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Lab session # 4
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1 Question 1

Removing an edge adds a connected component. After removing randmly two edges there is therefore 3 connected components.

2 Question 2

Let's give a counterexample. We consider two graphs G_1 and G_2.
G_1 is a cycle graph og size 5 $c_1 \leftrightarrow c_2 \leftrightarrow c_3 \leftrightarrow c_4 \leftrightarrow c_5$.
G_2 is made of two connected components : a closed triplet (three nodes three edges) $t_1 \leftrightarrow t_2 \leftrightarrow t_3 \leftrightarrow t_1$ and a couple (two nodes one edge) $d_1 \leftrightarrow d_2$.
The degree distribution is in both case 22211.
Suppose G_1 and G_2 are isomorphic and let f be the bijective mapping. Since d_1 and d_2 are the only nodes to have same degree than c_1, we must have $f(c_1) \in \{d_1, d_2\}$.
Then since $c_1 \leftrightarrow c_2$ we must have $f(c_1) \leftrightarrow f(c_2)$ and $f(c_2) \in \{d_1, d_2\}$.
Repeating the last argument we have that $f(c_3) \in \{d_1, d_2\}$ and therefore f is not a bijection. This is absurd.

3 Question 3

A complete graph of n nodes has $\frac{n(n-1)}{2}$ edges. This means that G is a complete graph with one edge missing. A complete graph has $\binom{n}{3}$ closed triplets, therefore G has $\binom{n}{3} - (n-2)$ closed triplets as a missing edge removes $n-2$ triangles. The number of open triplets stays $\binom{n}{3}$ even with a missing edge so the clustering coefficient is $1 - \frac{n-2}{\binom{n}{3}} = 1 - \frac{6}{n(n-1)}$

4 Question 4

According to proposition 2 page 4 of article [1], the multiplicity k of the eigenvalue 0 of L equals the number of connected components $A1, ..., Ak$ in the graph. The eigenspace of eigenvalue 0 is spanned by the indicator vectors $\mathbb{1}_{A1}, ..., \mathbb{1}_{Ak}$ of those components.

5 Question 5

The K-means algorithm is stochastic. It starts with a random set of data points as initial centroids. Since spectral clustering uses K-means, it is stochastic too.

6 Question 6

a) $m = 8,\ l_1 = 4,\ l_2 = 3,\ d_1 = 9,\ d_2 = 7$ therefore the modularity is $\frac{4}{8} - (\frac{9}{16})^2 + \frac{3}{8} - (\frac{7}{16})^2 \backsimeq 0,3672$

b) $m = 8,\ l_1 = 1,\ l_2 = 2,\ l_3 = 1,\ d_1 = 4,\ d_2 = 8,\ d_3 = 4$ therefore the modularity is $\frac{1}{8} - (\frac{4}{16})^2 + \frac{2}{8} - (\frac{8}{16})^2 + \frac{1}{8} - (\frac{4}{16})^2 = 0,125$

7 Question 7

$\phi(P_4) = (4, 3, 2, 1)$ and $\phi(K_4) = (6, 0, 0, 0)$ therefore $k(P_4, P_4) = 16 + 9 + 4 + 1 = 30$, $k(P_4, K_4) = 24$, $k(K_4, K_4) = 36$

References

[1] Ulrike Von Luxburg. A tutorial on spectral clustering. *Statistics and computing*, 17(4):395–416, 2007.