Topics in Macro B: Homework 4

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1 True or False

1.1 Borrowing constraint

Think of an individual who lives forever and has to decide how much to save every period (the only available asset to him is a one-period risk-free bond). He faces an exogenous borrowing constraint of $B > 0$ (i.e., he is only allowed to hold negative assets down to $-B$). When the borrowing constraintis not binding, the consumption Euler equation holds with equality. Hence, their consumption-savings decision is not influenced by the borrowing constraint.

Answer: False due to Case 3. Markets are incomplete. Suppose this is a one-agent model. Thus, we have three cases:

- Case 1: $\beta(1+r) < 1$ without uncertainty. Suppose the borrowing constraint is not binding, so if u is concave, we have $u'(c_t) = \beta(1+r)u'(c_{t+1})$ And the consumption-savings decision is not influenced by the borrowing constraint.
- Case 2: $\beta(1+r) = 1$ with uncertainty. Same as before.
- Case 3: $\beta(1+r) < 1$ with uncertainty. In this case, we have that the optima savings rule $A'(Z; B)$ implicitly depends on B (as Z depends on B). Therefore, in this case his consumption-savings decision is actually influenced by the borrowing constraint.

1.2 Complete vs Incomplete Markets I

In an endowment economy with complete markets, the interest rate is always s.t. $\beta(1+r) = 1$ in equilibrium. Your answer should depend on average endowments.

Answer: It depends on the endowments. Suppose that endowments $\{y_t\}_{t=0}^{\infty}$ are uncertain. Let the setting be the same as in PIH. Suppose there is an hypothetical budget constraint $B > 0$ that is not realised. Then, if we are in an economy such that for all t

$$\frac{1}{t+1}\sum_{t=1}^{T} y_j > B + \frac{1}{t+1}\sum_{j=0}^{t} c_j$$

We can support the consumption stream without having to hit the budget constraint. Therefore, if this particular condition holds, we can have an "as if" complete markets model where $\beta(1+r) < 1$.

1.3 Complete vs Incomplete Markets II

In a production economy with incomplete markets,the interest rate is always s.t. $b(1+r) < 1$ in a steady state with constant output.

Answer: True. Suppose we are in a steady state with constant output. Then, $y' = y = \bar{y}$. Therefore, since we are in a production economy, it follows that capital demand will also be constant given that $\bar{y} = f(k_t)$ for all t, so $\bar{k} = k_t$ for all t. Therefore, k, y are fixed in time and thus

$$w = f(k) - kf'(k), \quad R = f'(k) = r + \delta$$

Given the no arbitrage condition we can think of capital supply as assets. Therefore, the existence of the firm creates a demand for consumers to supply more capital, so they can avoid hitting their borrowing constraint. However, firm demand for capital is limited and thus it must be that the equilibrium interest rate r is below $\frac{1}{\beta} - 1$, which implies that $\beta(1+r) < 1$.

1.4 Incomplete Markets with Endowments vs Production

Suppose we are in a steady state of a production economy where aggregate production is Cobb-Douglas in capital and labor. Compare the interest rate in this economy with an endowment economy where the idiosyncratic endowments are exactly equal to the idiosyncratic wages received in the production economy. In steady state, the interest rate is always higher in the endowment economy.

Answer: False. It is the othr way around, the interest rate is always higher in the production economy. This is because in a production economy with Cobb-Douglas, we can write the endowment wage in steady state as:

$$w = (1-\alpha)\left(\frac{R^P}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} = (1-\alpha)\left(\frac{r^d + \delta}{\alpha}\right)^{\frac{\alpha}{1-\alpha}}$$

which includes the interest rate determined by capital demand r^d. Imposing market clearing, we have that the overall capital (savings) supply must equal capital demand, given by:

$$A(r^s) = \int a dF(a, \epsilon_i) = K(r^d)$$

which will implicitly define an equilibrium interest rate $(r^*)^p$, where p superscript denotes that it arises from a production economy. In the $r - A(r)$ space, it is the intersection between capital supply an demand.

However, in an endowment economy, we have that the aggregate savings must be

$$A(r^s) = \int a dF(a, y_i) = 0$$

And therefore, in the $r - A(r)$ space, interest rate $(r^*)^e$, where the e superscript denotes that it is the endowment economy interest rate, will be given by the intersection between $A(r)$ and the line defined by $A(r) = 0$. Since idiosyncratic endowments are exactly equal to idiosyncratic wages, $A^p(r) = A^e(r)$, and thus we can see that $(r^*)^e < (r^*)^p$, as in Figure 1.

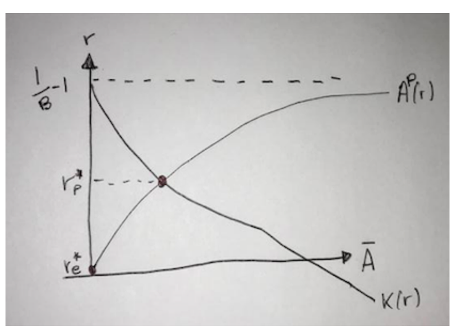

Figure 1: r-A(r) space

2 Non-AR(1) Markov Chain

For this exercise I provide a companion Matlab file `Nonar1chain.m`.

(a) For this exercise, I use the built-in Matlab function `logninv` in order to obtain 5 grid points with values $\epsilon = [0.7546, 1.6090, 2.7183, 4.5924, 9.7919]$. Each is calculated so $F_\epsilon(\epsilon_1) = 0.1, F_\epsilon(\epsilon_2) = 0.3, F_\epsilon(\epsilon_3) = 0.5, F_\epsilon(\epsilon_4) = 0.7, F_\epsilon(\epsilon_5) = 0.9$.

(b) The Markov transition matrix is defined in part (c) and it is given by

$$\mathbf{\Pi} = \begin{pmatrix} 0.92 & 0.02 & 0.02 & 0.02 & 0.02 \\ 0.02 & 0.92 & 0.02 & 0.02 & 0.02 \\ 0.02 & 0.02 & 0.92 & 0.02 & 0.02 \\ 0.02 & 0.02 & 0.02 & 0.92 & 0.02 \\ 0.02 & 0.02 & 0.02 & 0.02 & 0.92 \end{pmatrix}$$

To numerically construct a 5 times 5 matrix that satisfies such a Markov transition, I simulate a sequence of states using the specification of probabilities given for a total of $N = 9999999$ draws and then from this sequence I estimate the Markov transition matrix.

(c) We have to compute the invariant distribution analytically in terms of γ and π. This is the solution to the problem

$$\boldsymbol{f}\mathbf{\Pi} = \boldsymbol{f} \tag{1}$$

where, noting that in the initial distribution $\pi_i = \pi = 0.2$

$$\mathbf{\Pi} = \begin{pmatrix} \gamma + (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi \\ (1-\gamma)\pi & \gamma + (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi \\ (1-\gamma)\pi & (1-\gamma)\pi & \gamma + (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi \\ (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi & \gamma + (1-\gamma)\pi & (1-\gamma)\pi \\ (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi & (1-\gamma)\pi & \gamma + (1-\gamma)\pi \end{pmatrix}$$

and $\boldsymbol{f} = [f_1, f_2, f_3, f_4, f_5]$

First of all, denote $\pi + (1-\gamma)\pi = a$ and $b = (1-\gamma)\pi$

Then we have:

$$\mathbf{\Pi} = \begin{pmatrix} a & b & b & b & b \\ b & a & b & b & b \\ b & b & a & b & b \\ b & b & b & a & b \\ b & b & b & b & a \end{pmatrix}$$

The problem from 1 becomes:

$$\begin{aligned} af_1 + b(f_2 + f_3 + f_4 + f_5) &= f_1 \\ af_2 + b(f_1 + f_3 + f_4 + f_5) &= f_2 \\ af_3 + b(f_1 + f_2 + f_4 + f_5) &= f_3 \\ af_4 + b(f_1 + f_2 + f_3 + f_5) &= f_4 \\ af_5 + b(f_1 + f_2 + f_3 + f_4) &= f_5 \end{aligned}$$

And it follows that by symmetry of the equations that $f_i = f = \pi$ for $i = 1, \ldots, 5$.

(d) By the law of large numbers, and assuming a large number of individuals, we can think of the initial distribution as the proportion of individuals in a given state, and thus just iterate $f_{t+1} = \Pi f_t$, starting at $f_{1,0} = 1$ and $f_{i,0} = 0$, until convergence. The stationary distribution is given by $f = [0.2, 0.2, 0.2, 0.2, 0.2]$ after 110 iterations.

3 Heterogeneous human capital and growth

We focus on part (d), that is, must $\beta(1+r) < 1$ in equilibrium?

The question boils down to see if markets are incomplete here, that is, if there is some lack of insurance that makes individuals save more than necessary with respect to the full insurance case. In this case the constant probability of survival γ makes the market incomplete as individuals have to take into account in the future the probability $(1-\gamma)$ that they lose their accumulated human capital. Since returns to assets and human capital will be equalised in equilibrium (see previous problem set), we can see that $\int h dF(h, \epsilon_i) = K$ as in the Aiyagari model (given heterogeneous agents), and the same logic applies. Therefore, $\beta(1+r) < 1$ in equilibrium.