Aula_EQM

Slides

1) X_1 e X_2 a.a. X, $E(X)=0$ e $Var(X)=1$

$$M_1 = \frac{4X_1+6X_2}{10} \qquad M_2 = \frac{X_1+X_2}{3}$$

$$E(M_1) = \frac{4}{10}E(\cancel{X_1})^{0} + \frac{6}{10}E(\cancel{X_2})^{0} = 0$$

$$E(M_2) = \frac{1}{3}E(\cancel{X_1})^{0} + \frac{1}{3}E(\cancel{X_2})^{0} = 0$$

estimadores não viesados

$$Var(M_1) = EQM(M_1) = \frac{16}{100}Var(\cancel{X_1})^{1} + \frac{36}{100}Var(\cancel{X_2})^{1}$$

$$= \frac{52}{100}$$

$$Var(M_2) = EQM(M_2) = \frac{1}{9}Var(\cancel{X_1})^{1} + \frac{1}{9}Var(\cancel{X_2})^{1} = 2/9$$

$$\boxed{EQM(M_1) > EQM(M_2).}$$

2

$X_1, \dots, X_n$ aa X, $E(X) = \mu = 0$ e

$Var(X) = \sigma^2 = 1$. Considere $n = 10$.

$$M_1 = \frac{\sum_{i=1}^{n} X_i}{n-2} \qquad M_2 = \frac{1}{2} X_1 + \frac{\sum_{i=2}^{n} X_i}{n-1}$$

$$E(M_1) = \frac{\sum_{i=1}^{n} E(X_i)}{n-2} = \frac{0}{n-2} = 0.$$

$EQM(M_1) < EQM(M_2)$

$$E(M_2) = 0$$

$$Var(M_1) = \frac{1}{(n-2)^2} \sum_{i=1}^{n} Var(X_i) = \frac{n}{(n-2)^2} = \frac{10}{8^2} = \frac{10}{64}$$

$$Var(M_2) = \frac{1}{4} Var(X_1) + \frac{1}{(n-1)^2} \sum_{i=2}^{n} Var(X_i)$$

$$= \frac{1}{4} + \frac{1}{(n-1)^2}(n-1) = \frac{1}{4} + \frac{1}{n-1} = \frac{1}{4} + \frac{1}{9} = \frac{9+4}{36} = 13/36$$

Considere $X_1, \dots, X_n$ aa $X \sim N(\mu, \sigma^2)$. Qual estimador para σ^2 é preferível em termos de EQM?

$\hat{\sigma}^2$ ou S^2, onde $\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ e

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$?

Sabemos que $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ e portanto,

$E(S^2) = \sigma^2$ e $Var(S^2) = \frac{2\sigma^4}{n-1}$

↳ estimador não viesado de σ^2.

Note que $\hat{\sigma}^2 = \frac{n-1}{n} S^2$ e assim,

$E(\hat{\sigma}^2) = \frac{n-1}{n}\sigma^2$ e $Var(\hat{\sigma}^2) = \left(\frac{n-1}{n}\right)^2 \frac{2\sigma^4}{n-1} = \frac{2\sigma^4(n-1)}{n^2}$.

↳ estimador viesado de σ^2

$EQM(S^2) = Var(S^2) = \frac{2\sigma^4}{n-1}$

$EQM(\hat{\sigma}^2) = Var(\hat{\sigma}^2) + \text{vício}^2$, onde $\text{vício} = \frac{n-1}{n}\sigma^2 - \sigma^2 = \left[\frac{n-1-1}{n}\right]\sigma^2$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1} \quad , \text{ então}$$

$$E\left[\frac{(n-1)S^2}{\sigma^2}\right] = (n-1) \Rightarrow$$

$$\frac{(n-1)}{\sigma^2} E\left[S^2\right] = (n-1) \Rightarrow$$

$$E[S^2] = \frac{\cancel{(n-1)}\sigma^2}{\cancel{(n-1)}} = \sigma^2 \quad //$$

E,

$$Var\left[\frac{(n-1)S^2}{\sigma^2}\right] = 2(n-1) \Rightarrow$$

$$\frac{(n-1)^2}{\sigma^4} Var(S^2) = 2(n-1) \Rightarrow$$

$$Var(S^2) = \frac{2\cancel{(n-1)}\sigma^4}{(n-1)^{\cancel{2}}} = \frac{2\sigma^4}{(n-1)} \quad //$$

$$= \left[\frac{n-1-n}{n}\right]\sigma^2 = -\frac{1}{n}\sigma^2.$$

$$EQM(\hat{\sigma}^2) = \frac{2\sigma^4(n-1)}{n^2} + \left(\frac{1}{n}\sigma^2\right)^2 =$$

$$\frac{2\sigma^4(n-1)}{n^2} + \frac{1}{n^2}\sigma^4 = \frac{\sigma^4}{n^2}\left[2(n-1)+1\right] =$$

$$\frac{\sigma^4}{n^2}\left[2n-2+1\right] = \frac{(2n-1)\sigma^4}{n^2}.$$

Note que $\hat{\sigma}^2$, apesar de viciado, apresenta um EQM menor que o EQM do estimador S^2.