Help sheet

1. CDF, $\xi \sim F(x) = P(\xi < x)$

1. $F(x)$ left continuous
2. $F(x)$ monoton increasing
3. $\lim_{x\to-\infty} F(x) = 0, \quad \lim_{x\to\infty} F(x) = 1$

$P(a \le \xi < b) = F(b) - F(a)$

2. PDF, $\xi \sim f(x)$

$F(x) = \int_{-\infty}^{x} f(t)dt \qquad F'(x) = f(x)$

1. $f(x)$ Borel-measurable
2. $f(x) \ge 0$
3. $\int_{-\infty}^{\infty} f(x)dx = 1$

$P(a < \xi < b) = \int_a^b f(x)dx$

3. Expectation, variance properties

ξ: random variable, $\qquad a, b \in \mathbb{R}$

$\mathbb{E}(a\xi + b) = a \cdot \mathbb{E}\xi + b \qquad \mathbb{D}^2(a\xi + b) = a^2 \cdot \mathbb{D}^2\xi$

4. Discrete distributions

4.1. Binomial distribution, $\xi \sim Binom(n, p) \qquad n \in \mathbb{N},\ 0 < p < 1$

$P(\xi = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, \ldots, n$

$\mathbb{E}\xi = np \qquad \mathbb{D}^2\xi = np(1-p)$

4.2. Poisson distribution, $\xi \sim Poiss(\lambda) \qquad \lambda > 0$

$P(\xi = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, \ldots$

$\mathbb{E}\xi = \lambda \qquad \mathbb{D}^2\xi = \lambda$

5. Absolute continuous distributions

5.1. Uniform distribution, $\xi \sim Uni(a,b)$ $\qquad a < b$

$f(x) = \begin{cases} 0, & x \notin [a,b] \\ \frac{1}{b-a}, & x \in [a,b] \end{cases}$ $\qquad F(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a < x < b \\ 1, & x > b \end{cases}$

$\mathbb{E}\xi = \frac{a+b}{2} \qquad \mathbb{D}^2\xi = \frac{(b-a)^2}{12}$

5.2. Exponential distribution, $\xi \sim Exp(\lambda)$ $\qquad \lambda > 0$

$f(x) = \begin{cases} 0, & x < 0 \\ \lambda e^{-\lambda x}, & x > 0 \end{cases}$ $\qquad F(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\lambda x}, & x > 0 \end{cases}$

$\mathbb{E}\xi = \frac{1}{\lambda} \qquad \mathbb{D}^2\xi = \frac{1}{\lambda^2}$

5.3. Normal distribution, $\xi \sim \mathcal{N}(\mu, \sigma^2)$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$

$\mathbb{E}\xi = \mu \qquad \mathbb{D}^2\xi = \sigma^2$

Standardization: $\frac{\xi - \mathbb{E}\xi}{\sqrt{\mathbb{D}^2\xi}} = \frac{\xi - \mu}{\sigma} \sim \mathcal{N}(0,1)$ standard normal

Standard normal CDF: $\phi(x)$ $\qquad$ /symmetric: $\phi(x) = 1 - \phi(-x)$/
matlab: normcdf()

6. Chebisev's inequality, $\xi \geq 0$

$$P(|\xi - \mathbb{E}\xi| < k) > 1 - \frac{\mathbb{D}^2\xi}{k^2} \qquad \Longleftrightarrow \qquad P(|\xi - \mathbb{E}\xi| \geq k) \leq \frac{\mathbb{D}^2\xi}{k^2}$$

7. Statistics

$X_1, \ldots, X_n$: sample

sample mean: $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$ $\qquad$ matlab: mean()
uncorrected variance: $s^2 = \frac{1}{n}\sum_{i=1}^n (X_i - \bar{X})^2 = \frac{1}{n}\sum_{i=1}^n X_i^2 - \bar{X}^2$
(corrected) variance: $s^{*2} = \frac{1}{n-1}\sum_{i=1}^n (X_i - \bar{X})^2 = \frac{n}{n-1}s^2$ $\qquad$ matlab: var()
(corrected) standard deviation: $s^* = \sqrt{s^{*2}}$ $\qquad$ matlab: std()

$X_1^*, \ldots, X_n^*$: ordered sample (mon. inc.)

empirical CDF (ECDF): $F^*(x) = \begin{cases} 0, & x \leq X_1^* \\ \frac{k}{n}, & X_k^* < x \leq X_{k+1}^* \quad k = 1, \ldots, n-1 \\ 1, & x > X_n^* \end{cases}$

q-quantile: $Q_q^* = X_{[s]}^* + \{s\}(X_{[s]+1}^* - X_{[s]}^*)$ $\qquad s = q \cdot (n+1)$ $\qquad$ [.]:integer, {.}:fraction