# ECHO-QGP manual Version 1.5.0- $\alpha$

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# Chapter 1

# Introduction

WARNING: document in progress, incomplete and not reviewed!

### 1.1 What is ECHO-QGP

ECHO-QGP is a program used to model heavy-ions collisions. It solves numerically the viscous hydrodinamic equations in the Israel-Stewart theoretical frame [1] [2] in 2+1D or 3+1D in Bjorken coordinates, from the formation of quark-gluon plasma up to the freeze-out stage. The user can set up different initial conditions (Optical and Monte Carlo Glauber, as well as custom initial energy density profile) and he/she can also choose between both analytic and tabulated equations of state.

The programming language chosen for ECHO-QGP is FORTRAN 2008.

#### 1.2 License

The code is released under the GPL v. 2.0; please, read the file LICENSE.TXT contained into the source directory or have a look at: https://www.gnu.org/licenses/gpl-2.0.html.

# 1.3 Funding

The development of ECHO-QGP has been supported by the Italian Ministry of Education and Research grant PRIN 2009 "Il Quark–Gluon Plasma e le collisioni nucleari di alta energia", by the INFN project RM31 and by funding from HIC for FAIR.

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#### 1.4 Main limitations - what the code cannot do

The main limitations of the code are:

- The code can describe one fluid only, expanding in "vacuum" (see next point about the "vacuum" term)
- The code needs a minimum baseline for the value of the energy density
  of the order of 1 KeV for the inviscid case, and 0.1 1 MeV for the
  viscous case
- The code can run either dissipative hydro or ideal (i.e. non resistive)
   MHD simulations, but at the moment it cannot run dissipative MHD simulations
- The time derivatives have a limited ( $\sim$  first order) accuracy
- Baryon charge evolution is properly treated only in the inviscid case; in the viscous case it only plays a numerical role, but it doesn't have a physical meaning since there are some missing terms in the equations describing its evolution
- To ensure the stability of the program, it is necessary to switch off the viscosity effects under a certain temperature (usually, less than 100MeV, i.e. well below the freezeout temperature, where the hydrodynamical description is not valid anymore)
- If initial conditions are not very "smooth", the program may crash before reaching the freezeout temperature (unfortunately, currently this happens quite often with Glauber-Monte Carlo initial conditions)
- The correct parsing of the configuration parameters requires not to alter the structure of the files param.dat and settings.txt
- The hypersurface computation always runs over a single core: it is not parallelized
- The grid partitioning in runs exploiting MPI is limited to the x-axis only and the user can exploit a number of CPUs not exceeding one third of the cells along the x axis
- I/O actions are performed by only a single processor
- The code does not yet take into account a post-hydro after-burner (any transport code or resonance decayer, rescatterings, etc)

# Chapter 2

# How to set up and run simulations

## 2.1 Requirements

Some requirements are assumed in order to use ECHO-QGP v1.5:

- a working GNU/Linux environment
- a fortran 2008 compiler
- the make utility
- the tar utility
- the gzip utility

The current version was developed and tested with:

Ubuntu 16.04 amd64 equipped with the GCC 5.4.0 compiler

The makefile provided within the package contains some of the most frequent flags used for specific architectures, so that an expert user can modify it at pleasure.

Additionally, some the (optional) postprocessing tools require GDL (http://www.gnudatalanguage.org) or  $IDL^{\circledR}$  (by EXELIS VIS) to be installed.

Root access is not needed to compile or run the program.

#### 2.1.1 Parallel run requirements

To execute parallel runs of ECHO-QGP on multiple cores machines, an MPI library is also needed. The code has been tested using only the OpenMPI implementation of the Message Passing Interface (MPI) standard, version 1.6.5 (all versions from 1.3.x to 1.8.x should also work, but they were not tested).

### 2.2 Download of ECHO-QGP

The latest version is downloadable from the official ECHO-QGP website: http://theory.fi.infn.it/echoqgp

From now on, <*version*> occurrences refer to the version of the code on which we are working on (e.g. ECHO-QGP-<*version*>.tar.gz could be ECHO-QGP-1.0.03 or ECHO-QGP-1.5.11).

The version id is composed by three numbers: a change in the first number will correspond to deep changes in the structure of the code or in the physics that it handles, a change in the second number will reflect significant improvements or minor new features, changes in the third number are reserved to bug-fixes only.

Only bug-fixing versions will not introduce incompatibilities with previous versions, while first and second digit new versions probably it will.

#### 2.2.1 Check of the integrity of the file

It is possible to check the source code tarball integrity: it is sufficient to download the corresponding md5sum checksum file and issue the command:

```
md5sum -c ECHO-QGP-<version>.tar.gz.md5sum
```

which returns, on positive match:

```
ECHO-QGP-<version >.tar.gz: OK
```

For any other return state, the user should download again the package.

#### 2.2.2 Uncompress the folder

To unpack the compressed archive containing the sources, just issue the command:

```
tar xzf ECHO-QGP-<version >.tar.gz
```

The uncompressed folder will contain: some sub-directories for utilities and post-hydro programs, some text files and all the files building ECHO-QGP itself. In detail, the bundle is composed as follows:

**analysis** *folder* Contains the post-hydro tools for the particle spectra production (see 7)

doc folder Contains the latest version of this manual and the Latest version of th

**eos\_data** *folder* Contains some essential files to run the simulations (equation of state, particle list, chemical potential, temperature definition ...)

LICENSE.TXT text file License under wich ECHO-QGP is released.

makefile The instructions to build the program using GNU Make

param.dat text file Configuration file of ECHO-QGP

random\_seed.dat An integer number to provide an initial seed for the Random Number Generator (used with Glauber-Monte Carlo initial conditions) (see 4.4)

**tests** *folder* Contains the essential informations and data to reproduce the examples described in the tutorials chapter (see 6) of this manual

**tools** *folder* Contains some tools to analyze the results of the simulations (see 5)

The remaining files included are the Fortran files containing the modules of ECHO-QGP. We give here an extremely synthetic description of each one of them:

common.f90 contains all the global variables and some utility functions

echo.f90 main file

**eos.f90** handles the Equation of State and the interplay among the thermodynamic variables

evolve.f90 Computes the time evolution of the quark-gluon plasma

**glaubermc.f90** Prepares the initial state for the hydro evolution with the Glauber Monte Carlo model

holib.f90 Library with high order reconstruction algorithms

hypersurface.f90 Computes the thermal freezeout hypersurface

init.f90 Initialization file

out.f90 Handles the output

parallel\_mpi.f90 Handles the parallelization

parallel\_nompi.f90 Dummy file for the serial run

**system.f90** Prepares the lattice and the metrics, handles the conversions between primitive and conserved variables

viscous.f90 Computes the viscous corrections

work.f90 Riemann solver and interfaces to high order libraries

# 2.3 Configure ECHO-QGP

The configuration file param.dat can be edited with any text editor (e.g. vim, gedit, geany, kate...), but it is **very important NOT TO ALTER the structure of the file** (number of columns before the = symbol or before the! symbol). The change in the parameters in param.dat are read at runtime, so it is not necessary to compile again the program.

There are some parameters that, as for now, are not configurable from the param.dat file, as they are declared in the Fortran files and their modification requires to recompile the program. See section 4.7 for further details.

# 2.4 How to build and run ECHO-QGP

To build the program, enter the source directory and issue the command:

make

The command make reads the instructions on how to compile from the file makefile.

The make command can be invoked with five different targets:

make to build the program for serial runs on a single processor

**make par** to build the program parallel runs on multi-cores systems using the MPI library

**make tools** to build the post-processing utilities contained into the tools directory

make clean to remove the files produced during a previous compilation (executable, object files, modules and so on)

**make cleanall** to remove both the files produced by the compiler and the files produced by an ECHO-QGP run

An expert user can edit the makefile to change or choose a proper compiling option, suitable for its own compiler and architecture.

On a successful build, the executable file echo.exe can be found in the directory. To run the program, simply issue:

```
./echo.exe
```

#### 2.4.1 Build and run on multi-processors architectures

If the user has a computer with many CPUs and the MPI library installed, he/she can speed up the simulations launching parallel runs. In order to compile ECHO-QGP with MPI, just issue the command:

```
make par
```

and then to run ECHO-QGP:

```
mpirun -np <number of processors> ./echo.exe
(e.g. mpirun -np 4 ./echo.exe)
```

where the -np flag sets up the number of processors to be used (to be more precise, the number of processes, but usually the best choice is to insert the number of CPUs available). Please, note that the minimum allowed number of cells along x is the number of processors divided by 3.

The output files are stored in the outrooo1 subdirectory. The output directory can be changed passing the -o flag to ECHO-QGP:

```
/echo.exe -o new_name
```

The name of the output directory must be 8 characters long.

When performing Glauber-Monte Carlo simulations, the suffix numbers of output directories are automatically assigned: they will be equal to the index numbers identifying the selected events (for example, if the users chooses

to run events with id 113 and 114 in the sequence generated by ECHO-QGP, then the output directories will be outr0113 and outr0114).

When correctly executed, ECHO-QGP prints a summary of the grid, the parameters and the algorithms employed, and then a short message each time the output is written and a final message about time elapsed. You can see an example of a successful running output (up to the second time-step in the listing 2.1)

```
ECHO-QGP STARTED
   nrk
      **** Settings and parameters: ****
       *** optical-geometrical Glauber initial
               Using Bjorken coordinates
            This is a viscous simulation
             Bulk viscosity is neglected
This simulation uses \mbox{MPI} and the number
Evolved shear viscous tensor components: xx, yy, zz, xy, xz
tt, tx, ty, tz and zz are obtained imposing orth. and null
   trace
                         Grid parameters:
    x, y and z (or eta) number of cells:
                                                 51
                                                            51
                            x range (fm):
                                                -9.000
   9.000
                            y range (fm):
                                                -9.000
   9.000
                  z (or eta) range (fm):
                                                -9.000
   9.000
                                                 0.353
                       step x,y,eta (fm):
   0.353
                0.353
                         Time parameters:
                           starting time:
                                               1.000
                                               1.400
                             ending time:
              ending temeperature (MeV):
                                             130.000
                        maximum timestep:
                                               0.005
                                               0.080
eta/s parameter for shear viscosity tens
                                               0.095
Temperature limit for smoothing viscosit
                              eq of state
                                                   3
    numerical derivatives with anal. eos
                                                   0
                     Nucleus parameters:
                                            197.000
                              proj. mass
                              radius (fm)
                                               6.380
                         W.-S. width (fm)
                                               0.535
```

```
sqrt(s) (GeV)
                                             200.000
                                       Y_b
                                              5.361
                       cross section (mb)
                                               4.200
                    impact parameter (fm)
                                               5.000
               initial hardness parameter
                                               0.150
       central energy density (GeV/fm^3)
                                              20.000
                       enezero (GeV/fm^3)
                                            0.10E-02
                        przero (GeV/fm^3)
                                            0.22E-03
                          central density
                                               0.100
pp rapidity distribution shift (deta - e
                                               1.000
pp rapidity distribution width (sigeta p
                                               1.500
        ueta A coeff. so that u^e = A * x:
                                               0.000
       Computing freeze-out hypersurface
Hypersurface computation based on temper
                                             140.000
               Freezeout threshold (MeV):
time interval between hypersurfaces comp
                                               0.050
              Other numerical parameters:
   Courant-Fr.-Lew. condition parameter:
                                               0.200
                Reconstruction algorithm:
                                                         MPE5
                                 Limiter:
                                                            MM2
          z-step thickness function (fm)
                                               0.001
          r-step thickness function (fm)
                                               0.001
                                            0.10E-06
                     integration accuracy
                       Output parameters:
          interval between log updating:
                                               0.010
       interval between output printing:
                                               0.050
                        output precision:
                                              double - 8 bytes
 Variables printed in the output files:
 density
 v x
 vу
 ٧z
 pressure
 energy density
 temperature
 entropy density
 bulk viscosity
 pi^tt
 pi^tx
 pi^ty
 pi^tz
 pi^xy
 pi^xz
 pi^yz
 pi^xx
 pi^yy
 pi^zz
 u0 or gamma Lorentz factor
```

```
dutdt
   duxdx
   duydy
   duzdz
   theta or expansion rate
   vorticities will also be printed into separated output
     files
  Calculating thickness function...
  check nucleus mass 196.94642
   Done!
   *******
101 RUN NUMBER 1 - RUN 1 OF 1
   Initializations done...
   mkdir outr0001/
   Grid computed...
Files for hypersurface computation written...
   Summary of variables written...
    Time: 1.00000000 - out0001.dat 1.61100000 secs
107
   Copying param.dat into the output directory...
Pressure treshold (GeV/fm<sup>3</sup>): 2.9468165948506055E-003
     Energy density treshold (Gev/fm^3): 1.3051308079431541
     E-002
     Time: 1.05000000 - out0002.dat 1.01000000 secs
```

Listing 2.1: Output of ECHO-QGP

# **Chapter 3**

# **ECHO-QGP** configuration

The execution of ECHO-QGP can be configured:

- by changing the default values of the parameters inside the file common.f90 and recompiling the program
- by modifying a parameter file(by default: *param.dat*), which is read by ECHO-QGP when it starts
- by passing the values of the parameters as command line arguments when launching the program

The parameters set in the parameter file have the precedence over the default parameters, but, in turn, they are superseeded by the parameters set as command line arguments.

When ECHO-QGP starts, it prints a summary of the values of the most important parameters. For each parameter, the number at the beginning of the line where its value is printed indicates where it has been set.

- 0: default value inside common.f90
- 1: parameter file
- 2: command line argument
- 3: automatically changed by the program because inconsistent with other parameters

# 3.1 Configuration using a file

By default, the name of the file containing the preferred values of the parameters is "param.dat", but it is possible to specify another name invoking ECHO-QGP with the argument *-PARAM FILE < name of the parameter file >*.

Changing a parameter in such file does not imply to compile again ECHO-QGP, because the file is read during the execution of the program.

While in earlier (1.x) versions of ECHO-QGP it was mandatory to preserve the structure of the file, now the order and even the presence in the file of the various options is not important anymore. All possible options have already a default value assigned in *common.f90* which is used whenever an option is commented or removed. The lines can be commented out writing at their beginning! or # or  $\$  Empty lines are allowed.

Since ECHO-QGP version 2.0, the properties of a nucleus (radius, charge...) are stored into a separate file (by default, "nuclear data.dat'').

An example of param.dat is reported in the listing 3.1. This file has different sections, related to the various modules of the program.

```
! kind of initialization
  !O=Geometric Glauber, 1=2D shock tube, 2=1D viscous shear
     flow test, 3=Glauber-MonteCarlo, 4=viscous Gubser flow,
     5=tabulated initial energy, 6=Alfven2D, 7=3D blast wave
  !8=1D Bjorken MHD flow, 9=3D mhd shock, 10=1D Lyutikov, 11=1
     D Beraudo-Haddadi, 12=rotor test, 13=0rszag-Tang test,
     14=generic debugging test, 15=external file with conserv
     . variables
  INIT_TYPE=3
 ! kind of simulation
  !system coordinates: 1=Minkowski, 2=Bjorken
  COORD . . . . = 2
 !it takes into account viscous effects: 0=n0, 1=yes
  VISCOUS . . = 0
 !if O it cuts off bulk viscosity
 BULK . . . . = 0
! if not 0 it runs in MHD mode
 M H D . . . . . = 1
 !if 1 it applies the Dedner's method to enforce the
     solenoidal condition
 DIVCLEAN . = 1
22 !parameter which tunes the dumping of the divergence of B
  GLM_PARAM=5.
  !option to reduce the suppress the initial B field where the
      energy density is very low
 DUMP_IN_B=O
 !energy density threshold under which the initial B field is
      suppressed
 B_DUM_EN.=0.02
```

```
!maximum ratio between magnetic and thermal pressure where
     the initial B field is suppressed
 Bp_ov_Tp.=0.01
tensor components are reduced
  CUT_TEMP .=0.08
36
  ! grid parameters
|! number of cells along x direction
 NX . . . . . . = 121
44 !number of cells along y direction
 NY . . . . . . = 121
| ! number of cells along z (Minkowski) or eta (Bjorken)
     direction
 NZ....=141
44 ! minimum value for x
  XMIN . . . . . = -15
46 ! maximum value for x
  XMAX . . . . . = 15
48 ! minimum value for y
  YMIN . . . . . = -15
5d ! maximum value for y
  YMAX . . . . . = 15
 \mid!minimum value for z (Minkowski) or eta (Bjorken)
      coordinates
  ZMIN.... = -14.
 !maximum value for z (Minkowski) or eta (Bjorken)
     coordinates
  ZMAX....=14.
  ! time parameters
| start simulation proper time (in fm/c)
  TSTART . . . = 0.5
eq !stop simulation proper time (in fm/c)
 TSTOP . . . . = 0 . 5
4 simulation ends when maximum temperature in GeV is less
     than TEMP\_END
  TEMP_END.=0.4
  ! output parameters
[ !proper time interval between log updating (it should be
      greater than MAXDT)
  DTL0G . . . . = 0 . 01
[ !proper time interval between output printings (it should be
      greater than MAXDT)
  DTOUT . . . . = 0 . 1
output precision: it can be 4 (bytes) - i.e. single - or 8
     (bytes) - i.e. double - precision
 OUTP_PREC=8
| maximum timestep
 MAXDT . . . . = 0 . 01
74 !restart possibilities: O=never, 1=from last output
 RESTART . . = O
```

```
! algorithm parameters
  !Courant-Fr.-Lew. condition
  CFL . . . . . = 0 . 1
80 !reconstruction algorithm
  ! REC_ALGO . = "MPE5"
82 ! when a superluminal velocity is obtained, the speed is
      reduced to this value (in c units)
  ! MAXSPEED . = 0.995
84
  !initialization with external files
86 !file containing initial B field
  B_in_FILE=none
ss !file containing initial conservative variables u(kvx:kpr)
  EX_C_FILE = none
90
  ! beam parameters
! name of the file containing the data about nuclei
  !NUC_DATAF="nuclear_data.dat"
94 symbol of the colliding ions (max 5 characters)
  NUCLEUS . . = Au
96 !sqrt(s_NN) (GeV)
  RADS . . . . . = 200.
  !total inelastic cross section (mb)
  SIGMA_IN.=42.
10d !impact parameter (fm)
  B . . . . . . . = 10.
102 ! initial conditions
  !initial condition: 0 (energy) / 1 (entropy)
104 IENENTR . . = 0
  !initial hardness
106 AH . . . . . . . = 0 . 15
  !energy dens. or entropy dens. (depending on IENENTR) at the
       origin (GeV fm^-3), w. 2D/3D diff. init.
  !minimum value of energy density (GeV/fm^3) for the initial
      energy density profile
11d ENEZERO . . = 0.002
  !charge density at the origin (fm^-3)
112 RHOCENTER = 0.1
  !shift of the pp rapidity distribution (Hirano)
114 DETA . . . . . = 1 . 2
  !width of the pp rapidity distribution (Hirano)
116 SIGETA . . . = 0 . 6
  ! viscosity parameters (active only when {\tt VISCOUS} parameter
118 !eta/entropy dens. parameter (natural units) for shear
      viscosity tensor - in the NS case simply {\tt ETA}
  ETA_S . . . . = 0 . 1
| relaxation time coefficient for viscosity
  | TAU_PI_C.=3.
!shear viscous tensor component to be derived from the other
       with trace conditions (zz or no)
 || TRACE_IMP=zz
```

```
124 ! equation of state
  !1=analytic eos p=e/3, 2=analytic eos ideal gas law, 3=
      tabulated eos, 4=tabulated from analytic eos
126 EOS . . . . . = 3
  !name of the file containing the tabulated eos (if used) -
      max 15 characters
128 TABEOSFILE = qcdIEOSO.dat
  !1= with an. eos it uses num derivatives to find other
      quantities, 0 = user must provide explicit deriv.
13d NUM_DER . . = 0
  !algorithm to be used in the MHD case to retrieve the
      primitive variables
132 ALG_SOLV . = 1
134 ! section for Glauber MonteCarlo initial conditions
  !number of nuclear configurations
136 NCONF . . . . = 10
  !number of impact parameters per configuration
138 NBCOLL . . . = 1
  !fixed impact parameter (1) or not (0) - default is 0
140 FIXED_B . . = 1
  !identifies the event from which to start among the
      generated events
142 EV_START.=1
  !identifies the event at which to stop among the generated
      events
144 EV_STOP . . = 1
  !model parameters (taken from Eskola et al., PRC83, 034901)
146 KAPPA . . . . = 19.0
  !smearing parameter
148 SIG.....=0.8
  !kind of collision: 1=AA, 2=dA, 3=pA
150 COLLISION = 1
  ! section for freezout hypersurface computation
152 !O disable / 1 enable hypersurface computation
  HYP_COMPU=1
154 !O freezout based on temperature - 1 freezeout based on
      energy density
  FREEZKIND = 1
!freezeout threshold: temperature (GeV) or energy density (
      GeV/fm^3)
  FREEZEVAL = . 5
158 !time interval between hypersurfaces computations
  HYPSURFTI = 0.1
16d ! section for initialization with an energy or entropy
      density distribution (depending on IENENTR param)
      tabulated into a file
  !name of the file with the tabulated energy or entropy
      density distribution (max 18 characters)
162 IN_D_FILE=ed.dat
  ! section for simulations with "tilted" initial energy
      density profile
164 !eta_m to produce initial en. dens. tilting as in http://
      arxiv.org/pdf/1501.04468v2.pdf, disabled if <0
```

```
| ETAM_TILT = -1
166 !for tests with initial u^eta!=0
  UETA_COEF=0.
168 ! section for mhd simulations
  !source of the initial B field (1=both classical and chiral,
      2=classical only, 3=chiral only)
170 MAGFIELDT = 3
  !electrical conducitvity (in GeV)
172 EL_COND..=0.0058
  !chiral conducitvity (in GeV)
174 CHIR_COND = 0.0015
  !initial B amplification factor
B_{amp_fac=1.
178 *************
  ! Printed variables in the output files (please, select 1 (
      ON) or O (OFF) for each variable)
180 density..=1
  v x . . . . . . = 1
182 vy . . . . . = 1
  vz....=1
184 pressure . = 1
  ene_dens.=1
186 temper . . . = 1
  entr_dens=1
188 bulk_visc=1
  pi^tt....=1
190 pi^tx....=1
  pi^ty...=1
192 pi^tz....=1
  | pi^xy...=1
194 pi^xz....=1
  |pi^yz...=1
196 pi^xx....=1
  | pi^yy...=1
198 pi^zz....=1
  gamma . . . . = 1
200 bx . . . . . = 1
  by . . . . . . = 1
202 bz . . . . . = 1
  ex....=1
204 ey . . . . . = 1
  ez....=1
204 !prints the derivatives of variables into separate output
      files (and on f.o. hypersurface)
  derivativ=0
| !prints hydrodynamical directed and elliptic flows and
      eccenticity in ascii files
  || flows....=0
```

Listing 3.1: Example of param.dat

#### 3.2 Kind of initialization

This section is devoted to the pre-hydro modules, and it can set-up the initial energy density (or entropy density) profile in such a way that it reproduces a known test, or situation. See chapters 4 and 6 for the various initializations that can be reproduced with ECHO-QGP.

**INIT\_TYPE** integer- flag. It is the flag selecting among different initial profiles, for a variety of cases. This is a development version and probably many options (written in orange) will be removed in the next stable public release. The default values of the various options are written in red. At the moment, the allowed possibilities are (see next chapter 4) (the options in orange are candidates for removal):

- 0. Glauber-geometric initialization (default)
- 1. 2D shock tube test
- 2. 1D viscous shear flow test
- 3. Glauber-Monte Carlo initialization
- 4. viscous Gubser's fluid test
- 5. Initialization with a tabulated energy or entropy density profile
- 6. Alfvén 2D test
- 7. 3D blast wave test
- 8. 1D Bjorken MHD flow
- 9. 3D mhd shock
- 10. 1D Lyutikov
- 11. 1D Beraudo-Haddadi
- 12. Rotor test
- 13. Orszag-Tang test
- 14. Rotor test
- 15. generic debugging test
- 16. Initialization using an external file with conservative variables (e.g. with  $T^{\mu\nu}$  computed using UrQMD)

### 3.3 Kind of simulation

This section is devoted the set-up of the simulation, such as choosing the coordinates (see also 4).

COORD integer- flag. It sets the metric used:

1. Minkowski coordinate system with metric diag(-1,1,1,1)

- 2. Bjorken coordinate system, with metric diag(-1,1,1, $\tau^2$ ) (default)
- **VISCOUS** integer- flag. Enables (1) or disables (0) the viscous correction to the evolution equations default: 0
- BULK integer- flag. Enables (1) or disables (0) the bulk viscosity contribution default: 0 In the first case the parameter  $\zeta=2\eta(1/3-cs^2)$ , in the last  $\zeta=10^{-40}$
- MHD integer- flag. Enables (1) or disable (0) the evolution of the magnetic field, i.e. using 0 it does not consider any magnetic field. At the moment, only ideal non dissipative magnetohydrodynamical simulations are possible default: 0
- **CUT\_TEMP** real, flag and cutoff. If > 0 then under the temperature CUT\_VALUE (expressed in GeV) the viscous tensor components are "smoothed", i.e. their magnitude is reduced default: 0.08 This section is devoted the set-up of the lattice

### 3.4 Grid parameters:

```
NX integer- value. The number of cells along x direction (def: 120)
NY integer- value. The number of cells along y direction (def: 120)
NZ integer- value. The number of cells along eta direction. To set up 2D+1 simulations, just impose NZ=1 (def: 120)
XMIN real- value. The minimum value for x (def: -12.)
XMAX real- value. The maximum value for x (def: 12.)
YMIN real- value. The minimum value for y (def: -12.)
YMAX real- value. The maximum value for y (def: 12.)
ZMIN real- value. The minimum value for z (or η) (def: -12.)
ZMAX real- value. The maximum value for z (or η) (def: 12.)
```

# 3.5 Time parameters

The simulation starts at a given proper time and can be terminated by two different criteria: when it reaches a threshold temperature or at a given proper time.

- **TSTART** real- value. Proper time at which the simulation starts ( $\tau_0$  in fm/c) (def: 1.)
- **TSTOP** real- value. Proper time at which the simulation ends (in fm/c) (def: 11.)
- **TEMP\_END** real- flag and cutoff. The simulation ends when the maximum temperature in GeV reaches a value less than TEMP\_END. If this parameter is set to 0, then this feature is turned off. (def: 0.)

### 3.6 Output parameters

This section specifies the output frequency and criteria.

- **DTLOG** real- value. Proper time interval between log updating (it should be greater than MAXDT) (def: 0.01)
- **DTOUT** real-value. Proper time interval between output printings (it should be greater than MAXDT) (def: 1.)
- **OUTP\_PREC** integer- flag. The user can choose between 4 (single) or 8 (double) bytes precision when printing output values. Note that in the particle-production tools, this choice is made at build time. (def: 8)
- MAXDT real- value. Maximum time-step allowed for the time-integration. (def: 0.002)
- **RESTART** integer- flag. Enables (1) or disables (0) the possibility to restart a simulation from the last output file. If enabled, it forces the frequency of output printing and freeze-out hypersurface computation to be done at the same time. (def: 0)

# 3.7 Algorithm parameters

- **CFL** real- value. The Courant–Friedrichs–Lewy condition (def: 0.4)
- REC\_ALGO character- flag. Choice of reconstruction algorithm (also used for computing spatial derivatives of velocities for obtaining shear viscous tensor components). Possibile choices are: TVD2, CENO3, WENO3, WENO5, PPM4, MPE3, MPE5, MPE7. (def: MPE5)
- MAXSPEED real-value. If, during the computations, a superluminal velocity is found, ECHO-QGP reduces the velocity components so that their module is equal to MAXSPEED. (def: 0.995)

ALG\_SOLV integer- flag. Choice of the inversion algorithm to retrieve primitive variables for the MHD case (with pure hydro it uses a dedicated algorithm). Possible options are: 1 (default) Brent method solving a 3×3 system, 2 Brent method solving a nested 2×2 system, 3 Broyden's algorithm, 4 solver for the ideal gas EOS (automatically chosen if EOS=2), 5 solver specific for the photon gas EOS, 6 analytic subdeterminants method.

bound\_con integer- flag. Boundary conditions: 0 = reflective, 2 = outflow
 (def: 2)

### 3.8 Collision parameters

In this section ECHO-QGP reads all the parameters characterizing the collision: the specie of the nuclei, the beam energy . . .

**NUCLEUS** character- flag. It sets the specie of colliding nuclei, and it corresponds to the symbol of the atom (e.g. Au or Pb). The essential parameters corresponding to each nucleus are written in a the separate file *nucleus.dat*, i.e. mass in a.m.u., radius in fm, Wood-Saxon width in fm and normal nuclear density in  $fm^{-3}$ . The user can add other species simply appending other data with the same format at the end of the file. (def: Au)

**RADS** real- value. Total energy per nucleon in the center of mass frame  $(\sqrt{s_{NN}})$ , in GeV (def: 200.)

SIGMA\_IN real- value. The total inelastic nucleon-nucleon cross section, in mb (def: 42.)

**B** real- value. The impact parameter b, expressed in fm. (def: 5.)

# 3.9 Initial conditions parameters for Glauber initializations

**IENENTR** integer- flag. Allows to use either energy (0) or entropy (1) density for the profile used in the initial stage modeling (def: 0)

**AH** real-value. Hardness parameter  $\alpha_{BC} \in [0, 1]$ , in equation 4.1 (def: 0.15)

**ECENTER** real-value. Central (x = y = z = b = 0) value for the field used in the initial profile (def: 40.), i.e.:

For IENENTR=0, ECENTER is the energy density in GeV/fm<sup>3</sup> For IENENTR=1, ECENTER is the entropy density in fm<sup>-3</sup>

- **ENEZERO** real- value. Minimum allowed value for the energy density, in GeV/fm<sup>3</sup>, at program starting (def: 0.001)
- **RHOCENTER** real-value. Central (x = y = z = b = 0) value for the baryon charge density in  $fm^{-3}$ . *Please, note that in the current version, this parameter does not have any physical meaning*: it is exploited as a tracer (i.e. as a mathematical trick to simplify the evolution equations of shear viscous tensor components). In order to perform viscous simulations RHOCENTER must be different from zero. (def: 1.)
- **DETA** real- value. Shift of the pp-rapidity distribution ( $\eta_{flat}$  in equation 4.1) (def: 2.)
- **SIGETA** real- value. Width of the pp-rapidity distribution ( $\sigma_{\eta}$  in equation 4.1) (def: 1.)
- **NUC\_DATAF** character- value. Name of the file containing the data about the nuclei. (def: nuclear\_data.dat)

### 3.10 Viscosity parameters

These parameters are only considered when the flag VISCOSITY=1.

- **ETA\_S** real- value. It is the shear viscosity coefficient divided by the entropy density:  $\eta/s$  (in natural units). It is employed in the calculation of the shear viscous tensor, where s is the entropy density. (def: 0.08)
- **TAU\_PI\_C** real- value. The parameter which controls the  $\tau_{\pi}$ , the relaxation time, in the formula:  $\tau_{\pi} = TAU\_PI\_C\frac{ETA\_S}{T}$ , where T is the temperature (def: 5.)
- **TRACE\_IMP** character- flag. Discriminates the equation used to close the system. It can assume two different values (def: zz):
  - **TRACE\_IMP=no** The components  $\pi^{xx}$ ,  $\pi^{yy}$ ,  $\pi^{zz}$ ,  $\pi^{xy}$ ,  $\pi^{xz}$ ,  $\pi^{yz}$  of the shear viscous tensor are evolved, while the  $\pi^{tt}$ ,  $\pi^{tx}$ ,  $\pi^{ty}$ ,  $\pi^{tz}$  components are retrieved imposing the orthogonality condition;
  - **TRACE\_IMP=zz** the  $\pi^{zz}$  component is obtained imposing the tracelessness condition (in addition to the orthogonality condition)

# 3.11 Equation of State parameters

**EOS** integer- flag. Discriminates the Equation of State used by ECHO-QGP. The options are:

- 1. The photon gas EOS  $\mathcal{P} = \frac{e}{3}$ . (default)
- 2. The ideal gas EOS  $\mathcal{P}=(\Gamma-1)(e-\rho)$ , where the adiabatic index  $\Gamma=\frac{4}{3}$ . This EOS is used mainly for tests and comparisons with other RMHD codes for astrophysics. When using this EOS, the  $\rho$  means mass density and not baryon number density.
- 3. ECHO-QGP uses a tabulated equation of state, specified in the file TAB EOS FILE.

When selectiong the options 1 or 2, the user needs to define its own relation for the temperature by editing the files  $eos\_data/temperature.def$  The relations among the pressure (P), the charge density  $(\rho)$  and energy density (e) are explicitly written in the files .def contained in the folder  $eos\_data$ , with suffixes 1 or 2:

- pressure\_vs\_rh\_en.def Dependence of the pressure (P,  $GeV/fm^3$ ) on the energy density (e,  $GeV/fm^3$ )
- temperature.def Dependence of the temperature (T, GeV) on the energy density  $(e, \text{GeV/fm}^3)$
- energy\_den.def (Optional, only if NUM\_DER=0) the dependence of energy density on pressure
- dprdrh.def (Optional, only if NUM\_DER=0) the partial derivative of
   pressure respect to density
- dprden.def (Optional, only if NUM\_DER=0) the partial derivative of
   pressure respect to energy density

All the \*.def files in the eos\_data folder can be edited as the user wishes, as long as the variable and parameters names are preserved. Any change in these files requires to issue again the command make.

- TAB\_EOS\_FILE character(max 15 characters). Name of the file containing the tabulated EoS (if EOS=3), to be located in the eos\_data directory. (def: qcdIEOS0.dat) ECHO-QGP allows the use of any tabulated EoS of this kind, if provided by the user in the following format:
  - The first row containing the number N of entries of the file.
  - N rows, each one containing: the temperature T in GeV, the energy density in  $GeV/T^4$ , the pressure in  $GeV/T^4$ , the square speed of sound  ${c_s}^2 \equiv dP/de$ ; all separated by white spaces

In the bundle, there are four available tabulated EoS files:

**qcdIEOS.dat** EoS of ref. [3], arising from a weak-coupling QCD calculation with realistic quark masses

**qcdIEOS0.dat** Nearly identical to qcdIEOS.dat, except for the first entry for T=0 (added to solve some stability problems at low temperatures, far below the temperature range of existence of the quark-gluon plasma).

**pce.dat** EoS of ref. [4], based on partial chemical equilibrium obtained by matching a Hadron-Resonance-Gas EoS (HRG EoS) at low temperature with the continuum-extrapolated lattice-QCD results by the Budapest-Wuppertal collaboration [5]. The HRG EoS was obtained by summing the contributions of all hadrons and resonances in the PDG up to a mass of 2 GeV:  $P = \sum_r P_r$ .

**pce0.dat** The same as pce.dat, but with an additional entry for T=0.

NUM\_DER integer- flag. Enables (1) or disables (0) the automatic computation of the derivatives for the thermodynamic variables, in case of an analytic EoS. Works with EOS=1 or EOS=2. When this option is turned off, user must provide explicit derivatives in the files eos\_data/energy\_den.def and eos\_data/part\_der\_pr\_vs\_rh\_en.def. (def: 0)

# 3.12 Glauber Monte Carlo initial conditions parameters

NCONF integer-value. Number of nuclear configurations (def: 100)

**NBCOLL** integer- value. Number of impact parameters per configuration (def: 1)

**FIXED\_B** integer- flag. If 1, it runs simulations with fixed impact parameter. (def: 0)

**EV\_START** integer- ID. ID number of the event from which to start among the generated events (def: 1)

**EV\_STOP** integer- ID. ID number of the event from which to stop among the generated events (def: 5)

KAPPA real-value. Value of the K parameter in:

$$e(\tau_0, \mathbf{x}) = \frac{K}{2\pi\sigma} \left\{ (1 - \alpha) \sum_{i=1}^{N_{\text{part}}} \exp\left[-\frac{(\mathbf{x} - \mathbf{x}_i^{\text{part}})^2}{2\sigma^2}\right] + \alpha \sum_{i=1}^{N_{\text{coll}}} \exp\left[-\frac{(\mathbf{x} - \mathbf{x}_i^{\text{coll}})^2}{2\sigma^2}\right] \right\}.$$

For further details see [6, 7]. (def: 19.0)

**SIG** real- value. Value of the  $\sigma$  (smearing) parameter in the above equation. For further details see [6,7]. (def: 0.8)

**COLLISION** integer- flag. Discriminates the kind of collision

- 1=AA (nucleus-nucleus) (default)
- 2=dA (deuton-nucleus)
- 3=pA (proton-nucleus)
- **EPROT** real- value. In pA collisions, it is the energy of the proton beam. (def: 200)
- AVG\_IC integer- flag. If 0 it runs events by events simulation, if 1 it runs just one simulations, but with initial conditions obtained averaging the initial conditions of many events. (def: 0)
- MIN\_PART integer- value. The minimum number of participant nucleons to accept the event. (def: 18)
- PW real- value. In MHD simulations, the fraction of the magnetic field created by the participants which is taken into account to compute the initial magnetic field. (def: 0.5)
- **EL\_COND** real- value. In MHD simulations, the value of the electrical conductivity  $\sigma$  in GeV. (def: 0.0058)
- **CHIR\_COND** real- value. In MHD simulations, the value of the chiral conductivity  $\sigma_{\gamma}$  in GeV. (def: 0.0015)
- **MAGFIELDT** integer- flag. Contributions to the initial magnetic field: 1 = both classical and chiral, 2 = classical only (default), 3 = chiral only

# 3.13 Generic magnetic field options

- **B\_amp\_fac** real- value. In MHD simulations, it multiplies the initial B field components by this factor. (def: 1.)
- **DUMP\_IN\_B** integer- flag. In MHD simulations, it suppresses (1) or not (0) the initial B field. (def: 1.)
- **B\_DUM\_EN** real- value. The energy density threshold to apply the B field suppression (below this value B is suppressed, above it is not). (def: 0.05)
- **Bp\_ov\_Tp** real- value. In case of magnetic field suppression, it is the value of the ratio between magnetic and thermal pressure. (def: 0.01)
- **DIVCLEAN** integer- flag. In MHD simulations, it turns off (0) or on (1) the Dedner's (or generalized lagrangian multiplier) divergence cleaning method. (def: 0)

- **GLM\_ALPHA** real- value. Parameter of the divergence cleaning method. (def: 5.)
- **B\_in\_FILE** character- value. Name of the external B field with the initial B field (ascii format, each line contains bx, by and bz, the external loop is on z and the interanl loop is on x). (def: initialB.dat)

# 3.14 Decoupling hypersurface parameters

The task of the routine hypersuface.f90 included in ECHO-QGP is to find, store and print the coordinates of the hypersurface detected during the hydrodinamic evolution.

**HYP\_COMPU** integer- flag. Disables (0) or enables (1) the computation of the decoupling hypersurface. If set to 0, it saves computational time but it does not perform any decoupling technique.(def: 0)

**FREEZKIND** integer-flag. Criterion for the detection of the hypersurface.

- 0. Isothermal hypersurface Uses a threshold temperature (FREEZE-VAL=  $T_{fo}$ , GeV)
- 1. Constant energy density hypersurface Uses a threshold energy density (FREEZEVAL=  $e_{fo}$ , GeV/fm<sup>3</sup>) (default)

**FREEZEVAL** Decoupling field threshold: temperature (GeV) or energy density ( $GeV/fm^3$ ) (def: 0.5)

HYPSURFTI real-value. Time interval (in fm/c) of the grid passed to the hypersurface-finding routine. Sometimes to treat shocks it is useful to set a very dense grid, but in order to compute the hypersurface a lesser dense grid is needed, so it would be a waste of computing time to perform the check at every hydro step. (def: 0.1)

# 3.15 Initialization with a custom energy or entropy density profile

ECHO-QGP can use as a starting profile  $(\tau=\tau_0)$  a custom 2D energy density or entropy density profile at mid-rapidity, with the field values at any cell center. The structure of the file must be: x coordinate, y coordinate, energy density value in GeV/fm $^3$  or entropy density value in fm $^{-3}$ . The field values of the tabulated file will be interpolated on the grid chosen in param.dat if the grid on which they lie does not match with the grid of ECHO-QGP.

- IN\_D\_FILE character(max 60 characters). For initialization 5: name of the file with the tabulated energy or entropy density distribution (def: ed.dat)
- **EX\_C\_FILE** character(max 60 characters). For the initialization using a file with the values of the conserved variables. (def: cons.dat)

# 3.16 Section for simulations with "tilted" initial energy density profile

These options work only with Geometrical Glauber initial conditions.

**ETAM\_TILT** real-value. Parameter to produce a "tilting" in the initial energy distribution as in ref. [8]; if the parameter is set to a negative number, no tilting is introduced (def: -1)

**UETA\_COEF** Parameter used for tests with initial conditions different from the BIC ( $u^{\eta} = u^x = u^y = 0$ ). Here  $u^{\eta} \neq 0$ , and  $u^{\eta}(x) = \text{UETA\_COEF} \cdot x$  (the impact parameter b lies on the x axis). (def: 0.)

# 3.17 Output files and printed variables

outdir character- value. Custom part of the name of the output directory.

This section simply contains a list of all the fields that the simulation can print in the output files. The user can enable (1) or disable (0) the output of any variable. The computational time also depends on how many variables are printed. They all are real- flags - with the precision defined by the flag OUTP\_PREC.

```
density (baryon charge density) (def: 1)

vx (v^x contravariant) (def: 1)

vy (v^y contravariant) (def: 1)

vz (v^z or v_\eta contravariant) (def: 1)

pressure (p, expressed in GeV/fm<sup>3</sup>) (def: 1)

ene_dens (e, energy density, expressed in GeV/fm<sup>3</sup>) (def: 1)

temper (T, temperature, expressed in GeV) (def: 0)

entr dens (s, entropy density, expressed in fm<sup>-3</sup>) (def: 0)
```

- **bulk\_visc** Π (bulk viscosity, only active when VISCOUS parameter is set to 1) (def: 0)
- pitt  $\pi^{tt}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **pitx**  $\pi^{tx}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- pity  $\pi^{ty}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **pitz**  $\pi^{tz}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **pixy**  $\pi^{xy}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **pixz**  $\pi^{xz}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **piyz**  $\pi^{yz}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **pixx**  $\pi^{xx}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **piyy**  $\pi^{yy}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- pizz  $\pi^{zz}$  component of shear viscous tensor, active only when VISCOUS parameter is set to 1 (def: 0)
- **gamma**  $\gamma$  Lorentz factor (also  $v_t$ ) (def: 0)
- **bx** ( $B^x$  contravariant) (active only when MHD is set t 1) (def: 1)
- by ( $B^y$  contravariant) (active only when MHD is set t 1) (def. 1)
- **bz** ( $B^z$  or  $B^\eta$  contravariant) (active only when MHD is set t 1) (def. 1)
- ex ( $E^x$  contravariant) (active only when MHD is set t 1) (def: 1)
- ey ( $E^y$  contravariant) (active only when MHD is set t 1) (def: 1)
- ez ( $E^z$  or  $E^{\eta}$  contravariant) (active only when MHD is set t 1) (def: 1)
- derivative Prints the derivatives of velocities and temperature vs  $\tau, x, y$  and  $\eta$  at  $\tau$  intervals given by parameter DTOUT into separate output ASCII files der0001.dat, der0002.dat, ... and, if the computation of f.o. hypersurface is active, also on f.o. hypersurface, producing the file hypersurf\_deriv.txt (def: 0)

flows Prints the hydrodynamical eccentricities on the transverse plane vs  $\eta$  at  $\tau$  intervals given by parameter DTOUT into separate output ASCII files df0001.dat, ep0001.dat, ec0001.dat, df0002.dat, ep0002.dat, ec0002.dat, . . . . . (def: 0)

The computations are valid only in the inviscid case.

The definition we used for the spatial eccentricity eco... files is:

$$\varepsilon_c(\eta)_{const.\tau} = \frac{\int dx dy (x - x_0)^2 - (y - y_0)^2}{\int dx dy (x + x_0)^2 - (y + y_0)^2}$$

the definition for the df0... files with average flow in x direction is:

$$\langle v_x \rangle (\eta) = \frac{\int dx dy \gamma v_x \epsilon}{\int dx dy \gamma \epsilon}$$

while the definition [9] for the momentum eccentricity ep0... files is:

$$\varepsilon_p(\eta)_{const.\tau} = \frac{\int dx dy (\epsilon + p) \cdot \gamma^2 \cdot (v_x^2 - v_y^2)}{\int dx dy (\epsilon + p) \cdot \gamma^2 \cdot (v_x^2 + v_y^2) + 2 \cdot p}$$

where  $\epsilon$  is the energy density,  $\gamma$  is the Lorentz factor and p is the pressure.

# 3.18 Essential parameters for projectile nuclei

It is possible to choose the parameters for specific nuclei and/or to give the essential parameters (i.e. mass in a.m.u., radius in fm, Wood-Saxon width in fm and normal nuclear density in fm<sup>-3</sup>), writing them in a file (by default, the file *nuclear\_data.dat* is used). If a new projectile nucleus is wanted, it is sufficient to type (or copy and paste) a new line, where the 5 characters at the beginning are reserved for the name and the rest of the line structure is preserved.

# 3.19 Arguments passing

ECHO-QGP accept as an argument all the configuration options listed in the previous section, with a leading minus character (.e.g. -DTOUT). In addition, it accepts the following arguments:

- -v, -version it prints version information and exits
- -r, -restart it restarts from last saved frame Currently not operational
- -PARAM\_FILE character- value. An alternative name for the parameter file. (def: param.dat)

# Chapter 4

# Available initialization alternatives

As its hydrodynamic starting point, ECHO-QGP can set up different initial energy density (or entropy density) profiles, based on various widespread models or on some of the most common numerical tests.

The ones currently available in the code are listed below and they are selected through the parameter INIT\_TYPE in the configuration file param.dat.

#### 4.1 Initialization 0 - Geometric Glauber

Here we compute the initial state for the energy density profile in the Optical Glauber Model frame [10].

We define the usual thickness function as:

$$T(x,y) = \int_{-\infty}^{\infty} \frac{\rho_0}{1 + e^{\frac{\sqrt{x^2 + y^2 + z^2} - r}{\delta}}} dz$$

where  $\rho_0$ ,  $\delta$  and r are the normal nuclear density, the Wood-Saxon width and the radius of the nucleus (all of them defined at the end of param.dat file). Then we define:

$$T_{+}(\mathbf{x}_T) = T(\mathbf{x}_T + \mathbf{b}/2)$$
  $T_{-}(\mathbf{x}_T) = T(\mathbf{x}_T - \mathbf{b}/2)$ 

where  $\mathbf{x}_T = (x,y)$  is the vector of the transverse plane coordinates and  $\mathbf{b}$  is the impact parameter vector, connecting the centers of the two nuclei. In our conventional cartesian reference frame, the  $\mathbf{b}$  vector is oriented along the positive x axis and the two nuclei have initial momentum along the z axis (whence the reaction plane is the xz plane).  $\mathbf{b}$  is assigned with the B parameter.

Given:  $\sigma$  as the total nucleon-nucleon inelastic cross section (corresponding

to the parameter SIGMA\_IN), m as the mass number of the colling nuclei and T(x,y) as the thickness function, we define:

$$T_1(x,y) = T_+ \left(1 - \left(1 - \frac{\sigma T_-}{m}\right)^m\right) \qquad T_2(x,y) = T_- \left(1 - \left(1 - \frac{\sigma T_+}{m}\right)^m\right)$$

while the number of binary collisions is:

$$n_{BC}(x,y) = \sigma T_{+}(x,y) T_{-}(x,y)$$

We define the wounded nucleons weight function  $W_N$  as:

$$W_N(x, y, \eta) = 2 \left( T_1(x, y) f_-(\eta) + T_2(x, y) f_+(\eta) \right)$$

where:

$$f_{-}(\eta) = \begin{cases} 1 & \eta < -\eta_m \\ \frac{-\eta + \eta_m}{2\eta_m} & -\eta_m \le \eta \le \eta_m \\ 0 & \eta > \eta_m \end{cases}$$

and

$$f_{+}(\eta) = \begin{cases} 0 & \eta < -\eta_m \\ \frac{\eta + \eta_m}{2\eta_m} & -\eta_m \le \eta \le \eta_m \\ 1 & \eta > \eta_m \end{cases}$$

Finally, the initial proper energy density distribution is assumed to be:

$$\varepsilon(x, y, \eta) = \varepsilon_0 W(x, y, \eta) H(\eta),$$

where  $\varepsilon_0$  is assigned with the parameter ECENTER and the total weight function  $W(x, y, \eta)$  is defined as:

$$W(x,y,\eta) = \frac{(1-\alpha)W_N(x,y,\eta) + \alpha n_{BC}(x,y)}{(1-\alpha)W_N(0,0,0) + \alpha n_{BC}(0,0)}.$$

where  $\alpha$  is the AH parameter and:

$$H(\eta) = \exp\left(-\frac{\tilde{\eta}^2}{2\sigma_n^2}\theta(\tilde{\eta})\right)$$
  $\tilde{\eta} = |\eta - \eta_0| - \eta_{flat}/2$ 

where  $\eta_0 = 0$  if the parameter ETAM\_TILT > 0 ("tilted initial energy profile"), otherwise:

$$\eta_0 = \frac{1}{2} \ln \left( \frac{1 + \beta_0}{1 - \beta_0} \right)$$

where

$$\beta_0 = \frac{T_- - T_+}{T_- + T_+}$$

Viscous tensor components are initialized as:

$$\pi^{11} = g_{11} \frac{2 \eta}{3 \tau_0}$$

$$\pi^{22} = g_{22} \frac{2 \eta}{3 \tau_0}$$

$$\pi^{33} = -g_{33} \frac{4 \eta}{3 \tau_0}$$

$$\Pi = -\frac{\zeta}{\tau_0}$$

All other components are set to 0.

#### 4.2 Initialization 1 - 2D shock tube

Initial condition is a constant pressure profile  $p_1$  for  $y \le -x$  and  $p_2$  otherwise. All other variables are set to 0. The inital values of the variables can be changed only editing the init.f90 file (and recompiling ECHO-QGP).

#### 4.3 Initialization 2 - 1D viscous shear flow

These initial conditions are designed to perform the (1+1)-D test in Minkowski Cartesian coordinates described in [6] and [11].

Pressure and density have constant values (density  $\rho=1$  and pressure p=0.25 so that, using ideal equation of state energy density e=3p=0.75

and 
$$e+p=1$$
), and  $v^y=v0\cdot \mathrm{erf}\left(\frac{x}{2\sqrt{t\cdot\eta/s}}\right)$ , where the constant  $v0\ll c$ 

and t is time (with this choice of the various parameters, t should be 1 at the beginning of the simulation).

#### 4.4 Initialization 3 - Glauber-MonteCarlo

This initialization is based on a simple Glauber MonteCarlo model, designed as follows [7] [6]:

- A Woods-Saxon distribution is sampled to create an initial nuclear configuration. The transverse positions of the nucleons of the two colliding nuclei are reshuffled into their center-of-mass frame.
- For each configuration a random impact parameter  $b \in [0, b_{max}]$  is extracted from the distribution  $dP = 2\pi b db$ . A collision happens if the positions  $x_i$  of the nucleon i (from nucleus A) and  $x_j$  of nuclean j (from nucleus B) are such that:  $(x_i x_j)^2 + (y_i y_j)^2 < \sigma_{NN}/\pi$ . The parameter min\_participants (contained into file glaubermc.f90) establishes the minimum number of participant nucleons to accept or reject the

event. The user, with the parameter NCONF, can choose how many configurations to compute.

• To each participant and collision is associated an energy density distribution described by a gaussian function with variance  $\sigma$ ; these distributions are then summed together:

$$e(\tau_0, \boldsymbol{x}) = \frac{K}{2\pi\sigma} \left\{ (1 - \alpha) \sum_{i=1}^{N_{part}} \exp\left[ -\frac{(\boldsymbol{x} - \boldsymbol{x}_i^{part})^2}{2\sigma^2} \right] \right. \\ \left. + \alpha \sum_{i=1}^{N_{coll}} \exp\left[ -\frac{(\boldsymbol{x} - \boldsymbol{x}_i^{coll})^2}{2\sigma^2} \right] \right\}.$$

• The profile  $e(\tau_0, x)$  is then multiplied by

$$f(\eta) = \begin{cases} 0 & |\eta| \ge y_{beam} \\ \exp\left(-\frac{\tilde{\eta}^2}{2\sigma_{\eta}^2}\right) & \eta_{flat} \le |\eta| < y_{beam} \\ 1 & |\eta| < \eta_{flat} \end{cases}$$

where  $\tilde{\eta}=|\eta|-\eta_{flat}/2$ ,  $\eta_{flat}$  is given, in param.dat, by the DETA parameter,  $\sigma_{\eta}$  by SIGETA and  $y_{beam}=\log\frac{\sqrt{S_{NN}}}{m_N}$ 

• Viscous tensor components are initialized as in initialization 0.

We advice that, at the time of the release of ECHO-QGP 1.5.0- $\alpha$ , initializations with p+A and d+A collisions have not been properly tested.

#### 4.5 Initialization 4 - Viscous Gubser Flow

Viscous Gubser flow with initial values provided by a tabulated file named Initial\_Profile\_GubserFlow.dat contained into the main ECHO-QGP source tree. This is a 2D+1 test in Bjorken coordinates. See the proper section 6.3 in the tutorials chapter for further informations.

# 4.6 Initialization 5 - tabulated initial energy or entropy density profile

This is a 2D+1 initialization. Inital energy density profile (if parameter IENENTR is set to 0) or entropy density profile (if parameter IENENTR is set to 1) are read from file named IN\_D\_FILE. Viscous tensor components are initialized as in initialization 0.

### 4.7 Other parameters inside the code

The user can change some parameters included inside the code.

After changing one of these parameters, the user has to recompile the code before using the program.

If the user changes files with definitions like <code>eos\_data/temperature.def</code>, which are inserted into other source files by an *include* directive, he/she has also to clean the source tree with *make clean* or *make cleanall* before recompiling the program with *make* or *make par*.

**lambda0, lambda1, lambda2** Parameters located into system.f90, inside the subroutine *system\_sources*, they enable (if set to 1.) or disable (if set to 0.) the contributions to the source terms of viscous tensor components of: the  $-\frac{4}{3}\pi^{\mu\nu}\theta$  term, the second order terms, the vorticity terms.

iseed Parameter saved into the file random\_seed.dat. When using Glauber-MC initialization procedure, if ECHO-QGP finds this file, it reads the integer value it contains for seeding the random number generator, thus reproducing the last energy density profile. If the user removes this file, ECHO-QGP uses the integer value obtained from a <code>system\_clock</code> call as the new random seed <code>iseed</code> and it saves its value inside the file <code>random\_seed.dat</code>.

limiters The tvd2, ceno3 and ppm4 reconstruction algorithms may use MM2 or MC2 as possible limiters. The pre-defined algorithm is MM2, but the user can change it to MC2 commenting/uncommenting just a few lines inside holib.f90 following the instructions inserted into the code.

If the user wishes to change how the values of viscous tensor are initialized, he/she has to modify the final part of the subroutine *viscous\_initio*, contained into file viscous.f90.

If the user wishes to change the initial  $u^{\eta}$  velocity field, he/she has to modify the subroutine  $calc\_vel\_long\_nz\_ueta$ , contained into the file init.f90.

The parameter  $\eta_0$  as defined into 4.1 can also be changed editing the function H of eta into the file init.f90.

## Chapter 5

# Post-processing tools

Inside the tools directory there are several utilities to manipulate the results to make easy to visualize them using programs like grace, IDL/gdl or gnuplot:

### 5.1 IDL/GDL scripts

**readvar.pro** IDL/GDL script that reads the output of echo-qgp and stores the variables it into 3-dimensional arrays, the grid data into 3 arrays x, y and z. The files grid\_summary.dat and param.dat must be in the same directory with data where this script is launched.

readvartime IDL/GDL script that reads the output files of echo-qgp and the values of a chosen variable at a chosen point on the grid in a chosen range of output files are printed into a chosen file togheter with the time at which they refer. The files grid\_summary.dat and param.dat must be in the same directory with data where this script is launched.

**readder.pro** IDL/GDL script that reads the output files of echo-qgp containing data about variable derivatives at fixed  $\tau$ . The files <code>grid\_summary.dat</code> and <code>param.dat</code> must be in the same directory with data where this script is launched.

**ce.pro** IDL/GDL script that, for the inviscid case and Bjorken coordinates only, computes the total energy  $\overline{E_{tot}}$  and the total angular momentum  $J^{zx}$  along the y direction over the computational grid using the formulas:

$$E_{tot} = \int \tau(\cosh T^{00} - \sinh T^{z0}) d\eta dx dy$$

$$J^{zx} = \int \tau(\cosh \eta (zT^{0x} - xT^{0z}) - \sinh \eta (zT^{zx} - xT^{zz})) d\eta dx dy$$

Since the energy density value defined by the ENEZERO parameter is added to each cells of the grid at the beginning of each run with Glauber initial conditions, the values of these "basement" energy density and the corresponding pressure can be assigned to the <code>limit\_en</code> and <code>limit\_pr</code> variables (in the first lines of the script) before running the script, so that they will be subtracted. These values of <code>limit\_en</code> and <code>limit\_pr</code> variables are printend to the standard output by ECHO-QGP at the beginning of each run.

Please, note that this utility is based on a raw summation of the values inside the cells and, depending on what are the grid dimensions, the grid resolution and the value of ENEZERO, it may give inaccurate or even wrong results.

You are invited to always check the results changing grid dimensions, grid resolution and the value of ENEZERO.

#### 5.2 Fortran utilities

**readx.f90** this small program can be individually compiled with the command: *gfortran -o readx.exe -fdefault-real-8 readx.f90*.

The synopsys of the program is:

./readx.exe number\_of\_start\_frame number\_of\_end\_frame

If selected in the param.dat file (and, for viscous tensor components, if
we run a simulation with viscosity), there will be the following output

files (nnnn refers to the four digits number identifying the frame):

- EPSnnnn.dat the energy density
- PRnnnn.dat the pressure
- VXnnnn.dat the velocity along x direction
- VYnnnn.dat the velocity along y direction
- VZnnnn.dat the velocity along  $\eta$  direction
- RHOnnnn.dat the charge density
- Snnnn.dat the entropy density
- Tnnnn.dat the temperature
- bulknnnn.dat the bulk viscosity
- ttnnnn.dat the  $\pi^{tt}$  shear viscous tensor component
- txnnnn.dat the  $\pi^{tx}$  shear viscous tensor component
- tynnnn.dat the  $\pi^{ty}$  shear viscous tensor component
- tznnnn.dat the  $\pi^{tz}$  shear viscous tensor component
- xynnnn.dat the  $\pi^{xy}$  shear viscous tensor component
- xznnnn.dat the  $\pi^{xz}$  shear viscous tensor component

- yznnnn.dat the  $\pi^{yz}$  shear viscous tensor component
- xxnnnn.dat the  $\pi^{xx}$  shear viscous tensor component
- yynnnn.dat the  $\pi^{yy}$  shear viscous tensor component
- zznnnn.dat the  $\pi^{zz}$  shear viscous tensor component
- x.dat, y.dat, eta.dat, tau.dat the grid coordinates

**fromecho.f90** this small program can be individually compiled with the command: gfortran -o fromecho.exe -fdefault-real-8 fromecho.f90 .

The synopsis of the program is:

./fromecho input\_file output\_file x|y|z.

The program takes as input one of the output files of readx and select the values of the given variables along the x, y or z direction from the 0 up to the right border, printing them into output file.

For example:

./fromecho.exe PR0001.dat pressure.txt x

it will print into the file pressure.txt the values of pressure along x from 0 up to the border of the grid. The ouput file is a two column ascii file, in the first column there are the coordinate values and in the second column the variable values.

**fromecho2d.f90** this utility extracts a 2D slice parallel to one of the coordinate axis from a 3D set of data. It can be individually compiled with the command: *gfortran -o fromecho2D.exe -fdefault-real-8 fromecho2D.f90* and its synopsis is:

/fromecho input output x|y|z x|y|z (optional: x|y|z value, default is 0) where input is one of the files containing single variable data produced by *readx*.

Usage example:

./fromecho prova input prova output x y

This extracts data from file prova\_inputs and writes into file output\_file the values on the x-y plane for  $z\!=\!0$ 

Usage example:

./fromecho prova input prova output z x 5

This extracts data from file prova\_inputs and writes into file output\_file the values on the z-x plane for y=5 For the optional 5th argument, actually it is selected the grid value which is closest to the chosen value, i.e. no interpolations are made.

The ascii output file contains the coordinate value on the first axis, the coordinate values on the second axis and the variable value in that point of the slice. Assuming that these values are x,y and f(x,y), the output file is written in this way:

```
x(1) y(1) f(x(1),y(1))
x(1) y(2) f(x(1),y(2))
x(1) y(3) f(x(1),y(3))
...
x(1) y(ny) f(x(1),y(ny))
blank line
x(2) y(1) f(x(2),y(1))
x(2) y(2) f(x(2),y(2))
...
x(2) y(ny) f(x(2),y(ny))
blank line
x(3) y(1) f(x(3),y(1))
x(3) y(2) f(x(3),y(2))
...
x(nx-1) y(ny) f(x(nx-1),y(ny))
```

The blank lines are inserted to make easier to plot the output file with *gnuplot*.

**timev.f90** this small program can be individually compiled with the command: *gfortran -o timev.exe -fdefault-real-8 timev.f90*.

Usage example:

```
./timev.exe T 1 137 51 40 37 pippo
```

It prints into the file named "pippo" two columns of values: the first one contains the time, the second one the values of the variable (usually T stands for temperature) at cell of indexes x 51, y 40 and z 37 stored in the files from T0001.dat to T0137.dat

**search.sh** This very small and very simple bash script searches and highlights a given string inside all source code files. Example: ./search.sh allocate

## 5.3 The particle spectra production tool

The tool for producing particle spectra is contained into the analysis directory and can be compiled simply with the *make* command. This program is able to use OpenMP for speeding up computations in system with multicore processors with shared memory (and, of course, with OpenMP installed). To take advantage of OpenMP, you have to edit the makefile adding the compilation flag *-fopenmp* and execute the make command (you just have to comment/uncomment a couple of lines at the beginning of the makefile). If

compilation is successfull, export the environment variables used by OpenMP (unless they have already been set up in advance), e.g., for a 4 cores cpu:

```
export OMP_NUM_THREADS=4
export OMP_SCHEDULE=GUIDED,4
```

At the end of the compilation stage, you'll get an executable named thermal.exe. When OpenMP is used, if thermal.exe is the only relevant running application, the common *top* utility will report for thermal.exe a %CPU usage almost equal to OMP NUM THREADS times 100.

The configuration file is settings.txt. The output directory where the program stores the results must be created before launching it.

For a much more detailed description of this tool, please, read chapter 7.

## Chapter 6

## **Tutorials**

In this chapter we will give some examples of how to use the code.

The tutorials will not cover all the features of ECHO-QGP, nevertheless they should give to the user a working knowledge sufficient to perform many basic tasks

Usage examples of external programs like *gnuplot* or the *gnudatalanguage* will be provided, too, hoping that they will be useful to young students and to other people not familiar with them.

The tests directory contains the configurations files and some plots (together with related datafiles) of the results of the numerical simulations presented in this chapter.

#### 6.1 A 3D+1 simulation with Glauber model - test A

Unpack the code with:

```
tar xf echo-qgp-<version >.tar.gz
```

Then, enter into the code directory:

```
cd echo-qgp-<version>
```

If you wish, you can modify the makefile to best exploit the capabilities of your processor or for debugging purposes. Default optimization flag -O2 should fit well for most cases.

If your cpu has only one core or if you wish to run ECHO-QGP serially, to compile just issue:

make

otherwise, with a multi-core machine with MPI installed, issue:

```
make par
```

Now, open the file param.dat with your favorite text editor (e.g. vim, emacs, gedit, geany, kwrite, kate...).

Let's assume that we wish to run a 3D+1 simulation of the hydrodynamical evolution of the QGP after the collision of two gold nuclei at an  $\sqrt(s_{NN})=200\,\mathrm{GeV}$  using Geometrical Glauber Initial conditions with an impact parameter of 7 fm, taking into account shear viscosity effects with an  $\eta/s=0.08.$  We choose  $140\,\mathrm{MeV}$  as freeze-out temperature. The details of ECHO-QGP configuration are all written into the param. dat file inside the folder tests/testA. Assuming that we have got a two core computer, we'll use:

```
mpirun -np 2 ./echo.exe
```

or, if we have got only a single core machine (and so, obviously, we didn't choose to use mpi):

```
./echo.exe
```

If we wish to run the program even after we log out from the computer (very common situation when we use an ssh connection), we may use nohup:

```
nohup mpirun -np 2 ./echo.exe&
```

With *nohup*, the messages printed by echo-qgp on the screen will be written into a file named nohup.out.

The output data files will be written into the outr0001 directory; inside that directory a log file showing the progress of the computation will be updated every DTLOG  $\tau$  intervals (in this example run, each 0.01 fm/c).

Let's suppose, now, that the run ended and we wish to analyze the results. First, we need to compile the fortran postprocessing utility with the command:

```
make tools
```

and then we have to copy the produced executable files (with the .exe extension) and the GDL scripts into the output directory:

```
cp tools/*.exe tools/*.pro outr0001
```

Now, let's move into the output directory:

```
cd outr0001
```

If you have installed IDL or its free clone named gnudatalanguage (at least, version 0.9.3, its website is: http://gnudatalanguage.sourceforge.net), you can use some scripts to make some sketchy, but quick plots. Let's see how. First, launch gdl:

```
gdl
```

You'll get an interactive shell.

With the script *readvar.pro* you will be able to read the variables of the output and manipulate them interactively.

First, we check if initialization was correct, so we read the first output file:

```
GDL> readvar, 0001
% Compiled module: READVAR.
This program reads the output of echo-qgp
Reading grid summary.dat
            101
nx
    :
            101
    :
ny
            101
nz
    :
           0.277228 - ystep:
                                    0.277228 - zstep:
xstep:
   0.277228
           -14.0000 – ymin:
                                                          -14.0000
xmin:
                                  -14.0000 – zmin:
            14.0000 - ymax:
xmax:
                                   14.0000 - zmax:
                                                           14.0000
Reading grid.dat
% Compiled module: SKIP LUN.
Run with viscosity
Output has 8 bytes precision
Variables to be read:
density
vx
vy
pressure
energy density
temperature
entropy density
bulk viscosity
pi^tt
pi^tx
pi^ty
pi^tz
pi^xy
pi^xz
pi^yz
pi^xx
```

```
pi^yy
pi^zz
gamma
Number of variables is:
                                20
Reading file: out0001.dat
time: 1.00000000
time (or tau) is stored into variable t=
                                                  1.0000000
Arrays available: x,y and z of type double with
                                                         101
                                                                 101
       101 elements
Arrays available with their maximum, mean and minimum values:
% Compiled module: MEAN.
          0.10000000
                            0.10000000
                                             0.10000000
          0.0000000
                            0.0000000
                                             0.0000000
          0.0000000
                            0.0000000
                                             0.0000000
vy:
          0.0000000
                            0.0000000
                                             0.0000000
vz:
pr:
          3.9807911
                         0.030049485
                                        6.1038367e-05
en:
          15.303008
                           0.13728677
                                         0.00030000000
temp:
           0.30648078
                            0.057427603
                                             0.047645369
                62.920094
                                0.69550263
                                               0.0075776172
entropy:
          0.0000000
                            0.0000000
                                             0.0000000
bu:
          0.0000000
                            0.0000000
                                             0.0000000
tt:
tx:
          0.0000000
                            0.0000000
                                             0.0000000
          0.0000000
                            0.0000000
                                             0.0000000
ty:
          0.0000000
                            0.0000000
                                             0.0000000
          0.0000000
                            0.0000000
                                             0.0000000
xy:
xz:
          0.0000000
                            0.0000000
                                             0.0000000
yz:
          0.0000000
                            0.0000000
                                             0.0000000
         0.66217778
                        0.0073195439
                                        7.9747652e-05
xx:
                                         7.9747652e-05
         0.66217778
                        0.0073195439
yy:
     -0.00015949530
                         -0.014639088
zz:
                                            -1.3243556
v0:
          1.0000000
                            1.000000
                                             1.0000000
```

It is usually a wise practice not only to check what ECHO-QGP prints to the standard output, but also that the simulation has been properly initialized and the gdl scripts offer a good method to test it.

We can easily check, for example, grid dimensions, grid resolution and what variables will be printed into the output files. Also, we can check that initial velocities are all 0 and that the minimum value of  $\pi^{\eta\eta}$  is twice the maximum value of  $\pi^{xx}$  and  $\pi^{yy}$  with changed sign, while other shear viscous tensor components are 0.

It is beyond the scope of this tutorial to teach how to use gdl (or IDL), however we'll give some additional basic information to show how to make some sketchy plots. Further informations can be easily found on the web.<sup>1</sup>

Using gdl it possible to store the data as multidimensional arrays into the RAM of the computer and then extract, print and plot some sections of them.

<sup>&</sup>lt;sup>1</sup>See, for example:

http://www.ita.uni-heidelberg.de/~dullemond/lectures/num\_fluid\_2009/App\_B.pdf or http://www.sgeier.net/tools/GDL-intro.pdf

In gdl array indexes start from 0 and intervals are denoted by colons; the arguments of the functions are separated by commas. So, for example, to print to print the first four values of the x array:

```
GDL> print, x(0:3)
-13.861386 -13.584158 -13.306931 -13.029703
```

To make things faster, you can move across the history of commands using the keyboard arrows and re-edit the commands.

Dimensions inside multi-dimensional arrays are separated by commas. The asterisk means "all elements contained into that dimension". We can assign the values of a slice of an already existing array to a new array, for example the following energy\_for\_z\_0 array contains the value of energy density for  $\eta=0$ :

```
GDL> energy_for_z_0=reform(en(*,*,50))
GDL> help, energy_for_z_0
ENERGY_FOR_Z_0 DOUBLE = Array[101, 101]
```

The help command tells what kind of values its argument is; please, remind that gdl is case insensitive (.i.e. "a" and "A" are the same variable).

Here we used *reform* to cut off the dimension with only 1 element. In this particular case, actually it was not needed, but it is recommended to always use it when you wish to cut slices with less dimensions to avoid to get 3D objects instead as 2D, as in this case:

```
GDL> energy_try=en(*,50,*)
GDL> help, energy_try
ENERGY_TRY DOUBLE = Array[101, 1, 101]
```

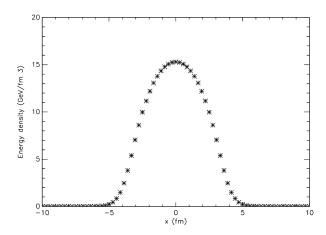
To make a plot, we can type:

```
GDL> plot, x, energy_for_z_0(_{\star},50), xtitle="x (fm)", ytitle=" Energy density (GeV/fm^{3})", psym=2, xrange=[-10,10]
```

and a plot like that in Fig. 6.1 will appear on a separate window (actually, here we show a postscript figure created with a couple of additional commands, see further in this text; plots on the screen usually have b/w colors inverted).

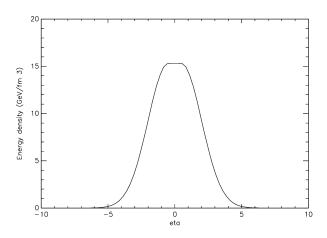
If we didn't plot the x array together with the *en* slice, then gdl on the x axis would have used only the index numbers of the elements of the slice of the *en* array.

Let's see now how the energy density profile looks like moving along the  $\eta$  direction at x=y=0 (Fig. 6.2):



**Figure 6.1:** Plot of energy density in  $GeV/fm^3$  at  $\tau = 1 \, fm/c$  for  $y = \eta = 0$ .

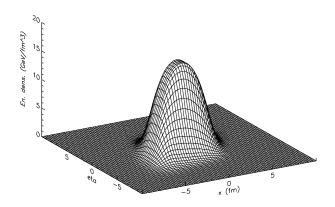
```
GDL> plot, z, en(50,50,*), xtitle="eta", ytitle="Energy density ( <math>GeV/fm^3)", xrange=[-10,10]
```



**Figure 6.2:** Plot of energy density in  $GeV/fm^3$  at  $\tau = 1 \, fm/c$  for x = y = 0.

Now let's make a surface plot of the energy density along the  $x-\eta$  plane for y=0 (Fig. 6.3):

```
GDL> surface, reform(en(*,50,*)), x, z, xtitle="x (fm)", ytitle=" eta", ztitle="En. dens. (GeV/fm^3)", xrange=[-10,10], yrange =[-10,10]
```



**Figure 6.3:** Surface plot of energy density in  $GeV/fm^3$  at  $\tau = 1 fm/c$  for y = 0.

Let's replot the same quantities, but using a contour plot (Fig. 6.5):

```
GDL> set_plot, 'ps'
GDL> device, filename="plot4.ps"
GDL> contour, reform(en(*,50,*)), x, z, xtitle="x (fm)", ytitle="
    eta", xrange=[-6,6],yrange=[-5,5], LEVELS = [0.0, 2.5, 5.,
    7.5, 10., 12.5, 15, 17.5, 20.], C_LABELS = [1, 0, 1, 0, 1, 0,
    1, 0, 1]
GDL> device, /close
```

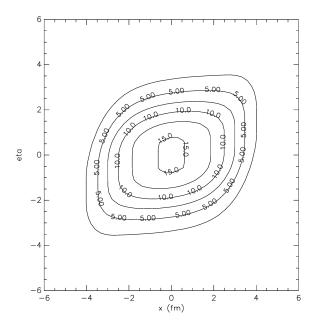
Now, let's have a look at what is the situation at a later time, using the fortran utilities.

Output data are written into binary files; the *readx.exe* utility extracts these data into separate files for each variable in ascii format.

Calling readx.exe without any argument prints an help message:

```
./readx.exe
Please, insert the range of echo-qgp output files from which you
   want to extract values.
(Optionally, you can also write the name of the directory where
   the output files will be written)
(default: postproc/readx)
```

If we are interested only in output at  $\tau=10.0\,fm/c$ , we can look at the time.dat file and, after reading that corresponding output data number is 91, we can type:



**Figure 6.4:** Contour plot of energy density in  $GeV/fm^3$  at  $\tau=1\,fm/c$  for y=0.

```
./readx.exe 91 91
time= 9.99999999999922
```

If we wished to extract all output data from  $\tau=4.5\,fm/c$  to  $\tau=4.8\,fm/c$  we would use:

Now, we copy the utility fromecho.exe into the new created postproc/readx directory and then we enter into it:

```
cp fromecho.exe postproc/readx cd postproc/readx
```

Calling fromecho.exe without any arguments prints an help message:

```
./fromecho.exe
Synopsis:
To extract data from the left to the right border of the grid
   along x, y or z direction:
./fromecho input_file output_file x|y|z
To extract data using a specific range along one direction and
   fixing the other ones at a point:
./fromecho input_file output_file x|y|z x-indx-start x-indx-end y
   -indx-start y-indx-end z-indx-start z-indx-end
Please, note that starting and ending indexes of the fixed
   directions must be equal
Usage example: ./fromecho prova input prova output y 33 33 2 150
   44 44
This extracts data from file prova_inputs and write the values
   along y direction from point with index 2 to
point with index 150, fixing x at index 33 and z at index 44
```

We extract the value of temperature at  $\tau=10\,fm/c$  along the x axis for  $y=\eta=0$ :

```
./fromecho.exe T0091.dat temperature_x_tau_10.0.dat x
```

Now extract the value of temperature at  $\tau = 10 \, fm/c$  along the  $\eta$  axis for x = y = 0, for all the  $\eta$  range (reminding that now indexes go from 1 to 101):

```
./fromecho.exe T0091.dat temperature_z_tau_10.0.dat z 51 51 51 51 1 101
```

All the files created by fromecho.exe consists of two columns: the first one is the coordinate position, the second one the value of the variable. This data format can be easily read by many plotting software.

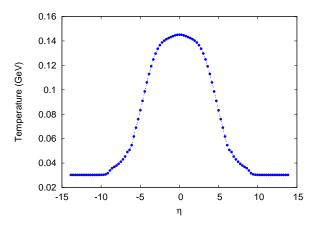
For example, using gnuplot (Fig. 6.5):

```
gnuplot
 GNUPLOT
                             last modified 2012-03-04
 Version 4.6 patchlevel 0
 Build System: Linux x86 64
  Copyright (C) 1986-1993, 1998, 2004, 2007-2012
 Thomas Williams, Colin Kelley and many others
  gnuplot home:
                   http://www.gnuplot.info
 faq, bugs, etc:
                   type "help FAQ"
 immediate help:
                   type "help" (plot window: hit 'h')
Terminal type set to 'wxt'
gnuplot> set term pos eps col enh font "Helvetica, 22"
Terminal type set to 'postscript'
```

```
Options are 'eps enhanced defaultplex \
leveldefault color colortext \
dashed dashlength 1.0 linewidth 1.0 butt noclip \
nobackground \
palfuncparam 2000,0.003 \
"Helvetica" 22 fontscale 1.0 '
gnuplot> set out "temperature.eps"
gnuplot> set ylabel "Temperature (GeV)"
gnuplot> set xlabel "{/Symbol h}"
gnuplot> plot "./temperature_z_tau_10.0.dat" with linespoints lt 7 linecolor 3 notitle
```

We remind that the image files in Encapsulated Postscript (.eps) format can be converted into Portable Document Format (.pdf) with the utility ps2pdf, e.g.:

```
ps2pdf -dEPSCrop temperature.eps
```



**Figure 6.5:** Test A - Temperature profile at  $\tau = 10 \ fm/c$  for  $y = \eta = 0$ .

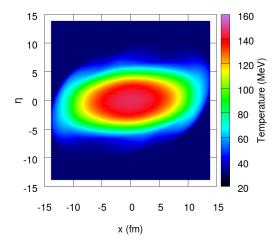
We can use the utility *fromecho2d.exe* to extract values for a 2D slice of the grid, for example:

```
./fromecho2d.exe T0091.dat tempxz91.dat x z
```

Then, we can use a gnuplot script like the following to plot the data:

```
# Set terminal and output
set terminal pngcairo size 900, 900 enhanced font 'Helvetica,22'
set encoding utf8
set output 'contour91xz.png'
```

```
# Set plot properties
set size square
set pm3d
unset surface
set view map
set key outside
set pm3d interpolate 0,0 # interpolate the color
# Set the color palette
set palette model RGB defined ( 0 'black', 1 'blue', 2 'cyan',3 '
   green',4 'yellow', 5 'red', 6 'purple')
# Se the axes
set xlabel 'x (fm)'
set ylabel "{/Symbol h}"
set cblabel "Temperature (MeV)"
# Draw the plot
splot 'tempxz91.dat' u 1:2:($3*1000.) notitle
```



**Figure 6.6:** Test A - Contour of temperature (MeV) at  $\tau = 10 \ fm/c$  for y = 0.

A copy of the plotted datafile tempxz91.dat can be found in tests/testA. Now let's see how to compute the thermal particle spectra. From the ECHO-QGP main directory of sources we enter the analysis subdirectory and we compile the tools:

make

We remind that, to make use of OPENMP, the user has to edit the makefile and export some environment variable, see 5.3 for more details. Using the following configuration file settings.txt:

```
! This is the file containg the settings for the routine producing
     the spectra
! *** ECHO-RELATED
                                            ! 1+1, 2+1, 3+1
\dim \ldots = 3
   dimension for the simulation
                                            ! Is it a viscous hydro
visco hyd=1
    simulation? 0=no, 1=yes
visco spe=1
                                            ! viscosity corrections
   to the distribution function? 0=no, 1=yes
                                            ! Evaluate vorticity on
vorticity=0
   hypersurface
nout.... = 0
                                            ! total number of
   outputs (if 0 reads all available)
! *** PARTICLE-RELATED
                                            ! points for the
npt \dots = 40
   transverse momentum
                                            ! min transverse
ptmin... = 0.0
   momentum
ptmax... = 4.0
                                            ! max transverse
   momentum
nphi.... = 36
                                            ! points for the polar
   angle
phimin... = 0.0
                                            ! min polar angle
phimax...=6.28318530718
                                            ! max polar angle
                                              points for the
nrap.... = 41
   particle rapidity
rapmin... = -4.0
                                            ! min rapidity
rapmax...=4.0
                                            ! max rapidity
listorseq=2
                                            ! 0- all; 1-sequence
   ID_start-ID_stop; 2-list; 3-stable
                                            ! ID of first particle
ID_start.=211
   to be considered
ID_stop.. = -10213
                                            ! ID of last particle to
    be considered
part_list=2
                                            ! particles in the list
211
3122
                                            ! 1=on 0=off
antibar..=1
                                            ! read chemical
chempot..=1
   potential from ../eos_data/chemical_potential.txt; 0 ste mu=0
! *** INPUT
inputdir.=../outr0001/
                                            ! input directory
ioutdir..=../outr0001/spectra/
                                            ! output directory
file .... = out
                                            ! name of the ECHO
   outout file
```

```
! *** RANDOM SEED (saved anyway)
seed \dots = 0
                                            ! if 0 generates random
   seed, otherwise use that
ptbox.... = 3.0
                                            ! upper limit for mc box
                                            ! if 0 is 2*pi
phibox...=0
ybox.... = 00.0
                                            ! upper limit for mc box
! *** HISTOGRAM-RELATED
mxv_pt... = 3.0
                                            ! max value for pt
mnv_pt...=0.0
                                            ! min value for pt
mxv_ang..=6.28318
                                            ! max value for phi
mnv ang..=0.0
                                            ! min value for phi
mxv y \dots = 0.0
                                            ! max value for y
mnv y... = -0.0
                                            ! min value for y
binpt....=50
binphi...=1
binrap...=1
                                            !
6.28318530718
```

we can compute the thermal spectra of  $\pi^+$ ,  $\Lambda^0$  and  $\overline{\Lambda^0}$ , taking into account the viscous corrections to the distribution function.

Please, remember to create the outr0001/spectra directory before running the tool.

After executing the program:

```
./thermal.exe
```

we can enter into outr0001/spectra and make some plots:

```
gnuplot plot_pionplus_spectra.gp
```

The plot files can be quickly converted from EPS to PDF with

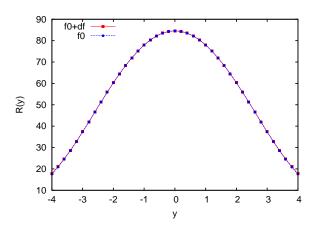
```
for i in *.eps; do ps2pdf -dEPSCrop $i; done
```

(See figures 6.7,6.8,6.9,6.10).

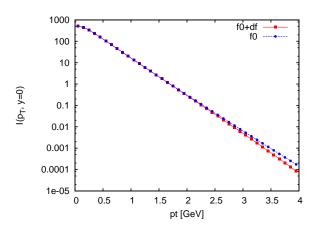
#### 6.2 2D shock tube - test B

This is a simple test of a 2D shock wave.

The param.dat files of this test can be found into the tests/testB directory. The initial state is simply costitued by a sharp jump in pressure when crossing the y=-x locus, whose values can be adjusted editing the init.f90 file a few lines after the condition:



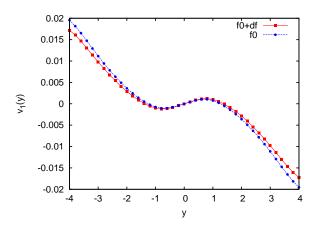
**Figure 6.7:** Test A - dNdy of  $\pi^+$  particles. Red color means that viscous corrections to thermal distribution have been taken into account, blue color means not.



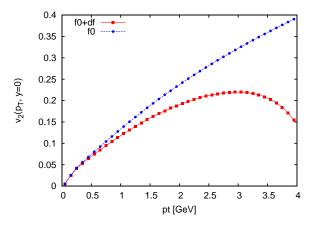
**Figure 6.8:** Test A -  $I(p_T,y=0)$  of  $\pi^+$  particles. Red color means that viscous corrections to thermal distribution have been taken into account, blue color means not.

```
else if ( init_type .eq. SHOCK_TUBE_2D) then
```

```
if(y(iy) .lt. -x(ix)) then
    vv(kpr) = 5.401411
else
    vv(kpr) = 0.337588
end if
```



**Figure 6.9:** Test A - Directed flow of  $\pi^+$  particles. Red color means that viscous corrections to thermal distribution have been taken into account, blue color means not.



**Figure 6.10:** Test A - Elliptic flow of  $\pi^+$  particles. Red color means that viscous corrections to thermal distribution have been taken into account, blue color means not.

After compiling ECHO-QGP, we run ECHO-QGP a few times using different values for  $\eta/s$ , changing the ETA\_S parameter into param.dat. We can also execute an inviscid simulation setting the parameter VISCOUS..=0. Please, take care to not overwrite the results of the various simulations, either renaming the output directory after the end of each simulation or using the - $\sigma$  flag when launching the program (e.g. ./echo.exe - $\sigma$  out\_2Dtest).

First, we check if the initial conditions are correct.

We copy the tools/readvar.pro gdl script into the output directory (e.g. outr0001, but it could have a different name) and then we use it:

```
cd outr0001
gdl
```

```
GDL> readvar, 0001
```

We can make a sketchy plot simply typing:

```
GDL> plot, x, pr(*,100)
```

This is fine to make a simple check, but, if we wish to prepare a better quality plot, we can export the data to a file and then use it another tool (e.g. gnuplot, grace and many others). To perform this task, we can use the fortran postprocessing tools or gdl itself, preparaing, for example, a script like (file tests/testB/tofile.pro):

```
openw, 12, "initial_pressure_testB.dat"

;please, change the for-cycle interval accordingly with your grid
;remind that array indexes in gnudatalanguage start from 0
;for example, if you have a 201x201 grid, initial and final values
    are 0 and 200
;and, if the grid is symmetric respect to the origin, the index
    corresponding to 0 is 100
for i=0,200 do begin
        printf, 12, x(i), pr(i,100)
endfor

free_lun, 12

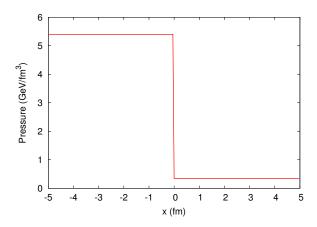
END
```

If we run the script with:

```
GDL> .r tofile
```

we should get an ascii file called <code>initial\_pressure\_testB.dat</code> which we can plot using another program (6.11): Assuming to have executed four simulations, one inviscid and the other three with  $\eta/s=0.001,\,0.01,\,0.1,$  after compiling the fortran tools and copying them into the output directory, we can extract the data at the end of the run ( $\tau=4\,\mathrm{fm/c}$ ) with:

```
./readx.exe 31 31
```



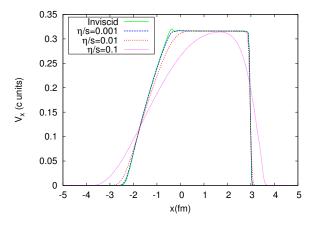
**Figure 6.11:** Test B - Initial pressure profile at  $\tau = 1, fm/c$  for y = 0.

followed by (into the postproc/readx directory):

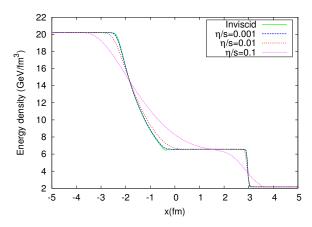
```
./fromecho.exe EPS0031.dat en0_1.dat x 1 201 101 101 1 1 ./fromecho.exe VX0031.dat vx0_1.dat x 1 201 101 101 1 1
```

In the previous example, the name of the output files refer to energy density and  $V_x$  of the run with  $\eta/s=0.1$ .

In the tests/testB directory there is a simple gnuplot script to plot the results after gathering them in a common directory (Fig. 6.12 and 6.13).



**Figure 6.12:** Test B - Comparison of  $V_x$  profiles at  $\tau = 4$ , fm/c for y = 0.



**Figure 6.13:** Test B - Comparison of energy density profiles at  $\tau =$ 4, fm/c for y = 0.

#### 1D viscous shear flow test - test C 6.3

This is merely the execution of the test described in section 4.3 and [6]. The file param.dat to set up the simulation can be found in tests/testC. The results can be examined with the usual tools, following the procedures already shown in the previous examples:

```
./readx.exe 91 91
./fromecho.exe VY0091.dat vy.dat x 1 301 1 1 1 1
```

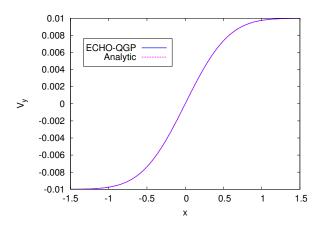
A simple gnuplot script (plot1D.gp) to plot the results is also included.

#### 6.4 Gubser flow test - test D

The description of this test can be found on [12] [8], here we'll simply summarize what do to perform it.

First, we need a set of initial values, which can be found into the tests/testD directory. This file, named Initial\_Profile\_GubserFlow.dat, has been produced with Maple 12 executing the worksheet initial\_conditions\_maple12.mw contained into the same directory. The settings inside param.dat must match those of the tabulated initial conditions (e.g. the grid) and also  $\tau_{\pi}$ ,  $\eta/s$  and the relation between temperature and energy density must correspond to what chosen for computing the input file with initial conditions. The file tests/testD/param.dat works with the provided intial conditions.

To complete the setup, the user has to comment/uncomment the temperature



**Figure 6.14:** Test C - Comparison between the analytical solution and ECHO-QGP at  $\tau=10$  fm/c.

relation with energy density inside the eos\_data/temperature.def file, clean the sources with *make clean* and recompile ECHO-QGP. If the grid used for computing the initial conditions is changed, then it is required to change not only the lines defining the grid into param.dat, but also the checks contained into init.f90 after the line with comment "! GUBSERCHECKS".

Since in this test we wish to compare the evolution of some variables with the semianalytical solutions obtained numerically with some external code, we need also these solutions. In the tests/testD directory we provide the solutions computed with *Maple 12* at  $\tau=4$  fm/c; the Maple worksheet can be reused, with only minor changes, to compute the solutions at different times and/or with different grid resolutions.

After the end of the run, we can check the results. There are many ways to do it, here we'll use gdl and gnuplot.

```
cd outr0001
```

We can use the shell commands inside *gnudatalanguage* adding a dollar (\$) sign before issuing the command. For example, to list the files inside the current directory to check if the script readvar.pro is there we can type:

```
GDL> $1s -1 | grep readvar
-rw-r-r--- 1 g g 12562 mag 9 02:42 readvar.pro
```

Now we read the output at  $\tau = 4$  fm/c (please, remind that you can look at time.dat to see what is the output number corresponding to that time).

```
GDL> readvar, 0031
% Compiled module: READVAR.
This program reads the output of echo-qgp
Reading grid_summary.dat
            401
    :
            401
ny
nz
              1
           0.0500000 - ystep:
                                    0.0500000 - zstep:
xstep:
   2.00000
                                                           -1.00000
xmin:
           -10.0250 – ymin:
                                   -10.0250 – zmin:
                                                            1.00000
xmax:
            10.0250 - ymax:
                                    10.0250 - zmax:
Reading grid.dat
% Compiled module: SKIP_LUN.
Run with viscosity
                  8 bytes precision
Output has
Variables to be read:
density
VX
vy
VZ
pressure
energy density
temperature
entropy density
bulk viscosity
pi^tt
pi^tx
pi^ty
pi^tz
pi^xy
pi^xz
pi^yz
pi^xx
pi^yy
pi^zz
gamma
Number of variables is:
                               2.0
Reading file: out0031.dat
time: 4.00000000
time (or tau) is stored into variable t=
                                                4.0000000
Arrays available: x,y and z of type double with
                                                               401
                                                       401
         1 elements
Arrays available with their maximum, mean and minimum values:
% Compiled module: MEAN.
          0.21052070
                           0.12870059
                                         0.0034551016
rho:
         0.97009619
                      8.2463097e-16
                                         -0.97009619
vx:
vy:
         0.97009619
                     -6.2895014e-15
                                         -0.97009619
vz:
          0.0000000
                           0.0000000
                                           0.0000000
pr:
       0.0074253124
                       0.0010125367
                                       1.0013452e-05
en:
        0.022275937
                       0.0030376102
                                       3.0040355e-05
          0.059237610
                           0.027047653
                                           0.011351802
temp:
```

```
entropy:
              0.50139177
                             0.088629927
                                             0.0035284097
bu:
          0.0000000
                          0.0000000
                                           0.0000000
tt:
          0.0000000
                      -0.0011994727
                                      -0.0089583879
       0.0093095075 -3.2743781e-18
tx:
                                      -0.0093095075
       0.0093095075 -6.1862789e - 18
                                      -0.0093095075
ty:
tz:
          0.0000000
                          0.0000000
                                          0.0000000
       0.0044716459
                     1.7072896e-18
                                      -0.0044716459
xy:
                                          0.0000000
xz:
          0.0000000
                          0.0000000
          0.0000000
                          0.0000000
                                           0.0000000
yz:
      1.1768752e-05
                    -0.00081962219
                                      -0.0096843503
xx:
      1.1768752e-05
                    -0.00081962219
                                       -0.0096843503
yy:
zz:
      0.00010049364
                    2.7411661e-05
                                      -3.8899064e-07
v0:
          4.1203721
                          1.9250202
                                           1.0000000
```

Now we read the tabulated file with the solutions computed by Maple using the script read\_M12.pro, in the tests/testD directory:

The script reads the file maple\_gub\_full\_y\_eq\_x\_tau\_4.0; the user can easily modify the worksheet to read other files with the same structure. Now we prepare the data for plotting. We are interested in the values with coordinates x=y, so first we declare an array to store these values:

```
GDL> a=dblarr(401)
```

Then we fill the array, in this example with the values of  $u^x = \gamma v^x$ :

```
GDL> for i = 0.400 do a(i) = v0(i, i) * vx(i, i)
```

And now we print the values to compare into the file ux-comparison.dat:

```
GDL> openw, 12, "ux-comparison.dat" GDL> for i=0,400 do printf, 12, x(i), a(i), mux12(i) GDL> free_lun, 12
```

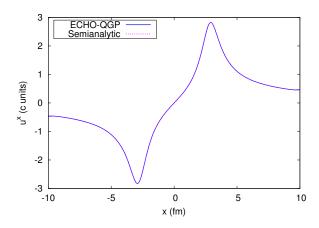
Now we can make the plot with gnuplot:

```
set term pos eps col enh font "Helvetica,22"
set out "ux.eps"
set xlabel "x (fm)"
set ylabel "u^x (c units)"
set key top left box
```

```
plot "ux_comparison.dat" u 1:2 w l lt 1 lc 3 lw 3 title "ECHO-QGP", "ux_comparison.dat" u 1:3 w l lt 3 lc 4 lw 3 title "Semianalytic"
```

The commands are gathered into the plot-ux.gp script under tests/testD, which can be executed with:

```
gnuplot plot-ux.gp
```



**Figure 6.15:** Test D - Comparison between the semianalytical solution and ECHO-QGP results at  $\tau = 4$  fm/c.

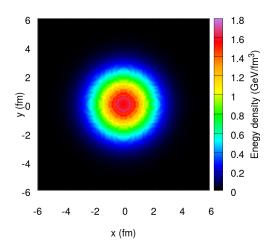
Into the tests/testD directory there are a few more plots of other variables, obtained with the same procedure.

#### 6.5 Initialization with a tabulated file - test E

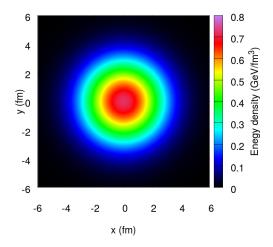
In this kind of initialization, the initial entropy density distribution is provided with an ascii tabulated file.

Changing the IENENTR parameter from 1 to 0, it is also possible to initialize ECHO-QGP using an energy density distribution. The name of the input file can be chosen with the IN D FILE parameter.

The input file for this example is created with a short python script (init\_tab.py) that computes a simple gaussian distribution. The script and the param.dat file to configure ECHO-QGP are into the tests/testE directory. See figures 6.16 and 6.17.



*Figure 6.16:* Test E - Initial energy density distribution at  $\tau = 1$  fm/c.



**Figure 6.17:** Test E - Energy density distribution at  $\tau = 5$  fm/c.

#### 6.6 Glauber Monte Carlo initialization - test F

In the tests/testF directory can be found a param.dat file to initialize a Glabuer-Monte Carlo simulation.

The random seed determines what will be the sequence of events, so, using the same random seed, saved into the file random\_seed.dat, it is possible to reproduce the same initial series of events.

Other parameters can be ajdusted editing the first lines of glaubermc.f90. In this example, we try to produce a series of events from a sequence of 500 nuclear configurations, each with 20 different impact parameters, getting 1881 events which satisfy our selection criteria (i.e., a minimum of 18 participants, as specified with the option  $min\_participants=18$  at the beginning of file glaubermc.f90). The results are stored into the file partcoll.dat. From this list, we select the event with id 67, but we can choose a sequence of events instead of a single one.

Now we plot the initial energy density distribution. The procedure, for each plot, is the same, apart, obviously, for the names of the files. For the event with id 67:

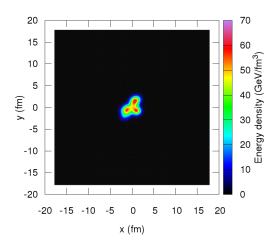
we compile the fortran prostprocessing tools:

make tools
cp tools/readx.exe outr0067
cd outr0067
We are interested in the first and last output files, so:
./readx 1 1
./readx 94 94
cp/tools/fromecho*.exe postproc/readx
cd postproc/readx

```
./fromecho2d.exe EPS0001.dat en out1.dat x y
```

```
gnuplot contour_out1.gp
```

The Fig. 6.18 shows the initial energy distribution on the  $\eta=0$  (x,y) plane at  $\tau=1$  fm/c. Now we look at the energy density profile as a function of  $\eta$  for



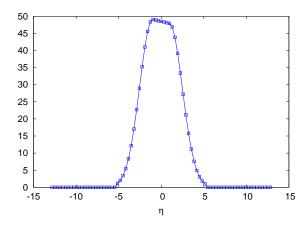
**Figure 6.18:** Test F - Initial energy density distribution at  $\tau = 1$  fm/c with Glauber-MonteCarlo initialization.

```
x = y = 0 (Fig. 6.19):
```

./fromecho.exe EPS0001.dat energy\_vs\_eta\_tau1.dat z 56 56 56 56 1 81

```
gnuplot plot_en_vs_eta.gp
```

Now we plot the initial energy distribution on the  $\eta=0$  (x,y) plane at  $\tau=10.22$  fm/c (last output frame before the end of the simulation, i.e. when the maximum temperature on the grid was less than 137 MeV) (Fig. 6.20).



**Figure 6.19:** Test F - Initial energy density vs  $\eta$  at  $\tau=1$  fm/c and x=y=0 (Glauber-MonteCarlo initialization).

```
./fromecho2d.exe EPS0094.dat en_out94.dat x y
```

gnuplot contour\_out94.gp

### 6.7 Run with "tilted" initial conditions - test G

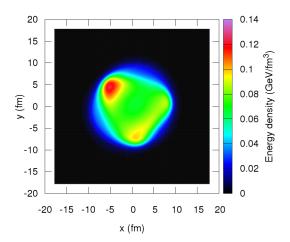
In this example we run a simulation with Geometric Glauber initial conditions, without viscous effects, but with a "tilted" energy density distribution. The files related to this example can be found into the tests/testG directory. Using the same procedure shown in the previous example (test F), we draw a contour plot of the initial energy distribution, to visually verify the effect of "tilting" (Fig. 6.21).

Now, since this is a simulation without viscosity effects, we use the GDL postprocessing utilities to verify the conservation of total energy and angular momentum during the fluid evolution.

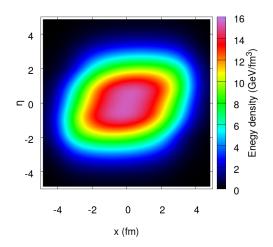
So, after copying from the tools directory into the outr0001 directory the readvar.pro and ce.pro scripts, we launch *gdl* and we run them:

```
GDI> readvar, 0001
```

GDL> ce



**Figure 6.20:** Test F - Initial energy density distribution at  $\tau = 10.22$  fm/c with Glauber-MonteCarlo initialization.



**Figure 6.21:** Test G - Initial energy density distribution at  $\tau = 1$  fm/c.

getting, for tau = 1 fm/c:

Total energy: 12400.207

J^y: -5838.3068

then, to see what are the total energy and the angular momentum  $J^y$  at  $\tau=11~{\rm fm/c}$ :

readvar, 0101

ce

getting:

Total energy: 12371.264

J^y: -5834.8883

So, in this run, total energy was conserved within a  $\sim 0.2\%$  while , for  $J^y,$  within a  $\sim 0.06\%.$ 

## Chapter 7

## Particle spectra production

We always refer to the Minkowski and Bjorken metrics respectively as

$$g_M^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \qquad g^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & & -1 & \\ & & & -\frac{1}{\tau^2} \end{pmatrix}$$
(7.1)

We convert the description from hydro to particles, using the Cooper-Frye prescription. Referring directly to the article by Cooper and Frye [13]:

$$E\frac{\mathrm{d}^{3}N_{i}}{\mathrm{d}p^{3}} = \frac{d^{3}N_{i}}{dyp_{T}dp_{T}d\phi} = \frac{g_{i}}{(2\pi)^{3}} \int_{\Sigma} \frac{p^{\mu}d^{3}\Sigma_{\mu}}{\exp\frac{u^{\mu}p_{\mu} - \mu_{i}}{T_{\mathrm{FO}}} \pm 1}$$
(7.2)

The index i refers to the nature of the particle. Note that this time the rapidity y refers to the produced particle, and not the fluid! The decomposition of the particle four-momentum must be consistent with the Milne coordinates:

$$p^{\mu} = \begin{pmatrix} m_{\mathrm{T}} \cosh(y - \eta_s) \\ p_{\mathrm{T}} \cos \phi \\ p_{\mathrm{T}} \sin \phi \\ \frac{m_{\mathrm{T}}}{\tau} \sinh(y - \eta_s) \end{pmatrix} \qquad m_{\mathrm{T}} = \sqrt{m_i^2 + p_{\mathrm{T}}^2}$$

$$p_{\mathrm{T}} = \sqrt{p_x^2 + p_y^2}$$

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)$$

$$(7.3)$$

The current version of the particle spectra production routines **only works in Milne (Bjorken) coordinates**.

## 7.1 hypersuface.f90 in ECHO-QGP

The task of the routine hypersuface.f90 included in ECHO-QGP is to find, store and print the coordinates of the hypersurface detected during the hydrodinamic evolution. The user can choose the hypersurface to be isothermal or constant energy, using the switch HYP\_COMPU in the param.dat file.

The routine checks in every space-time direction if two adjacent cells are one above and one below the criterion. Let US refer to the cells with the index i, being the direction positive with crescent i (es.  $x_{i+1} > x_i$ ); and  $F_i, F_{i+1}, F^*$ respectively the selected field (temperature or energy density) in the adjacent cells and of the constant hypersuface. Then we know that the hypersuface lies between the considered cells if

$$(F_{i+1} - F^*)(F^* - F_i) \ge 0$$

When this happens we store all the needed information, namely

#### in the ideal case

an array with 13 entries:

#### in the viscous case

In addition to the ideal case, an-

- 1.  $\tau$  or t
- **2.** *x*
- 3. *y*
- 4.  $\eta$  or z
- **5.** *ρ*
- 6.  $v^x$
- 7.  $v^y$
- 8.  $v^{\eta}$  or  $v^z$
- 9. p
- 10.  $dV^{\perp \tau}$  or  $dV^{\perp t}$
- 11.  $dV^{\perp x}$
- 12.  $dV^{\perp y}$

other array with 12 entries

- 14. bulk
- 15.  $\pi^{xy}$
- 16.  $\pi^{xz}$  or  $\pi^{x\eta}$
- 17.  $\pi^{yz}$  or  $\pi^{y\eta}$
- 18.  $\pi^{xx}$
- 19.  $\pi^{yy}$
- 20.  $\pi^{zz}$  or  $\pi^{\eta\eta}$
- 21.  $\pi^{tt}$  or  $\pi^{\tau\tau}$
- 22.  $\pi^{tx}$  or  $\pi^{\tau x}$
- 23.  $\pi^{ty}$  or  $\pi^{\tau y}$
- 24.  $\pi^{tz}$  or  $\pi^{\tau\eta}$
- 25.  $(e+p)T^2$
- 13.  $dV^{\perp \eta}$  or  $dV^{\perp z}$

the variables are computed interpolating linearly, using a weight w:

$$w = \frac{F_i - F^*}{F_i - F_{i+1}}$$

The volume element in the bjorken case is defined as

$$\mathrm{d}V = \begin{cases} \mathrm{d}V^{\perp \tau} = s^{\tau} \tau \Delta x \Delta y \Delta \eta & (\mathrm{fm}^3) \\ \mathrm{d}V^{\perp x} = s^{x} \tau \Delta \tau \Delta y \Delta \eta & (\mathrm{fm}^3) \\ \mathrm{d}V^{\perp y} = s^{y} \tau \Delta \tau \Delta x \Delta \eta & (\mathrm{fm}^3) \\ \mathrm{d}V^{\perp \eta} = \frac{s^{\eta}}{\tau} \Delta \tau \Delta x \Delta y & (\mathrm{fm}^2) \end{cases} \quad \text{with} \quad s^{\mu} = -sign\left(\frac{\partial T}{\partial x^{\mu}}\right)$$

note that currently for most cells just one of the  $\mathrm{d}V$  component is not vanishing.

The above described array is printed in the file hypersuface.txt in the output folder. This file contains as a first row the selected switch (namely FREEZKIND ), the temperature (in GeV) of the hypersuface and its energy density (GeVfm<sup>-3</sup>). After the first row it contains a block for each time-step. The block head is an integer: it is the number of frozen cells contained which corresponds to the amount of lines to be read before finding another block. Each one of these lines is a frozen cell, where the column index refers to the array "hypersurface" described above. For the ideal case the columns are 13, for the viscous one the columns are 25.

### 7.2 Particle spectra routines

All the spectra routines are contained in the folder analisi. There are four separate programs:

thermal which can be build with the command make thermal

utils\_therm which can be build with the command make utils\_them

MCgen which can be build with the command make MCgen

utils histogram which can be build with the command make utils\_histogram

Issuing just the make command, just thermal and MCgen are produced.

thermal produces the averaged spectra of the selected particle specie, whereas MCgen produces a ramdom sample of such particles. This last program needs some of the output files of the former, but since the computational load for thermal is heavy, the user can exploit utils\_therm to produce just the files needed by the Monte Carlo generator, saving time. utils\_histogram, instead allows the user to re-arrange in different binnings the histogram produced by MCgen

All these programs need the file settings.txt to be carefully filled in.

## 7.3 How to configure the particle production

All the programs contained in the folder analisi use as configuration file settings.txt, although not all parameters are effectively used in each program. In the listing 7.1 an example of the configuration file is provided. As we did in ECHO-QGP configuration file, also this settings.txt is divided into sections and most important needs its structure not to be altered.

```
! This is the file containg the settings for the routine...
  ! *** ECHO-RELATED
  dim....=3
                                                 ! 1+1, 2+1, 3+...
  visco_hyd=1
                                                 ! Is it a visc...
  visco_spe=1
                                                 ! viscosity co...
  vorticity=1
                                                 ! Evaluate vor...
  nout . . . . = 0
                                                 ! total number...
  ! *** PARTICLE-RELATED
 npt . . . . . = 41
                                                 ! points for t...
  ptmin....=0.0
                                                 ! min transver...
 | ptmax . . . . = 4 . 0
                                                 ! max transver...
  nphi....=36
                                                 ! points for t...
                                                 ! min polar an...
13 phimin...=0.0
  phimax...=6.28318530718
                                                ! max polar an...
15 nrap . . . . = 41
                                                 ! points for t...
  rapmin . . . = -4.0
                                                 ! min rapidity
17 rapmax . . . = 4.0
                                                 ! max rapidity
  listorseq=0
                                                 ! 0- all; 1-se...
19 ID_start.=211
                                                 ! ID of first ...
  ID_stop..=-10213
                                                 ! ID of last p...
21 | part_list = 0
                                                 ! particles in...
  antibar..=1
                                                 ! 1=on 0=off
23 chempot . . = 1
  ! *** INPUT
inputdir.=../out001/
                                                 ! hypersurface...
  ioutdir..=../out001/spectra/
                                                 ! output paren...
mcoutdir.=../out001/mc001/
                                                 ! output child...
  file....=out
                                                 ! name of the \dots
29 ! *** RANDOM SEED (saved anyway)
                                                 ! if 0 generat...
  seed . . . . . = 0
31 ptbox...=3.0
                                                 ! upper limit ...
  phibox . . . = 0
                                                 ! if 0 is 2*pi
33 ybox . . . . = 6.0
                                                 ! upper limit ...
  oversmpl.=1.0
                                                 ! should be 1....
35 ! *** HISTOGRAM-RELATED
 | mxv_pt...=4.0
                                                 ! max value fo...
37 mnv_pt...=0.0
                                                 ! min value fo...
                                                 ! max value fo...
 mxv_ang..=6.28318
39 mnv_ang..=0.0
                                                 ! min value fo...
 | mxv_y . . . . = 3 . 0
                                                 ! max value fo...
41 mn v _ y . . . . = -3.0
                                                 ! min value fo...
  binpt . . . . = 10
43 | binphi . . . = 1
  binrap . . . = 11
45 6.28318530718
```

**Listing 7.1:** Example of settings.txt

Despite all programs read the settings file, not all the variables are being used in each.

#### 7.3.1 ECHO-QGP related parameters

This section contains the parametrs and the flags related to the hydrodynamic part of the simulation, such as the

- dim integer- flag. Sets the dimensionality of the ECHO-QGP simulation. Allowed values are 2 or 3, for (1+2)-D and (3+1)-D simulations respectively. If the cross-check with the output with ECHO-QGP gives a different dimensionality the code quits with en error message.
- **visco\_hyd** integer- flag. Passes to the program the information about the ECHO-QGP simulation: must be 1 if the simulation was a viscous one, 0 if it was ideal. Note that the file containing the hypersurface information and the array listed in section 7.1 are allocated accordingly.
- visco\_spe integer- flag. Enables(1) or disables(0) the viscous contributions
  to the distribution function in the spectra computation. It can only be
  activated when visco\_hyd=1
- vorticity integer- flag. Enables(1) or disables(0) the computation of thermal vorticity over the decoupling hypersurface. It needs the file hypersurf\_deriv.txt produced by ECHO-QGP in the same directory containing hypersurface.txt
- **nout** integer- flag. It is the number of time-steps to be taken into account. If set to 0 the program reads all the time steps produced by ECHO-QGP, if not the program asks the user if the number of output is correct and wants a Y/N answer from the keyboard, followed by the return command to continue. This variable is very useful for debugging.

#### 7.3.2 Particle related parameters

This section contains the parametrs related to the produced particles, such as the species and the momentum.

The programs in this section use concatenated loops over the momentum components, where we refer to the momentum components referring to equations 7.3, and using the transverse momentum  $p_{\rm T}$ , the polar angle  $\phi$  and the particle rapidity y as the indipendent variables. The three-dimensional grid in momentum is thus composed by  ${\tt npt} \times {\tt nphi} \times {\tt nrap}$  ( $N_{p_{\rm T}} \times N_{\phi} \times N_{\rm y}$ ). The points in each one of the three components are computed following the simple scheme

```
dpt=(ptmax-ptmin)/npt
do i=1, npt
pt(i)=ptmin+(i-0.5)*dpt
end do
```

```
dphi=(phimax-phimin)/nphi
do i=1, nphi
    phi(i)=phimin+(i-1.0)*
        dphi
4 ! we would like start
        with 0
end do
```

The particle rapidity initialization is very similar to the ones listed above, except the fact that the code checks whether the range is symmetric and tries to adjust  $N_{\rm v}$  (adding a point) to have the value 0 too.

```
npt integer-value. Number of transverse momentum (p_T) points ptmin real-value. Minimum of the range in transverse momentum (p_{T\min}; GeV) ptmax real-value. Maximum of the range in transverse momentum (p_{T\max}; GeV) nphi integer-value. Number of polar angle (\phi) points phimin real-value. Minimum of the range in polar angle (\phi_{\min}) phimax real-value. Maximum of the range in polar angle (\phi_{\max}) nrap integer-value. Number of rapidity (y) points rapmin real-value. Minimum of the range in rapidity (y_{\min}) rapmax real-value. Maximum of the range in rapidity (y_{\max})
```

**listorseq** integer- flag. It is the switch that allows the user to change the set of particles to be used. All the programs can use three different sets of particles.

In the folder <code>eos\_data</code> must be present the file <code>pdglist.txt</code>, which lists all the particle which can be included in the computation (one can see a human-readable table in the Appendix ??). The code uses such table to read the particles features. The first integer number is the ID number of the particle, which we use in the settings file to identify it. listorseq can assume following values:

- 0. Uses ALL the particle listed in the file pdglist.txt
- 1. Uses a the particles included in the list, between the ID corresponding to ID\_START and the ID corresponding to ID\_STOP
- 2. Uses a list of particles, which IDs <u>must be listed in the settings file</u>, just after the part\_list line

- For the sake of clearness, we show in this section three examples of the using of these options in the subsection 7.6
- ID\_start integer- particle ID. Only used if listorseq=1, is the first particle
   in the range the user wants to used
- ID\_stop integer- particle ID. Only used if listorseq=1, is the last particle
   in the range the user wants to used
- part\_list integer- value. Only used if listorseq=2, is the total number of
   particles listed in the settings file, which must correspond to the lines
   with the IDs after this parameter.
- antibar integer- flag. Since in the pdglist.txt file there is no antibaryon listed, this switch enables (1) or disables (0) the production of the antibaryons corresponding to the listed baryons.
- chempot integer- flag. If set to 0 it forces the chemical potentials of all considered particles to be 0. If set to 1, the program reads the chemical potential corresponding to the given temperature from the file chemical\_potential.txt contained in the eos\_data folder.

### 7.4 Input-Ouput

The section of the file settings.txt dedicated to the input/output addresses simply contains paths and names:

inputdir character\*64 - path. Path of the folder containing the file hypersurface.txt

- ioutdir character\*64 path. Path in which all the files produced by thermal
   (and by utils\_therm) are stored. The same path also contains the info
   used by MCgen. If the folder exists already, the content is replaced, while
   if the folder does not exist, a new one is created.
- mcoutdir character\*64 path. Path in which all the files produced by MCgen (and by utils\_histogram) are stored. If the folder exists already, the content is replaced, while if the folder does not exist, a new one is created.

### 7.5 Parameters for the Monte Carlo production

In this section the user can find all the parameters for the Monte Carlo generation of particles.

seed integer- flag and value. If 0 is given, the seed is automatically generated and stored in the file report\_MCgen.txt (placed in the folder specified by the path mcoutdir) **ptbox** real- value. Upper limit for the random sampling (from a uniform distribution) of the transverse momentum ( $p_T$ , GeV).

**phibox** real- flag and value. Upper limit for the random sampling (from a uniform distribution) of the polar angle  $(\phi)$ . If set to 0 the upper limit is automatically set to  $2\pi$ 

**ybox** real- value. Upper limit for the random sampling (from a uniform distribution) of the rapidity (y).

The user must be careful in the choice of ptbox, phibox and ybox: given three random numbers  $r_1, r_2, r_3 \in [0; 1]$  the momentum is sampled with the formula:

$$p_{\rm T} = r_1 \cdot {\tt ptbox} \tag{7.4}$$

$$\phi = r_2 \cdot \mathtt{phibox} \tag{7.5}$$

$$y = r_3 \cdot 2 \cdot ybox - ybox \tag{7.6}$$

so a bad choice of the upper limits can lead either to a wrong sampling ensable or a bottleneck in the code.

**oversmpl** real- flag and value. This parameter allows the user to perform an oversampled (if the parameter is greater than 1) simulation, as well as an undersampled (if the parameter is less than 1) simulation. The histograms computation automatically takes care of this parameter renormalizing them. This parameter is mainly useful during the debugging stage.

#### Histogram related parameters

When the Monte Carlo production of the particles is terminated, some histograms are automatically calculated, in order to be compared with the averaged spectra. Such histograms calculation can also be performed independently with the program utils\_histogram. This section specifies the binning of such histograms.

mxv\_pt real- value. Maximum value in the transverse momentum histogram
mnv\_pt real- value. Minimum value in the transverse momentum histogram
mxv\_ang real- value. Maximum value in the polar angle histogram
mnv\_ang real- value. Minimum value in the polar angle histogram
mxv\_y real- value. Maximum value in the particle rapidity histogram
mnv\_y real- value. Minimum value in the particle rapidity histogram

binpt integer- value. Number of bins in the transverse momentum histogram

binphi integer- value. Number of bins in the polar angle histogram

binrap integer-value. Number of bins in the particle rapidity histogram

### 7.6 Examples of particles subset:

The IDs of the particles are listed into the file eos\_data/pdglist.txt.

#### 7.6.1 Producing all available particles: listorseq=0

We provide here an example of the particle section in the setup file, for a simulation including *ALL the particles*, comprehensive of antibaryons. We also set as active in the computation, the appropriate and the chemical potential in settings.txt

Please note in 7.2 that there is NO line after the part\_list line, and the variable part\_list itself is set to 0.

```
! *** PARTICLE-RELATED
  npt . . . . . = 41
                                                ! points for t...
  ptmin....=0.0
                                                ! min transver...
 ptmax . . . . = 4.0
                                                ! max transver...
 nphi....=36
                                                ! points for t...
  phimin...=0.0
                                                ! min polar an...
 phimax...=6.28318530718
                                                ! max polar an...
  nrap . . . . . = 41
                                                ! points for t...
  rapmin . . . = -4.0
                                                ! min rapidity
  rapmax . . . = 4.0
                                                ! max rapidity
 listorseq=0
                                                ! 0- all; 1-se...
                                                ! ID of first ...
 ID_start.=0
13 ID_stop..=0
                                                ! ID of last p...
 part_list=0
                                                ! particles in...
 antibar..=1
                                                ! 1=on 0=off
 chempot..=1
```

**Listing 7.2:** Example of the section in settings.txt producing all the available particles.

#### **7.6.2 Producing an interval of listed particles:** listorseq=1

Let's assume that the user wants to produce all the particle listed in *pdglist.txt* between two given IDs, say  $\pi^+$  (with ID 211) and  $\eta$  (with ID 221), which are:  $\pi^+, \pi^0, \pi^-, K^+, K^-, K^0, \overline{K^0}, \eta$ . The fastest way to configure *settings.txt* is to set listorseq=1, ID\_start=211 and ID\_stop=221. Here we provide an

example setup for a simulation producing such ensemble, without the production of antibaryons, and with the appropriate chemical potential. Please note that still there is NO line after the part\_list line, and the variable part\_list itself is set to 0.

```
! *** PARTICLE - RELATED
 npt . . . . . = 41
                                                ! points for t...
 ptmin....=0.0
                                                ! min transver...
 ptmax...=4.0
                                                ! max transver...
                                                ! points for t...
 nphi....=36
 phimin...=0.0
                                               ! min polar an...
 phimax...=6.28318530718
                                               ! max polar an...
 | nrap . . . . = 41
                                               ! points for t...
| rapmin . . . = -4.0
                                                ! min rapidity
 | rapmax . . . = 4 . 0
                                                ! max rapidity
11 listorseq=1
                                                ! 0- all; 1-se...
  ID_start.=211
                                                ! ID of first ...
13 ID_stop..=221
                                                ! ID of last p...
 | part_list = 0
                                                ! particles in...
15 antibar..=0
                                                ! 1 = on 0 = off
 chempot..=16
```

**Listing 7.3:** Example of the section in settings.txt producing a custom sequence of particles.

#### 7.6.3 Producing a detailed list of particles listorseq=2

Within this configuration, the user can pick any list of particles to produce, and specify their ID in any order. Let's assume that the user wants to produce  $\pi^+$  (ID 211),  $K^-$  (ID -321),  $\rho^0$  (ID 113),  $\omega$  (ID 223) p (ID 2212) and  $\bar{p}$  (not listed); for a total amount of 5 listed particle and 1 anti-baryon. In the settings.txt file the variable part\_list must contain the numeber of listed particles (part\_list=5) and the IDs of the particles must be placed one-perline just after this parameter (as shown in the listings 7.4).

```
! *** PARTICLE-RELATED
  npt . . . . . = 41
                                                 ! points for t...
 | ptmin . . . . = 0 . 0
                                                 ! min transver...
 ptmax . . . . = 4.0
                                                 ! max transver...
 nphi....=36
                                                 ! points for t...
 | phimin . . . = 0 . 0
                                                ! min polar an...
 phimax...=6.28318530718
                                                ! max polar an...
                                                ! points for t...
  nrap....=41
| rapmin . . . = -4.0
                                                 ! min rapidity
  rapmax...=4.0
                                                 ! max rapidity
11 listorseq=2
                                                 ! 0- all; 1-se...
  ID_start.=211
                                                 ! ID of first ...
13 ID_stop..=221
                                                 ! ID of last p...
  part_list=5
                                                 ! particles in...
15 211
```

```
-321
17 113
223
18 2212
antibar..=0 ! 1=on 0=off
chempot..=16
```

**Listing 7.4:** Example of the section in settings.txt producing a custom list of particles.

#### 7.6.4 Producing the standard subset of particleslistorseq=3

Example setup for a simulation including the standard subset of particles:  $\pi^+,\pi^0,\pi^-,K^+,K^0,\bar{K^0},K^-,\eta,\omega,p,n,\Lambda,\Sigma^+,\Sigma^0,\Sigma^-,\Xi^0,\Xi^-.$ 

Please note that still there is NO line after the part\_list line, and the variable part\_list itself is set to 0.

```
! *** PARTICLE-RELATED
  npt . . . . . = 41
                                                 ! points for t...
  ptmin....=0.0
                                                 ! min transver...
  ptmax . . . . = 4.0
                                                 ! max transver...
  nphi....=36
                                                 ! points for t...
 | phimin . . . = 0 . 0
                                                 ! min polar an...
 phimax...=6.28318530718
                                                 ! max polar an...
 nrap....=41
                                                 ! points for t...
 rapmin...=-4.0
                                                 ! min rapidity
10 rapmax . . . = 4.0
                                                 ! max rapidity
 listorseq=0
                                                 ! 0- all; 1-se...
12 ID_start.=0
                                                 ! ID of first ...
  ID_stop..=0
                                                 ! ID of last p...
14 part_list = 0
                                                 ! particles in...
  antibar..=1
                                                 ! 1 = on 0 = of f
16 chempot . . = 0
```

**Listing 7.5:** Example of settings.txt producing the standard subset of particles.

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