Mathematics for IT 2 — Practice Questions

Polynomials & Vectors

A. Polynomial Functions

A1. Polynomial algebra — addition and subtraction

Add or subtract and simplify:

1. $(-9x^3 + 7x^2 - 5x + 3) + (13x^3 + 2x^2 - 8x - 6)$
2. $(7x^3 - 8x^2 + 9x - 6) - (2x^3 - 6x^2 - 3x + 9)$

A2. Polynomial algebra — multiplication

Expand and simplify:

1. $(-9x^3 + 7x^2 - 5x + 3)(8x - 6)$
2. $(9x - 6)(2x^3 - 6x^2 - 3x + 9)$

A3. Long division (remainder and factor theorems)

Determine the quotient and remainder:

1. $\dfrac{5x^3 - 4x^2 - 3x + 6}{x - 2}$
2. $\dfrac{4x^3 - 3x^2 + 5x - 3}{x - 4}$
3. $\dfrac{x^3 - 2x^2 - 3x + 5}{x - 5}$
4. $\dfrac{2x^3 + 3x^2 - x + 4}{x + 2}$
5. $\dfrac{3x^3 - 11x^2 + 10x - 12}{x - 3}$

A4. Factorisation

Factor completely:

1. $18x + 27$
2. $3x^2 + 6x$
3. $9x^4 - 18x^3 + 27x^2$
4. $x(x + 5) + 3(x + 5)$
5. $x^3 + 2x^2 - 4x - 8$
6. $x^2y - 16y + 32 - 2x^2$
7. $2x^3 - 8a^2x + 24x^2 + 72x$

B. Vectors

B1. Components and operations in $\mathbb{R}^2$

Let $z_1 = 2\,\mathbf{i} + 4\,\mathbf{j}$ and $z_2 = 5\,\mathbf{i} + 2\,\mathbf{j}$. Find:

1. $z_1 + z_2$
2. $z_2 - z_1$

B2. Vector combinations in $\mathbb{R}^3$

Let $\vec{u} = 2\,\mathbf{i} - 3\,\mathbf{j} + 4\,\mathbf{k}$, $\vec{v} = 3\,\mathbf{i} + \mathbf{j} - 2\,\mathbf{k}$, $\vec{w} = \mathbf{i} + 5\,\mathbf{j} + 3\,\mathbf{k}$. Compute:

1. $\vec{u} + \vec{v}$
2. $2\vec{u} - 3\vec{v} + 4\vec{w}$

B3. Dot product (compute the value)

1. For $u = (1, -2, 3)$ and $v = (4, 5, -1)$, find $u \cdot v$.
2. For $u = (2, -3, 4)$ and $v = (-1, 2, 1)$, find $u \cdot v$.
3. For $u = (1, 2, 3, 4)$ and $v = (6, k, -8, 2)$, find k such that $u \cdot v = 0$.

B4. Inner products among three vectors

Given $u = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $v = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, $w = 4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$, find:

1. $u \cdot v, \quad u \cdot w, \quad v \cdot w$

B5. Norms (lengths) and unit vectors

1. Find $\|u\|$ for $u = (1, -2, -4, 5, 3)$.
2. Given $u = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $v = 2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, $w = 4\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$, compute $\|u\|$, $\|v\|$, $\|w\|$.

B6. Mixed practice (vectors u, v, w as in B4)

1. $2u - 3v$
2. $3u + 4w - 2v$
3. $\|u\|$, $\|v\|$, $\|w\|$

B7. Distance, angle and projection

Let $u = (1, -2, 3)$ and $v = (2, 4, 5)$. Compute:

1. $d(u, v) = \|u - v\|$
2. $\cos\theta = \dfrac{u \cdot v}{\|u\|\|v\|}$
3. $\mathrm{proj}_v(u) = \dfrac{u \cdot v}{\|v\|^2}\, v$

B8. Magnitude in $\mathbb{R}^3$

Find $|\vec{v}|$ for $\vec{v} = 3\,\mathbf{i} - 4\,\mathbf{j} + 12\,\mathbf{k}$.

B10. Vector exercises — Part 1

1. Find the norm and the direction cosines of each of the vectors $3\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$, $\mathbf{i} - 5\mathbf{j} - 8\mathbf{k}$, $6\mathbf{i} - 2\mathbf{j} + 12\mathbf{k}$. Also find the modulus and the direction cosines of their sum.

2. If $\mathbf{a} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, determine the scalar and vector products, and the angle between $\mathbf{a}$ and $\mathbf{b}$.

3. If $\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, determine (a) $\overrightarrow{OA} \cdot \overrightarrow{OB}$, (b) the cosine of the angle between them.

4. Find the cosine of the angle between $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$.

5. Find the scalar product $\mathbf{a} \cdot \mathbf{b}$ and the vector product $\mathbf{a} \times \mathbf{b}$ for:

   (a) $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

   (b) $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$