Lista 4

Exercício 1. Suponha que, para todo x, $|g(x)| \leq x^4$. Calcule $\lim\limits_{x \to 0} \dfrac{g(x)}{x}$.

Exercício 2. Seja f definida em $\mathbb{R}$ e tal que, para todo x, $|f(x)-3| \leq 2|x-1|$. Calcule $\lim\limits_{x \to 1} f(x)$. Justifique.

Exercício 3. Calcule, caso exista, $\lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$ em que f é dada por

(a) $f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{se} \quad x \neq 0 \\ 0 & \text{se} \quad x = 0 \end{cases}$
(b) $f(x) = \begin{cases} x \sin \frac{1}{x} & \text{se} \quad x \neq 0 \\ 0 & \text{se} \quad x = 0 \end{cases}$

Exercício 4. Calcule.

(a) $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$

(e) $\lim\limits_{x \to 0} \dfrac{x}{\sin x}$

(b) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x}$

(f) $\lim\limits_{x \to 0} \dfrac{\sin 3x}{x - \pi}$

(c) $\lim\limits_{x \to 0} \dfrac{\tan 3x}{\sin 4x}$

(g) $\lim\limits_{x \to 0} \dfrac{x^2}{\sin x}$

(d) $\lim\limits_{x \to 0} \dfrac{1 - \sin x}{2x - \pi}$

(h) $\lim\limits_{x \to 0} \dfrac{x^2}{\sin x}$

Exercício 5. Calcule.

(a) $\lim\limits_{x \to +\infty} \dfrac{1}{x^2}$

(i) $\lim\limits_{x \to -\infty} \dfrac{1}{x^3}$

(b) $\lim\limits_{x \to +\infty} \left[5 + \dfrac{1}{x} + \dfrac{3}{x^2} \right]$

(j) $\lim\limits_{x \to +\infty} \left[2 - \dfrac{1}{x} \right]$

(c) $\lim\limits_{x \to +\infty} \dfrac{2x + 1}{x + 3}$

(k) $\lim\limits_{x \to -\infty} \dfrac{2x + 1}{x + 3}$

(d) $\displaystyle\lim_{x\to-\infty} \frac{x^2 - 2x + 3}{3x^2 + x + 1}$

(l) $\displaystyle\lim_{x\to+\infty} \frac{5x^4 - 2x + 1}{4x^4 + 3x + 2}$

(e) $\displaystyle\lim_{x\to+\infty} \frac{x}{x^2 + 3x + 1}$

(m) $\displaystyle\lim_{x\to-\infty} \frac{2x^3 + 1}{x^4 + 2x + 3}$

(f) $\displaystyle\lim_{x\to+\infty} \sqrt[3]{5 + \frac{2}{x}}$

(n) $\displaystyle\lim_{x\to-\infty} \sqrt[3]{\frac{x}{x^2 + 3}}$

(g) $\displaystyle\lim_{x\to+\infty} \frac{\sqrt{x^2 + 1}}{3x + 1}$

(o) $\displaystyle\lim_{x\to+\infty} \frac{\sqrt[3]{x^3 + 2x - 1}}{\sqrt{x^2 + x + 1}}$

(h) $\displaystyle\lim_{x\to+\infty} \frac{\sqrt{x} + \sqrt[3]{x}}{x^2 + 3}$

(p) $\displaystyle\lim_{x\to+\infty} \left[x - \sqrt{x^2 + 1}\right]$