MATE6611_Project

9/25/2020

4.2

c Show that $\mathbb{E}[Y] = \frac{1}{\theta}$ and that $Var(Y) = \frac{1}{\theta^2}$

Calculating the mean:

$$\mathbb{E}[Y] = \int_0^\infty y\theta e^{-y\theta} dy$$

Using integration by parts, let $u = y$, $dv = \theta e^{-y\theta dy}$, then

$$\mathbb{E}[Y] = -ye^{-y\theta}\Big|_0^\infty + \int_0^\infty e^{-e\theta} dy = 0 + \frac{-e^{-y\theta}}{\theta}\Big|_0^\infty = \frac{1}{\theta}$$

Calculating variance:

$$Var(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2 = \int_0^\infty y^2\theta e^{-y\theta} dy - \theta^{-2}$$

Using integration by parts, let $u = y^2$ and $dv = \theta e^{-y\theta} dy$, then

$$Var(Y) = -y^2 e^{-y\theta}\Big|_0^\infty + 2\int_0^\infty ye^{-y\theta} dy - \theta^{-2} = -\frac{1}{\theta^2} + \frac{2}{\theta}\int_0^\infty y\theta e^{-y\theta} dy = -\frac{1}{\theta^2} + \frac{2}{\theta}\mathbb{E}[Y] = -\frac{1}{\theta^2} + \frac{2}{\theta^2} = \frac{1}{\theta^2}$$