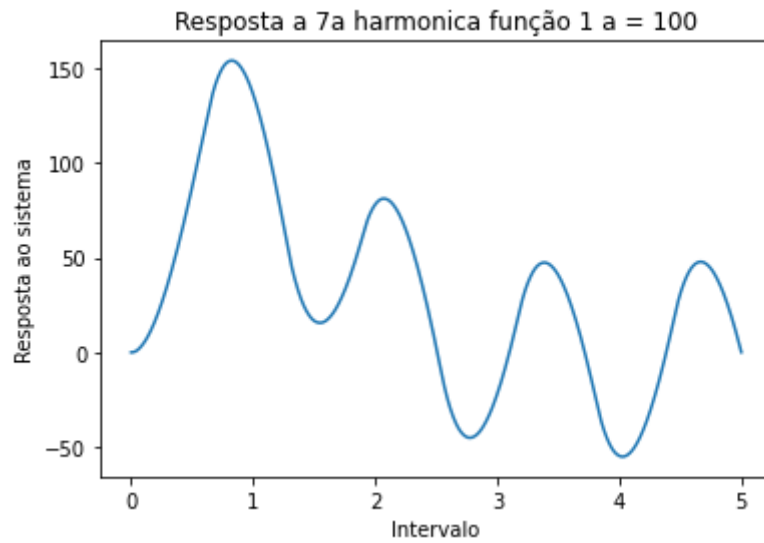
Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmônicas







3.2 Função 2

A segunda função de transferência é $H(s) = (s+10^4) \cdot \frac{1}{s^2+20\beta s+100}$

Analogamente a função 1 a EDO da função 2 será:

$$y''(t) + 20\beta y'(t) + 100y(t) = x'(t) + 10^4 x(t)$$

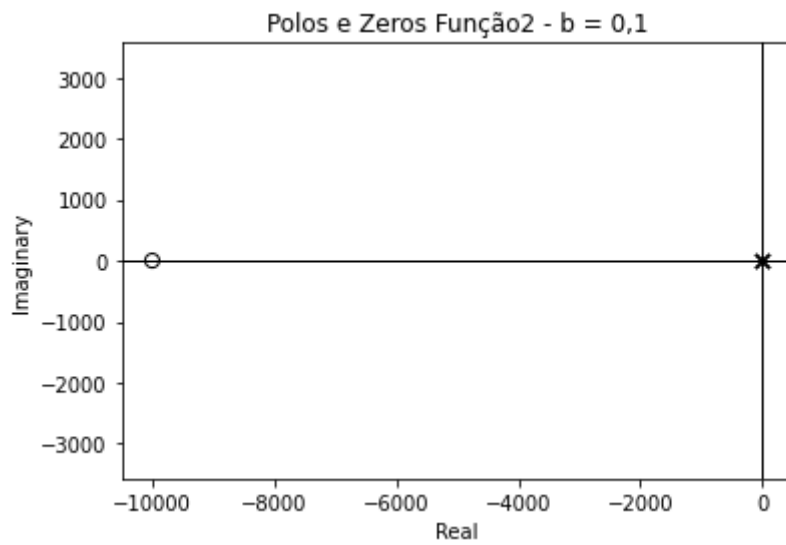
3.2.1 Para $\beta = 0.1$

Letra A: EDO

A função de transferência é $H(s) = (s+10^4) \cdot \frac{1}{s^2+2s+100}$

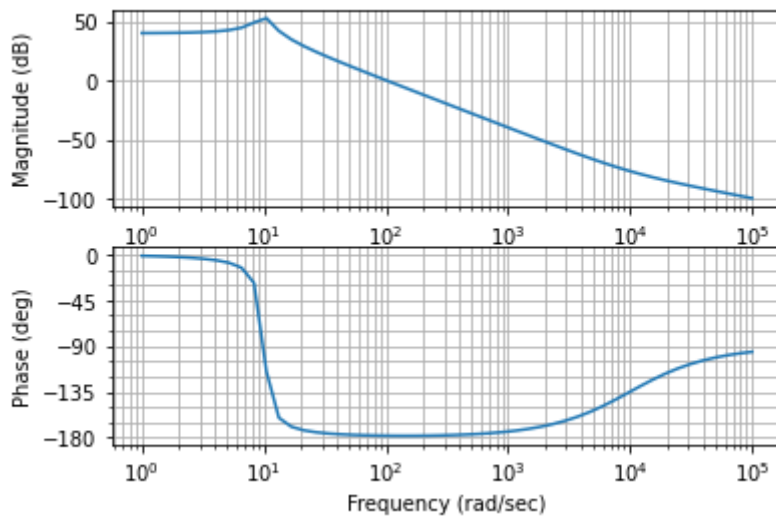
$$y''(t) + 2y'(t) + 100y(t) = x'(t) + 10^4 x(t)$$

Letra B: Diagrama de Polos e zeros

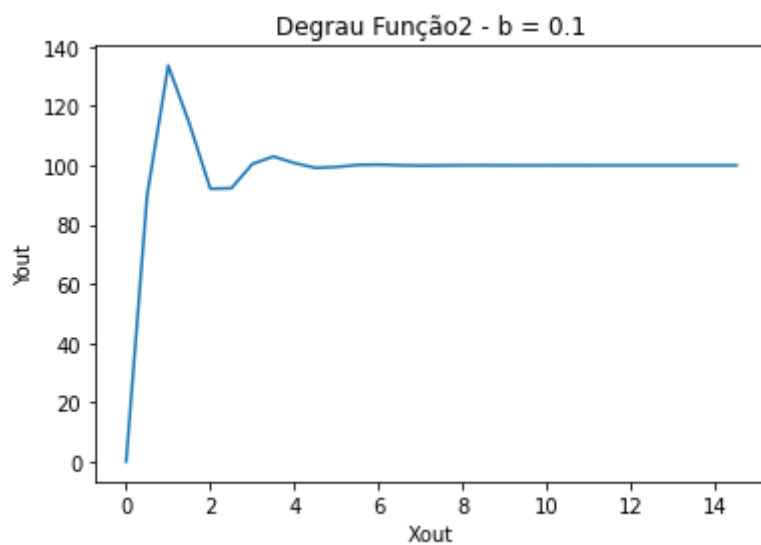


Letra C: Diagrama de Bode

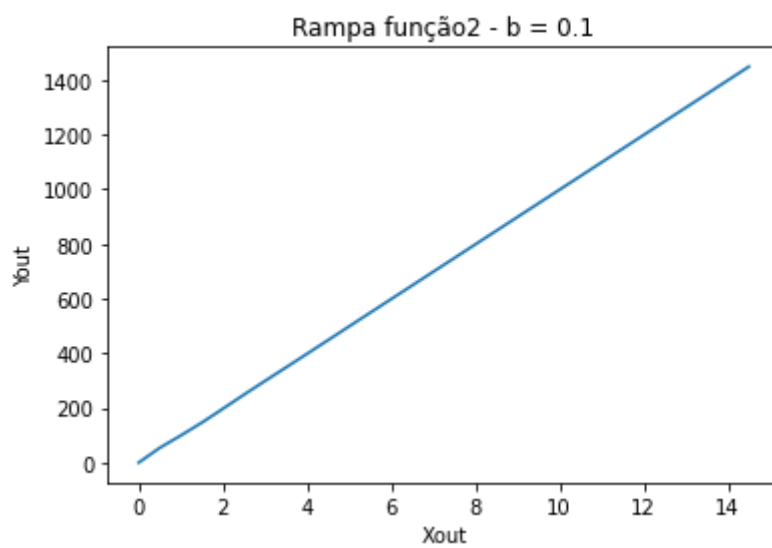
Diagrama de Bode Função2, $b = 0,1$



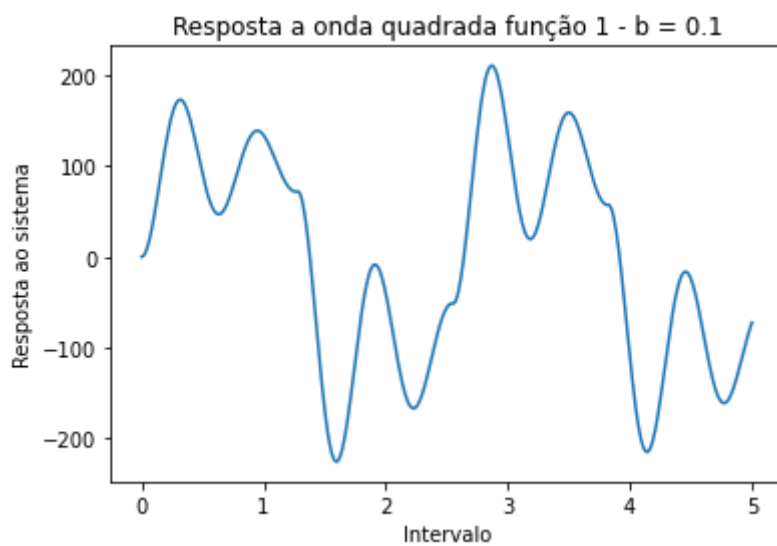
Letra D: Resposta ao degrau unitário



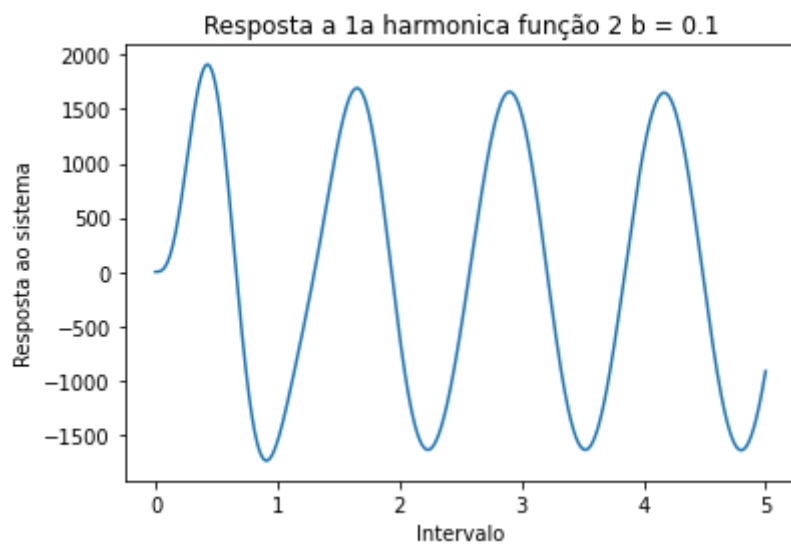
Letra E: Resposta a rampa unitária

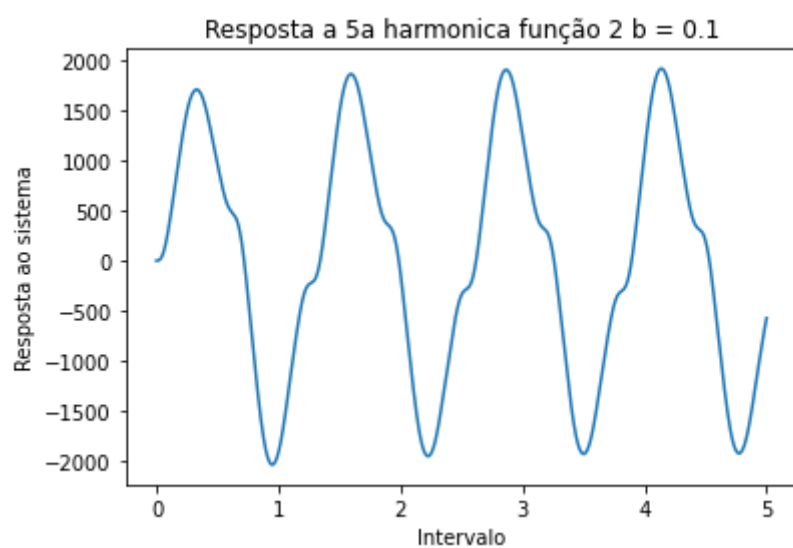
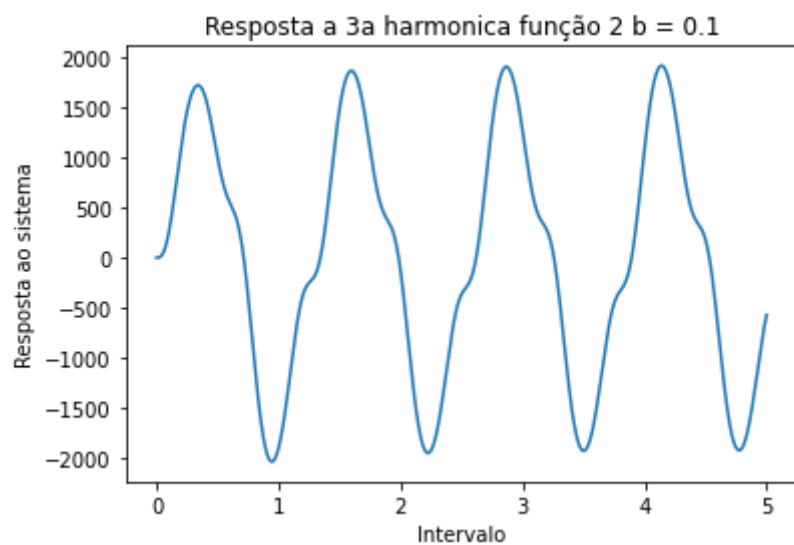


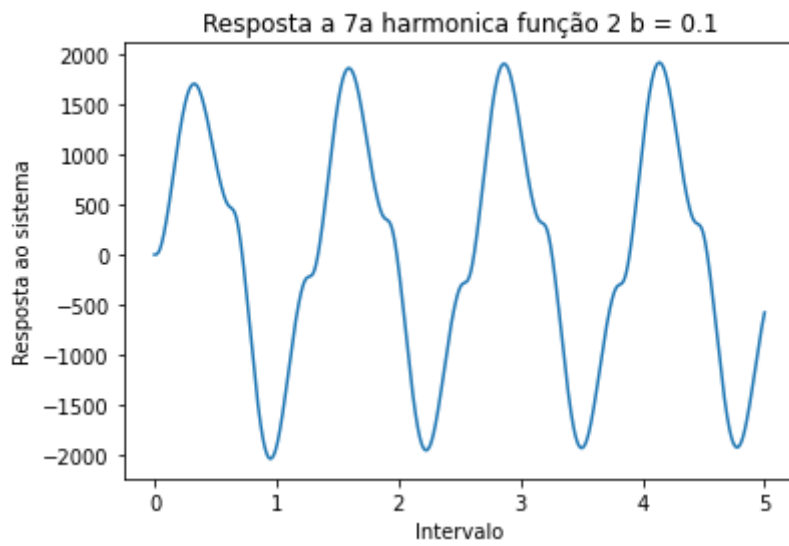
Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmonicas





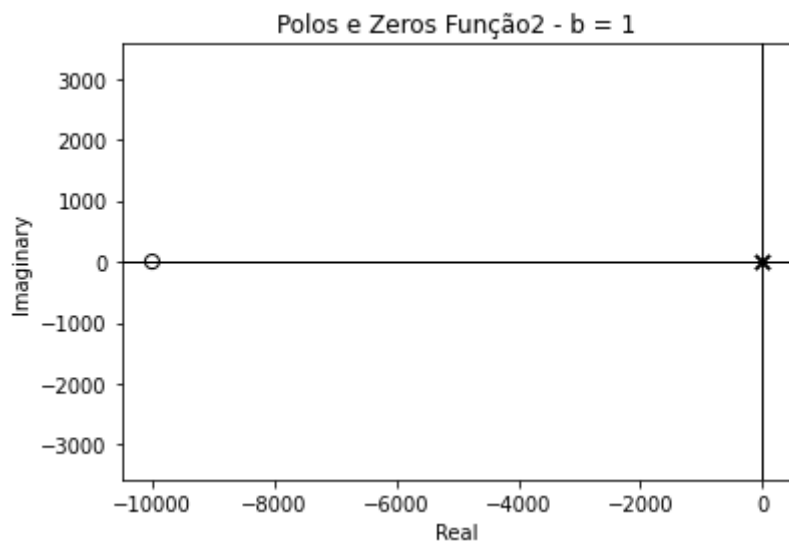


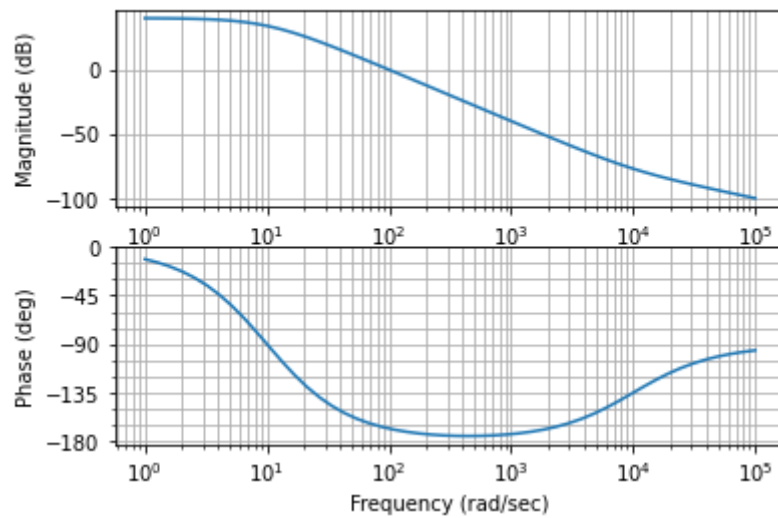
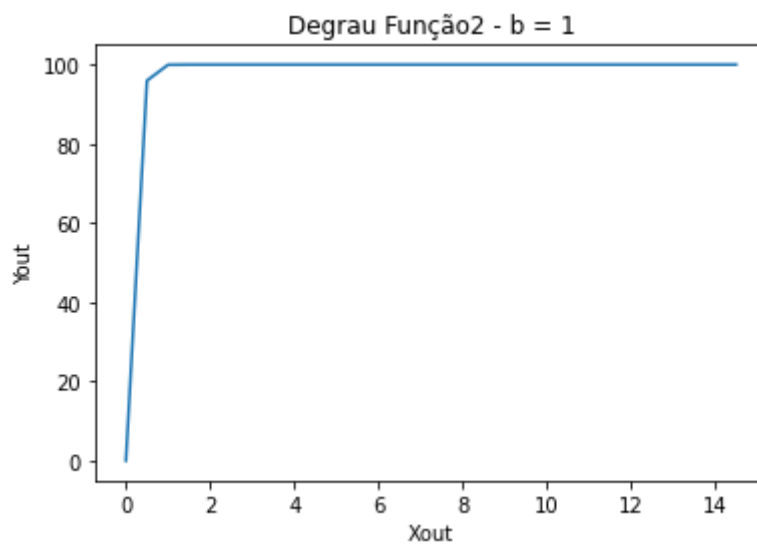
3.2.2 Para $\beta = 1$

Letra A: EDO A função de transferência é $H(s) = (s+10^4) \cdot \frac{1}{s^2+20s+100}$

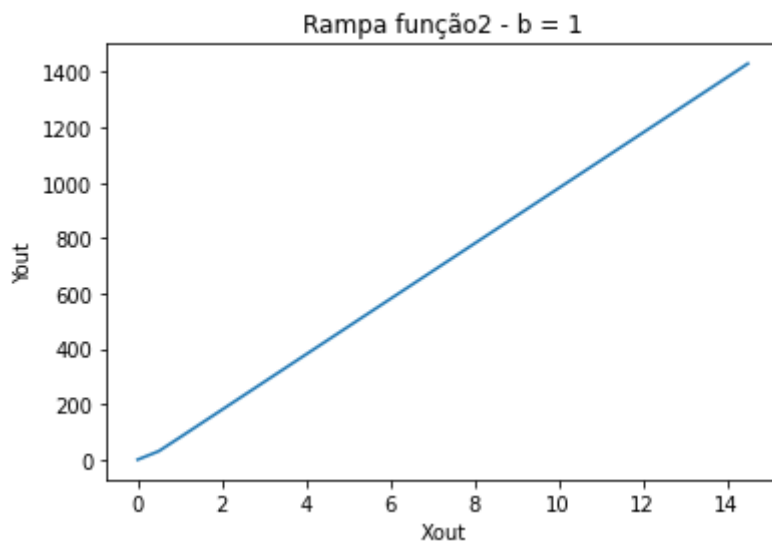
$$y''(t) + 20y'(t) + 100y(t) = x'(t) + 10^4 x(t)$$

Letra B: Diagrama de Polos e zeros

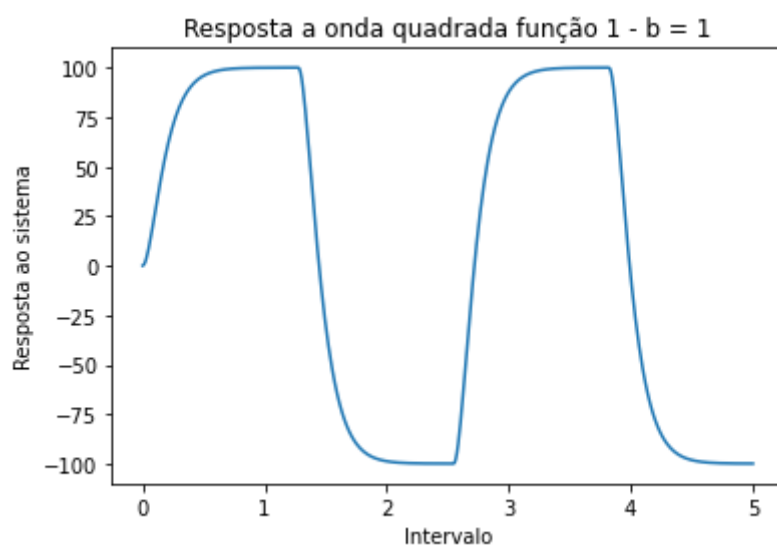


Letra C: Diagrama de Bode**Diagrama de Bode Função2, $b = 1$** **Letra D: Resposta ao degrau unitário**

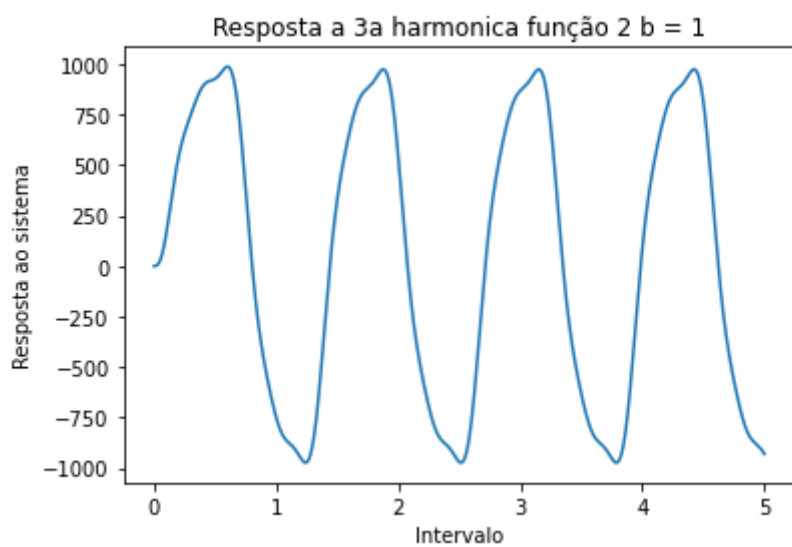
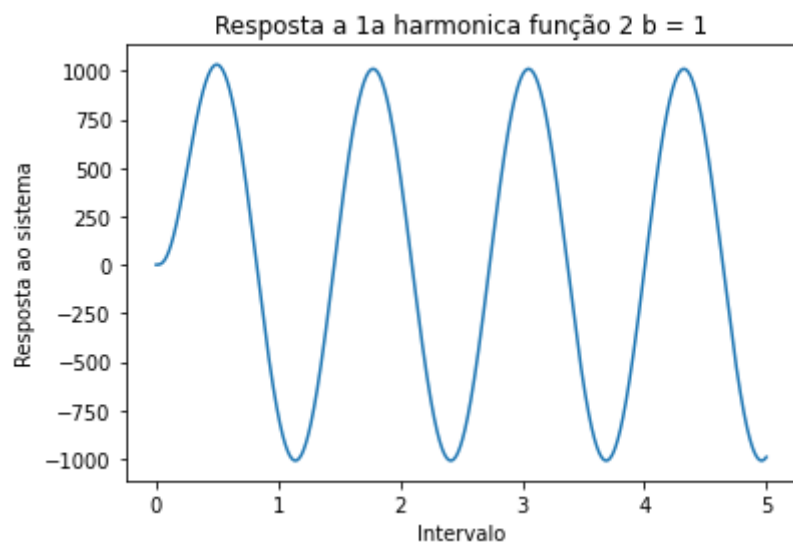
Letra E: Resposta a rampa unitária

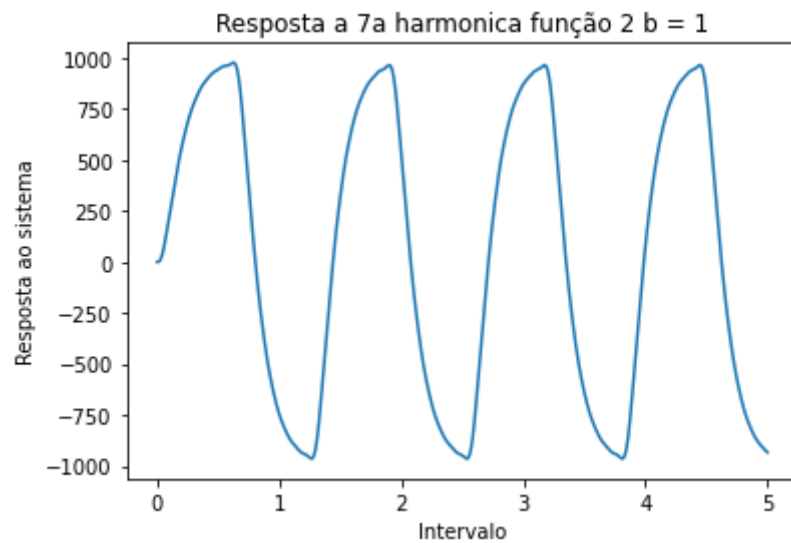
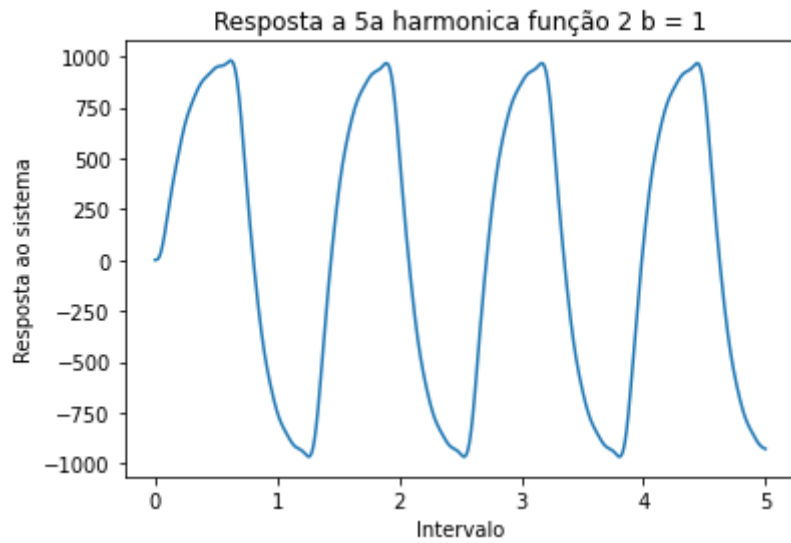


Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmônicas

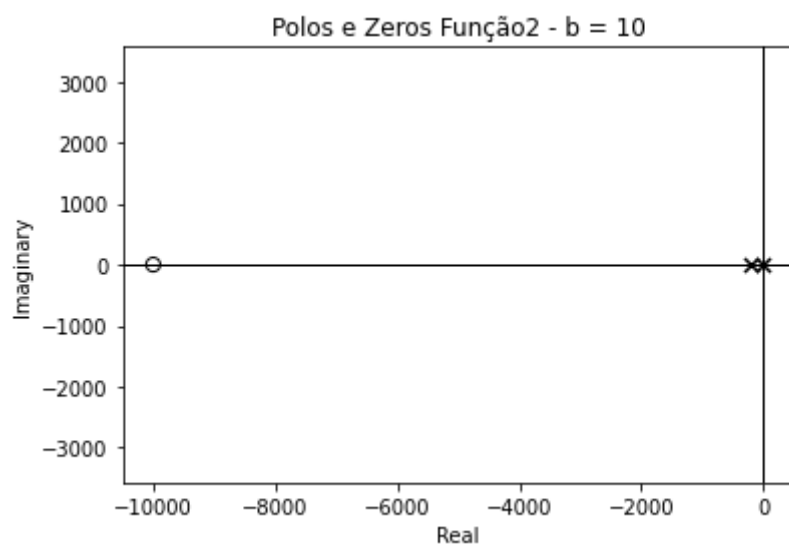




3.2.3 Para $\beta = 10$

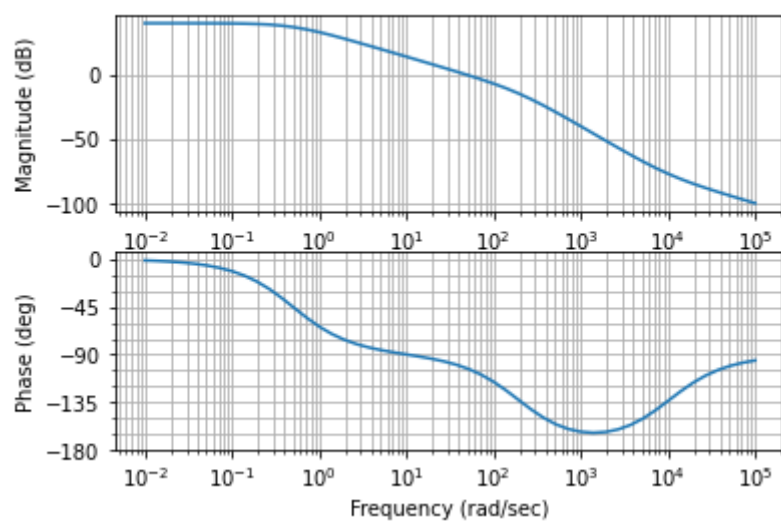
Letra A: EDO A função de transferência é $H(s) = (s+10^4) \cdot \frac{1}{s^2+200s+100}$
 $y''(t)+200y'(t)+100y(t) = x'(t)+10^4x(t)$

Letra B: Diagrama de Polos e zeros

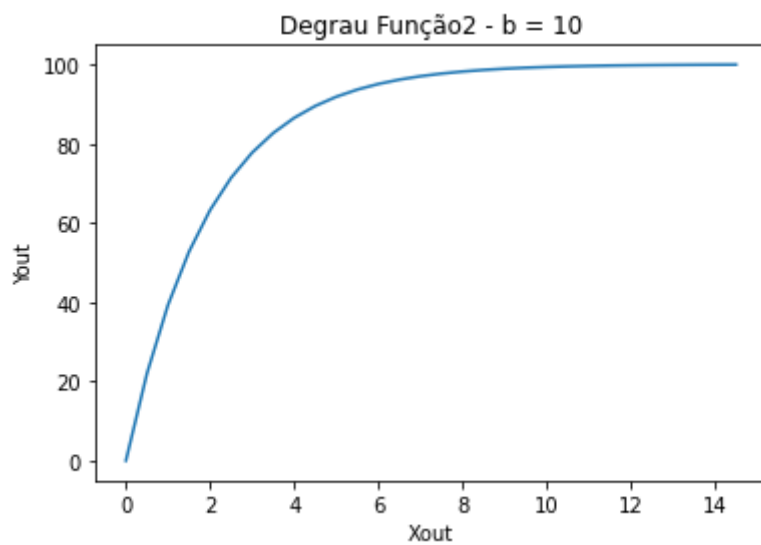


Letra C: Diagrama de Bode

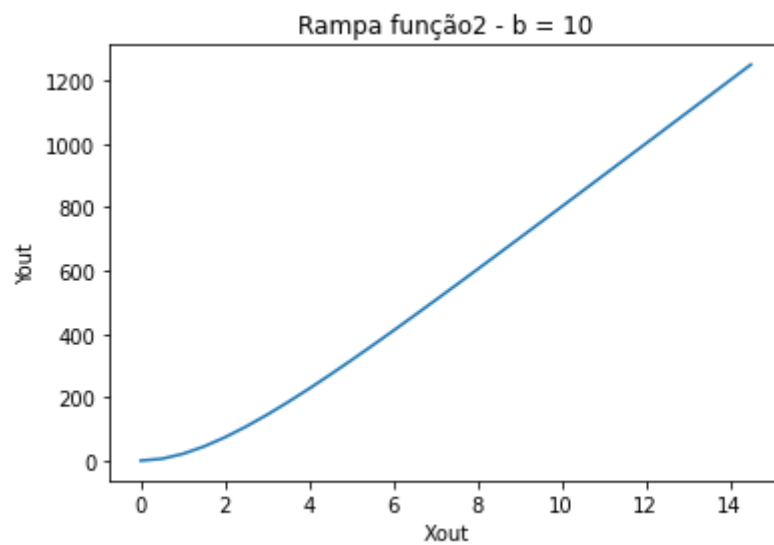
Diagrama de Bode Função2, $b = 10$



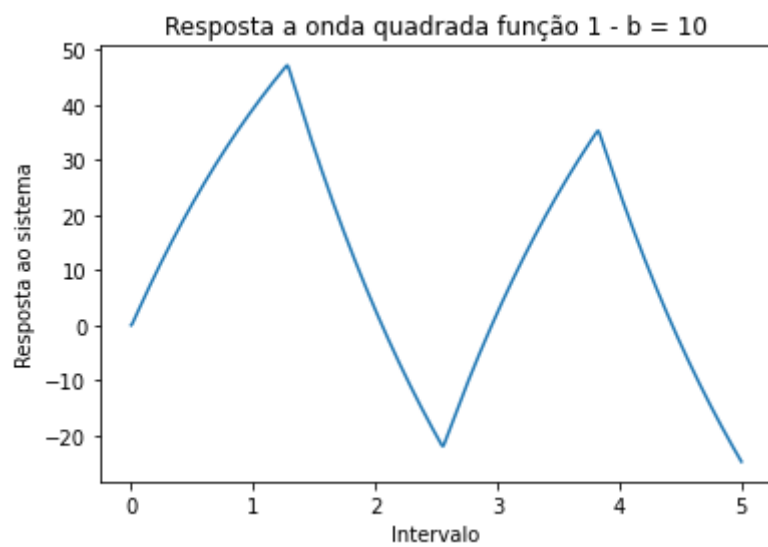
Letra D: Resposta ao degrau unitário



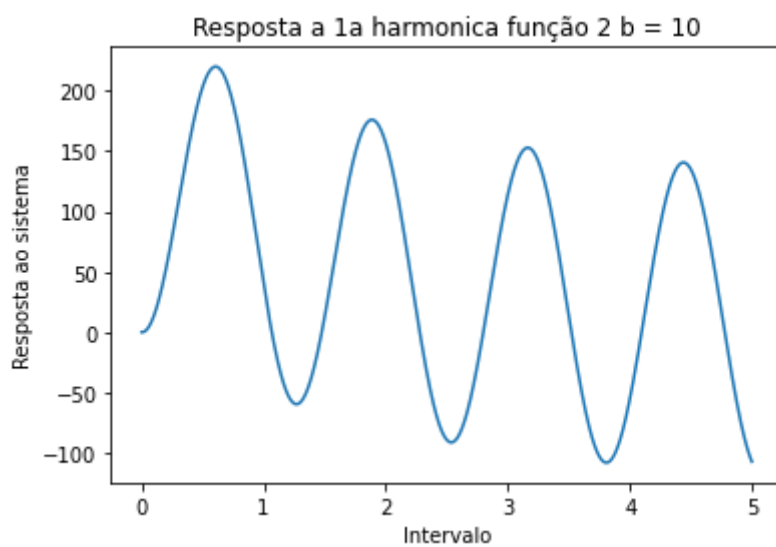
Letra E: Resposta a rampa unitária

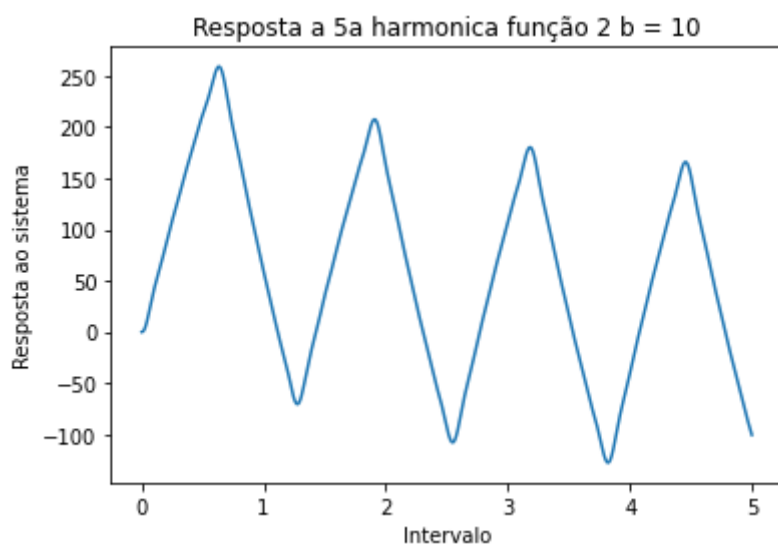
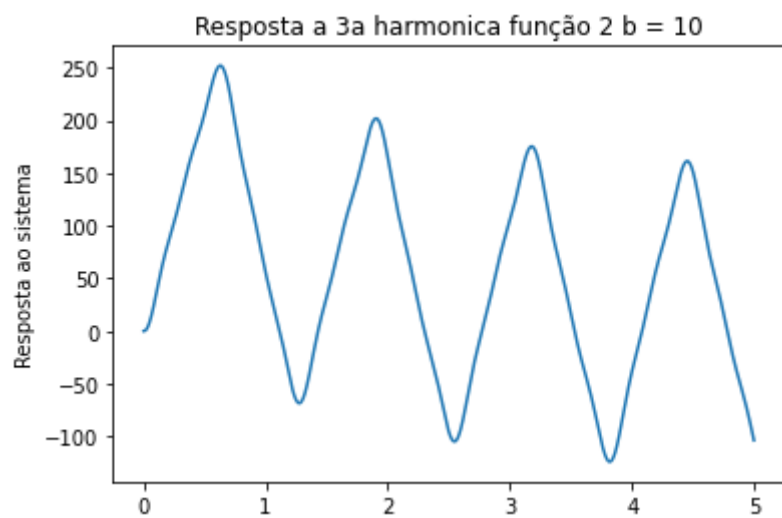


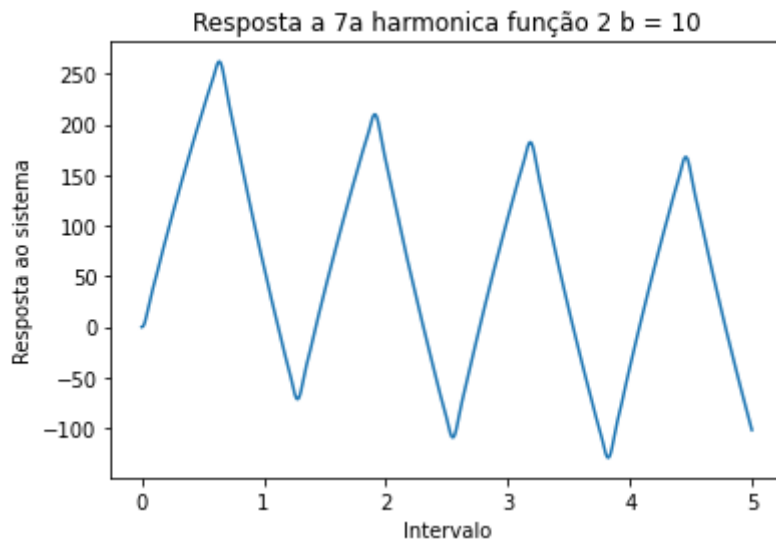
Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmonicas





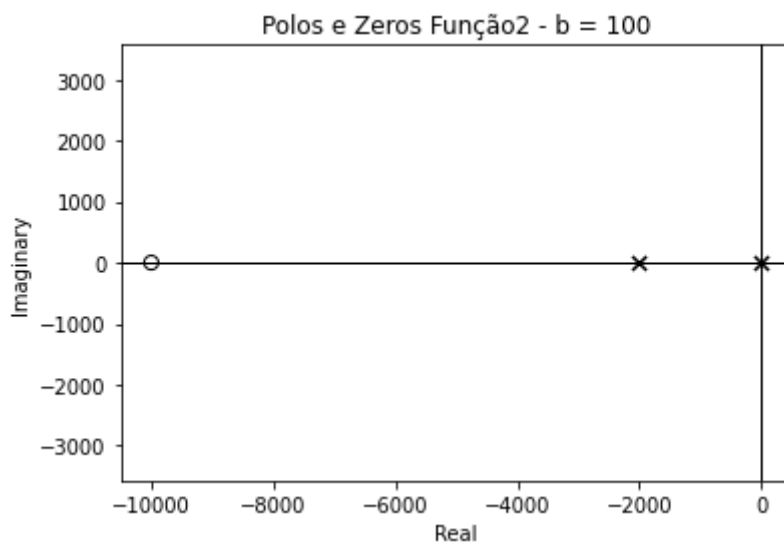


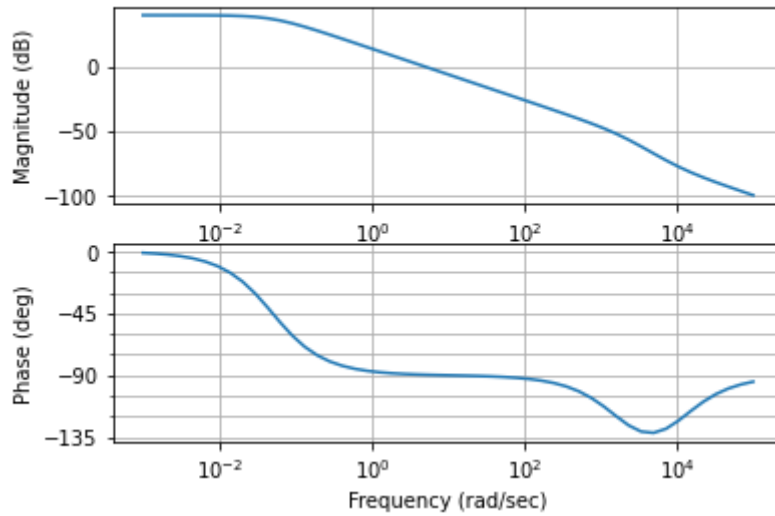
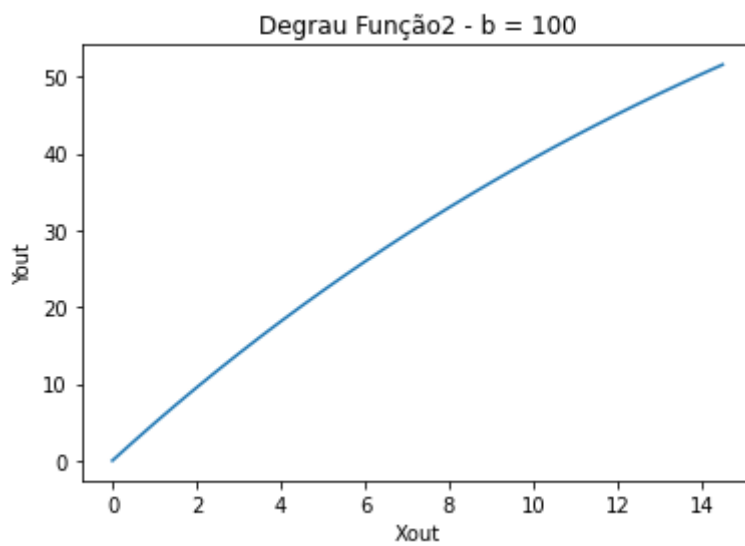
3.2.4 Para $\beta = 100$

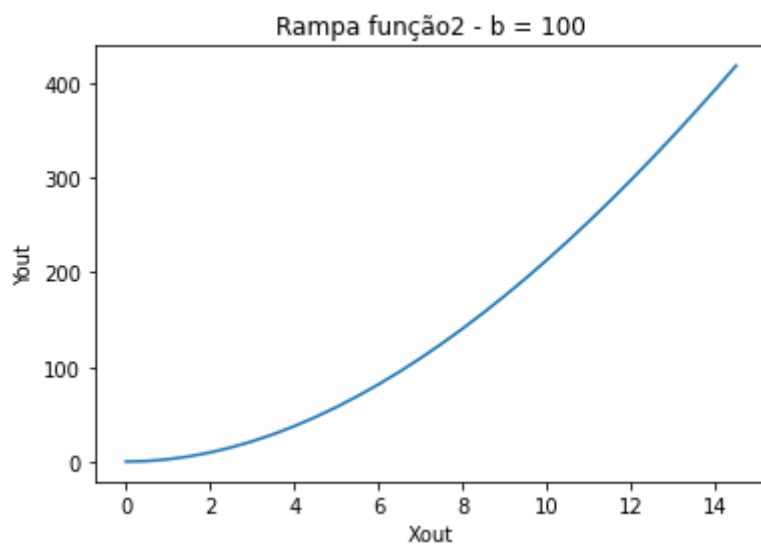
Letra A: EDO A função de transferência é $H(s) = (s+10^4) \cdot \frac{1}{s^2+2000s+100}$

$$y''(t)+2000y'(t)+100y(t) = x'(t)+10^4x(t)$$

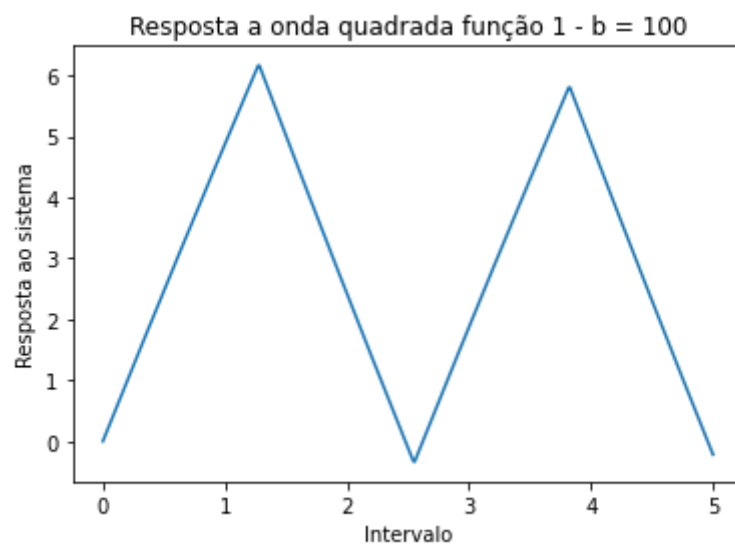
Letra B: Diagrama de Polos e zeros



Letra C: Diagrama de BodeDiagrama de Bode Função2, $b = 100$ **Letra D:** Resposta ao degrau unitário**Letra E:** Resposta a rampa unitária



Letra F: Resposta a onda quadrada



Letra G, H, I e J: Harmonicas

