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Autovalores e Autovetores de Matrizes Reais Simétricas - O Algoritmo QR - Parte 2

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Sumário

1 Introdução

No Exercício-Programa 1, programamos, dentre outras tarefas, o algoritmo QR para matrizes tridiagonais simétricas.

Neste exercício-programa utilizaremos as transformações de Householder para fazer a redução de uma matriz simétrica a uma matriz tridiagonal simétrica semelhante. Dado $w \in R^n$ podemos definir a transformação de Householder como $Hw: \ R^n \rightarrow R^n$ dada por $Hw = I - \frac{2w \cdot w^T}{w \cdot w}$, em que cada $x \in R^n$ associa $Hw_x = x - 2\frac{w \cdot x}{w \cdot w}x$ e determina a reflexão do vetor x. Dado dois vetores x e y em R^n e não nulos, podemos considerar que $Hw_x = \lambda y$ com $\lambda \in R^n$, tomando $w = x + \alpha y$, em que $\alpha = \pm\frac{\|x\|}{\|y\|}$.

Para transformar uma matriz A_{nxn} em $T = HAH^T$, em que T é uma matriz tridiagonal e H é o produto das transformações $Hw_{n-2}, \ Hw_{n-3}$ e etc, é preciso primeiro aplicar a primeira transformação Hw_1 e zerar os elementos da primeira coluna de A a partir da terceira linha e, para isso, iremos definir $a_1^{-T} = (0, \ A_{2,1}, \ A_{3,1}, \ \ldots, \ A_{n-1}$ e

$$Hw_1 = \begin{bmatrix} \mathbf{1} & \mathbf{0}^T \\ \mathbf{0} & H\overline{w}_1 \end{bmatrix}$$

e ao aplicar essa matriz de transformação, obteremos a matriz:

$$H_{w_1}A = \begin{bmatrix} x & x & x & \ldots & x \\ x & x & x & \ldots & x \\ 0 & x & x & \ldots & x \\ . & . & . & \ldots & . \\ . & . & . & \ldots & . \\ 0 & x & x & \ldots & x \end{bmatrix}$$

Esse processo é aplicado até que a matriz se torne tridiagonal. Logo, na i-ésima transformação teremos

$$Hw_1 = \begin{bmatrix} \mathbf{I}_i & \mathbf{0}^T \\ \mathbf{0} & H\overline{w}_i \end{bmatrix}$$

e uma matriz tridiagonal simétrica, pronta para ser processada pelo algoritmo QR.

2 Metodologia

2.1 Questões A e B

Nas tarefas A e B iremos aplicar a transformação de Householder às matrizes dadas por a) e b) para encontrar seus autovalores e autovetores, e também verificar se $Av = \lambda v$ para os respectivos pares e se são ortogonais. As matrizes se encontram nos arquivos input-a e input-b, respectivamente.

$$A = \begin{pmatrix} 2 & 4 & 1 & 1 \\ 4 & 2 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$

a)

Com os autovalores sendo $7,\ 2,\ -1,\ -2$.

$$A = \begin{pmatrix} n & n-1 & n-2 & \cdots & 2 & 1 \\ n-1 & n-1 & n-2 & \cdots & 2 & 1 \\ n-2 & n-2 & n-2 & \cdots & 2 & 1 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 2 & 2 & 2 & \cdots & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 1 \end{pmatrix}$$

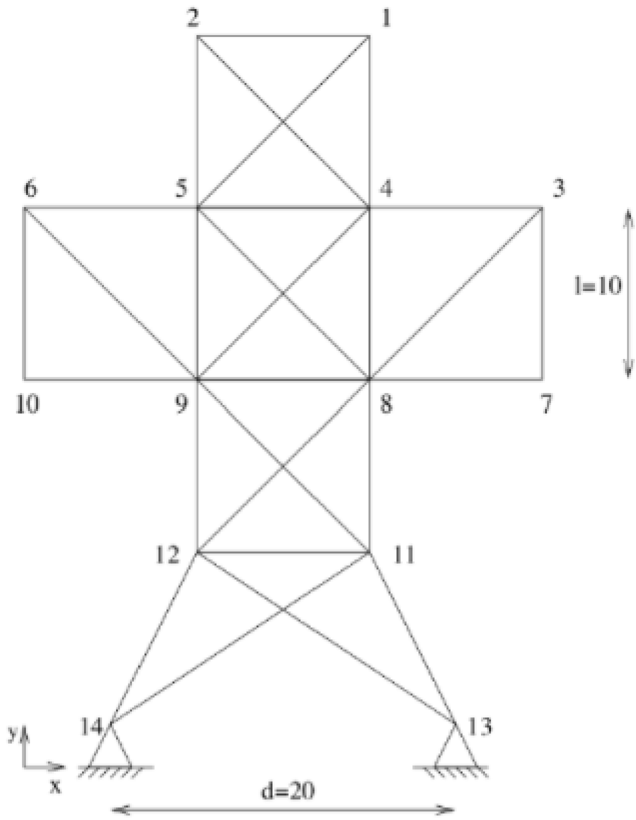
b)

Com os autovalores sendo definidos por:

$$\lambda_i = \frac{1}{2}\left[1 - cos\frac{(2i-1)\pi}{2n+1}\right]^{-1},\ i = 1, 2, ..., n.$$

2.2 Questão C

Essa questão lida com a aplicação da transformação de Householder em uma treliça simples. Assim, é pedido para que nós calculemos as frequências e os modos de vibração em um sistema de treliças planas, como demonstrado na figura abaixo.



Dados:

$\rho = 7,8 \times 10^3\ kg/m^3$

$A = 10^{-1}\ m^2$

$E = 200\ GPa$

$L = 10\ m$

$barras = 28$

$ns = 12$

$pontos\ de\ apoio = 2$

3 Discussão dos resultados

3.1 Questões A e B

Na questão A, com a matriz vinda do input-a, obtivemos os seguintes resultados:

A matriz inicial, após passar pelo algoritmo de Householder, ficou da seguinte forma:

```
Matriz tridiagonal simétrica equivalente:
[[2.00000 4.24264 0.00000 0.00000]
 [4.24264 3.00000 1.41421 0.00000]
 [0.00000 1.41421 2.00000 0.00000]
 [0.00000 0.00000 0.00000 -1.00000]]
```

E a matriz H^T ficou:

```
Matriz H^t:
[[1.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -2.965e+20 1.327e+25 1.456e+24]
 [0.000e+00 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.093e+31 3.093e+31 2.357e+25 -1.055e+30 -1.158e+29]
 [0.000e+00 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 1.970e+25 -8.821e+29 -9.676e+28]
 [0.000e+00 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.247e+25 5.582e+29 6.123e+28]]
```

Com isso, temos uma matriz própria para ser utilizada pelo algoritmo QR. Passando a matriz tridiagonal simétrica equivalente pelo algoritmo QR, obtivemos os seguintes autovalores:

```
Auto-valor [1]:
7.0
Auto-valor [2]:
2.0
Auto-valor [3]:
2.0
Auto-valor [4]:
-1.0
```

E os seguintes autovetores:

```
Auto-vetores:
[[9.776e-01 -1.101e-10 -2.103e-01 -7.305e-05]
 [1.983e-01 -3.333e-01 9.217e-01 3.201e-04]
 [4.957e-02 6.667e-01 2.302e-01 7.072e-01]
 [4.957e-02 6.667e-01 2.307e-01 -7.070e-01]]
Número de iteracoes com deslocamento espectral: 4.0
```

Os autovalores coincidem com os dados pelo enunciado.

Já na matriz vinda do input-b, obtivemos os seguintes resultados:
A matriz inicial, após passar pelo algoritmo de Householder, ficou da seguinte forma:

```
Matriz tridiagonal simétrica equivalente:
[[20.00000 49.69909 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [49.69909 152.20000 16.52453 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 16.52453 17.02222 4.74992 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 4.74992 6.62393 2.21346 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 2.21346 3.56561 1.26177 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 1.26177 2.25490 0.80323 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.80323 1.57333 0.54753 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.54753 1.17379 0.39047 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.39047 0.91954 0.28712 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.28712 0.74775 0.21550 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.21550 0.62624 0.16383 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.16383 0.53713 0.12534 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.12534 0.46984 0.09589 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.09589 0.41779 0.07286 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.07286 0.37670 0.05451 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.05451 0.34369 0.03964 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.03964 0.31677 0.02743 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.02743 0.29454 0.01727 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.01727 0.27596 0.00868]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00868 0.26027]]
```

E a matriz H^T ficou:

```
Matriz H^t:
[[1.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00 0.000e+00]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -1.115e+33 -8.500e+26 3.806e+31 4.175e+30]
 [0.000e+00 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -3.890e+26 -2.965e+20 1.327e+25 1.456e+24]
 [0.000e+00 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.092e+31 3.093e+31 3.093e+31 2.357e+25 -1.055e+30 -1.158e+29]
 [0.000e+00 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 2.585e+31 1.970e+25 -8.821e+29 -9.676e+28]
 [0.000e+00 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.636e+31 -1.247e+25 5.582e+29 6.123e+28]]
```

Com isso, temos uma matriz própria para ser utilizada pelo algoritmo QR. Passando a matriz tridiagonal simétrica equivalente pelo algoritmo QR, obtivemos os seguintes autovalores:

```
Auto-valor [1]:
170.40427
Auto-valor [2]:
19.0081
Auto-valor [3]:
6.89678
Auto-valor [4]:
3.56048
Auto-valor [5]:
2.18681
Auto-valor [6]:
1.41556
Auto-valor [7]:
1.0791
Auto-valor [8]:
0.84146
Auto-valor [9]:
0.67787
Auto-valor [10]:
0.56314
```

```
Auto-valor [11]:
0.4803
Auto-valor [12]:
0.41904
Auto-valor [13]:
0.37295
Auto-valor [14]:
0.33791
Auto-valor [15]:
0.31111
Auto-valor [16]:
0.29059
Auto-valor [17]:
0.27504
Auto-valor [18]:
0.25246
Auto-valor [19]:
0.25333
Auto-valor [20]:
0.25596
```

E os seguintes autovetores:

```
Auto-vetores:
[[1.000e+00 -4.744e-20 2.009e-29 -6.337e-36 1.189e-38 2.755e-40 2.597e-41 1.493e-42 3.840e-44 5.987e-46 6.885e-48 6.290e-50 4.614e-52 2.618e-54 1.050e-56 2.448e-59 1.696e-62 2.583e-66 9.478e-70 -5.925e-75]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.174e+30]
 [-5.291e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.175e+30]
 [-5.292e+13 -1.115e+33 -1.115e+33 -1.113e+33 1.091e+33 1.031e+33 1.152e+33 1.154e+33 1.128e+33 1.120e+33 1.118e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.117e+33 1.116e+33 1.848e+29 -3.806e+31 -4.175e+30]
 [-1.846e+07 -3.890e+26 -3.890e+26 -3.883e+26 3.806e+26 3.596e+26 4.017e+26 4.026e+26 3.934e+26 3.907e+26 3.900e+26 3.897e+26 3.897e+26 3.897e+26 3.897e+26 3.897e+26 3.893e+26 6.446e+22 -1.327e+25 -1.456e+24]
 [1.467e+12 3.092e+31 3.092e+31 3.087e+31 -3.025e+31 -2.859e+31 -3.193e+31 -3.200e+31 -3.127e+31 -3.106e+31 -3.100e+31 -3.098e+31 -3.098e+31 -3.098e+31 -3.098e+31 -3.098e+31 -3.095e+31 -5.124e+27 1.055e+30 1.157e+29]
 [1.226e+12 2.585e+31 2.585e+31 2.580e+31 -2.529e+31 -2.390e+31 -2.669e+31 -2.675e+31 -2.614e+31 -2.596e+31 -2.591e+31 -2.590e+31 -2.589e+31 -2.589e+31 -2.589e+31 -2.589e+31 -2.587e+31 -4.284e+27 8.821e+29 9.676e+28]
 [-7.761e+11 -1.636e+31 -1.636e+31 -1.633e+31 1.600e+31 1.512e+31 1.689e+31 1.693e+31 1.654e+31 1.643e+31 1.640e+31 1.639e+31 1.639e+31 1.639e+31 1.639e+31 1.639e+31 1.637e+31 2.711e+27 -5.582e+29 -6.123e+28]]
Número de iteracoes com deslocamento espectral: 21.0
```

3.2 Questão C

Os autovalores também coincidem com os autovalores calculados utilizando a fórmula do enunciado.

Na questão c, utilizamos os dados do arquivo input-c, que continha algumas informações gerais nas duas primeiras linhas, e no restante do arquivo informações sobre as barras da treliça. Com isso, calculamos essencialmente duas matrizes, a matriz K, com a contribuição de cada barra para a matriz de rigidez total, e a matriz M, com a massa de cada um dos nós. Infelizmente a matriz M do nosso código teve de ser escrita com os valores calculados à mão. Munidos dessas duas matrizes, pudemos calcular a matriz $\tilde{K}$, que segue a seguinte fórmula: $\tilde{K} = M^{-\frac{1}{2}} K M^{-\frac{1}{2}}$

Essa matriz será inserida no algoritmo Householder e posteriormente inserida no algoritmo QR para que tenhamos os autovalores dela. A raiz quadrada dos autovalores serão as frequências de vibração.

A matriz $\tilde{K}$ ficou:

```
Matriz K~
[[209667.81469 -162894.20765 0.00000 -21323.51075 21819.60167 -47475.18687 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [-162894.20765 482682.65372 -69516.19315 -31969.72788 -133168.03038 -16731.75705 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 -69516.19315 330543.16937 -108572.78073 41958.21289 -120045.96700 -39400.73559 19859.77131 -30554.71905 0.00000 0.00000 0.00000]
 [-21323.51075 -31969.72788 -108572.78073 346173.93430 -116204.76405 0.00000 55587.23123 -89960.61142 -20952.95203 -43861.21692 0.00000 0.00000]
 [21819.60167 6610.60186 0.00000 -116204.76405 406611.38470 -164338.66188 0.00000 -20575.50729 -85706.07140 -15458.07977 0.00000 0.00000]
 [-47475.18687 60167.37280 -120045.96700 0.00000 -164338.66188 603558.00840 -90827.17124 6016.83950 -29632.00457 -184033.34908 44170.43386 0.00000]
 [0.00000 0.00000 -39400.73559 55587.23123 0.00000 -90827.17124 514236.36264 -176898.42910 7861.36994 101296.02655 -115145.77265 0.00000]
 [0.00000 0.00000 19859.77131 -89960.61142 -20575.50729 6016.83950 -176898.42910 267068.11180 -76602.48139 0.00000 -14248.09333 14579.57485]
 [0.00000 0.00000 -30554.71905 14042.95260 4254.54002 -29632.00457 0.00000 -76602.48139 412350.40213 -203293.95605 -21361.75754 -88981.15081]
 [0.00000 0.00000 0.00000 -43861.21692 55587.23123 -27185.92824 101296.02655 0.00000 -203293.95605 851857.69230 -48744.17385 56915.84911]
 [0.00000 0.00000 0.00000 0.00000 0.00000 44170.43386 -115145.77265 -14248.09333 -49789.55923 -48744.17385 137743.35335 -37465.12747]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 14579.57485 -9830.97490 56915.84911 -37465.12747 97193.24057]]
```

E sua matriz tridiagonal simétrica equivalente ficou:

```
Matriz tridiagonal simétrica equivalente:
[[209667.81469 172392.58505 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [172392.58505 522602.83323 168877.60774 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 168877.60774 545137.17679 282907.90174 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 282907.90174 606978.94851 267128.11136 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 267128.11136 578938.98619 205196.01992 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 205196.01992 314169.29811 135971.65525 0.00000 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 135971.65525 302420.88612 210238.80540 0.00000 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 210238.80540 498182.78420 138023.02945 0.00000 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 138023.02945 330474.13733 111222.75261 0.00000 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 111222.75261 287403.26404 78460.29603 0.00000]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 78460.29603 159637.81306 96178.48190]
 [0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 0.00000 96178.48190 304072.18570]]
```

Já as cinco menores frequências de vibração encontradas foram:

```
[1] Menor frequência de vibração
281.22517
[2] Menor frequência de vibração
281.24133
[3] Menor frequência de vibração
302.13807
[4] Menor frequência de vibração
405.83224
[5] Menor frequência de vibração
405.83263
```

4 Conclusão

Neste EP pudemos dar continuidade ao aprendizado iniciado no EP1, aprendendo uma nova ferramenta que nos possibilita formatar matrizes simétricas para serem utilizadas no algoritmo QR, além de encontrarmos mais uma utilidade para ambas as ferramentas, utilidade essa que é bem ampla e pode até ser usada por nós alunos em outras disciplinas dos nossos cursos.

Durante o desenvolvimento do EP encontramos diversas dificuldades, tanto relacionadas ao entendimento do enunciado, quanto à questões de programação. Mas no geral consideramos nosso desempenho satisfatório.

5 Bibliografia

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2. TREFETHEN, Lloyd. A.; BAU, David. Numerical linear algebra. Tailândia: Society for Industrial and Applied Mathematics, 1997.