

# MATE6611\_Project

9/25/2020

## 4.2

**c Show that  $\mathbb{E}[Y] = \frac{1}{\theta}$  and that  $Var(Y) = \frac{1}{\theta^2}$**

Calculating the mean:

$$\mathbb{E}[Y] = \int_0^{\infty} y\theta e^{-y\theta} dy$$

Using integration by parts, let  $u = y$ ,  $dv = \theta e^{-y\theta} dy$ , then

$$\mathbb{E}[Y] = -ye^{-y\theta} \Big|_0^{\infty} + \int_0^{\infty} e^{-y\theta} dy = 0 + \frac{-e^{-y\theta}}{\theta} \Big|_0^{\infty} = \frac{1}{\theta}$$

Calculating variance:

$$Var(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2 = \int_0^{\infty} y^2\theta e^{-y\theta} dy - \theta^{-2}$$

Using integration by parts, let  $u = y^2$  and  $dv = \theta e^{-y\theta} dy$ , then

$$Var(Y) = -y^2 e^{-y\theta} \Big|_0^{\infty} + 2 \int_0^{\infty} ye^{-y\theta} dy - \theta^{-2} = -\frac{1}{\theta^2} + \frac{2}{\theta} \int_0^{\infty} y\theta e^{-y\theta} dy = -\frac{1}{\theta^2} + \frac{2}{\theta} \mathbb{E}[Y] = -\frac{1}{\theta^2} + \frac{2}{\theta^2} = \frac{1}{\theta^2}$$